5 Methods and Philosophy of Statistical Process Control

Learning Objectives

1. Understand chance and assignable causes of variability in a process
2. Explain the statistical basis of the Shewhart control chart, including choice of sample size, control limits, and sampling interval
3. Explain the rational subgroup concept
4. Understand the basic tools of SPC; the histogram or stem-and-leaf plot, the check sheet, the Pareto chart, the cause-and-effect diagram, the defect concentration diagram, the scatter diagram, and the control chart
5. Explain phase I and phase II use of control charts
6. Explain how average run length is used as a performance measure for a control chart
7. Explain how sensitizing rules and pattern recognition are used in conjunction with control charts

5.1 Introduction

Statistical Process Control (SPC): SPC is a powerful collection of problem-solving tools useful in achieving process stability and improving capability through the reduction of variability. The seven major tools (also called *the magnificent seven*) of SPC are

1. Histogram or Stemplot
2. Check sheet
3. Pareto chart
4. Cause-and-effect diagram
5. Defect concentration diagram
6. Scatter diagram
7. Control chart

5.2 Chance and Assignable Causes of Quality Variation

Chance (or common) causes account for the uncontrollable, natural variation present in any repetitive process. A process that is operating with only chance causes of variation is said to be **in statistical control** or **in control**. The chance causes are an inherent part of the process.

Assignable (or special) causes are those whose effect can be detected and controlled. Assignable causes are not the part of chance causes. A process that is operating in the presence of assignable causes is said to be an **out-of-control process**.

Sources of assignable causes: Improperly adjusted or controlled machines, operator errors or defective raw materials etc. See the following Figure 5.1 for both chance and assignable causes of variation (discussion in the middle of page 181).

Note: The statistical process control (SPC) is useful to detect the occurrence of assignable causes of process shifts so that investigation of the process and corrective action may be undertaken before producing many non-conforming items. The ultimate goal of statistical process control is the elimination of variability in the process.

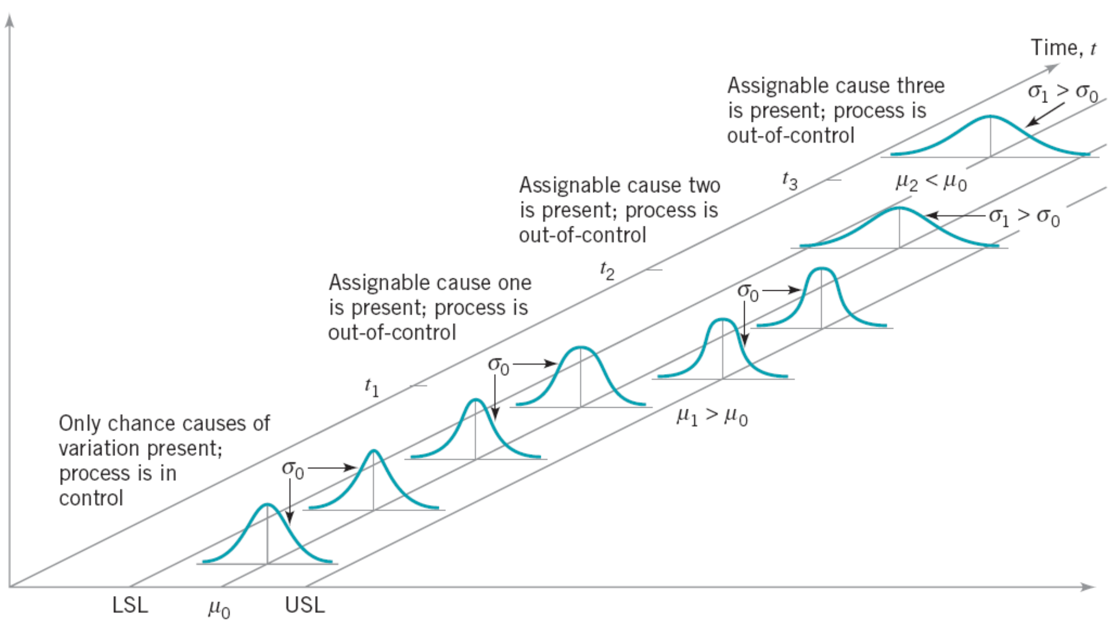


FIGURE 5.1 Chance and assignable causes of variation.

