Maintenance phase biochemistry and contractility combine to encode a dynamically stable polarity state in the *C elegans* zygote

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Cell polarization

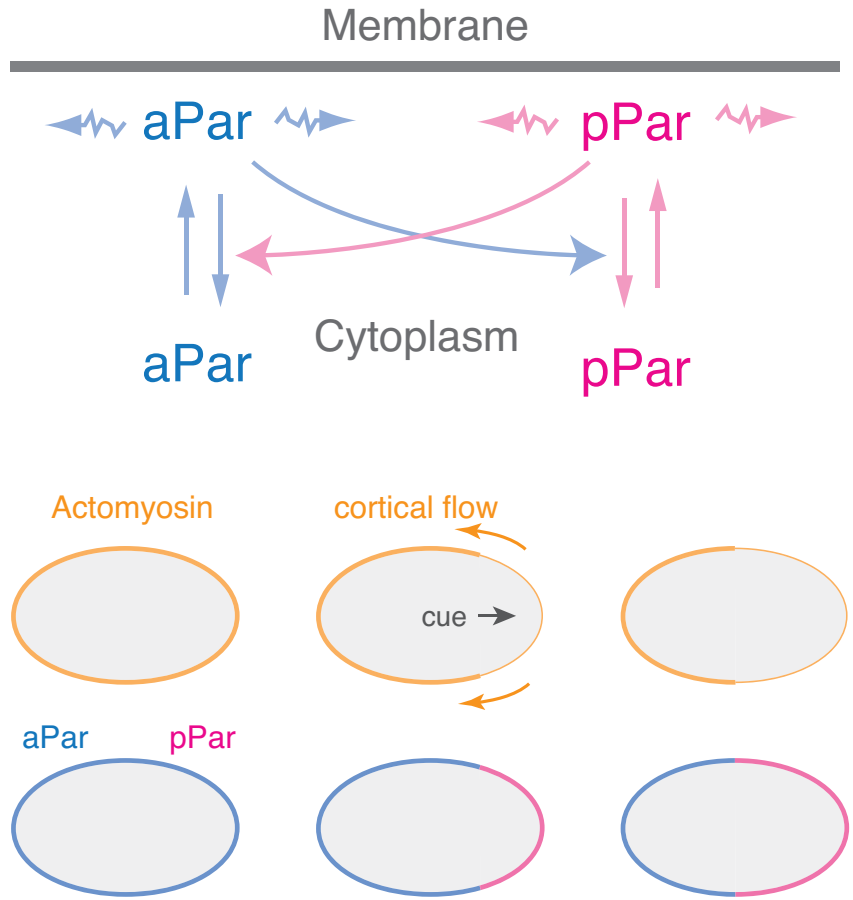
C elegans model system

Ingredients

- PAR proteins
  - aPARs (PAR-3, PAR-6, CDC-42)
  - pPARs (PAR-2, CHIN-1)
- Actomyosin flows

Wild type sequence

- Centrosomes $\rightarrow$ PAR-2 localized
- Sperm cue $\rightarrow$ Myosin inhibition
- Expansion of boundary to stable point ("establishment")
- "Maintenance:" boundary stays



Movie: *C elegans* wild type

The wild-type boundary is stable (the movie)

Use CDK-1 (RNAi) to expand maintenance phase; boundary just sits there

This slide: movie

The wild-type boundary is stable (the book)

