Attentional Multilayer Ensemble Graph Neural Networks for Skeleton-based Exercise Assessment

Abstract—**Musculoskeletal disorders represent a significant global health challenge, necessitating innovative solutions for effective rehabilitation exercise assessment. Existing systems leveraging 3D skeleton data from sensors like Kinect struggle with key challenges, including the inability to dynamically prioritize critical joints, model long-range dependencies, and robustly handle noise in skeletal features. To address these limitations, we propose the Attentional Multilayer Ensemble Graph Convolutional Network (AMLE-GCN), an advanced framework designed for comprehensive rehabilitation exercise evaluation. AMLE-GCN integrates a Multi-Head Attention Layer to dynamically emphasize spatial and temporal dependencies across position- and orientation-specific features, enabling robust and fine-grained assessments. The framework further employs a multi-level ensemble strategy, including feature-level, decision-level, and feature-fusion model-level ensembles, to enhance adaptability and generalization. Extensive evaluations on two benchmark datasets, UI-PRMD and KIMORE, demonstrate that AMLE-GCN achieves 89.6% accuracy on UI-PRMD and 86.2% accuracy on KIMORE, significantly outperforming existing methods. These results underscore its potential to transform home-based rehabilitation by providing precise, scalable, and adaptive assessments.**

Index Terms—**Place holder**

I. Introduction

Musculoskeletal disorders pose a significant global health challenge, affecting individual well-being and placing immense pressure on healthcare systems [1]. Therapeutic exercises prescribed by physical therapists are crucial for the rehabilitation of conditions such as back pain, sprains, and epicondylitis [2]. However, traditional hospital-based rehabilitation programs are expensive and inaccessible to many, particularly for patients with reduced working capacity [3]. As a cost-effective alternative, home-based rehabilitation programs have gained popularity but often suffer from poor patient adherence and lack accurate systems to assess the quality of exercises [4], [5]. Current home-based physical therapy systems using 3D skeleton data collected from motion sensors, such as Kinect, are limited in their ability to perform fine-grained assessments, especially in identifying subtle variations in joint movements during exercises [6]. Existing methods [7]–[9] often rely on single-modal skeleton features, leading to suboptimal utilization of positional and orientation data. Additionally, current evaluation standards rely on limited metrics, failing to adequately reflect prediction accuracy and hampering algorithmic development [10].

The increasing prevalence of home-based physical therapy underscores the urgent need for robust and intelligent assessment systems. Leveraging 3D skeleton data offers a unique opportunity to develop virtual therapists capable of addressing patient-specific barriers and motivating adherence to rehabilitation programs [11]. Despite this potential, current approaches fail to fully exploit the complementary relationship between positional and orientation data or address their inherent structural and physical differences [12]. This limitation highlights the need for computational frameworks that dynamically focus on relevant movements while capturing global and local skeletal features effectively. These challenges serve as the foundation for this research, driving the need for innovative models capable of enhancing the quality of rehabilitation exercise assessments.

To address these challenges, we propose the **Attentional Multilayer Ensemble Graph Convolutional Network (AMLE-GCN)**, as illustrated in Figure 1. The proposed framework utilizes Kinect v2 sensors to extract both skeleton and orientation features, which are subsequently processed through a graph construction module to create position- and orientation-specific graph representations. The system introduces a Multi-Head Attention Layer, which dynamically emphasizes critical joints and movements by attending to spatial and temporal dependencies across the input features. These enriched features are processed by independently trained position and orientation submodels, enabling a deeper understanding of global and local skeletal movements.

AMLE-GCN incorporates a robust multi-level ensemble strategy to enhance model performance and generalization. The system integrates predictions through feature-level, decision-level, and feature-fusion model-level ensembles, ensuring a comprehensive evaluation of rehabilitation exercises. The Multi-Head Attention Layer further enhances feature extraction by learning complex inter-dependencies between joints, allowing the model to capture subtle variations in movements critical for fine-grained assessments. The combination of these components ensures that AMLE-GCN is robust to noise, adaptable to diverse datasets, and capable of providing reliable assessments

