Universidade Estadual de Londrina
Centro de Ciências Exatas
Departamento Matemática

Lista 05

Dados de Identificação	
Professor:	Matheus Pimenta
Disciplina:	Cálculo I - 1MAT096
Aluno:	

Os exercícios em sua maioria foram retirados do livro: Um Curso de Cálculo - Vol. 1 - Autor:Guidorizzi, Hamilton Luiz (cap. 10, 11 e 12), a biblioteca possui diversos exemplares. Ao final do livro tem as respostas.

1. Determine a única função $y = y(x), x \in \mathbb{R}$, que satisfaça as condições dadas.
   (a) $\frac{\partial y}{\partial x} = 2y$ e $y(0) = 1$
   (b) $\frac{\partial y}{\partial x} = -y$ e $y(0) = -1$
   (c) $\frac{\partial y}{\partial x} = \frac{1}{2}y$ e $y(0) = 2$

2. Calcule:
   (a) $\int x dx$
   (b) $\int (3x + 1)dx$
   (c) $\int (x + \frac{1}{x})dx$
   (d) $\int (ax + b)dx$ onde a, b são escalares
   (e) $\int (3\sqrt[5]{x^2} + 3)dx$

3. Calcule:
   (a) $\int (x^3 + 2x + 3)dx$
   (b) $\int (x^2 + x + 1)dx$
   (c) $\int (x + \frac{1}{x^3})dx$
   (d) $\int (2 + \sqrt[4]{x})dx$
   (e) $\int \frac{x^2 + 1}{x} dx$