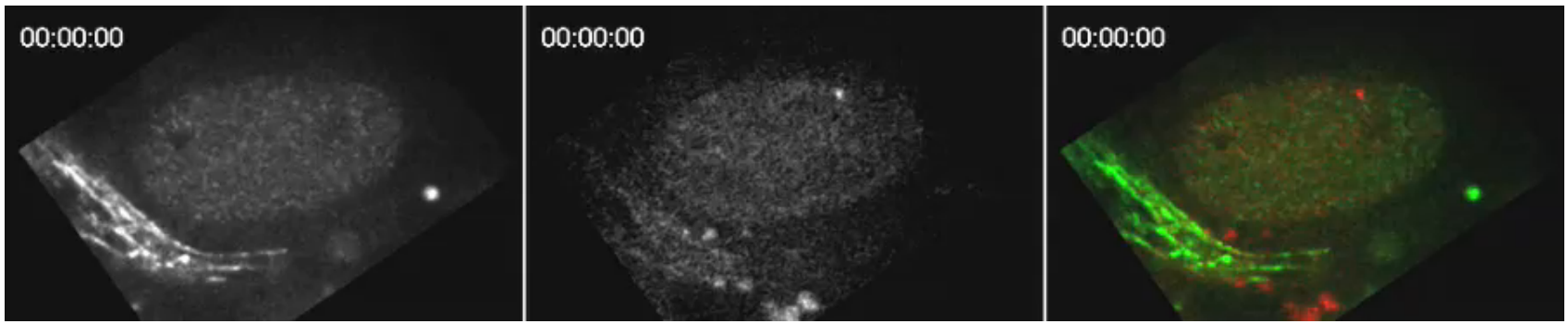
Plot of boundary position vs. time

"Maintenance phase"

Does "maintenance" maintain the current boundary in non wild-type embryos?

- Establishment requires actomyosin
- Knockdown rho during establishment $\rightarrow$ no flows
- Alternative: ECT-2 knockdown (GEF that activates Rho)
- Both cases: local zone of PAR-2 enrichment remains

NMY-2::GFP mCherry::PAR-2 Merge



Results in exactly the same boundary position!

- Requires PAR-2, MRCK

Zonies et al. *Development* (2010)

The main questions

How do the aPARs (PAR-3), pPARs (PAR-2) and actomyosin flows combine to yield a dynamically stable boundary position?

- Hypothesis 1: maintenance phase "rescue" = actomyosin instability (self-patterning) + mutual inhibition of PARs
  - Uniform state is unstable
  - Fundamentally different from wild type
- Hypothesis 2: "rescue" = pPAR/aPAR competition + pPAR inhibiting myosin
  - Uniform state stable
  - Small asymmetry amplified
  - Same as establishment phase mechanism

Experimental and modeling program

Hypothesis 1: maintenance phase "rescue" = actomyosin instability (self-patterning) + mutual inhibition of PARs

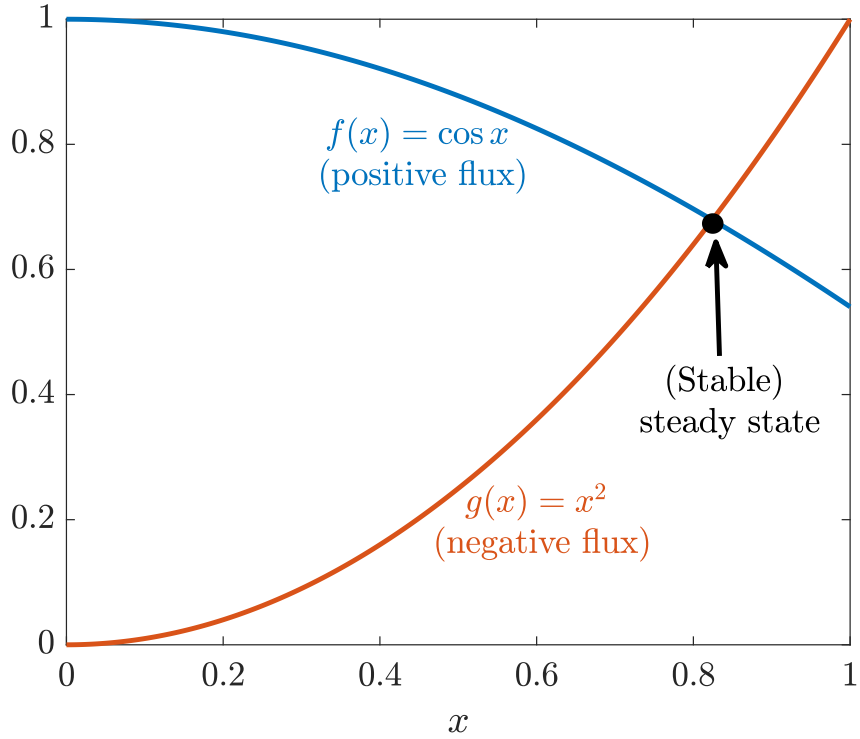
- Model myosin by itself
- Infer parameters from experiments
- Do we see self-amplification/instability?