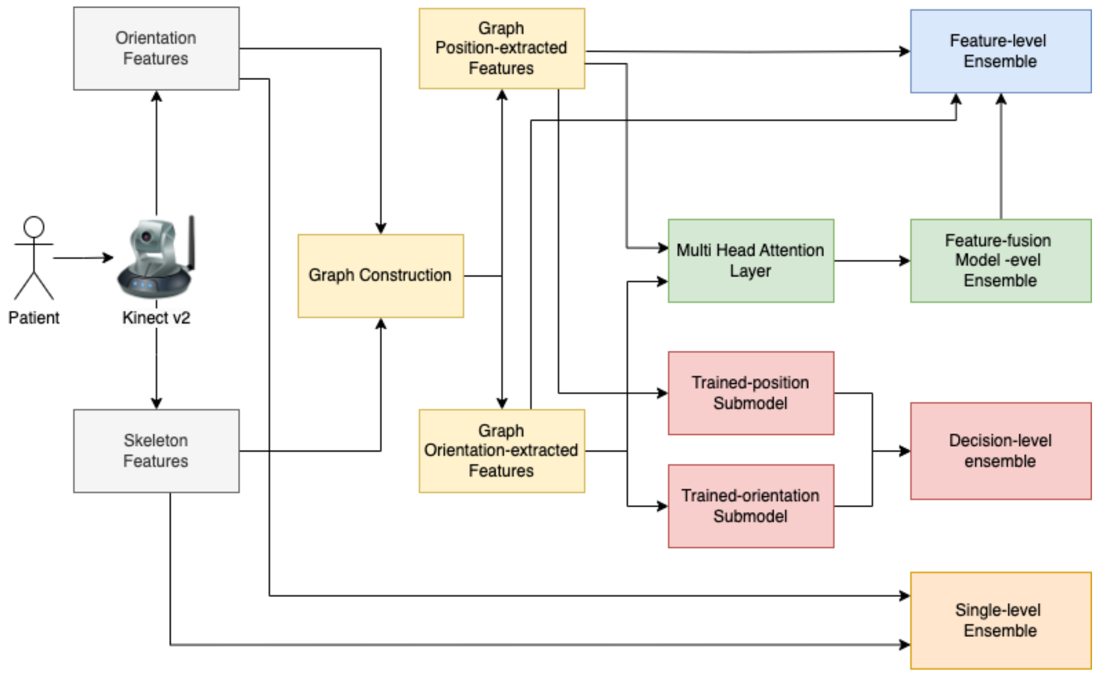

Fig. 1: Proposed architecture of the Attentional Multilayer Ensemble Graph Convolutional Network (AMLE-GCN). The model integrates position- and orientation-extracted features with a Multi-Head Attention Layer and a multi-level ensemble strategy for robust rehabilitation exercise assessment.

in real-world rehabilitation scenarios.

By leveraging advanced spatial and temporal attention mechanisms and ensemble learning, AMLE-GCN addresses key limitations of existing methods. Through extensive evaluations on the UI-PRMD and KIMORE datasets, our model achieves state-of-the-art accuracy, outperforming existing approaches by significant margins. This advancement demonstrates its potential to transform home-based rehabilitation by providing scalable, adaptive, and precise assessments.

The remainder of this paper is organized as follows: Section 2: Related Work discusses recent developments in skeleton-based rehabilitation systems and GCN advancements. Section 3: Methodology details the proposed AMLE-GCN framework, including its architecture and key components. Section 4: Dataset describes the datasets used in this study and the preprocessing steps. Section 5: Results presents the experimental findings and a comparative analysis with existing methods. Section 6: Conclusion summarizes the contributions and outlines future research directions.

II. Related Work

Graph convolutional networks (GCNs) have been widely adopted for skeleton-based tasks, including action recognition and rehabilitation exercise assessment [1], [9]. Despite their success, traditional GCNs face several limitations in capturing the spatial-temporal complexity of human movement in rehabilitation contexts. These models often rely on static adjacency matrices, treating all skeletal joints and connections equally, which fails to reflect the varying importance of joints during specific exercises [8]. Additionally, GCNs struggle to model long-range dependencies between distant joints, limiting their ability to capture global movement patterns essential for fine-grained assessments [7]. Furthermore, traditional GCNs are sensitive to noise in skeletal data and lack mechanisms to filter irrelevant or redundant information dynamically [13]. These challenges are particularly significant in rehabilitation scenarios, where subtle joint movements and temporal dynamics are crucial for evaluating exercise correctness and quality [6].

