```java
class Person {
    void speakTo(Person other) { System.out.println("kudos"); }
    void watch(SoccerPlayer other) { System.out.println("wow"); }
}

class Athlete extends Person {
    void speakTo(Athlete other) { System.out.println("take notes"); }
    void watch(Athlete other) { System.out.println("game on"); }
}

class SoccerPlayer extends Athlete {
    void speakTo(Athlete other) { System.out.println("respect"); }
    void speakTo(Person other) { System.out.println("hmph"); }
}
```

```java
Person itai = new Person();

SoccerPlayer shivani = new Person();

Athlete sohum = new SoccerPlayer();

Person jack = new Athlete();

Athlete anjali = new Athlete();

SoccerPlayer chirasree = new SoccerPlayer();
```

| Var | Static | Dynamic |
|---|---|---|
| itai | P | P |
| ~~shivani~~ | ~~SP~~ | ~~P~~ |
| sohum | A | SP |
| jack | P | A |
| anjali | A | A |
| chirasee | SP | SP |

```java
itai.watch(chirasree);

jack.watch(sohum);

itai.speakTo(sohum);

jack.speakTo(anjali);

anjali.speakTo(chirasree);

sohum.speakTo(itai);

chirasree.speakTo((SoccerPlayer) sohum);

sohum.watch(itai);

sohum.watch((Athlete) itai);

((Athlete) jack).speakTo(anjali);

((SoccerPlayer) jack).speakTo(chirasree);

((Person) chirasree).speakTo(itai);
```

Sp 18 MT2
Q8C

Suppose we have an algorithm with a runtime that is $\Theta(N^2 \log N)$ in all cases.

Which of these statements are definitely **true** about the runtime, definitely **false**, or there is **not enough information (NEI)**?

| | | | |
|---|---|---|---|
| $O(N^2 \log N)$ | ○ True | ○ False | ○ NEI |
| $\Omega(N^2 \log N)$ | ○ True | ○ False | ○ NEI |
| $O(N^3)$ | ○ True | ○ False | ○ NEI |
| $\Theta(N^2 \log 4 N)$ | ○ True | ○ False | ○ NEI |

$\log_4 N$

all logs are in same family of functions

logs of diff bases differ by a constant,

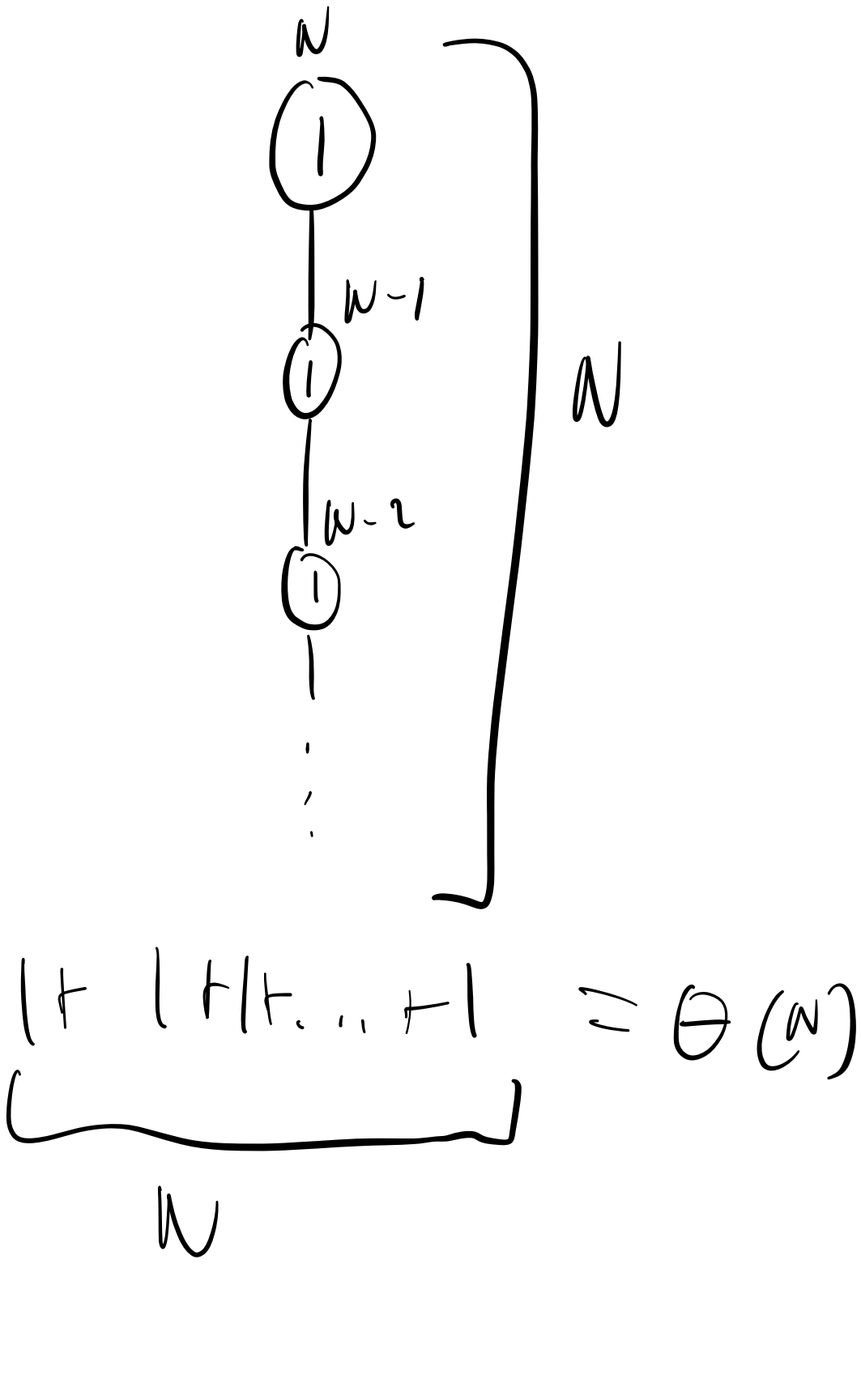
so can treat all logs the same.

```
public static void g4(int N) {
    if (N == 0) {
        return;
    }

    g4(N - 1);

    if (k(N)) {
        g4(N - 1);
    }
}
```