Hypothesis 2: "rescue" = pPAR/aPAR competition + pPAR inhibiting myosin

Mathematics of stability

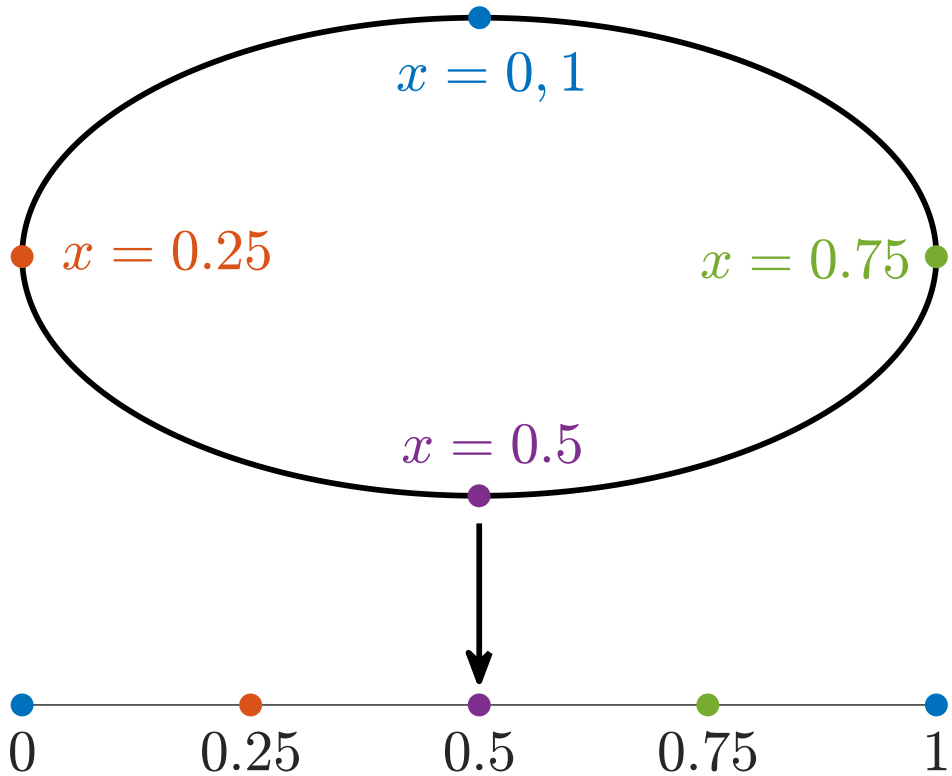
$$\frac{dx}{dt} = \underbrace{f(x)}_{\text{On rate}} - \underbrace{g(x)}_{\text{Off rate}}$$

- Steady states: $f(x_s) = g(x_s)$



- Stable steady state $f(x_s^+) < g(x_s^+) < 0$, $f(x_s^-) > g(x_s^-)$
- Perturb off the steady state and get pushed back to it

Model of the *C elegans* embryo



Myosin dynamics

$$\partial_t M + \underbrace{\partial_x (vM)}_{\text{Advection}} = \underbrace{D_M \partial_x^2 M}_{\text{Diffusion}} + \underbrace{k_M^{\text{on}} M_{\text{cyto}}}_{\text{On flux}} - \underbrace{k_M^{\text{off}} M}_{\text{Off flux}}$$

