1) what is the contrapositive, the converse and the inverse of the conditional statement?:-

"The home team wins whenever it is raining"

$p \to q$, q whenever p

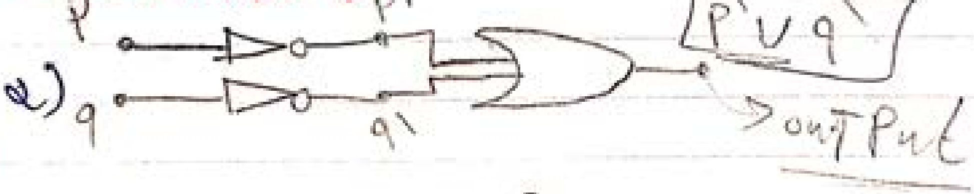
a) contrapositive:- if it's not raining then the home team loses. => $\neg q \to \neg p$

b) inverse:- if the home team loses, then it will not rain. => $\neg p \to \neg q$

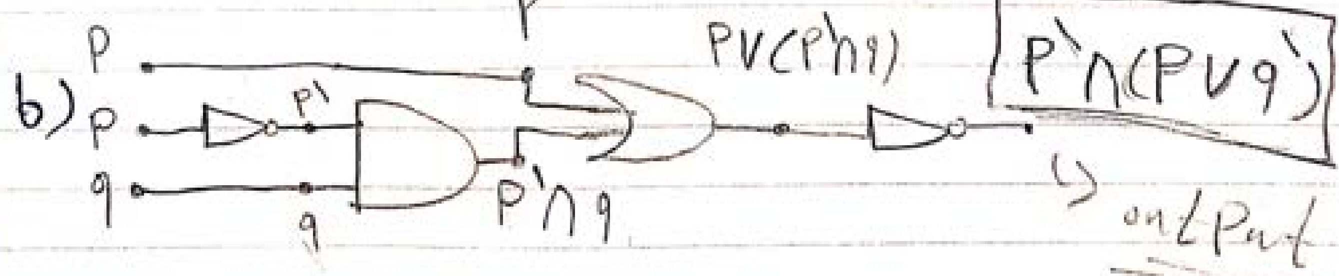
c) converse:- if the home team wins, then it's raining => $q \to p$

2) Find the output of each of these combinational circuits:-

a)



b)



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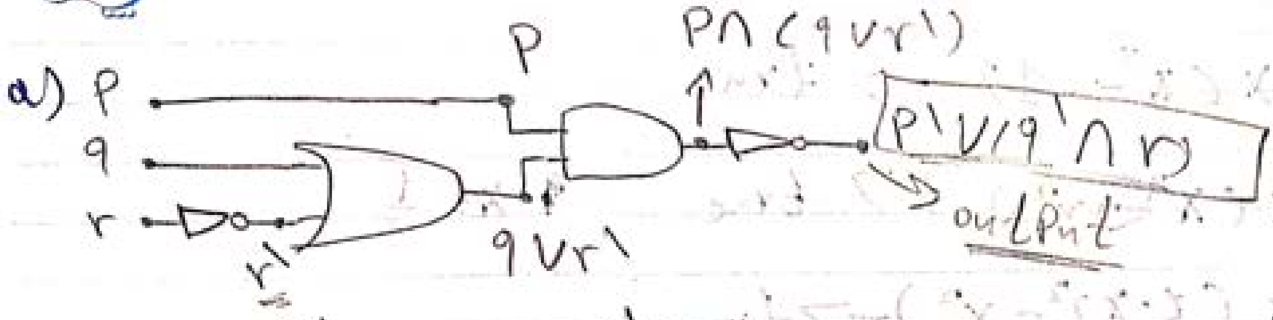
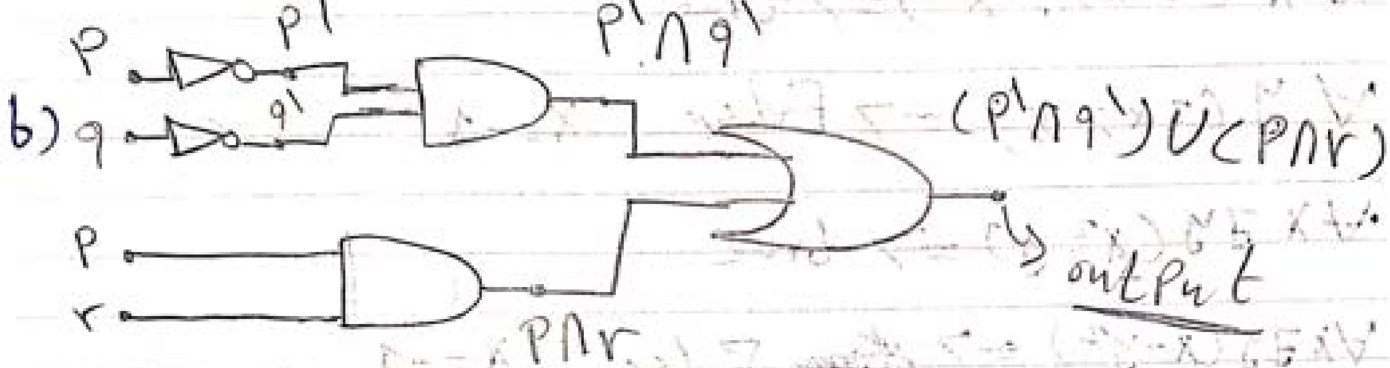
a)



b)



DATE 22/10/23 SUBJECT: Lec 2 - Quiz

Determine whether each of the following is a tautology or contradiction or neither :

