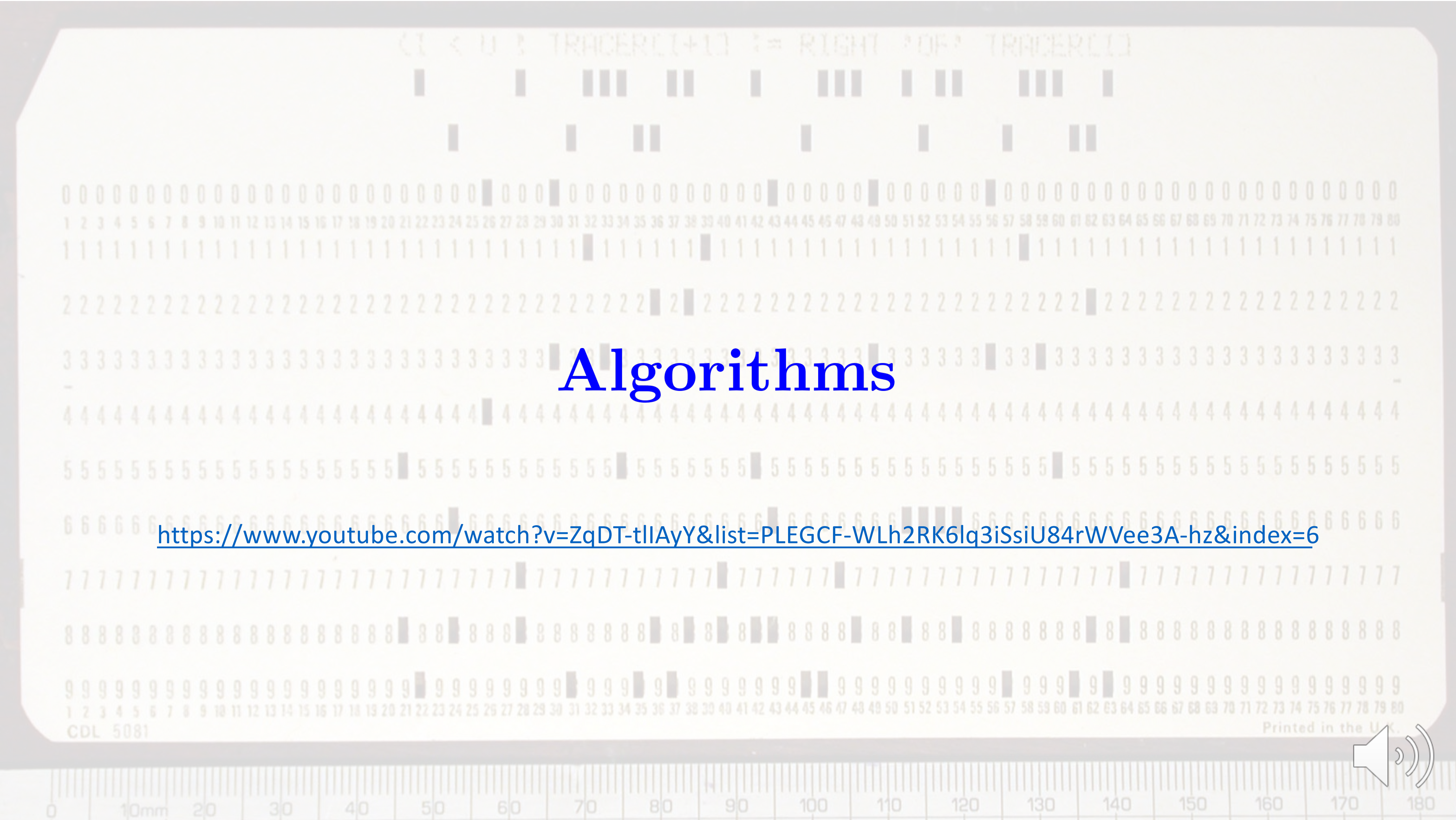


Algorithms

https://www.youtube.com/watch?v=ZqDT-tlIAyY&list=PLEGCF-WLh2RK6lq3iSsiU84rWVee3A-hz&index=6

Computational Complexity

P : set of decision problems for which there is a polynomial time algorithm

NP : We say that a decision problem is in NP if for any instance of the problem for which the answer is 'YES', there is *certificate* using which a *verifier* can verify this in polynomial time.

P stands for "polynomial" and NP stands for "nondeterministic polynomial".

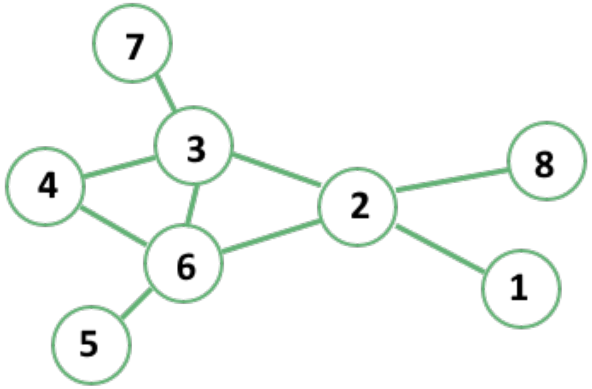
Computational Complexity

So, what are NP-Hard Problems?

believed to require exponential-time in the worst case

Definition: A problem is NP-hard if a polynomial-time algorithm solving it would refute the P≠NP conjecture.