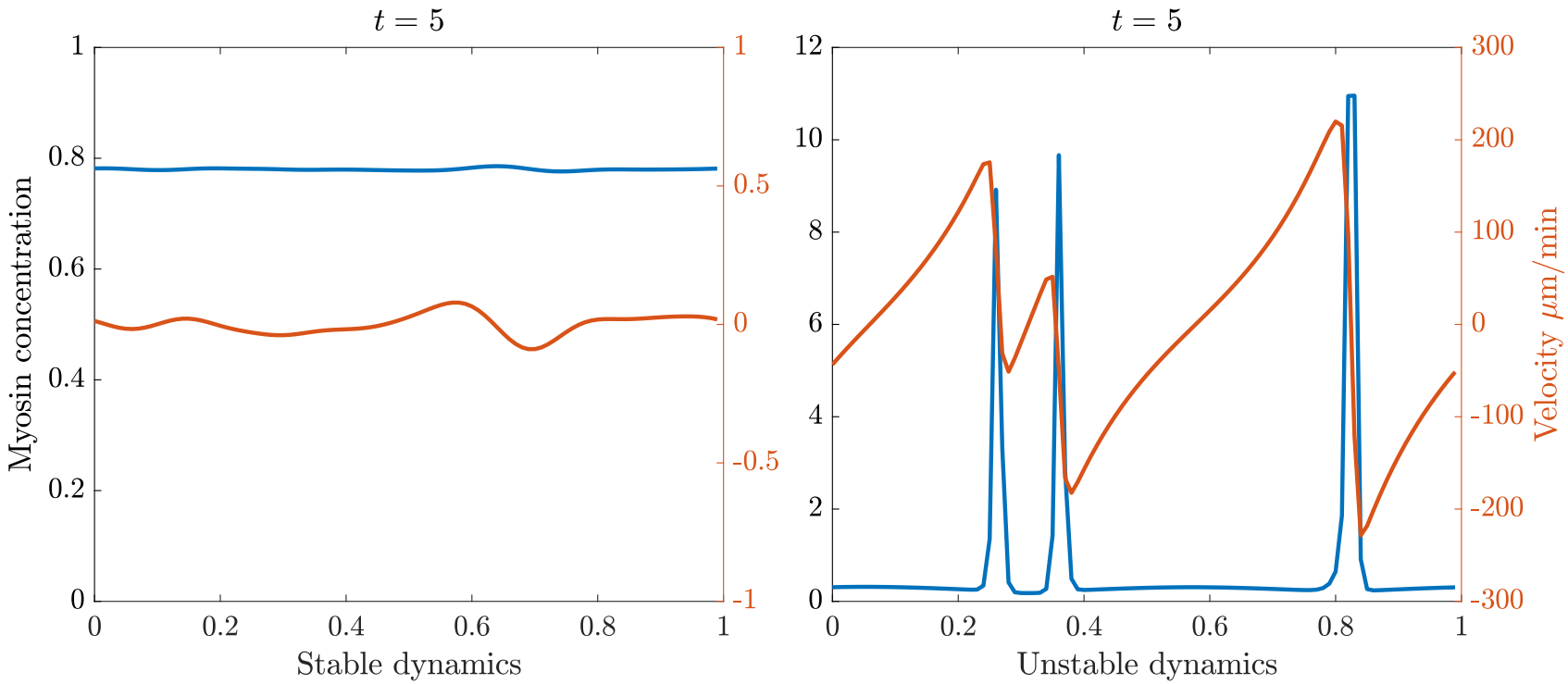
$$M_{\text{cyto}} = 1 - \int_0^1 M(x)\, dx$$

$$\underbrace{\gamma v}_{\text{Drag force}} = \underbrace{\eta \partial_x^2 v}_{\text{Viscous stress}} + \underbrace{\partial_x \sigma_a(M)}_{\text{Active stress}}$$

- Strong enough flows $\rightarrow$ self patterning
- Only unknown: stress vs. myosin relationship

