Homework 6

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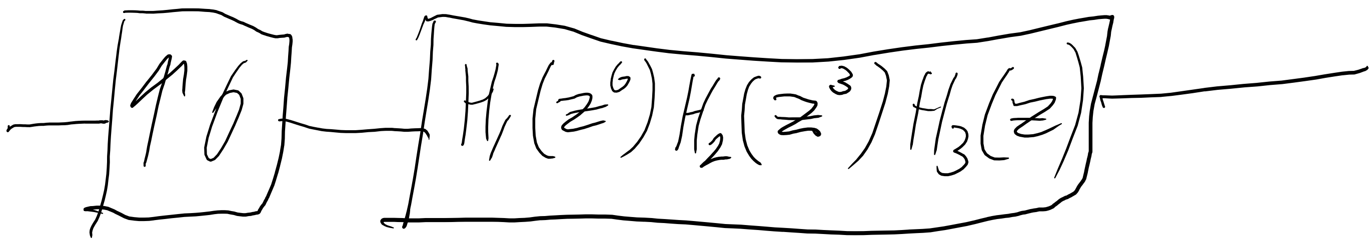
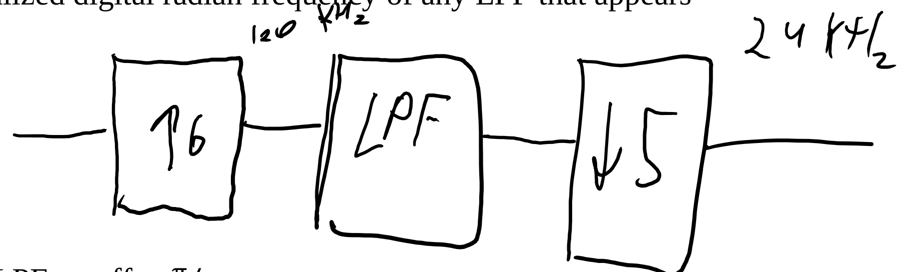
1. Refer to **Figure 1** which shows the system $H_3(\uparrow 3)H_2(\uparrow 2)H_1$. Using Noble identities, this can be expressed as either $(\uparrow M)G$ or $G(\uparrow M)$ for some LTI system $G(z)$ and decimation factor M. specify which is corrected and in particular specify $M, G(z)$, select the correct block diagram and write the correct operator notation for it. **Note:** Recall that for operator notation, ABx means first apply B, then apply A; in a diagram, with the signal flowing left to right, that means the B block is on the left, the A-block is on the right
   a. $\left(H_1(z^6)H_2(z^3)H_3(z)\right)(\uparrow 6)$

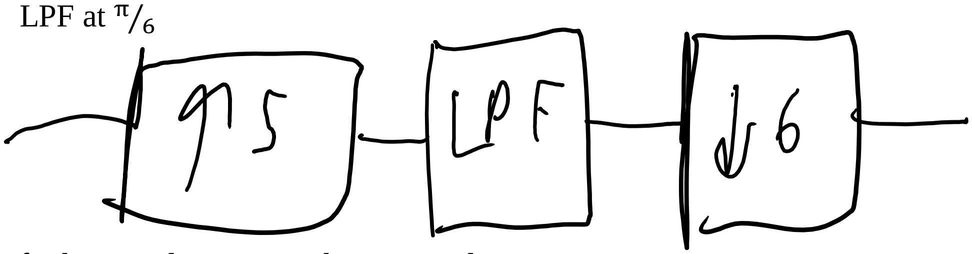


2. Suppose we want to perform sampling rate conversion to map data sampled at 20kHz to data sampled at 24kHz. Assuming the original analog signal is lowpass bandlimited so its information is completely contained in the 20kHz data we start with. Our goal is to prevent aliasing and imaging distortion at the output (even if it means loss of information, although we want to minimize any such loss). In addition, the upsampling and downsampling factors must be as small as possible
   a. What is the highest frequency that is present in the input signal?
      i. 10kHz
   b. Draw a block diagram showing the sequence of operations and specify the cutoff frequency in normalized digital radian frequency of any LPF that appears



      - LPF cutoff at $\pi/6$
   c. Specify the sampling rate that appears in intermediary stages between the 20kHz and 24kHz output
      i. 120kHz
   d. Specify the equivalent cutoff frequency of the LPF in Hertz (that is, given the digital cutoff frequency and the sampling rate that filter operates at, determine the cutoff in Hertz)
      i. $\omega = 2\pi \frac{f}{f_s} = \frac{\pi}{6}$
         1) $f = \frac{1}{12} f_s = \frac{120kHz}{12} = 10kHz$
   e. Does this system necessarily lead to a loss of information? If so, which frequency band needs to be eliminated?
      i. No loss of information since input is limited to 10kHz
3. Repeat the previous problem for the case of converting 24kHz to 20kHz
   a. What is the highest frequency that is present in the input signal?
      i. 12kHz

b. Draw a block diagram
   i. LPF at $^{\pi}/_{6}$



c. Specify the sampling rate in the intermediary stages
   i. 24*5 = 120 kHz

d. Specify the equivalent cutoff frequency of the LPF in Hertz
   i. 10kHz

e. Does the system necessarily lead to a loss of information? If so, which frequency band needs to be eliminated
   i. Yes. Frequency from 10kHz-12kHz needs to be eliminated

4. Refer to the analysis and synthesis filter banks shown in Figure 4.

b. If the input to the filter bank is X(z), the output has the form $T(z)X(z) + A(z)X(-z)$ where the PR condition requires $T(z) = cz^{-N}$ for some constant c and delay N, and A(z) = 0. These in turn can be expressed in terms of H0, H1, F0, F1. Compute T and A using time-domain operations, that is an expression such as U(z)V(z) can be obtained by convolving u[n], v[n]. By performing appropriate convolution and sums, you will end up with t[n], a[n] (the coefficients of the FIR filters T(z), A(z)). Then we need t[n] to have one non-zero coefficient and a[n] to be zero. Check this, and determine the error (i.e., the maximum magnitude of any of the coefficients that should be zero), and also find c and N
   i. $w_0[n] = \frac{1}{2}\left(h_0[n] * x[n] + (-1)^n h_0[n] * x[n]\right)$
   ii. $w_1[n] = \frac{1}{2}\left(h_1[n] * x[n] + (-1)^n h_1[n] * x[n]\right)$
   iii. $y[n] = f_0[n] * w_0[n] + f_1[n] * w_1[n]$
      1) $= f_0[n] * \frac{1}{2}\left(h_0[n] * x[n] + h_0[n] * (-1)^n x[n]\right) + f_1[n]$
         $* \frac{1}{2}\left(h_1[n] * x[n] + h_1[n] * (-1)^n x[n]\right)$
      2) $= \frac{1}{2}\Big(\left(f_0[n] * h_0[n] + f_1[n] * h_1[n]\right) * x[n] + \left(f_0[n] * h_0[n] + f_1[n] * h_1[n]\right)$
         $* (-1)^n x[n]\Big)$
   iv. $t[n] = \frac{1}{2}\left(f_0[n] * h_0[n] + f_1[n] * h_1[n]\right)$
   v. $a[n] = \frac{1}{2}\left(f_0[n] * h_0[n](-1^n) + f_1[n] * h_1[n](-1^n)\right)$
   vi. c = 2, N = 11