5.3 Statistical Basis for the Control Chart (CC)

Basic principles
Choice of control limits
Sample size and Sampling frequency
Rational subgroups
Analysis of Patterns on control charts
Sensitizing rules for the control charts

5.3.1 **Basic principles**

A typical control chart contains three horizontal lines: *Center line (CL), Upper control limit (UCL) and Lower control limit (LCL).*

See the following Figure 5.2.

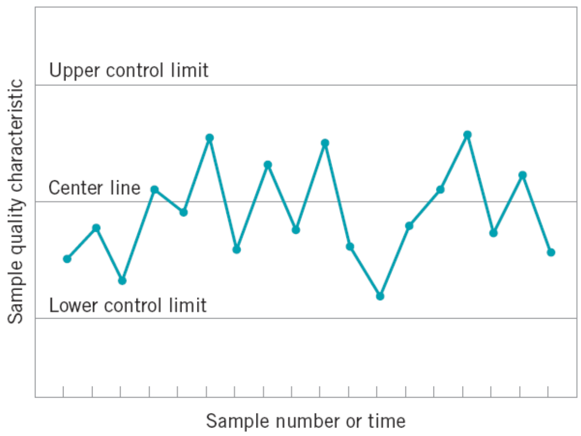


FIGURE 5.2 A typical control chart.

Center line: CL represents the average value of the quality characteristic corresponding to the in-control state.

As long as the points plot within the control limits, the process is assumed to be in control, and no action is necessary. Points plot outside of the control limits is interpret as evidence that the process is out of control and corrective action are required to find and eliminate the assignable cause(s).

Even all the points plot inside the control limits, if they behave in a systematic or nonrandom manner, then this could be an indication that the process is out of control. If the process is in control, all the plotted points should have an essentially random pattern.

Connection between hypotheses testing and control chart

If the $\bar{x}$ plots within the control limits, the process is in statistical control, do not reject null hypothesis. If the $\bar{x}$ plots outside the control limits, the process is out of statistical control, and reject null hypothesis.

Type I error (concluding the process is out of control when it is really in control). Type II error (concluding the process is in control when it is really out of control).

$\alpha = P(\text{Type I error}) = \text{Producer's Riak}$
$\beta = P(\text{Type II error}) = \text{Consumer's Riak}$

Model of the Control chart

Let w be a sample statistic that measures some quality characteristic of interest, and suppose,

$$E(w) = \mu_w$$
$$\mathrm{V(w)} = \sigma_w^2$$
$$SD(w) = \sigma_w$$

Then the UCL, center line, and LCL become:

$$\begin{aligned} UCL &= \mu_w + L\sigma_w \\ CL &= \mu_w \\ LCL &= \mu_w - L\sigma_w, \end{aligned} \qquad (1)$$

where L (usually 3) is the ``distance" of the control limits from the center line, expressed in standard deviation units. This is called Shewhart (Dr. Walter A. Shewhart) Control chart. Equation (1) is very useful.

Example: Consider the process mean, $\mu = 1.5$ mm and process standard deviation, $\sigma = 0.15$ mm. Now, if sample of size $n = 5$ are taken from this process, then

$$\mu_{\bar{x}} = \mu = 1.5, \quad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{0.15}{\sqrt{5}} = 0.0671$$

Then the 3σ control limits are respectively

$$UCL = 1.5 + 3 \times 0.0671 = 1.7013 \quad and \quad LCL = 1.5 - 3 \times 0.0671 = 1.2987.$$

After constructing the control limits, one might collect sample data and compute the sample statistics $\bar{x}$ or R and if the values of $\bar{x}$ (say) fall within the control limits and do not exhibit any systematic pattern we say the process is in statistical control at the level indicated by the chart. See the following Figure 5.4.