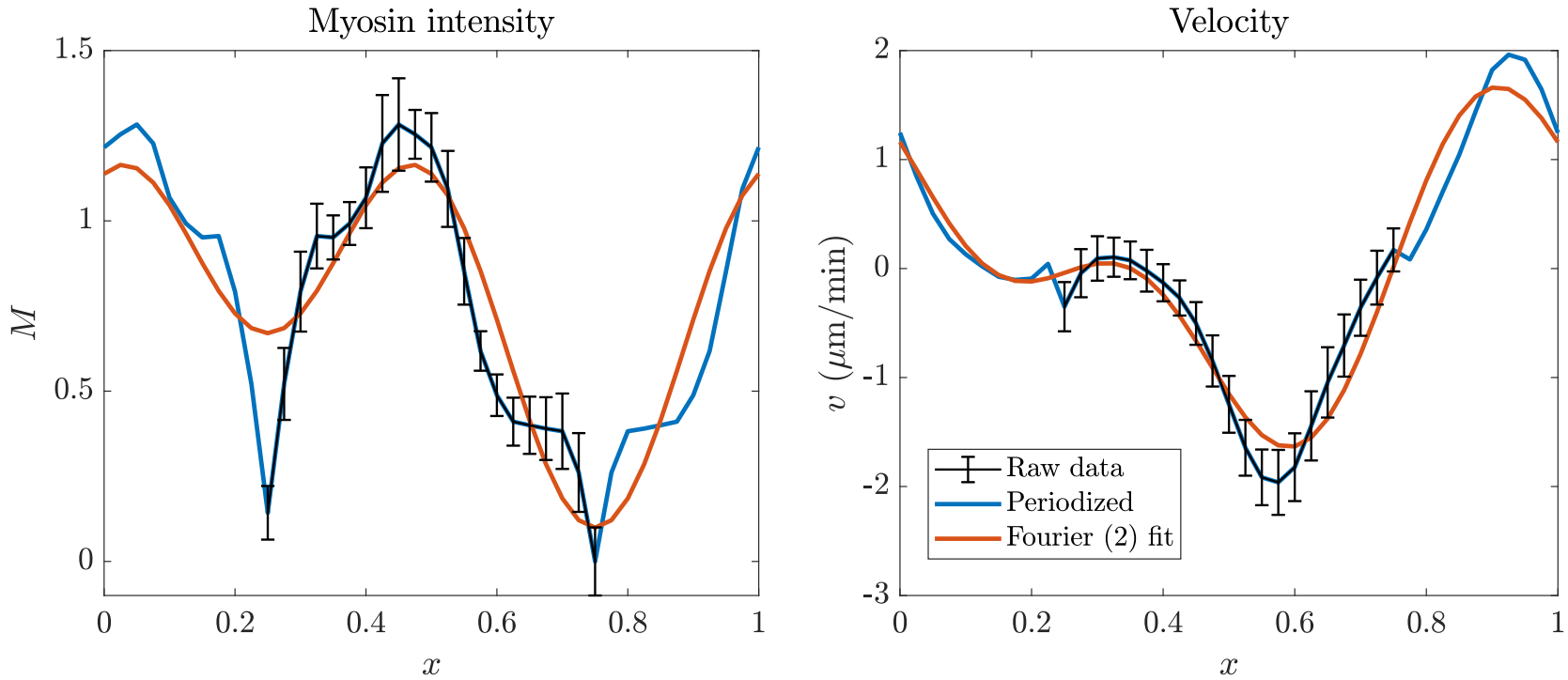
Bois et al. *PRL* (2011)