Best case: k(N) always false



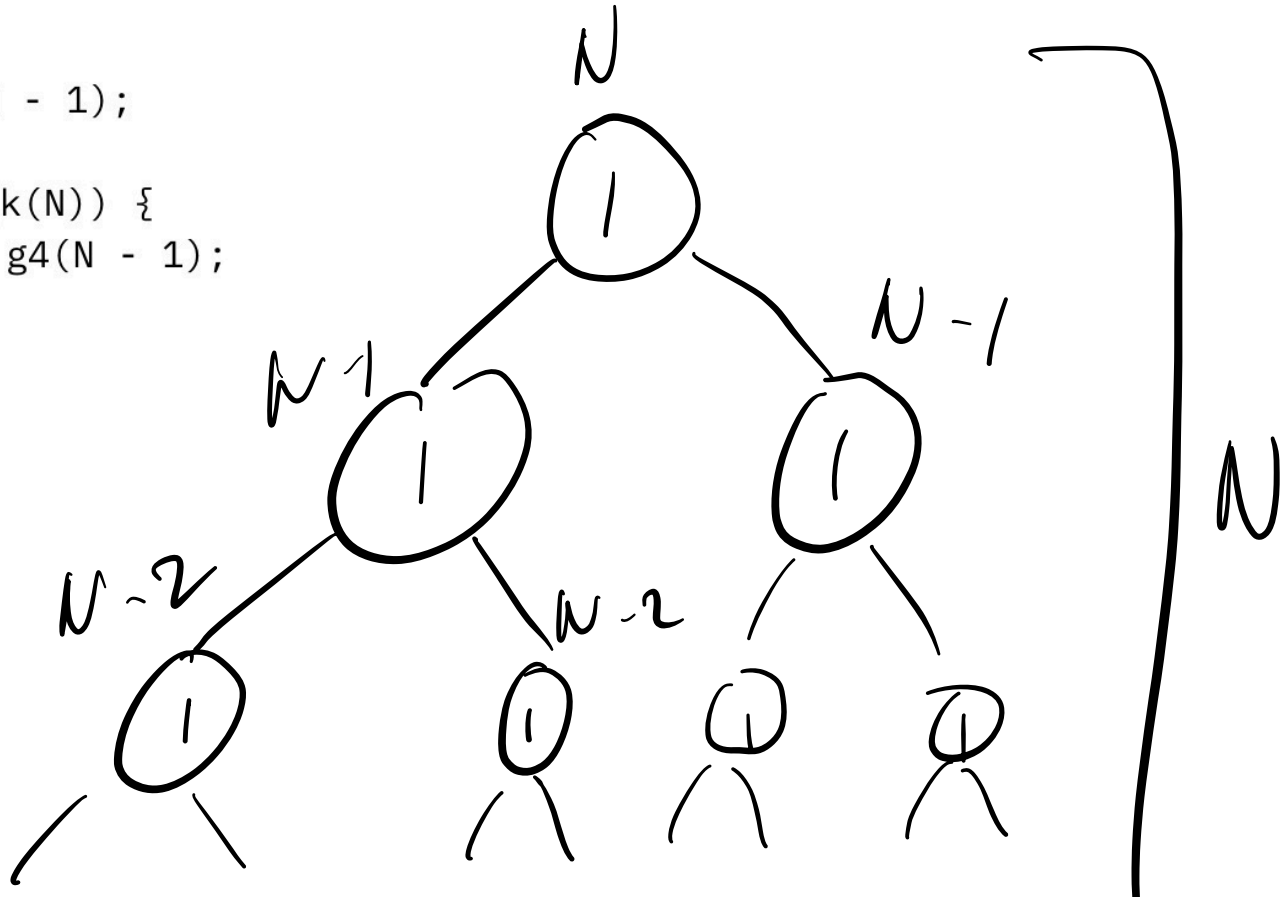

$1 + 1 + 1 + \ldots + 1 = \Theta(N)$

N

```
public static void g4(int N) {
    if (N == 0) {
        return;
    }

    g4(N - 1);

    if (k(N)) {
        g4(N - 1);
    }
}
```




$2^0 + 2^1 + 2^2 + \ldots + 2^N$