Convolutional neural networks (CNNs) have also been applied to physical rehabilitation tasks, but their ability to model skeletal data holistically is limited [5]. CNN-based approaches often fail to utilize the inherent spatial relationships between skeletal joints effectively, resulting in suboptimal representation of the body's structure [10]. Multi-stream methods, which divide the body into separate segments, inadequately capture interdependencies between body parts and interactions within each segment [11]. Additionally, these methods exhibit reduced robustness when faced with variability in patient movements, a common occurrence in real-world rehabilitation scenarios [14]. The lack of self-supervised learning techniques further hampers the adaptability of CNN-based models to diverse datasets and outlier behaviors [12]. Moreover, many CNN-based methods fail to incorporate spatial-temporal dynamics, which are critical for accurately assessing subtle joint movements and exercise quality in rehabilitation contexts [15].

Hidden Semi-Markov Models (HSMMs) have also been explored for rehabilitation exercise assessment and have demonstrated some effectiveness [3]. However, these systems suffer from several limitations. HSMM-based methods typically rely on manually selected features defined by physiotherapists, reducing their adaptability to diverse exercises and complex motion patterns [16]. While low-pass filters, such as the Butterworth filter, help mitigate noise in skeletal data, these systems remain sensitive to inaccuracies and occlusions, particularly when using data from motion sensors like Kinect v2 [17]. Temporal modeling in HSMMs often lacks the granularity required to capture subtle multi-joint dynamics, which are critical for comprehensive rehabilitation assessments [2]. Furthermore, scoring systems based on normalized log-likelihood and comparisons with Dynamic Time Warping (DTW) fail to accommodate patient-specific variations and broader rehabilitation scenarios [4]. These challenges are exacerbated by small datasets, such as the study with only 33 subjects performing five exercises, which limit the generalizability of findings [18]. Additionally, the reliance on manual clinical evaluation for validation introduces subjectivity, reducing the potential for fully automated systems [19].

In summary, existing methods across GCNs, CNNs, and HSMMs face challenges in addressing the nuanced spatial-temporal dynamics and variability of skeletal movements in rehabilitation contexts. These limitations highlight the need for advanced models that can dynamically prioritize critical joints and movements, filter noise effectively, and adapt to diverse datasets. Our work aims to address these gaps by introducing a novel framework that leverages attention mechanisms and ensemble learning to enhance robustness, accuracy, and adaptability in rehabilitation exercise assessment.

III. Methodology

A. Data Representation

Skeletal data is represented as graphs, where nodes correspond to skeletal joints and edges define the structural and temporal connections between them. This graph representation models both spatial relationships (e.g., bone connections) and temporal dependencies (e.g., joint movement sequences over time). Let the skeletal data be represented as $G = (V, E)$, where V is the set of nodes (skeletal joints), and E is the set of edges (connections between joints). The input data includes position features $\mathbf{P} \in \mathbb{R}^{N \times T \times D}$ and orientation features $\mathbf{O} \in \mathbb{R}^{N \times T \times D}$, where N is the number of joints, T is the sequence length, and D is the feature dimension.

B. Attention-Augmented Graph Convolutional Networks (AMLE-GCN)

The proposed AMLE-GCN framework enhances conventional graph convolutional networks (GCNs) by incorporating attention mechanisms. These mechanisms dynamically assign weights to graph nodes and edges, emphasizing the most relevant joints and their connections during specific exercises. This dynamic weighting improves the network's ability to model both local and global skeletal features, which are critical for fine-grained rehabilitation assessments.

1) Framework Design: The AMLE-GCN framework consists of the following components:

1) **Graph Construction:** The adjacency matrix $\mathbf{A}$ represents the graph structure, where $\mathbf{A} \in \mathbb{R}^{N \times N}$ encodes the spatial relationships between skeletal joints. Temporal dependencies are modeled through sequential graph representations over time steps T.
2) **Attention Mechanism:** The framework uses multi-head attention to dynamically weight nodes, edges, and time steps.
   - *Spatial Attention:* Multi-head spatial attention assigns importance to skeletal joints across multiple attention heads:

     $$\text{MultiHead}(\mathbf{Q}, \mathbf{K}, \mathbf{V}) = \text{Concat}(\text{head}_1, \ldots, \text{head}_h)$$

     where each head captures distinct spatial patterns.
   - *Temporal Attention:* Multi-head temporal attention focuses on critical time steps in movement sequences, computed similarly to spatial attention but over the temporal dimension.
   - *Joint Fusion:* Spatial and temporal attention outputs are combined:

     $$\mathbf{A}_{\text{joint}} = \alpha \cdot \mathbf{A}_{\text{spatial}} + (1 - \alpha) \cdot \mathbf{A}_{\text{temporal}}.$$