1- $P \to (P \vee q)$

P	q	$P \vee q$	$P \to (P \vee q)$
0	0	0	1
0	1	1	1
1	0	1	1
1	1	1	1

$\Rightarrow$ $\boxed{\text{tautology.}}$

2- $(P \to q) \wedge (\neg P \vee q)$

P	q	$P \to q$	$\neg P$	$\neg P \vee q$	$(P \to q) \wedge (\neg P \vee q)$
0	0	1	1	1	1
0	1	1	1	1	1
1	0	0	0	0	0
1	1	1	0	1	1

$\Rightarrow$ $\boxed{\text{neither}}$

3- $(P \vee q) \leftrightarrow (q \vee P)$

P	q	$P \vee q$	$P \vee q \leftrightarrow q \vee P$
0	0	0	1
0	1	1	1
1	0	1	1
1	1	1	1

$\boxed{\text{tautology}}$ $\Leftarrow$

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4- $(P \wedge q) \rightarrow P$

tautology

P	q	$P \wedge q$	$(P \wedge q) \rightarrow P$
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	1

5- $(P \wedge q) \wedge (P \vee q)$

neither

P	q	$P \wedge q$	$P \vee q$	$(P \wedge q) \wedge (P \vee q)$
0	0	0	0	0
0	1	0	1	0
1	0	0	1	0
1	1	1	1	1

6- $(P \rightarrow q) \rightarrow (P \wedge q)$

neither

P	q	$P \rightarrow q$	$P \wedge q$	$(P \rightarrow q) \rightarrow (P \wedge q)$
0	0	1	0	0
0	1	1	0	0
1	0	0	0	1
1	1	1	1	1

7- $(\neg P \wedge q) \wedge (P \vee \neg q)$

contradiction

P	q	$\neg P$	$\neg q$	$\neg P \wedge q$	$P \vee \neg q$	$(\neg P \wedge q) \wedge (P \vee \neg q)$
0	0	1	1	0	1	0
0	1	1	0	1	0	0
1	0	0	1	0	1	0
0	1	0	0	0	0	0

8- $(p \to \neg q) \vee (\neg r \to p)$

p	q	r	$\neg q$	$\neg r$	$p \to \neg q$	$\neg r \to p$	$(p \to \neg q) \vee (\neg r \to p)$
0	0	0	1	1	1	0	1
0	0	1	1	0	1	1	1
0	1	0	0	1	1	0	1
0	1	1	0	0	1	1	1
1	0	0	1	1	1	1	1
1	0	1	1	0	1	1	1
1	1	0	0	1	0	1	1
1	1	1	0	0	0	1	1

$\boxed{\text{tautology}}$

9- $[p \to (q \wedge r)] \leftrightarrow [(p \to q) \wedge (p \to r)]$

p	q	r	$q \wedge r$	$p \to (q \wedge r)$	$p \to q$	$p \to r$	$(p \to q) \wedge (p \to r)$	$[p \to (q \wedge r)] \leftrightarrow [(p \to q) \wedge (p \to r)]$
0	0	0	0	1	1	1	1	1
0	0	1	0	1	1	1	1	1
0	1	0	0	1	1	1	1	1
0	1	1	0	1	1	1	1	1
1	0	0	0	0	0	0	0	1
1	0	1	0	0	0	1	0	1
1	1	0	0	0	1	0	0	1
1	1	1	1	1	1	1	1	1

$\boxed{\text{tautology}}$

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10- $[(p \vee q) \to r] \oplus (\neg p \vee \neg q)$

p	q	r	$p \vee q$	$(p \vee q) \to r$	$\neg p$	$\neg q$	$\neg p \vee \neg q$	$[(p \vee q) \to r] \oplus (\neg p \vee \neg q)$
0	0	0	0	1	1	1	1	0
0	0	1	0	1	1	1	1	0
0	1	0	1	0	1	0	1	1
0	1	1	1	1	1	0	1	0
1	0	0	1	0	0	1	1	1
1	0	1	1	1	0	1	1	0
1	1	0	1	0	0	0	0	0
1	1	1	1	1	0	0	1	0

$\boxed{\text{neither}}$

Prove each of the following logical equivalences:-

1- $p \wedge [(p \vee q) \vee (p \vee r)] \equiv p$

$(p \wedge [p \vee q]) \vee (p \wedge [p \vee r])$ using [Absorption Law]

$= p \vee p = \boxed{p \equiv p}$

2- $q \wedge [(p \vee q) \wedge (\neg q \wedge \neg p)] \equiv q$

$q \wedge [(p \vee q) \wedge (q \vee p)] = q \wedge (p \vee q)$

$= \boxed{q \equiv q}$

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$P(x,y)$: $x / y = 1$, what is the truth value of the following :

1) $\forall x \forall y\, P(x,y)$ = False

2) $\forall x \exists y\, P(x,y)$ = False

3) $\exists x \forall y\, P(x,y)$ = False

4) $\exists x \exists y\, P(x,y)$ = true

~~$[\neg q \wedge (p \to q)] \to \neg p$~~

= ~~$[\neg q \wedge (\neg p \vee q)] \to \neg p$~~

= ~~$[\neg p] \to \neg p$ = tautology~~

$(\frac{1}{2})^4 = \frac{1}{16}$

$(\frac{1}{2})^2 = \frac{1}{4}$

= $p \vee (p \wedge q) \equiv p$

$= (p \ldots) \wedge (p \vee q)$

$(p \wedge q)$

$(p \wedge q)$

DATE 6/11/23 SUBJECT: Lec 3 - Quiz

1- Prove that if $n=ab$, $a, b > 0$, ∴ $a \le \sqrt{n}$ or $b \le \sqrt{n}$

⇒ assume that $n = 3$

∴ $3 = ab$, a might equal to 3 or 1, b is same as a case

∴ $\sqrt{n} = \sqrt{a}$, $\sqrt{3} = \sqrt{3}$, or $\sqrt{n} > \sqrt{b}$, $\sqrt{3} > \sqrt{1}$