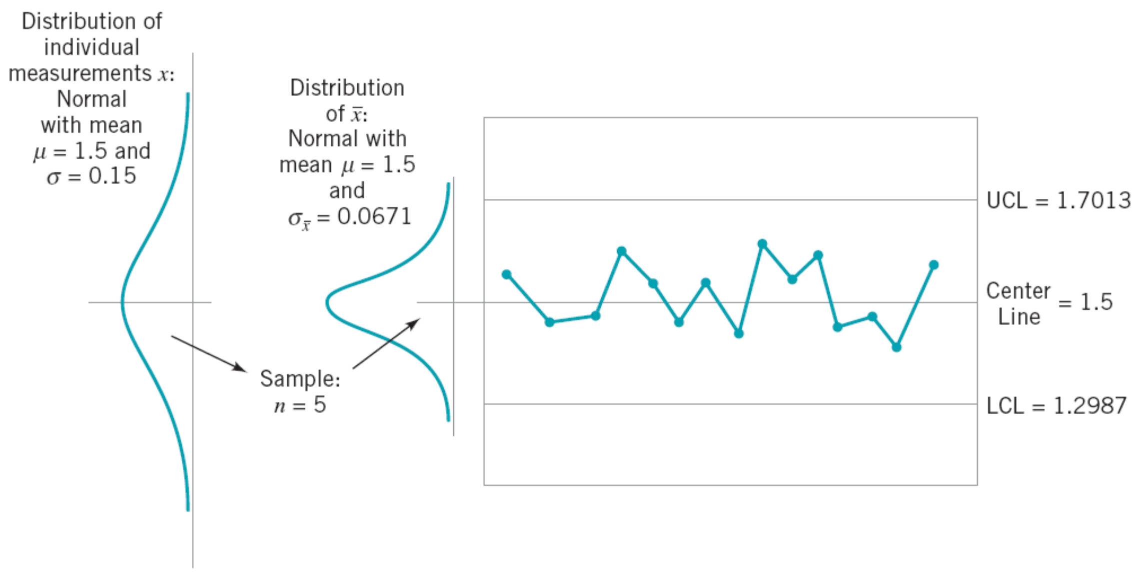


FIGURE 5.4 How the control chart works.

Improve the Process:

The most important use of control chart is to improve the process. See the following Figure 5.5

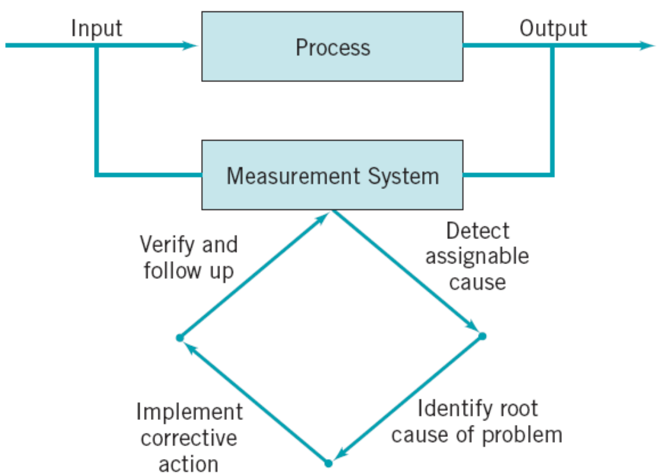


FIGURE 5.5 Process improvement using the control chart.

Types of control chart:

Variables control charts are discussed in chapter 6 and attribute control charts are discussed in chapter 7.

Control charts can be categorized in various ways, according to type of data, type of control, and type of application. Such classification can be of help in selecting an appropriate chart to use.

Five reasons for the popularity of control chart in USA (page 189)

1. Control charts are a proven technique for improving productivity.
2. Control charts are effective in defect prevention.
3. Control charts prevent unnecessary process adjustment
4. Control charts provide diagnostic information
5. Control charts provide information about the process capability.

5.3.2 Choice of Control Limits

By moving the control limits farther from the CL, we decrease the risk of type I error (producer risk) and by moving the control limits farther from the CL, we increase the risk of type II error (consumer). However, if we move the control limits closer to the CL, the opposite effect is obtain. That is, the risk of type I error is increased, while the risk of type II error is decreased.