     Fused features are integrated, pooled, and passed through a fully connected layer for final predictions.
3) **Graph Convolutions:** Graph convolutions are applied to the attention-weighted graphs. The graph convolution operation is defined as:

   $$\mathbf{H}^{(l+1)} = \sigma(\mathbf{D}^{-1/2}\mathbf{A}\mathbf{D}^{-1/2}\mathbf{H}^{(l)}\mathbf{W}^{(l)})$$

   where $\mathbf{H}^{(l)}$ is the input feature matrix at layer l, $\mathbf{W}^{(l)}$ is the learnable weight matrix, and $\mathbf{D}$ is the degree matrix.
4) **Fusion Strategy:** Features from position and orientation streams are fused using attention-based ensemble techniques. The fused representation is computed as:

   $$\mathbf{Z} = \alpha \cdot \mathbf{Z}_{\text{position}} + (1 - \alpha) \cdot \mathbf{Z}_{\text{orientation}}$$

   where α is a learnable weight parameter.

2) Training Process: The framework is trained using cross-entropy loss:

$$\mathcal{L} = -\frac{1}{M} \sum_{i=1}^{M} [y_i \log(\hat{y}_i) + (1 - y_i) \log(1 - \hat{y}_i)]$$

where M is the number of samples, y_i is the ground truth label, and $\hat{y}_i$ is the predicted probability. A five-fold cross-validation strategy is employed to ensure robustness and generalizability. Regularization techniques and dropout layers are incorporated to prevent overfitting.

C. Evaluation

The publicly accessible datasets UI-PRMD and KIMORE, which contain a variety of rehabilitation exercises, are used to assess the AMLE-GCN framework. The evaluation's main goal is to determine how well the framework predicts which exercise performances are correct and which are incorrect. Datasets are divided according to subject IDs in a five-fold cross-validation technique to guarantee robustness and generalisability.

IV. Dataset

To evaluate the proposed Attentional Multilayer Ensemble Graph Neural Networks (AMLE-GCN), three datasets were utilized, encompassing a variety of rehabilitation exercises to ensure the model's applicability in real-world scenarios.

A. UI-PRMD Dataset

The UI-PRMD (University of Idaho Physical Rehabilitation Movement Dataset) is a publicly available dataset specifically designed for analyzing rehabilitation exercises. It contains 10 rehabilitation exercises performed by 10 subjects, each repeating the exercises five times. The dataset provides detailed 3D joint position data, enabling evaluation of movement correctness and quality. Exercises such as "sit-to-stand" and "knee extensions," commonly prescribed in physical therapy, are included. The diversity of subjects and exercises makes this dataset highly reliable for skeletal movement analysis.

Fig. 2: Visualization of the UI-PRMD dataset, showing rehabilitation exercises captured in multiple poses and angles. These data represent key exercises for physical therapy and rehabilitation quality assessments.

B. KIMORE Dataset

The KIMORE (Kinect-based Movement Rehabilitation) dataset includes data from five rehabilitation exercises annotated by professional physiotherapists. It features 3D joint position data captured using Kinect sensors and corrective feedback indicating movement quality and correctness. This dataset is particularly valuable for testing the ability of models to detect subtle deviations in movement patterns, which are critical for effective rehabilitation assessments.

C. Dataset Preprocessing

To ensure consistency and enhance data quality, the following preprocessing steps were applied to all skeleton-based datasets:

- **Normalization:** Joint position and orientation features were normalized to account for variations in body size and sensor configurations, ensuring comparable feature representations across different subjects.
- **Noise Reduction:** A low-pass filter was applied to motion data to reduce sensor noise and enhance the reliability of captured skeletal trajectories.
- **Graph Representation:** Skeletal graphs were constructed, where each joint is represented as a node, and edges encode anatomical connections and temporal transitions between frames.
- **Cross-validation Partitioning:** A five-fold cross-validation strategy was employed, dividing the datasets into training, validation, and testing sets. This approach ensured robust evaluation of the model's predictive performance across subsets.
- **Consistency Checking:** The datasets were inspected for inconsistencies, such as erroneous or incomplete sequences, to ensure validity and reliability of the experimental results.