∴ $\sqrt{n} = \sqrt{ab}$, $\sqrt{n} = \sqrt{a} \cdot \sqrt{b}$,

$\boxed{∴ \sqrt{n} \ge \sqrt{b} \text{ and } \sqrt{n} \ge \sqrt{a}}$

2- Prove that for integer n, if n^2 is odd, ∴ n is odd

⇒ if n^2 is odd, ∴ $n^2 = 2K + 1$, $n = \sqrt{2K+1}$ ⇒ odd

∴ $\sqrt{odd}$ = odd, $\boxed{∴ \text{n is odd}}$

3- Show that the Proposition P(0) is true, where P(n) is

"if $\overset{p}{n>1}$, then $\overset{q}{n^2>n}$", Domain ∈ Z

⇒ $\forall n \, [P(n)]$, $n = 0$

∴ $P(0)$, $\underset{F}{0 > 1} \rightarrow \underset{F}{0 > 0}$, ∴ its true

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4- Let $P(n)$ be "if a and b are positive integers with $a \geq b$, then $a^n \geq b^m$", $D \in \mathbb{Z}^+$, show that $P(0)$ is true

p … q

$\therefore \boxed{\forall n\, P(n)\, [a^n \geq b \rightarrow a^n \geq b^m]}$ T

$\therefore P(0)$, $\because 0 \geq 0$ (T) $\rightarrow 0^n \geq 0^m$ ↳ $0 \times 0 \times 0 \times 0 \times \dots = 0$

↳ $0 \times 0 \times 0 \times \dots = 0$

$\therefore$ **$P(0)$ is true**

$P_1 \rightarrow (P_2 \leftrightarrow P_3)$

$P_1 \rightarrow (P_2 \rightarrow P_3 \wedge P_3 \rightarrow P_2)$

$\therefore P_1 \rightarrow (\neg P_2 \vee P_3 \wedge \neg P_3 \vee P_2)$

$\therefore P_1 \rightarrow (\neg P_2 \wedge \neg P_3 \vee 0 \vee 0 \vee P_3 \wedge P_2)$

$\therefore P_1 \rightarrow [\neg P_2 \wedge \neg P_3) \vee (P_3 \wedge P_2))$

$\therefore P_1 \rightarrow (\neg(P_2 \vee P_3) \vee (P_3 \wedge P_2))$

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3. Construct the Valid Argument

$$\begin{array}{l}(p \wedge q) \vee r \\ r \rightarrow s \\ \hline \therefore p \vee s\end{array}$$

1- $(p \wedge q) \vee r$ — Premise

2- $(p \vee r) \wedge (q \vee r)$ — distribution using ①

3- $(p \vee r)$ — simplification of ②

4- $r \rightarrow s$ — Premise

5- $\neg r \vee s$ — Logical equivilance

6- $p \vee s$ — resolution using ③ & ⑤

DATE 6 / 11 / 23 SUBJECT: Lecture 4 - Quiz

1- Use mathmatical induction to prove that $x^{2n} - y^{2n}$ is divisible by $x+y$ whenever n is a positive integer.

* Basic step :- $P(1) = x^2 - y^2 = (x+y)(x-y) \;/\; x+y$ ✓✓

* inductive step :- assume that $P(K)$ is true :-

$$\underline{x^{2K} - y^{2K} \;/\; x+y} \quad ✓✓$$

$\therefore$ for $K+1$:- $x^{2(K+1)} - y^{2(K+1)} = x^{2K+2} - y^{2K+2}$

$$= (x^{K+1} + y^{K+1})(x^{K+1} - y^{K+1})$$

[redacted]

$$= \underline{(x^2 - y^2)}(x^K - y^K) \;,\; \therefore x^2 - y^2 \;/\; x+y$$

$\therefore$ $\boxed{P(K+1) \text{ is true}}$

<u>note :-</u>

$\therefore \underline{(x^2 - y^2)(x^K - y^K)} = x^{2+K} - x^2 y^K - x^K y^2 + y^{2+K}$

$$= x^2(x^K - y^K) - y^2(x^K - y^K)$$

$$= \underline{(x^2 - y^2)(x^K - y^K)}$$

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2- Use mathmotical induction to prove that $3^n < n!$ For integer $n \geq 7$

* Basic step :- $P(7) \Rightarrow 3^7 < 7!$, $2187 < 5040$

↳ True

* inductive step :- assume $P(K)$ is true

" $3^K < K!$ ↗ $K \geq 7$

∴ for $K+1$:- if $3^K < K!$, ∴ $3^1 < K+1$

$3^{K+1} = 3 \cdot 3^K$, $3^{K+1} < 3\,K!$

∴ $3^{K+1} < K+1 \cdot K!$

$\boxed{∴ 3^{K+1} < (K+1)!}$, $P(K+1)$ is true

DATE 3/12/2023 SUBJECT: Lecture 5 - quiz

1. List the members of these sets:-

a) $\{x \mid x \text{ is a real number such that } x^2 = 1\} = \{-1, 1\}$

b) $\{x \mid x \text{ is a positive integer less than } 12\}$.

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$

c) $\{x \mid x \text{ is the square of an integer and } x < 100\}$ $\rightarrow x = y^2 = 1^2, 2^2, \ldots$

$x = y^2 < 100, \quad y < 10$

$= \{1, 4, 9, 16, 25, 36, 49, 64, 81\}$

$\cancel{x = \sqrt{2}}, \cancel{x = -\sqrt{2}}$

d) $\{x \mid x \text{ is an integer such that } x^2 = 2\} = \emptyset$

* Determine whether these statements are true or false.

a) $\emptyset \in \{\emptyset\}$ T

b) $\emptyset \in \{\emptyset, \{\emptyset\}\}$ T

c) $\{\emptyset\} \in \{\emptyset\}$ F

d) $\{\emptyset\} \in \{\{\emptyset\}\}$ T

2- use set builder notation to give a description of each of these sets

a) $\{0, 3, 6, 9, 12\} = \{3x \mid x$ is an integer less than 5 and greather than or equal to 0 $\}$

b) $\{-3,-2,-1,0,1,2,3\}$

$= \{x \mid x$ is an $\boxed{\text{Integer}}$ Less than 4 and greather than $-4\}$

