Binary Classification Using (tensor) LDA

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Outline

Linear Discriminant Analysis

Tensor Linear Discriminant Analysis

Linear Discriminant Analysis

- **Task**: Implement a binary classifier for car detection.
- Using (conventional) Linear Discriminant Analysis.

Steps

- Calculate mean μ
  - For class 0, class 1, and for the overall dataset
- Calculate covariance matrix
  - S_B and S_W
- Calculate projector matrix w

Determine w

- w is a projection vector that maximally separates the data

$$w = \arg\max_{w} \frac{w^T S_B w}{w^T S_w w} \tag{1}$$

Determine a projector w

$$y(X) = \begin{cases} +1, & \text{if } w^T x \geq \theta \\ -1, & \text{otherwise} \end{cases} \tag{2}$$

Plot from S_W

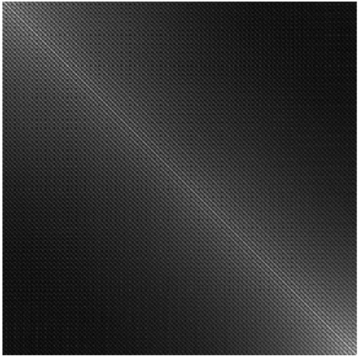

Figure 1: Plot from S_W

Plot from S_B

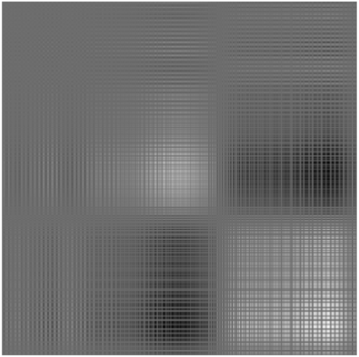

Figure 2: Plot from S_B

Plot from w

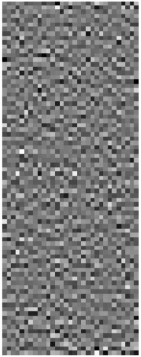

Figure 3: Plot from w

Plot from w

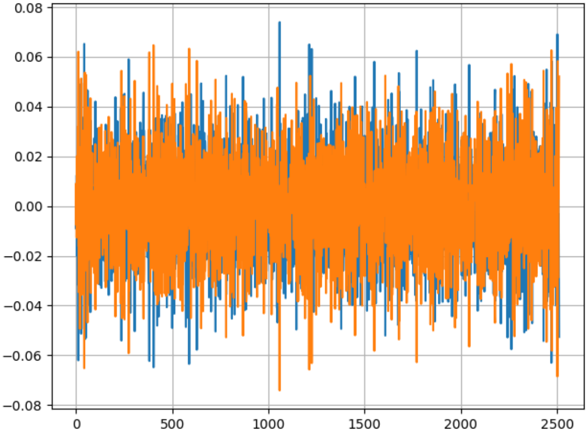

Figure 4: Plot from w

Result

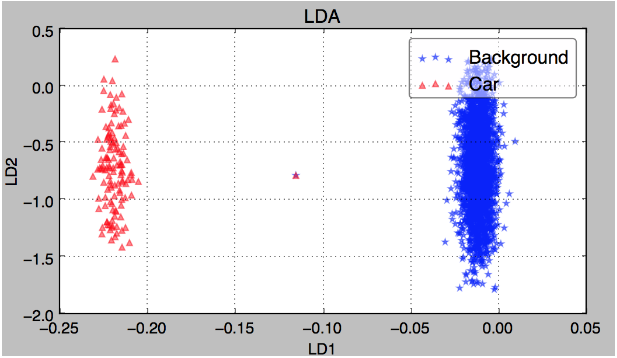

Figure 5: Result from LDA

Benchmarking

- S_W duration: 140.9035672799946
- S_B duration 0.48717122300877236
- W duration 50.67935458800639
- Prediction duration: 0.0007735701052006333

Precision and recall

$$Precision = \frac{tp}{tp + fp} \tag{3}$$

$$Recall = \frac{tp}{tp + fn} \tag{4}$$

Precision and recall

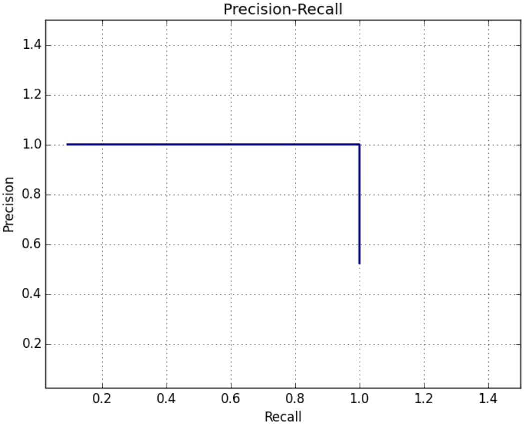

Figure 6: Precision and recall

Tensor Linear Discriminant Analysis

- **Task**: Implement a binary classifier for car detection.
- Using this time Tensor Linear Discriminant Analysis.

Tensor LDA properties

- Treat images as images not as vectors
- Loss information when images are flattened
- Fast learning
- Handles small sample size problem

Steps

- Read data
- Initialize vector u, and v
- Compute contractions u, and v for all images until difference is less than ϵ
  - $v(t) = \arg\min_{v} ||Xv - y||^2$
  - $u(t) = \arg\min_{u} ||Xu - y||^2$
  - u and v are kept orthogonal using Gram-Schmidt procedure
- Compute W
  - $W = u_1 v_1^T + ... + u_\rho v_\rho^T$
- Evaluate multiple classifiers on the training data
- Use best classifier fot test data

Tensor w

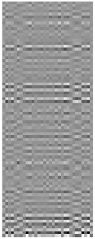

Figure 7: Plot from w in Tensor LDA

Tensor *w*

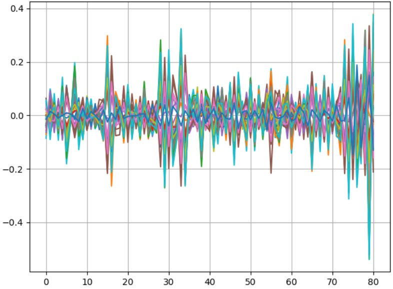

Figure 8: Plot from *w* in Tensor LDA

Tensor w

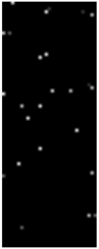

Figure 9: Plot for w in Tensor LDA

Precision and recall

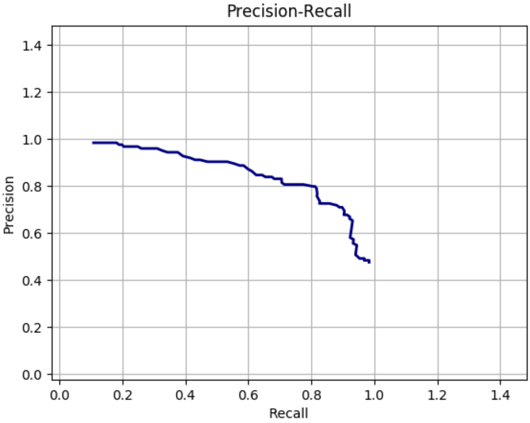

Figure 10: Precision and recall

Benchmarking

- Compute contraction duration: 26.600847046996932
- Evaluation duration: 4.858664466999471
- Prediction duration: 0.0010174380004173145