$1 + 2 + 4 + \ldots + 2^N = \Theta(2^N)$

$2^d$, $d$ = level

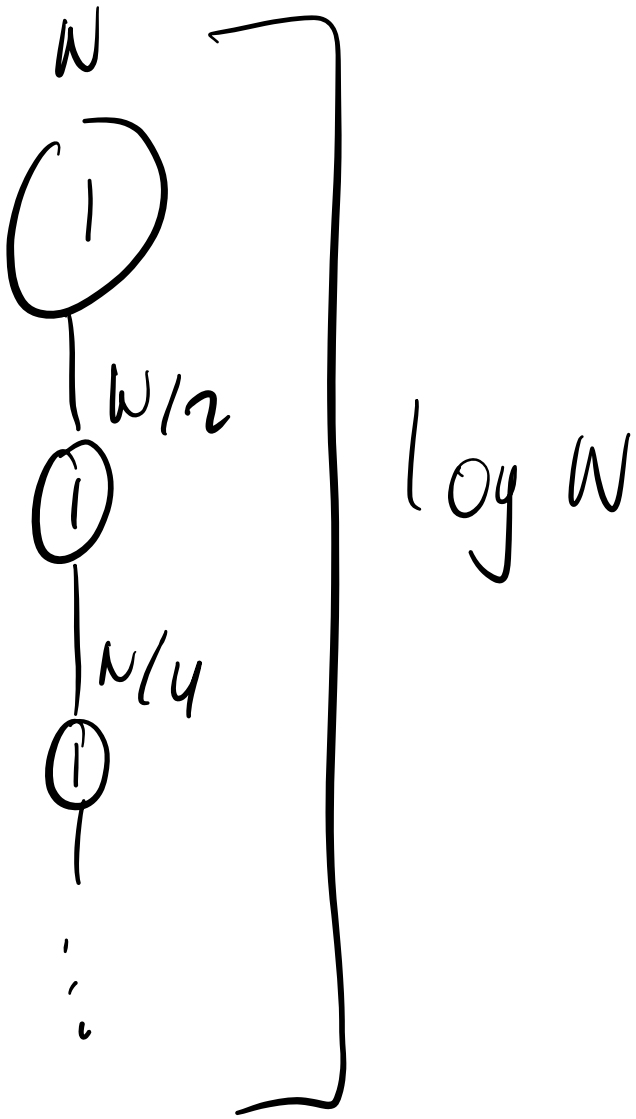
$1 + 2 + 4 + \ldots + f(N) = \Theta(f^N)$

```java
public static void g5(int N) {
    if (N == 0) {
        return;
    }

    g5(N / 2);

    if (k(N)) {
        g5(N / 2);
    }
}
```