4. Seja $\alpha \neq 0$ um real fixo. Verifique que:

(a) $\int \sin(\alpha x)dx = -\frac{1}{\alpha}\cos(\alpha x) + k$

(b) $\int \cos(\alpha x)dx = \frac{1}{\alpha}\sin(\alpha x) + k$

5. Calcule:

(a) $\int e^{2x}dx$

(b) $\int e^{-x}dx$

(c) $\int (x + 3e^x)dx$

(d) $\int \cos(3x)dx$

(e) $\int \sin(5x)dx$

(f) $\int (x^2 + \sin(x))dx$

6. Calcule:

(a) $\int \frac{e^x + e^{-x}}{2}dx$

(b) $\int (\sin(3x) + \cos(5x))dx$

(c) $\int \sin(\frac{x}{2})dx$

(d) $\int (\sqrt[3]{x} + \cos(3x))dx$

(e) $\int (1 - \cos(4x))dx$

(f) $\int \cos(\frac{x}{3})dx$

(g) $\int 5e^{7x}dx$

7. Determine a função $y = y(x), x \in \mathbb{R}$, tal que:

(a) $\frac{\partial y}{\partial x} = 3x - 1$ e $y(0) = 2$

(b) $\frac{\partial y}{\partial x} = \cos(x)$ e $y(0) = 0$

(c) $\frac{\partial y}{\partial x} = e^{-x}$ e $y(0) = 1$

(d) $\frac{\partial y}{\partial x} = \frac{1}{x^2}$ e $y(1) = 1$

(e) $\frac{\partial y}{\partial x} = x + \frac{1}{\sqrt{x}}$ e $y(1) = 0$

(f) $\frac{\partial y}{\partial x} = 2x - 1$ e $y(0) = 0$

8. Calcule:

(a) $\int_0^1 (x + 3)dx$

(b) $\int_0^4 \frac{1}{2}dx$

(c) $\int_1^3 dx$

(d) $\int_1^3 \frac{1}{x^3}dx$

(e) $\int_{-1}^1 (2x + 1)dx$

(f) $\int_0^1 (5x^3 - \frac{1}{2})dx$

9. Calcule:

(a) $\int_1^0 (2x + 3)dx$

(b) $\int_1^4 \frac{1}{\sqrt{x}}dx$

(c) $\int_1^2 (x^3 + x + \frac{1}{x^3})dx$

(d) $\int_0^4 \sqrt{x}dx$

(e) $\int_0^1 \sqrt[8]{x}dx$

(f) $\int_{-3}^3 x^3 dx$

10. Calcule:

(a) $\int_1^4 (5x + \sqrt{x})dx$

(b) $\int_1^2 \frac{1+x}{x^3}dx$

(c) $\int_1^4 \frac{1+x}{\sqrt{x}}dx$

(d) $\int_0^1 (x-3)^2 dx$

(e) $\int_1^2 \frac{1+t^2}{t^4}dt$

11. Calcule:

(a) $\int_{-1}^0 e^{-2x}dx$

(b) $\int_0^{\frac{\pi}{3}} (3 + \cos(3x))dx$

(c) $\int_0^1 \sin(5x)dx$

(d) $\int_0^1 2xe^{x^2}dx$

(e) $\int_0^{\frac{\pi}{4}} \sec^2(x)dx$

(f) $\int_0^{\frac{\pi}{4}} \tan^2(x)dx$

12. Calcule:

(a) $\int_0^{\frac{\pi}{3}} \cos(2x)dx$

(b) $\int_0^1 \frac{x^2}{1+x^3}dx$

(c) $\int_0^1 \frac{x^2}{(1+x^3)^2}dx$

(d) $\int_1^3 \frac{2}{5+3x}dx$

(e) $\int_{-1}^1 \sqrt[3]{x+1}dx$

(f) $\int_{-1}^0 x(x+1)^{100}dx$

(g) $\int_0^1 xe^{x^2}dx$

(h) $\int_{-1}^0 x\sqrt{x+1}dx$

13. Calcule:

(a) $\int_1^2 (x-2)^5 dx$

(b) $\int_0^1 (3x+1)^4 dx$

(c) $\int_0^1 \sqrt{3x+1}dx$

(d) $\int_{-3}^4 \sqrt[3]{5-x}dx$

(e) $\int_1^2 \frac{2}{(3x-2)^3}dx$

(f) $\int_0^1 \frac{1}{(x+1)^5}dx$

(g) $\int_{-2}^1 \frac{3}{4+x}dx$

14. Calcule a área do conjunto dado:

(a) $A = \{(x,y) \in \mathbb{R}^2; 1 \leq x \leq 2 \text{ e } 0 \leq y \leq \sqrt{x-1}\}$

(b) $B = \{(x,y) \in \mathbb{R}^2; 0 \leq x \leq 2 \text{ e } 0 \leq y \leq \frac{x}{1+x^2}\}$