V. Results

This section presents the evaluation results of the proposed Attention-Augmented Graph Convolutional Network (AMLE-GCN) framework on the UI-PRMD and KIMORE datasets. The performance metrics—accuracy, precision, recall, and F1-score—are used to evaluate the model's ability to distinguish correct from incorrect exercises and effectively assess movement quality.

A. Performance on UI-PRMD Dataset

The AMLE-GCN framework achieved an average accuracy of **87.36%** on the UI-PRMD dataset, with the Multi-Layer Ensemble (MLE) modality performing best, reaching **92.56%** for exercise E10. The Position (POS) and Orientation (ORI) streams contributed average accuracies of **76.87%** and **72.99%**, while FLE-1 and FLE-2 achieved **78.86%** and **73.59%**, respectively

Exercise-specific results show that AMLE-GCN effectively captured movement nuances, achieving high accuracy for exercises E2 (**90.91%**) and E6 (**91.35%**), which involve complex joint movements. However, exercises with subtle movements, such as E8, posed more challenges, with an MLE accuracy of **80.16%**. These results highlight the framework's generalization

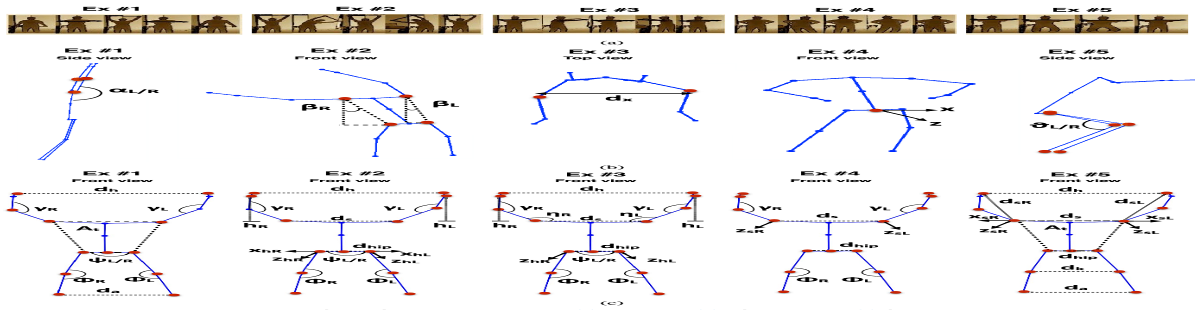

Fig. 3: Visualization of the KIMORE dataset, showing various rehabilitation exercises captured from multiple views (side, front, and top). Key joint angles and distances are annotated to represent movement patterns and physical parameters critical for assessing exercise quality.

TABLE I: Average prediction results implemented with state-of-the-art GCN models (see rows of Position and Orientation) and different ensemble methods of our EGCN implemented with different backbones, i.e., GCN, AGCN, and MS-G3D (Accuracy in %).

Method	**UI-PRMD (Kinect v2)**			**KIMORE**		
	GCN	**AGCN**	**MS-G3D**	**GCN**	**AGCN**	**MS-G3D**
Position Orientation	76.5	65.7	76.0	76.3	72.5	74.8
SLE	76.9	82.3	85.1	78.1	76.7	77.7
FLE-1	73.2	64.4	73.8	71.5	70.6	73.4
FLE-2	70.6	71.1	79.4	74.3	73.3	73.9
DLE-1	74.8	71.4	78.9	71.1	76.8	74.8
DLE-2	76.4	74.1	81.1	77.9	69.7	74.4
MLE	**86.9**	71.3	83.4	**80.1**	73.3	75.2
AMLE-GCN	**89**	70.2	81.3	**86**	70.8	75

ability across diverse exercise types while maintaining competitive accuracy.

B. Performance on KIMORE Dataset

On the KIMORE dataset, the AMLE-GCN achieved an average accuracy of **86.8%** using the MLE modality. Similar to the UI-PRMD dataset, the MLE approach outperformed other modalities, leveraging spatial and temporal skeletal features effectively. The POS and ORI streams contributed individual accuracies of **76.3%** and **77.5%**, respectively, reflecting their importance in capturing joint position and orientation dynamics.