Determine whether each of these statements is true or False :-

a) $x \in \{x\}$ T b) $\{x\} \subseteq \{x\}$ T

c) $\{x\} \in \{x\}$ F d) $\{x\} \in \{\{x\}\}$ T e) $\emptyset \subseteq \{x\}$ T

F) $\emptyset \in \{x\}$ F

Determine whether these statements are true or False :-

a) $\emptyset \in \{\emptyset\}$ T b) $\emptyset \in \{\emptyset, \{\emptyset\}\}$ T

c) $\{\emptyset\} \in \{\emptyset\}$ F d) $\{\emptyset\} \in \{\{\emptyset\}\}$ T

e) $\{\emptyset\} \subset \{\emptyset, \{\emptyset\}\}$ T F) $\{\{\emptyset\}\} \subset \{\emptyset, \{\emptyset\}\}$ T

g) $\{\{\emptyset\}\} \subset \{\{\emptyset\}, \{\emptyset\}\}$ F

what is the cardinality of each of these sets :-

a) $\{a\} \Rightarrow |\{a\}| = 1$ b) $\{\{a\}\} \Rightarrow |\{\{a\}\}| = 1$

c) $\{a, \{a\}\} \Rightarrow |\{a, \{a\}\}| = 2$

d) $\{a, \{a\}, \{a, \{a\}\}\} \Rightarrow |\{a, \{a\}, \{a, \{a\}\}\}| = 3$

* Prove that $P(A) \subseteq P(B)$ iff $A \subseteq B$:-

1- $P(A) \subseteq P(B) \rightarrow A \subseteq B$ 2- $A \subseteq B \rightarrow P(A) \subseteq P(B)$

* suppose $P(A) \subseteq P(B)$

$\therefore \forall x (x \in P(A) \rightarrow x \in P(B))$, $\because A \subseteq A$ & $B \subseteq B$

$\therefore A \in P(A)$ & $B \in P(B)$

$\therefore \forall x (x \in A \rightarrow x \in B)$ $\therefore \boxed{A \subseteq B}$ (1)

* suppose $A \subseteq B$,

$\therefore \forall x (x \in A \rightarrow x \in B)$, $\because A \subseteq A$ & $B \subseteq B$

$\therefore A \in P(A)$ & $B \in P(B)$

$\therefore \forall x (x \in P(A) \rightarrow x \in P(B))$, $\therefore \boxed{P(A) \subseteq P(B)}$ (2)

From (1), (2) :-

$P(A) \subseteq P(B) \longleftrightarrow A \subseteq B$

Let $A \div \{a, b, c, d, e\}$, $B = \{a, b, c, de, e, F, g, h\}$

Find :-

a) $A \vee B = \{X \mid X \in A \vee X \in B\} = \{a, b, c, de, F, g, h\}$

b) $A \cap B = \{X \mid X \in A \wedge X \in B\} = \{a, b, c, d, e\}$

c) $A - B = \{X \mid X \in A \wedge X \notin B\} = \emptyset$

d) $B - A = \{X \mid X \notin A \wedge X \in B\} = \{F, g, h\}$

what can you say about the sets A & B if you know that:

a) $A \vee B = A$? $\Rightarrow B \subset A \Rightarrow \forall x (x \in B \to x \in A) \wedge \exists x (x \in A \wedge x \notin B)$

b) $A \wedge B = A \Rightarrow A \subset B$

or $A \subseteq B \Rightarrow$ if $A \cap B = B$ & $A \cap B = A$
??

c) $A - B = A \Rightarrow A \cap B = \emptyset$, no elements intersect between A & B

d) $A \cap B = B \cap A \Rightarrow$ commutative Law

e) $A - B = B - A \Rightarrow A = B$

19) show that if A and B are sets, then :-

a) $A - B = A \cap \overline{B}$

$A-B = \{x \mid x \in A \wedge x \notin B\}$

$= \{x \mid x \in A \wedge x \in \overline{B}\}$

$= \underline{A \cap \overline{B}}$

$\boxed{\therefore A-B = A \cap \overline{B}}$

$A \cap \overline{B} = \{x \mid x \in A \wedge x \in \overline{B}\}$

$= \{x \mid x \in A \wedge x \notin B\}$

$= \underline{A-B}$

b) $A \cup (B-A) = A \cup B$

$\Leftrightarrow \; A \cup (B-A) = \{x \mid x \in A \vee (x \in B \wedge x \notin A)\}$

$\Leftrightarrow \; = \{x \mid (x \in A \vee x \in B) \wedge \underbrace{(x \in A \vee x \notin A)}_{1}\}$

$\Leftrightarrow \; = \{x \mid (x \in A \vee x \in B)\}$

$= \underline{A \cup B}$

$\Leftrightarrow \xrightarrow{\text{means}} \underline{\text{iff}}$

24- Let A, B and C be sets. show that :-

$(A-B)-C=(A-C)-(B-C)$

$=(A\wedge C^{\backprime})\wedge\neg(B\wedge C^{\backprime})$

$=(A\wedge C^{\backprime})\wedge(\neg B\vee C)$

$=$ ~~$(A\wedge B)$~~ $A\wedge C^{\backprime}\wedge B^{\backprime}\vee A\wedge C^{\backprime}\wedge C$