Best



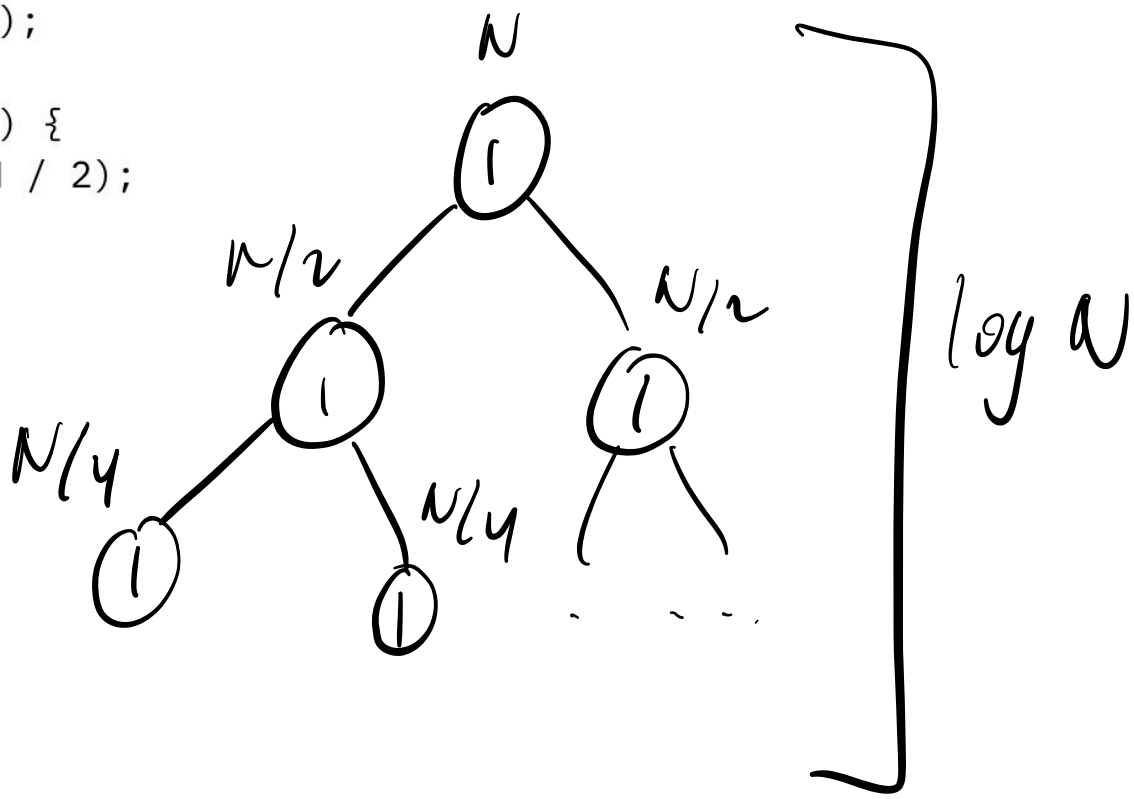

$\underbrace{1 + 1 + 1 + \ldots + 1}_{\log N \text{ terms}} = \Theta(\log N)$

```
public static void g5(int N) {
    if (N == 0) {
        return;
    }

    g5(N / 2);

    if (k(N)) {
        g5(N / 2);
    }
}
```

Worst case




$$1 + 2 + 4 + \ldots + N = \Theta(N)$$

$$2^{\log_2 N} = N$$

```
public static void func(int n) {
    for (int i = 0; i < n; i++) {
        for (int j = 0; j < n; j++) {
            if (j % 2 == 0) {
                n -= 1;
            }
        }
    }
}
```