Stable and unstable myosin dynamics



Unstable flow patterns characterized by unphysical speeds

Inferred myosin velocity



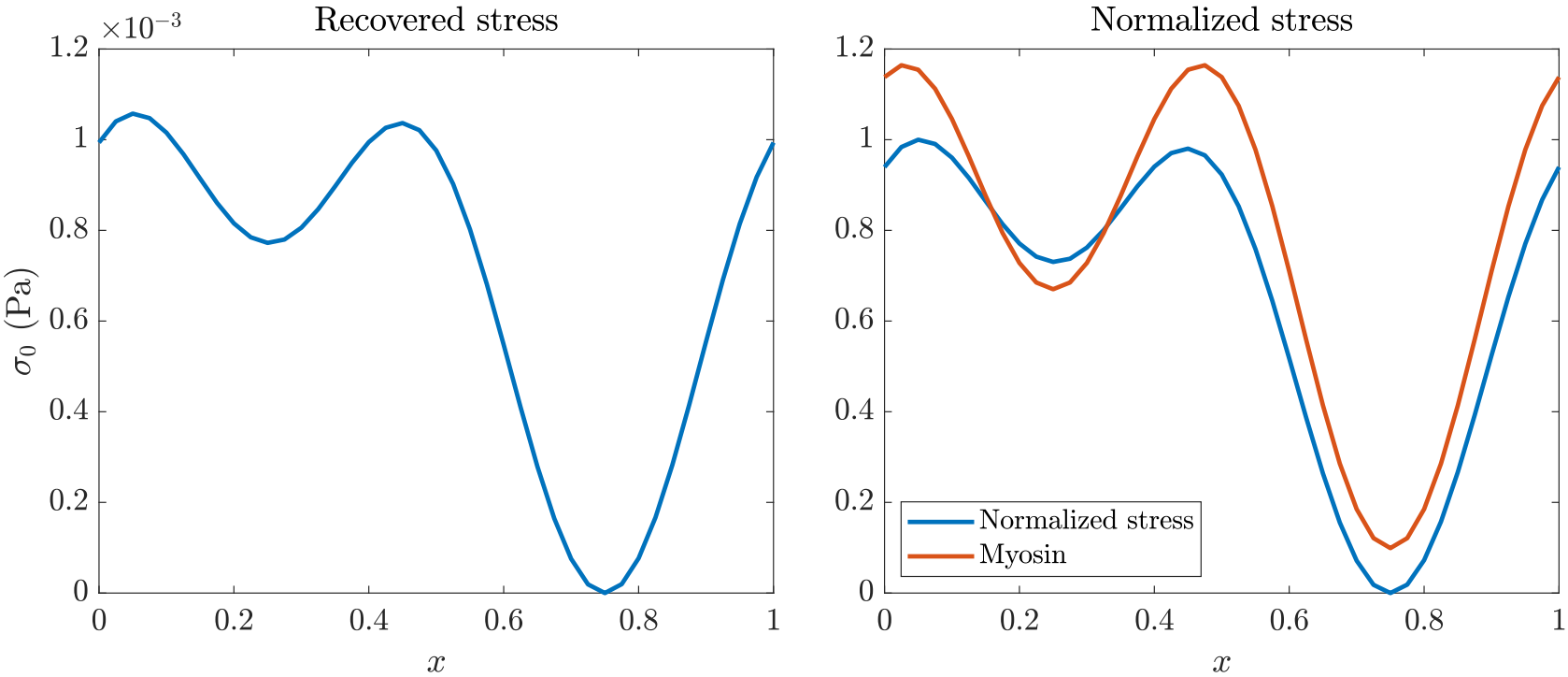
Solve

$$\gamma v = \eta \partial_x^2 v + \partial_x \sigma_a(M)$$

to get active stress $\sigma_a(M)$

Sailer et al. *Dev. Cell* (2010)

Inferred myosin dynamics



Active stress roughly proportional to myosin intensity

- $\sigma_a \approx \left(1.1 \times 10^{-3}\right) M$
- Not even close to strong enough for instability!
- Never see sharp peaks in arbitrary places
- Despite appearances, *no spontaneous symmetry breaking*
- System has 2 steady states

Experimental and modeling program

~~Hypothesis 1: maintenance phase "rescue" = actomyosin instability (self-patterning) + mutual inhibition of PARs~~