Computational Complexity



Let $G = (V, E)$ be an undirected graph.

A ***vertex cover*** of G is a subset $U \subseteq V$ of the vertices s.t. for each edge $(x, y) \in E$ at least one of the vertices in $\{x, y\}$ belongs to U.

What is the size of the smallest vertex cover? $3 \quad (\{2, 3, 6\})$

A formulation of the same problem is: Is there a cover of size<=k? (k$\in \{1, .., n\}$)

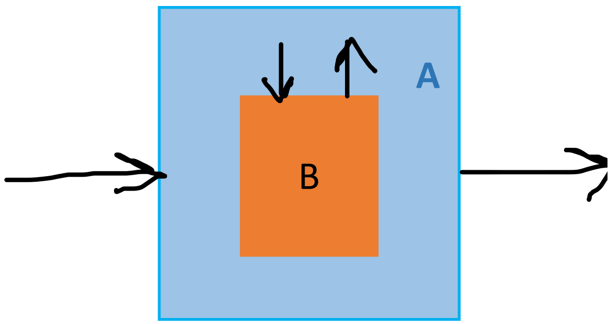
And an instance of the problem would be, for a given value of k, is there a solution? (O(n^k) complexity)

Given a graph G and a number k, is there a vertex cover of size $\leq k$ in G?

Computational Complexity

Reductions

Definition: A problem A is polynomial-time reducible to a problem B, if an algorithm that solves B can be *easily translated* to solve problem A.



easily translated:
Check picture. A should call B a polynomial number of times, and
The additional work outside B is also polynomial

Have we seen reductions in this course?

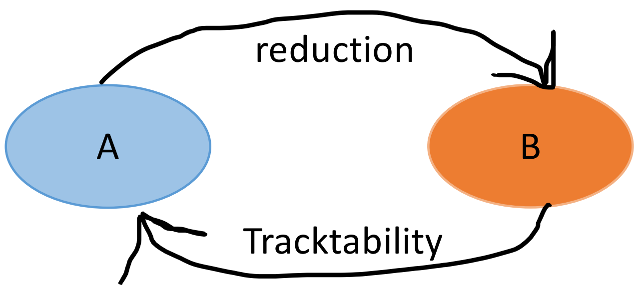
Median finding to sorting (O(nlogn))

All pair shortest path reduces to SSSP (n larger)

Computational Complexity

Notation. $A \leq_P B$

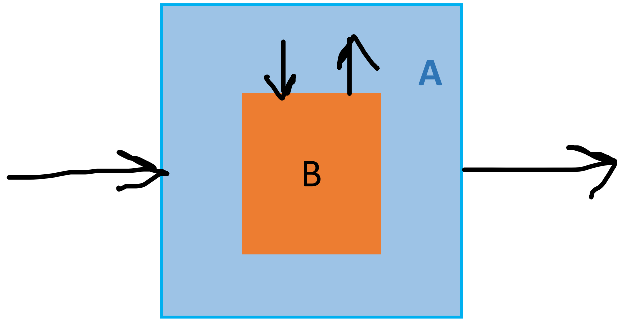
$\boldsymbol{A \leq_P B}$ **implies that:** If B has a polynomial time solution, than so does A



Note that if $Z \leq_P Y$ and $Y \leq_P X$ then $Z \leq_P X$.

Computational Complexity

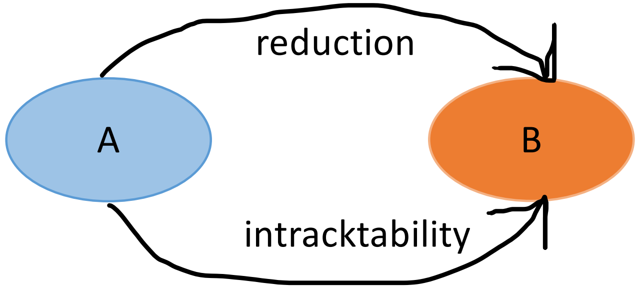
Reductions



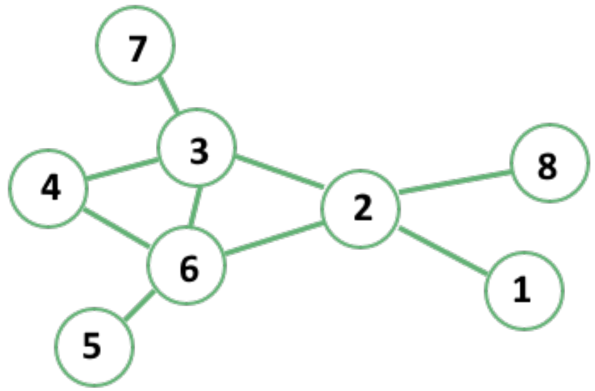
$A \leq_P B$

What if A is NP-hard?
If a problem A reduces to a problem B, and A is NP-hard, then B is NP-hard

How to prove that an algorithm B is NP-hard?
Choose a NP-hard problem A
Prove that A can be reduced to B
It helps to know some of these!