N  N

$[0 \ldots N] = N/2$ evens

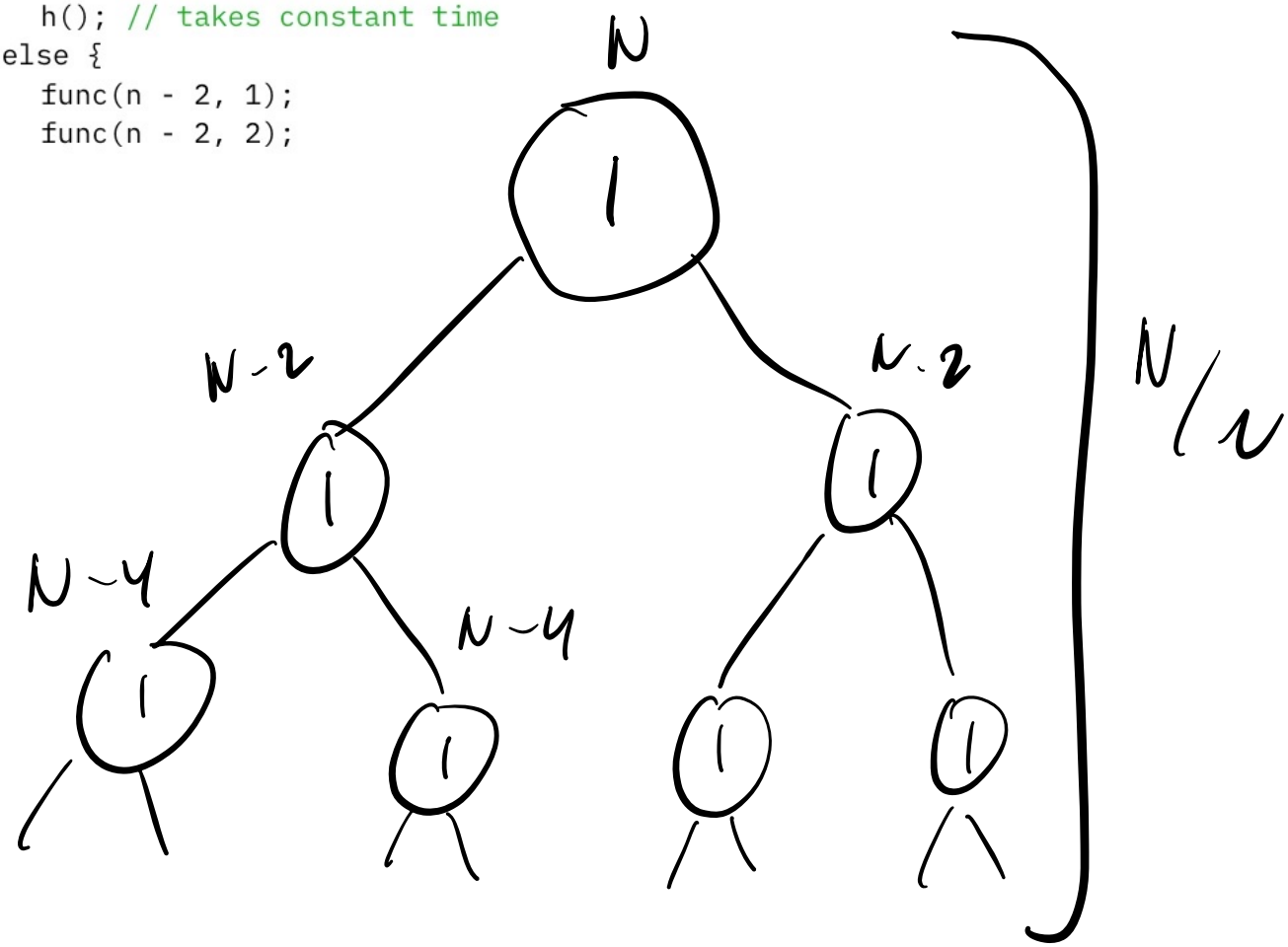
| inner loop runs | N |
|---|---|
| 1 | N/2 |
| 2 | N/4 |
| 3 | N/8 |
| ⋮ | ⋮ |
| N | N/N |

$$\frac{N}{2} + \frac{N}{4} + \frac{N}{8} + \ldots + \frac{N}{N}$$

$$N\left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots + \frac{1}{N}\right)$$

$$N\left(\frac{1}{2}\right) = \Theta(N)$$

```java
public static void func(int n, int k) {
    if (n <= 1) {
        h(); // takes constant time
    } else {
        func(n - 2, 1);
        func(n - 2, 2);
    }
}
```