$=A\wedge C^{\backprime}\wedge B^{\backprime}$

$=(A\wedge B^{\backprime})\wedge C^{\backprime}$

$=(A-B)-C=L.H.S$

$\{X\mid(X\in A\wedge X\notin B)\wedge X\notin C\}$

$\therefore\{X\mid(X\in A\wedge X\notin C)\wedge(X\notin B\wedge X\notin C)\}$

$\{X\mid(X\in A\wedge X\notin C)\wedge\overline{(X\in B\wedge X\notin C)}\}$

$\therefore\{X\mid(X\in A\wedge X\notin C)\wedge(X\notin B\vee X\in C)\}$

$=\{X\mid(X\in A\wedge X\notin C\wedge X\notin B)\vee\underbrace{(X\in A\wedge X\notin C\wedge X\notin C)}_{F}\}$

$=\{X\mid X\in A\wedge X\notin C\wedge X\notin B\}$

DATE 3/12/2023 SUBJECT: Lecture 6 - Quiz

3- Consider the function $F: R \to R$ defined as $F(x) = 3x - 7$, show that F is both injective & surjective

1- $\forall a \forall b \, (F(a) = F(b) \to a = b)$

$\cancel{3}a \cancel{-7} = \cancel{3}b \cancel{-7}$, $\therefore a = b$ ✓✓

2- $\forall b \exists a \, (b = F(a))$, $b = 3a - 7$, $a = \frac{b+7}{3}$ ✓

$\therefore a \in R$ ✓✓

Determine whether each of these functions is a bijection from R to R :-

a) $F(x) = 2x + 1$

$\forall a \forall b \, (F(a) = F(b) \to a = b)$

$\therefore 2a + 1 = 2b + 1$

$\therefore a = b$ ✓✓ (one-to-one)

$\forall b \exists a \, (F(a) = b)$, $\therefore 2a + 1 = b$, $a = \frac{b-1}{a}$

$a \in R$ ✓ (onto)

✓✓ $\therefore F(x) = 2x + 1$ is a bijection ✓✓

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b) $F(x) = x^2+1$ → $\forall a \forall b \,(F(a)=F(b) \to a=b)$

$\therefore a^2+1 = b^2+1$

$\therefore a = \pm b$ XX

(not 1-1)

$\therefore$ not a bijection

c) $F(x) = x^3$ → $\forall a \forall b \,(F(a)=F(b) \to a=b)$

$a^3 = b^3 \to \therefore a=b$ ✓✓

(1-1)

$\forall b \exists a \,(b=F(a))$

$\therefore b = a^3$, $a = \sqrt[3]{b}$ (onto)

$a \in R$

$\therefore F(x) = x^3$ is a bijection

Lecture 6
Quiz - cont'd

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*Let $F : R \to R$ be defined by $F(x) = 5x+8$, show that F is a bijection and find it's inverse.

1- (1-1)

$\forall_a \forall_b (F(a) = F(b) \to a = b)$

$\therefore \cancel{5}a + \cancel{8} = \cancel{5}b + \cancel{8}$

$\therefore a = b$ ✓

(1)

2- onto

$\forall_b \exists_a (F(a) = b)$

$\therefore 5a + 8 = b$

$\therefore a = \frac{b-8}{5}$ ✓ , $\therefore a \in R$

(2)

$\therefore$ (1), (2) :

$\therefore$ if $F^{-1}(x) = 5x+8$, $\therefore y = 5x+8$

$\therefore x = 5y+8$, $\boxed{y = \frac{x-8}{5}} \to F^{-1}$

1- Let F, g and h be the functions defined

$F : R \to R \quad F(x) = x^2 - 5$

$g : Z \to R \quad g(x) = \frac{5x}{x^2-2}$

$h : R \to Z \quad h(x) = \lfloor x \rfloor$

Find the value of each of the following

i) $(f\circ f)(2) = f(f(2)) = f(-1) = (-1)^2 - 5 = \boxed{-4}$

ii) $(g\circ h)(2.5) = g(h(2.5)) = g(2) = \frac{5\times \cancel{x}}{2^2 \cancel{x}} = \boxed{5}$

iii) $(f\circ g)(2) = f(g(2)) = \underbrace{f(5)}_{(25-5)} = \boxed{20}$

iv) $(h\circ h)(3.7) = h(h(3.7)) = h(3) = \boxed{3}$

v) $(h\circ g)(3) = h(g(3)) = h\left(\frac{15}{7}\right) = h\left(2+\frac{1}{7}\right)$
$= \boxed{2}$

vi) $(h\circ f)(1.5) = h(f(1.5)) = h(-2.75)$
$= \boxed{-2}$

vii) $(f\circ h)(1.5) = f(h(1.5)) = \underbrace{f(1)}_{(1^2-5)} = \boxed{-4}$

viii) $(g\circ h)(2) = g(h(2)) = g(2) = \boxed{5}$

what are the values of these sums?

a) $\sum_{k=1}^{5}(k+1) = \frac{5(6)}{2} + 5 = \boxed{20}$

b) $\sum_{0}(-2)^j = (-2)^0 + (-2)^1 + (-2)^2 + (-2)^3 + (-2)^4 = 1 \overset{-1}{\cancel{-2}} + 4 \overset{8}{\cancel{-8+16}}$
$=$ [redacted] $\boxed{11}$

c) $\sum_{i=1}^{10} 3 = 3\times 10 = \boxed{30}$

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d) $\sum_{j=0}^{8} (2^{j+1} - 2^{j}) = \sum_{j=0}^{8} 2^{j} (2-1)\ 1$

$= \sum_{j=0}^{8} 2^{j} = 1+2+4+8+16+32+64+128+256$

$= \boxed{511}$

Lec. 7 - Quiz

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List the ordered pairs in the relation R from $A = \{0,1,2,3,4\}$ to $B = \{0,1,2,3\}$, where $(a,b) \in R$ if and only if :-

a) $a = b \Rightarrow \{(0,0), (1,1), (2,2), (3,3)\}$

b) $a + b = 4 \Rightarrow \{(1,3), (3,1), (2,2), (4,0)\}$

c) $a > b \Rightarrow \{(1,0), (2,0), (2,1), (3,0), (3,1), (3,2), (4,0), (4,1), (4,2), (4,3)\}$

d) $a \mid b \Rightarrow \{(0,0), (1,0), (1,1), (1,2), (1,3), (2,2), (3,3), (2,0), (3,0), (4,0)\}$