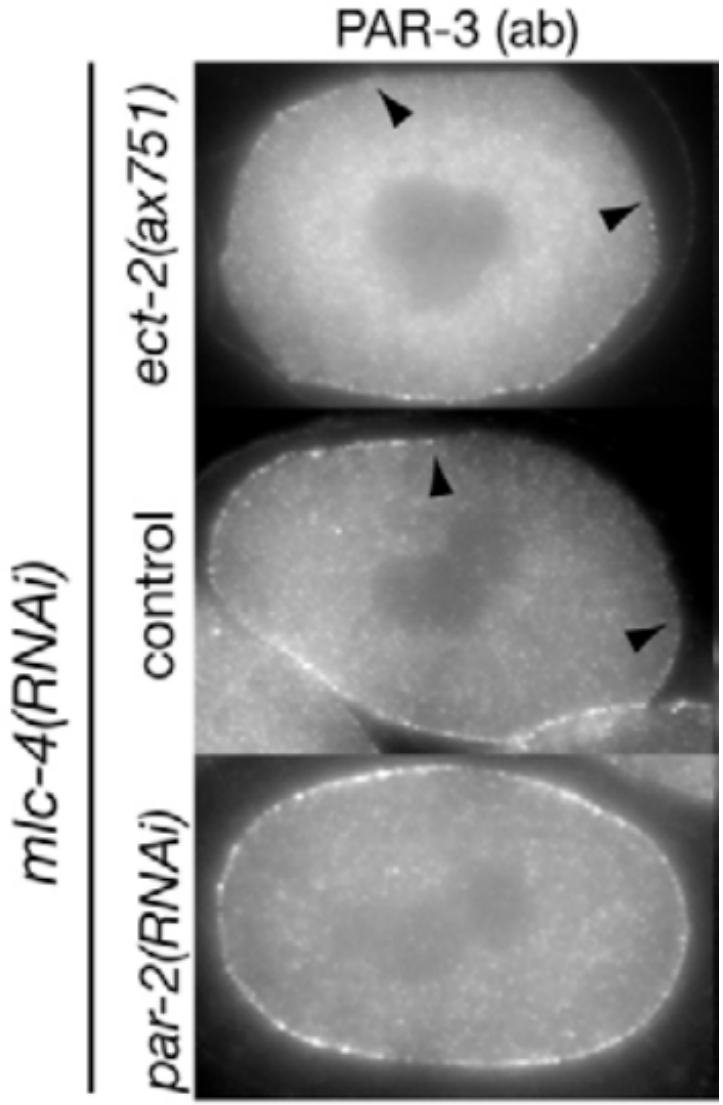
Hypothesis 2: "rescue" = pPAR/aPAR competition + pPAR inhibiting myosin

- Start with model of aPAR/pPAR competition
- Add myosin on top

PAR competition gives contracted boundary

PAR asymmetries stable by themselves without flows

- Wild-type embryos with MRCK knockout $\rightarrow$ boundary expands, but still asymmetric
- ECT-2 (RNAi) embryos: MLC-1 (RNAi) produces mutually inhibitory zones of PAR-2 and PAR-3
- Suggests that mutual antagonism encodes one steady state



Zonies et al. *Development* (2010)

First model for mutual inhibition

First model: mutual inhibition of proteins, mass-action kinetics

$$\partial_t A = D_P \partial_x^2 A + k_A^{\text{on}} A_{\text{cyto}} - k_A^{\text{off}} A - r_{\text{AP}} AP$$

$$\partial_t P = \underbrace{D_P \partial_x^2 P}_{\text{Diffusion}} + \underbrace{k_P^{\text{on}} P_{\text{cyto}} - k_P^{\text{off}} P}_{\text{Binding/unbinding}} \underbrace{- r_{\text{AP}} AP}_{\text{Mutual inhibition}}$$

- *One* stable steady state!
- Need a way to generate *bistability*

The Grill workaround

Polarization of PAR Proteins by Advective Triggering of a Pattern-Forming System

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In the *Caenorhabditis elegans* zygote, a conserved network of partitioning-defective (PAR) polarity proteins segregates into an anterior and a posterior domain, facilitated by flows of the cortical actomyosin meshwork. The physical mechanisms by which stable asymmetric PAR distributions arise from transient cortical flows remain unclear. We present evidence that PAR polarity arises from coupling of advective transport by the flowing cell cortex to a multistable PAR reaction-diffusion system. By inducing transient PAR segregation, advection serves as a mechanical trigger for the formation of a PAR pattern within an otherwise stably unpolarized system. We suggest that passive advective transport in an active and flowing material may be a general mechanism for mechanochemical pattern formation in developmental systems.

Developmental form emerges from coupling of pattern-forming biochemical networks with mechanical processes (*1*–*3*). In *Caenorhabditis elegans* zygotes, transient flows of a thin film of a mechanically active actomyosin cell cortex instruct the patterning of a conserved cell polarity pathway consisting of two groups of partitioning-defective (PAR) proteins that mutually exclude one another from the cell membrane (*4*–*14*). Initially, anterior PARs (aPARs: PAR-3, PAR-6, and atypical protein kinase C) cover the entire cell membrane. During polarization, flows of cortical actomyosin oriented away from the posterior-localized centrosome induces