15. Calcule:

(a) $\int_0^1 x\sqrt{x^2+3}dx$

(b) $\int_0^1 x(x^2+3)^5dx$

(c) $\int_0^1 x\sqrt{1-x^2}dx$

(d) $\int_{-1}^0 x^2e^{x^3}dx$

(e) $\int_0^1 \frac{1}{1+4s}ds$

(f) $\int_0^3 \frac{x}{\sqrt{x+1}}dx$

16. Calcule:

(a) $\int_{-1}^1 x^3(x^2+3)^{10}dx$

(b) $\int_0^{\frac{\pi}{3}} \sin(x)\cos^2(x)dx$

(c) $\int_0^{\frac{\pi}{6}} \cos(x)\sin^5(x)dx$

(d) $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sin(x)(1-\cos^2(x))dx$

(e) $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sin^3(x)dx$

17. Calcule e confira sua resposta por derivação quando possível:

(a) $\int 3dx$

(b) $\int x^5dx$

(c) $\int \sqrt{x}dx$

(d) $\int \sqrt[5]{x^2}dx$

(e) $\int \frac{1}{x^3}dx$

(f) $\int \frac{x+x^2}{x^2}dx$

(g) $\int \left(\cos(3x)+\frac{1}{2}\sin(4x)\right)dx$

(h) $\int \left(\frac{1}{3}\cos(2x)+\frac{1}{2}\cos(3x)\right)dx$

18. Calcule e confira sua resposta por derivação quando possível:

(a) $\int (\frac{1}{x}+\frac{1}{x^2})dx$

(b) $\int (e^x+4)dx$

(c) $\int e^{5x}dx$

(d) $\int (e^{2x}+e^{-x})dx$

(e) $\int (\frac{1}{x}+\frac{1}{e^x})dx$

(f) $\int_0^1 e^{2x}dx$

(g) $\int \left(2+\frac{1}{3}\sin(2x)\right)dx$

19. Calcule e confira sua resposta por derivação quando possível:

(a) $\int_1^2 \left(x+\frac{1}{x}\right)dx$

(b) $\int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-x^2}}dx$

(c) $\int \sin(2x)dx$

(d) $\int \cos(5x)dx$

(e) $\int \cos(\sqrt{3t})dt$

(f) $\int \left(\frac{1}{2}-\frac{1}{2}\cos(2x)\right)dx$

(g) $\int \left(\frac{1}{3}+\frac{5}{2}\cos(7x)\right)dx$

20. Calcule:

(a) $\int_0^{\frac{\pi}{3}} \sin(2x)dx$

(b) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos(\frac{\pi}{2})dx$

(c) $\int_0^{\frac{\pi}{3}} (\sin(3x) + \cos(3x))dx$

(d) $\int \tan(x)dx$

(e) $\int \tan^2(x)dx$

(f) $\int \sec^2(x)dx$

(g) $\int \sec(x)dx$

(h) $\int 3^x dx$

21. Calcule:

(a) $\int (5^x + e^{-x})dx$

(b) $\int \sin(6x)\cos(x)dx$

(c) $\int \sin(5x)\cos(x)dx$

(d) $\int \sin(3x)\cos(3x)dx$

(e) $\int \sin(3x)\sin(2x)dx$

(f) $\int \sin(x)\sin(3x)dx$

(g) $\int \sin(3x)\cos(2x)dx$

(h) $\int \cos(7x)\cos(3x)dx$

22. Calcule:

(a) $\int (3x-2)^3 dx$

(b) $\int \sqrt{3x-2}dx$

(c) $\int \frac{1}{3x-2}dx$

(d) $\int x\sin(x^2)dx$

(e) $\int xe^{x^2}dx$

(f) $\int x^2 e^{x^3}dx$

23. Calcule:

(a) $\int \cos^3(x)\sin(x)dx$

(b) $\int \sin^5(x)\cos(x)dx$

(c) $\int \frac{5}{4x+3}dx$

(d) $\int \frac{x}{1+4x^2}dx$

(e) $\int \frac{x}{(1+4x^2)^2}dx$

(f) $\int e^x\sqrt{1+e^x}dx$

24. Calcule:

(a) $\int_0^1 xe^{-x^2}dx$

(b) $\int_1^2 \frac{x}{1+3x^2}dx$

(c) $\int_{-\frac{3}{2}}^{-1} (2x+3)^1 00dx$

(d) $\int_0^{\sqrt{\pi}} x\sin(3x^2)dx$

(e) $\int_0^{\frac{\pi}{3}} \frac{\sin(x)}{\cos^2(x)}dx$

(f) $\int \sin^2(x)\cos(x)dx$

25. Calcule:

(a) $\int \sin(2x)\sqrt{5+\sin^2(x)}dx$

(b) $\int \cos^5(x)dx$

(c) $\int \tan(x)\sec^3(x)dx$

(d) $\int \sin(x)\sqrt{3+\cos(x)}dx$

(e) $\int \frac{2}{x_3}dx$

(f) $\int \left(x+\frac{3}{x-2}\right)dx$

26. Calcule:

(a) $\int \frac{1}{(x+1)(x-1)}dx$

(b) $\int \frac{2x+3}{x(x-2)}dx$

(c) $\int \frac{1}{x^2-4}dx$

(d) $\int \frac{5x+3}{x^2-3x+2}dx$

(e) $\int xe^x dx$

(f) $\int x\sin(x)dx$

27. Calcule:

(a) $\int x\ln(x)dx$

(b) $\int \ln(x)dx$

(c) $\int x^2\ln(x)dx$

(d) $\int x\sec^2(x)dx$

(e) $\int xe^{2x}dx$

(f) $\int e^x\cos(x)dx$

28. Calcule:

(a) $\int e^{-2x}\sin(x)dx$

(b) $\int x^3e^{x^2}dx$

(c) $\int x^3\cos(x^2)dx$

(d) $\int \frac{1}{\sqrt{4-x^2}}dx$

(e) $\int \frac{1}{4+x^2}dx$

(f) $\int \frac{x}{\sqrt{1-x^2}}dx$

29. Calcule:

(a) $\int \frac{x^2}{\sqrt{1-x^2}}dx$

(b) $\int \sqrt{9-4x^2}dx$

(c) $\int \frac{1}{x^2-4}dx$

(d) $\int \frac{x}{x^2-4}dx$

(e) $\int \frac{x+3}{x^2-x}dx$

30. Calcule:

(a) $\int \frac{x+1}{x(x-2)(x+3)}dx$

(b) $\int \frac{x+3}{x^3-2x^2-x+2}dx$

(c) $\displaystyle\int \frac{4}{x^3 - x^2 - 2x} dx$

(d) $\displaystyle\int \frac{x^3 + 1}{x^3 - x^2 - 2x} dx$

(e) $\displaystyle\int \sin(7x)\cos(2x) dx$

(f) $\displaystyle\int \sin(3x)\sin(5x) dx$

31. Calcule:

(a) $\displaystyle\int \cos(2x)\cos(x) dx$

(b) $\displaystyle\int \sin(x)\cos^2(x) dx$

(c) $\displaystyle\int \cos(x)\sin^4(x) dx$

(d) $\displaystyle\int \tan^5(x)\sec^2(x) dx$

(e) $\displaystyle\int \tan^3(2x)\sec(2x) dx$

(f) $\displaystyle\int \tan^6(x) dx$

32. Calcule a área da região limitada pelo gráfico de $f(x) = x^3$, pelo eixo x e pelas retas $x = -1$ e $x = 1$.

33. Calcule a área da região limitada pelas retas $x = 0, x = 1, y = 2$ e pelo gráfico de $y = x^2$.

34. Calcule a área do conjunto de todos os pontos (x, y) tais que: $x^2 \leq y \leq \sqrt{x}$

35. Calcule a área da região compreendida entre os gráficos de $y = x$ e $y = x^2$, com $0 \leq x \leq 2$.

36. Determine $y = y(x)$ que satisfaça:

(a) $\dfrac{\partial y}{\partial x} = e^y$ e $y(0) = 1$

(b) $\dfrac{\partial y}{\partial x} = 3y^2$ e $y(0) = \frac{1}{2}$