Let R be the relation represented by the matrix

$$M_R = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

Find the matrix representing

a) R^{-1}

b) $\overline{R}$

c) R^2

$$\boxed{b)\ \overline{R} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}} \qquad \boxed{a)\ R^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}}$$

$$c)\ R^2 = R \times R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \boxed{I_3}$$

1. For each of the following relations R on a set A, draw:

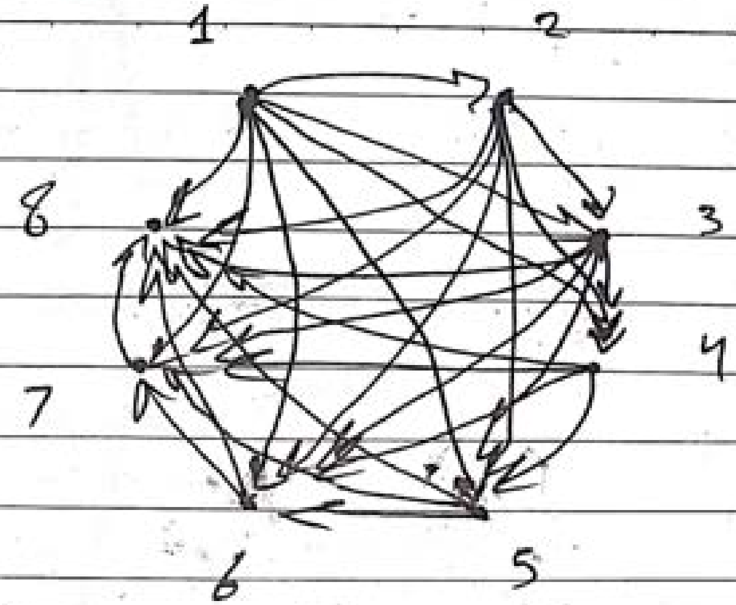
a) its coordinate grid diagram
b) its digraph
c) its matrix (0s & 1s)

i) $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$:- $a R b$ iff $a < b$

$\hookrightarrow = \{(1,2), (1,3), (1,4), (1,5), (1,6), (1,7), (1,8), (2,3), (2,4), (2,5), (2,6), (2,7), (2,8), (3,4), (3,5), (3,6), (3,7), (3,8), (4,5), (4,6), (4,7), (4,8), (5,6), (5,7), (5,8), (6,7), (6,8), (7,8)\}$

a)

b)