Computational Complexity



Let $G = (V, E)$ be an undirected graph.

An ***independent set*** in the graph is a subset $U \subseteq V$ of the vertices s.t. there is no edge in G between any pair of vertices in U.

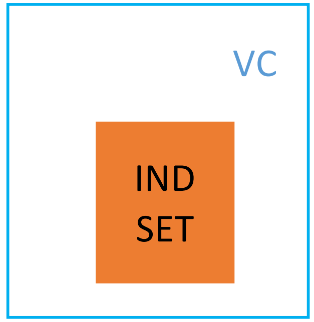
What is the size of the largest independent set? 5 $(\{1, 4, 5, 7, 8\})$

A ***vertex cover*** of G is a subset $U \subseteq V$ of the vertices s.t. for each edge $(x, y) \in E$ at least one of the vertices in $\{x, y\}$ belongs to U.

What is the size of the smallest vertex cover? 3 $(\{2, 3, 6\})$

Polytime reductions

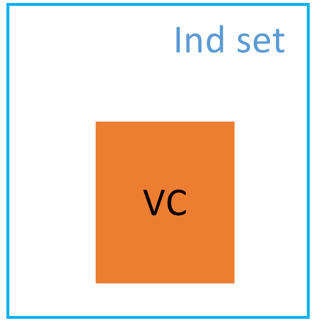
Is Vertex Cover $\leq_P$ Independent Set? *Yes*



1. Let U be the largest independent set of G
   Let X=V-U, then X is a vertex cover:
   let e: u→v be any edge in G, then u and v cannot both be in U
   Thus, at least one of u or v belong to X.

2. Moreover, X is the smallest cover.
   Assume there is a smaller cover X', and let U'=V-X'.
   We will prove that U' is an independent set:
      Assume there is an edge e between 2 elements a and b in U',
      then none of the nodes incident to e belong to X', a contradiction. Thus U' is an independent set,
   But U be the largest independent set of G, thus |U|=|U'|, and hence |X|=|X'|

Polytime reductions



Exercise

Is Independent Set $\leq_P$ Vertex Cover? *Yes*

- Choose a vertex cover S of G having size k. Take the set I of vertices not in S. There cannot be an edge between any two members of I, because if there were, S would not cover all of the edges. Hence, I must be an independent set. Since I has size n-k, G has an independent set of size n-k.
- It is easy to prove that, if S the smallest cover size, then I is the largest independent set

Polytime reductions

Is Vertex Cover $\leq_P$ Independent Set? *Yes*

Is Independent Set $\leq_P$ Vertex Cover? *Yes*

The complement of a minimum vertex cover is a maximum independent set and vice versa.

Homework

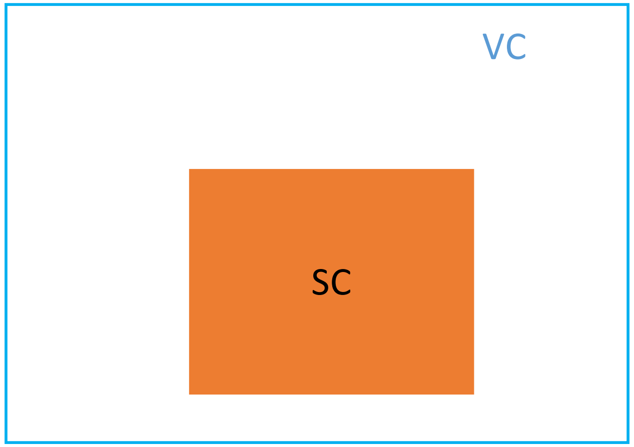
Is Vertex Cover $\leq_P$ Set Cover? *Yes*

Hint

Given a graph G=(V,E), we need to output a vertex cover, using

1. polynomial time calls to SC and
2. Any additional work required should be polynomial time

So we need to find a way to represent the VC problem as a SC problem, and thus benefit from its solution. How?



Computational Complexity

Suppose that $X \leq_P Y$. Which of the following statements are true?

Multiple Choice

A) If X can be solved in polynomial time, then so can Y.

B) X can be solved in poly time iff Y can be solved in poly time

C) If X cannot be solved in polynomial time, then neither can Y

D) If Y cannot be solved in polynomial time, then neither can X

E) If Y is NP-hard, so is X

F) If X is NP-hard, so is Y

G) If X is NP-complete, so is Y