A.31.

current population : $p(0)$
population after x years : $p(x)$

$$p(x+1) = p(x) + 0.017 \cdot p(x) = 1.017 \cdot p(x)$$

$$p(x) = 1.017^{x} \cdot p(0)$$

- find x where $1.017^{x} > 2$

$$x \cdot \log 1.017 > \log 2$$

$$x > 41.1189\cdots$$

- after 42 years the population has doubled.

2P

2p

- find x where $1.017^x > 2$

$$x \cdot \log 1.017 > \log 2$$

$$x > 41.1189\cdots$$

- after 42 years the population has doubled.

A.32. The growth rate on the Fibonacci sequence is in each step approximating to the golden ratio φ (1.61803), then growth rate $\propto \varphi$

The growth rate on exponential functions increases according to the value of the base; then:

$E_n = e^n$

Formal proof of φ

$\lim\limits_{n \to \infty} \frac{F_{n+1}}{F_n} = \varphi$

A function b^x is faster than Fibonacci sequence if $b > \varphi$

$\lim \frac{E_{n+1}}{E_n} = e$

A function b^x is rising at the same pace with F.S. if $b = \varphi$

$e > \varphi$

A function b^x is slower than F.S. if $b < \varphi$

A.33. is on the last page.

A.34. The Golden Spiral is a growing spiral with special pattern: when it turns its direction, it increases the size of ~~[illegible]~~ arm (edge) following the

A.33.

The longterm behavior of the given CA. : ~~Homogeneous (Class I).~~ Class 4, periodic also includes stable pattern.

With a value α greater than 0.33 (around)

: The values of cells soon become a stable and max state (65535) except a_0 and a_{100} (and some cells near a_0 and a_{100}).

e.g.) $\alpha = 0.34$

C.A.

[42000, 52963, 60183, 65087, 65535, ···· , 65535, 61927, 54679, 44217, 31156, 16265, 420]

⇒ with bigger α

the number of cells which have the state 65535 are more.

With a value α less than 0.33 (around)

: The values of cells soon become a stable and min state (0) except a_0 and a_{100} (and some cells near a_0 and a_{100}).

e.g.) $\alpha = 0.1$

[42000, 4725, 531, 59, 6, 0, ··· 0, 5, 47, 420]

⇒ with smaller α

the number of cells which have the state 0 are more.

if $b = \varphi$

$e \to \varphi$

A function b^x is slower than F.S. if $b < \varphi$

A.33. is on the last page.

A.34. The Golden Spiral is a growing spiral with special pattern: when it turns its direction, it increases the size of arm (edge) following the pattern of Fibonacci Sequence.

2p

A.36.
- Given 3.3 for a, x_i and $\hat{x}_i$ are oscillating between 0.479427 and 0.823603. (Class II)
- Given 3.51 for a, x_i and $\hat{x}_i$ are oscillating between the values 0.506713, 0.377722 and 0.825019, 0.877342. The pattern is like … 0.506713, 0.877722, 0.377722, 0.825019, … (Class II).
- Given 3.75 for a, x_i and $\hat{x}_i$ are still oscillating, but is difficult to see a pattern. (class IV).
- result of the function is depicted on the next page

4p

Cl: $\lim\limits_{n\in\mathbb{N}} \frac{F_{n+1}}{F_n} = \frac{1+\sqrt{5}}{2} =: \phi$

Pr: $\beta := \lim\limits_{n\in\mathbb{N}} \frac{F_{n+1}}{F_n} = \lim\limits_{n\in\mathbb{N}} \frac{F_{n+2}}{F_{n+1}}$

$$= \lim_{n\in\mathbb{N}} \left(\frac{F_{n+1}+F_n}{F_{n+1}}\right)$$

$$= \lim_{n\in\mathbb{N}} \left(1 + \frac{F_n}{F_{n+1}}\right)$$

$$= 1 + \beta^{-1}$$

$$\Rightarrow \beta = \phi$$

$$\cap$$

$$\frac{1 \pm \sqrt{5}}{2}$$