Exercise-specific analysis revealed that the model achieved the highest MLE accuracy for exercise Es5 at **90.6%**, demonstrating its capability to assess complex movements involving multiple joints. Slight performance drops were observed for exercises like Es2(L) and Es2(R), with accuracies of **83.2%** and **83.6%**, respectively, suggesting challenges in modeling bilateral movements. Nonetheless, the overall results confirm the AMLE-GCN's adaptability and superior performance in evaluating rehabilitation exercises.

VI. Discussion

The results highlight the effectiveness of integrating attention mechanisms into graph convolutional networks for rehabilitation exercise assessment. The AMLE-GCN framework demonstrated significant improvements in accuracy and robustness compared to baseline methods. The attention mechanisms dynamically emphasized critical joints and movements, enabling the model to better capture both global and local features. Moreover, the evaluation across multiple datasets underscores the framework's generalizability and its potential for practical deployment in real-world rehabilitation settings.

TABLE II: *Comparison of different ensemble strategies and single-modal methods on the UI-PRMD dataset. Accuracy in percentages. POS and ORI represent position and orientation, respectively.*

Exercise ID	**POS**	**ORI**	**SLE**	**FLE-1**	**FLE-2**	**DLE-1**	**DLE-2**	**MLE**
E1	73.33	72.78	69.44	78.89	76.11	76.67	67.78	**87.22**
E2	83.64	88.18	83.64	84.55	80.00	83.64	85.45	**90.91**
E3	67.65	55.88	71.57	56.86	64.71	60.78	54.90	**75.49**
E4	79.29	67.14	78.69	82.14	70.00	72.86	76.43	**86.43**
E5	80.95	69.05	81.63	82.14	76.79	82.14	76.79	**90.48**
E6	81.51	73.97	82.88	82.19	76.03	84.92	83.56	**91.35**
E7	87.30	76.19	84.92	90.48	69.84	88.10	88.10	**88.89**
E8	57.14	65.08	77.78	67.46	61.90	73.02	53.17	**80.16**
E9	71.67	83.33	75.83	82.50	66.67	83.33	79.17	**85.83**
E10	84.26	70.37	82.41	75.93	64.81	82.41	85.19	**92.56**
Average	**76.87**	**72.99**	**78.53**	**78.86**	**73.59**	**78.88**	**74.42**	**87.36**

TABLE III: *Comparison of different ensemble strategies and single-modal methods on the KIMORE dataset. Accuracy in percentages. POS and ORI represent position and orientation, respectively.*

Exercise ID	**POS**	**ORI**	**SLE**	**FLE-1**	**FLE-2**	**DLE-1**	**DLE-2**	**MLE**
Es1	78.0	77.7	85.5	69.0	72.6	72.6	86.7	**88.2**
Es2(L)	75.5	77.7	81.6	74.5	70.0	74.9	79.6	**83.2**
Es2(R)	71.1	80.1	77.7	76.6	76.2	80.1	81.5	**83.6**
Es3(L)	79.1	79.8	78.8	71.0	67.5	73.3	79.1	**81.9**
Es3(R)	81.0	80.3	87.1	75.1	73.8	73.7	79.4	**88.0**
Es4(L)	83.0	79.7	85.6	78.5	74.9	84.8	85.7	**88.9**
Es4(R)	80.6	79.1	87.3	76.4	76.8	81.8	89.1	**91.4**
Es5	74.1	77.7	86.3	74.1	66.7	74.1	83.9	**90.6**
Average	**76.3**	**77.5**	**83.0**	**73.1**	**72.3**	**77.4**	**82.8**	**86.8**

VII. Conclusion

This study presents the Attentional Multilayer Ensemble Graph Convolutional Network (AMLE-GCN) for skeleton-based rehabilitation exercise assessment. Leveraging spatial and temporal attention, it captures critical joints and movements while addressing limitations of traditional GCNs. Its multi-level ensemble strategy enhances adaptability and generalization. Evaluations on UI-PRMD and KIMORE datasets show state-of-the-art accuracy in assessing exercises, highlighting its potential for precise, scalable home-based rehabilitation. Future work includes expanding datasets, integrating diverse modalities, and exploring self-supervised learning.

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