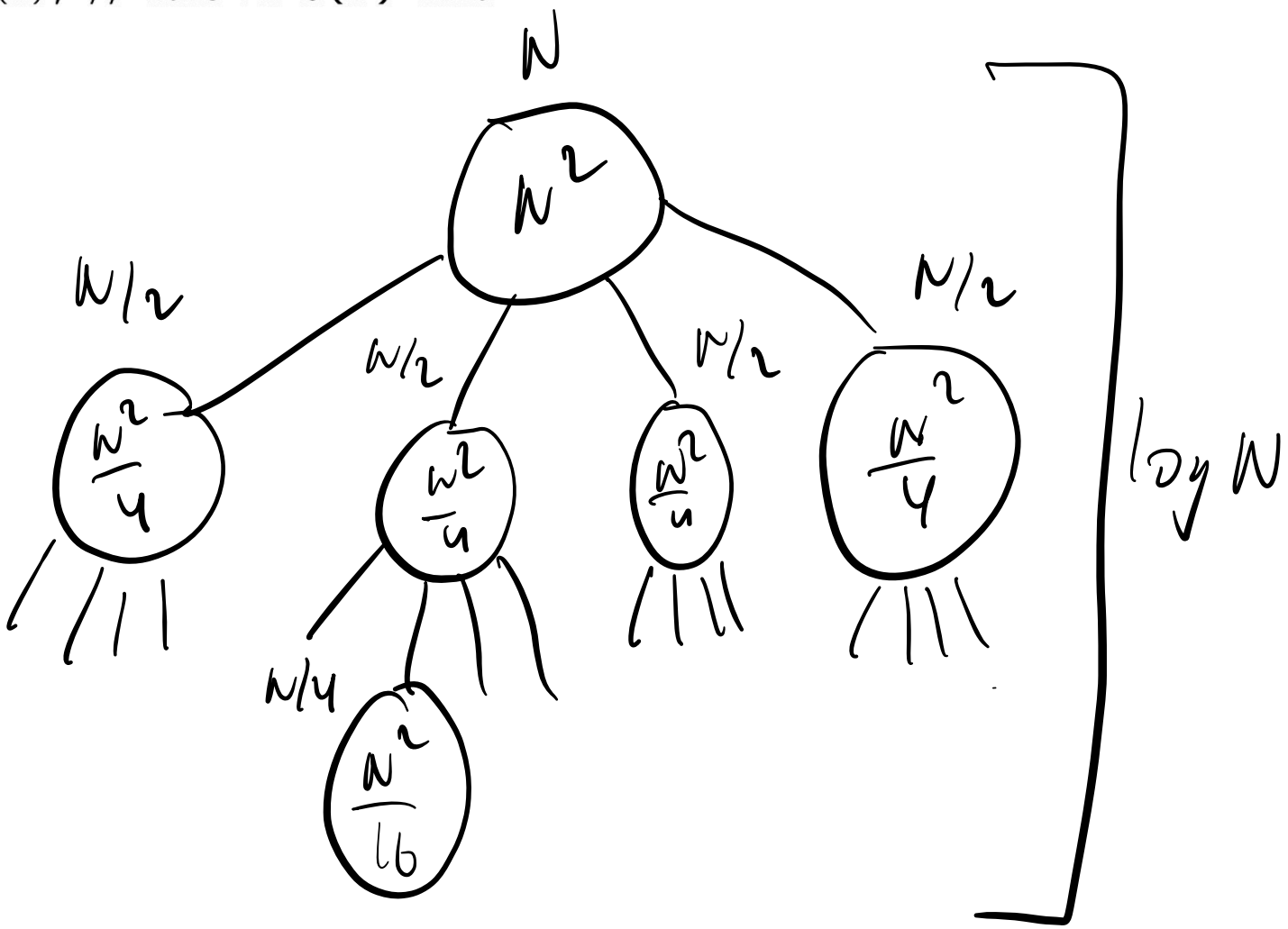



$$1 + 2 + 4 + \ldots + 2^{N/2} = 2^{N/2}$$

$$2^{k} = 2^{N/2} = \Theta(2^{N})$$

$$\left(2^{N/2}\right)^2 = 2^{N}$$

```
public static void f4(int N) {
    if (N == 0) {
        return;
    }
    f4(N / 2);
    f4(N / 2);
    f4(N / 2);
    f4(N / 2);
    g(N); // runs in Θ(N²) time
}
```




$$N^2 + 4\left(\frac{N^2}{4}\right) + 16\left(\frac{N^2}{16}\right) + \ldots +$$

$$N^2 + C_1\left(\frac{N^2}{C_1}\right) + C_2\left(\frac{N^2}{C_2}\right) + \ldots + C\left(\frac{N^2}{C_2}\right)$$

$$\underbrace{N^2 + C_1\left(\frac{N^2}{C_1}\right) + C_2\left(\frac{N^2}{C_2}\right) + \ldots + C\left(\frac{N^2}{C_2}\right)}_{\log N \text{ terms}}$$

$$\underbrace{N^2 + N^2 + N^2 + N^2 + \ldots + N^2}$$

$$\Theta\left(N^2 \log N\right)$$