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The Grill workaround

$$\partial_t A = D_A \partial_x^2 A - \partial_x(vA) + R_A$$

$$\partial_t P = D_P \partial_x^2 P - \partial_x(vP) + R_P \qquad (2)$$

where P is the local membrane concentration of pPARs, D_P denotes the diffusivity of membrane-bound pPARs, and R_A and R_P denote the reaction terms that describe membrane association and dissociation that now include interactions between the two species. We include reciprocal antagonistic feedback as detachment that depends on the concentration of the opposing species (Fig. 2A). Importantly, if this feedback is sufficiently nonlinear, mutual antagonism between A and P endows the system with a bistable character such that the membrane will tend to exist in one of two states: an anterior-like state, with $A > P$, or a posterior-like state with $A < P$ (see below) (*19*). Therefore,

$$R_A = k_{on,A} A_{cyto} - k_{off,A} A - k_{AP} P^{\alpha} A$$

$$R_P = k_{on,P} P_{cyto} - k_{off,P} P - k_{PA} A^{\beta} P \qquad (3)$$

where k_{AP} is a coefficient governing antagonism of A by P with α specifying stoichiometry and k_{PA} and β are similarly defined; $k_{on,P}$, $k_{off,P}$, and P_{cyto} are analogous to the corresponding A-specific variables. A similar framework without bistable character was proposed in (*20*), instead requiring local effects of the actin cortex on aPAR association rates for pattern formation.

Bistability in the reaction terms (Fig. 2B) has two important consequences that allow the system to account simultaneously for the unpolarized and polarized states of the embryo. First, bistability can drive the entire membrane of the system into one of two states: either an anterior-like homogeneous state, with aPARs enriched in the membrane and pPARs in the cytoplasm (similar to the *C. elegans* embryo before polarization), or the analogous posterior-like homogeneous state. Second, bistability also permits the system to support coexistence of distinct membrane domains in opposite states, separated in space and connected by boundary regions. A polarized system would contain exactly two such domains.

The Grill workaround

$$\partial_t A = D_A \partial_x^2 A - \partial_x(vA) + R_A$$

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where k_{AP} is a coefficient governing antagonism of A by P with α specifying stoichiometry and k_{PA} and β are similarly defined; $k_{on,P}$, $k_{off,P}$,

2.3 Bistability

For $\alpha \geq 1$, $\beta > 1$ or $\alpha > 1$, $\beta \geq 1$, Eqns. 3 permit bistability in the reaction terms.

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Bistability in the reaction terms (Fig. 2B) has two important consequences that allow the system to account simultaneously for the unpolarized and polarized states of the embryo. First, bistability can drive the entire membrane of the system into one of two states: either an anterior-like homogeneous state, with aPARs enriched in the membrane and pPARs in the cytoplasm (similar to the *C. elegans* embryo before polarization), or the analogous posterior-like homogeneous state. Second, bistability also permits the system to support coexistence of distinct membrane domains in opposite states, separated in space and connected by boundary regions. A polarized system would contain exactly two such domains.

The Charlie-Ed solution: PAR-3

Important experimental observations

- PAR-3 stable by itself
- All asymmetries are lost without it
- Higher recruitment rate when more monomers attached
- Oligomerization
- Evidence that feedback + oligomerization $\rightarrow$ stability

PAR-3 model

Pinned boundary position

Incorporating PAR-2

Mass action kinetics $\rightarrow$ different zones
Boundary can only make it so far! Need another piece

Experimental and modeling program

Hypothesis: "rescue" = pPAR/aPAR competition + pPAR inhibiting myosin

- Start with model of aPAR/pPAR competition
- Add myosin on top

Full model: PAR-2/PAR-3/Myosin

Dynamics of myosin

- Inhibited locally by pPARs
- Transports everything (including itself!)

Steady states

Contractility on/off

Start with mutual aPAR/pPAR inhibition. Let equilibrate. Add contractility. Let equilibrate. Turn off contractility. Should go back to first position

Remaining issues

What causes boundary to stop?

- This model:
- Experiment: something different?

Clear that flow profiles do not match

Branched actin as a brake on contractility

Cassandra's movies on arp 2/3 knockout
Idea: flow profile here should be same as original model
Then add branched actin to the model to complete the story