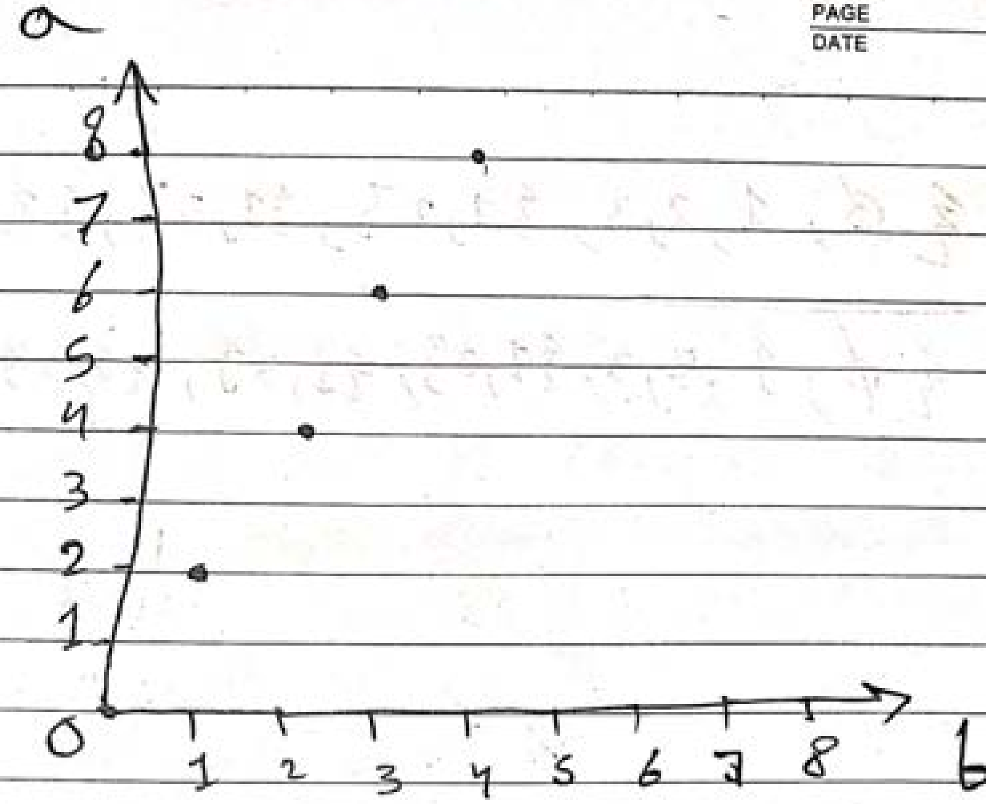


c) $M_R = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

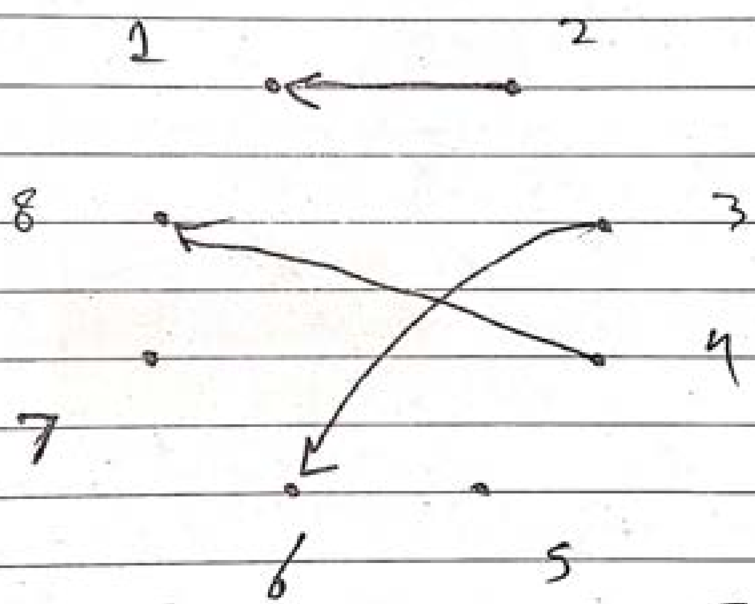
vii) $A = \{1,2,3,4,5,6,7,8\}$; aRb iff $a = 2b$

$R = \{(2,1),(4,2),(6,3),(8,4)\}$

a)



b)



c)

$$M_R = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\boxed{a \subseteq b = \forall_x (x \in a \to x \in b)}$$

(ix) $A = P\{1,2,3\}$, the Power set of $\{1,2,3\}$:

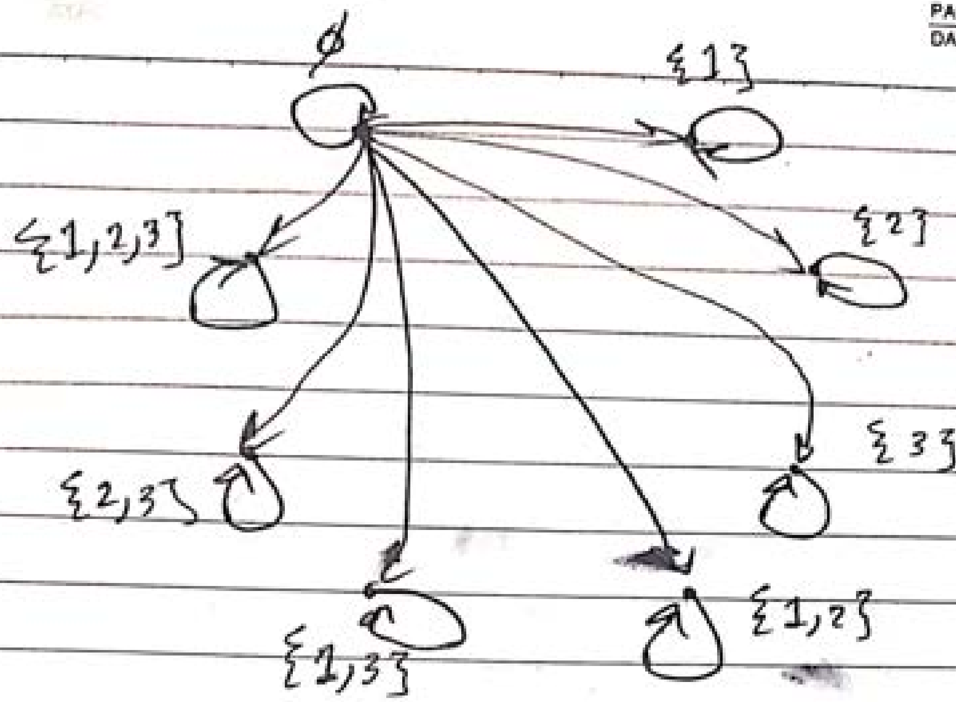
$a R b$ iff $a \subseteq b$

$R = \{ (\emptyset,\emptyset), (\emptyset,\{1\}), (\emptyset,\{2\}), (\emptyset,\{3\}), (\emptyset,\{1,2\}), (\emptyset,\{1,3\}), (\emptyset,\{2,3\}), (\emptyset,\{1,2,3\}), (\{1\},\{1\}), (\{2\},\{2\}), (\{3\},\{3\}), (\{1,2\},\{1,2\}), (\{1,3\},\{1,3\}), (\{2,3\},\{2,3\}), (\{1,2,3\},\{1,2,3\}) \}$

a)

a \ b	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$\emptyset$	X	X	X	X	X	X	X	X
$\{1\}$		X						
$\{2\}$			X					
$\{3\}$				X				
$\{1,2\}$					X			
$\{1,3\}$						X		
$\{2,3\}$							X	
$\{1,2,3\}$								X

b)



c)

$$M_R = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

2- the binary matrices M_R and M_S for two relations R and S respectively on the set A = {1,2,3,4,5} are :-

$$M_R = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{pmatrix} , M_S = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

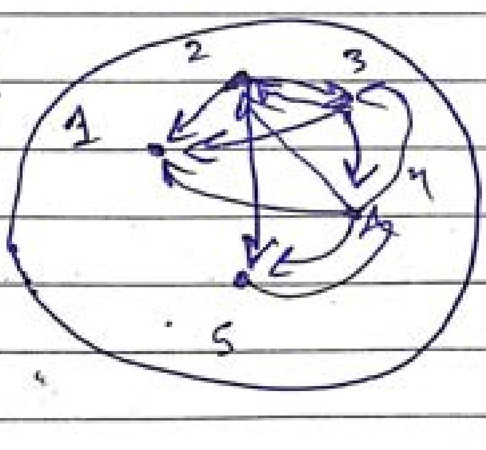
i)- List elements of R, S

= {(2,1), (2,3), (2,5), (3,1), (3,2), (3,4), (4,1), (4,2), (4,3), (4,5), (5,1), (5,4)}

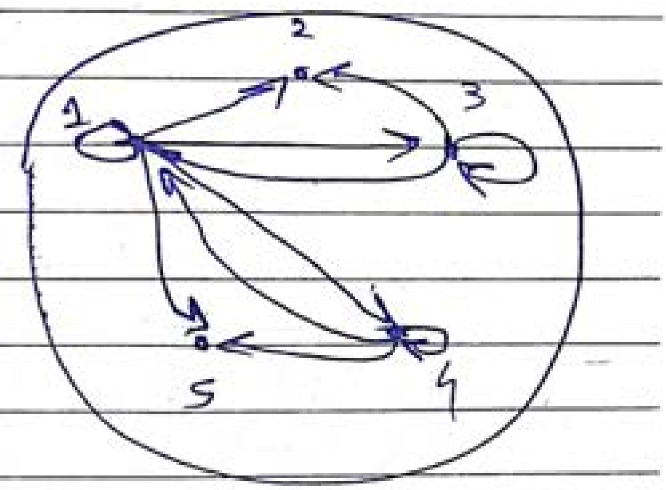
= {(1,1), (1,2), (1,3), (1,4), (1,5), (3,1), (3,2), (3,3), (4,1), (4,4), (4,5)}

ii)- draw digraph for each of them

R



S



Q4-2

3- let $A = Z^+ \times Z^+$ and R be the relation on A defined by $(a,b) R (c,d)$ if and only if $a+d = b+c$. show that R is reflexive, symmetric and transitive, but not anti symmetric

$(a,b) R (c,d) \Rightarrow a+d = b+c$

$\therefore (a,b) R (a,b) \Rightarrow a+b = b+a \Rightarrow$ reflexive ①

assume that $(a,b) R (c,d)$ ⟹ $(c,d) R (a,b)$

$\Rightarrow a+d = b+c \Leftrightarrow \boxed{c+b = d+a}$ ②

symmetric

assume that $(a,b) R (c,d)$ & $(c,d) R (x,y)$

we need to prove $\boxed{(a,b) R (x,y)} \Rightarrow a+y = b+x$

$\therefore a+d = b+c \quad , \quad c+y = d+x$

$a+c+y = d+x+a \leftarrow$ add a

$\therefore a+y = a+x+d-c$

$\therefore a+y = b+\cancel{c}+x-\cancel{c}$ ③

$\boxed{\therefore a+y = b+x} \Rightarrow$ transitive

التاريخ :

الموضوع :

From ② :- $(a,b) R (c,d)$ & $(c,d) R (a,b)$

∴ $(a,b) \neq (c,d)$

∴ not anti-symmetric

Let R be an equivalence relation on A and $x, y \in A$. Then $[x] = [y]$ iff xRy

1- $[x] = [y] \rightarrow xRy$, $\because R$ is reflexive

$\therefore xRx$, $\therefore x \in [x]$

$\because [x] = [y]$, $\therefore x \in [y]$

$\boxed{\therefore xRy}$

2- $xRy \rightarrow [x] = [y]$, assume element $a \in [x]$

$\therefore aRx$, $\because aRx \ \& \ xRy$

$\boxed{\therefore aRy}$ (transitive property)

$\therefore a \in [y]$, $\because a \in [x]$

$\boxed{\therefore [x] \subseteq [y]} \rightarrow (1)$

$xRy \rightarrow [x] = [y]$, assume $a \in [y]$, $\therefore aRy$

since R is symetric & xRy, $\therefore yRx$

$\because aRy \ \& \ yRx$, $\therefore aRx$, $\therefore a \in [x]$, $\because a \in [y]$

$\therefore [y] \subseteq [x] \rightarrow (2)$

التاريخ:

الموضوع:

From 1, 2 :-

$\therefore [x] = [y]$ if xRy

$$\boxed{\therefore xRy \to [x] = [y]}$$

Lecture 8 - Quiz

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Find a div m, and a mod m when:

a) $a = 228, m = 119$, $228 \bmod 119 = \boxed{109}$

$\Downarrow$

$228 = 119 \times 1 + 109$ $228 \text{ div } m = \boxed{1}$

b) $a = 9009, m = 223$ $9009 \bmod 223 = \boxed{89}$

$\Downarrow$

$9009 = 223 \times 40 + 89$

$9009 \text{ div } 223 = \boxed{40}$

c) $a = -10101, m = 333$ $-10101 \bmod 333 = \boxed{222}$

$\Downarrow$

$-10101 = -31 \times 333 + 222$ $-10101 \text{ div } 333 = \boxed{-31}$

d) $a = -765432, m = 38271$ $-765432 \bmod 38271 = \boxed{38259}$

$\Downarrow$

$-765432 = -21 \times 38271 + 38259$

$-765432 \text{ div } 38271 = \boxed{-21}$

PAGE
DATE

Find the integer a such that :

a) $a \equiv 43 \pmod{23}$ and $-22 \leq a \leq 0$

$\frac{a-43}{23}$

why $a \neq 43$
so that $\frac{43-43}{23} = 0$
because
$\boxed{-22 \leq a \leq 0}$

$\therefore a \equiv (43-23) \pmod{23}$
$\equiv 20 \pmod{23}$
$\equiv (20-23) \pmod{23}$
$\equiv -3 \bmod 23$

$\therefore \frac{a-(-3)}{23}$, $\boxed{\therefore a = -3}$, so $\frac{0}{23} = 0$, $r = 0$

b) $a \equiv 17 \pmod{29}$ and $-14 \leq a \leq 14$

$a \equiv (17-29) \pmod{29}$

$a \equiv -12 \pmod{29}$

$\boxed{\therefore a = -12}$

c) $a \equiv -11 \pmod{21}$ and $90 \leq a \leq 110$

$\therefore a \equiv 10 \pmod{21} \equiv 31 \pmod{21} \equiv 52 \pmod{21}$

$a \equiv 73 \pmod{21}$, $\therefore a \equiv 94 \pmod{21}$

$\boxed{\therefore a = 94}$