Example: Assume that the piston ring diameter is normally distributed, we find the probability of type I error is 0.0027. An incorrect out of control signal or false alarm will be generated in 27 out of 10,000 points. Moreover, the probability that a point taken when the process in control will exceed the 3-σ limits in one direction is 0.00135. Then the control limits for $\bar{x}$ chart would be

$$UCL = 1.5 + 3 \times 0.0671 = 1.7013 \quad and \quad LCL = 1.5 - 3 \times 0.0671 = 1.2987.$$

On the other hand if we specified a 0.001 type I error probability in one direction, then the appropriate multiple of the standard deviation would be 3.09. Then the control limits for $\bar{x}$ chart would be

$$UCL = 1.5 + 3.09 \times 0.0671 = 1.7073 \quad and \quad LCL = 1.5 - 3.09 \times 0.0671 = 1.2927.$$

Two σ, Three σ and Warning limits on Control Chart

3-σ are the usual action limits (outer limits): Any point plots outsides of this limit, a search for an assignable cause is made and corrective action is taken if necessary. 2-σ limits are called warning limits (inner limit):

If one or more points fall between the warning limits and control limits, which mean that the process may not be operating properly. See Figure 5.8, page 166.

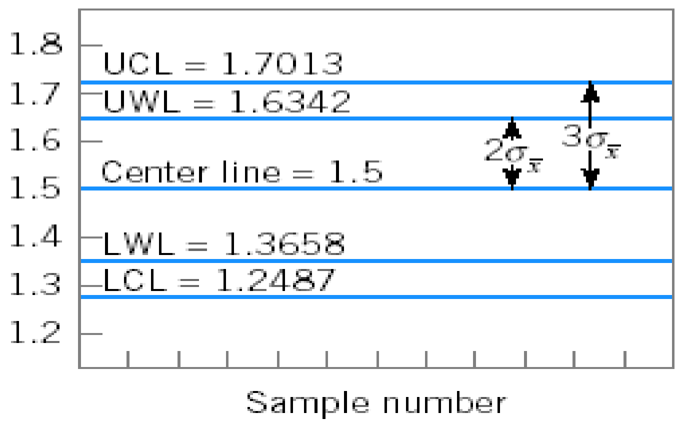


Figure 4-8 An $\bar{x}$ chart with two-sigma warning limits.

5.3.3 Sample size and Sampling frequency

In designing a control chart, we must specify both the sample size and frequency of the sampling. Generally, small sample at short intervals, or larger samples at longer interval. From Figure 5.9, page 191, the probability of detecting a shift from 1.50 icrons to 1.65 microns increases as the sample size n increases.

Average run length (ARL): ARL is the average number of points that must be plotted before a point indicates an out-of-control condition. If the process observations are uncorrelated, then for any Shewhart control chart, the ARL is defined as

$$ARL = \frac{1}{p},$$

where p is the probability that any point exceeds the control limits.

If $p = 0.0027$, then

$$ARL = \frac{1}{0.0027} = 370.37.$$

Conclusion: Even if the process remains in control, on the average, an out of control signal will generate every 370 samples.

Average time to signal (ATS)

$$ATS = ARL \times h,$$

where h =fixed interval of times.

Suppose we are sampling every hour, then

$$ATS = ARL \times 1 = 370.$$

There will have a false alarm about every 370 hours.

5.3.4 Rational Subgroups

Two general approach to constructing rational subgroup.

(1) Each sample consists of units that were produced at the same time (or closely together). Ideally we would like to take the consecutive units of production. It minimizes the chance of variability due to assignable causes within a sample, and it maximized the chance of variability between samples. See Figure 5.10, page 194.

(2) Each sample consists of product that are representative of all units that have been produced since the last sample was taken. Essentially each subgroup in a random sample of all process output over the sampling interval. This method is used when the control chart is employed to make decisions about the acceptance of all units of products that have been produced since the last sample. See Figure 5.11, page 194.

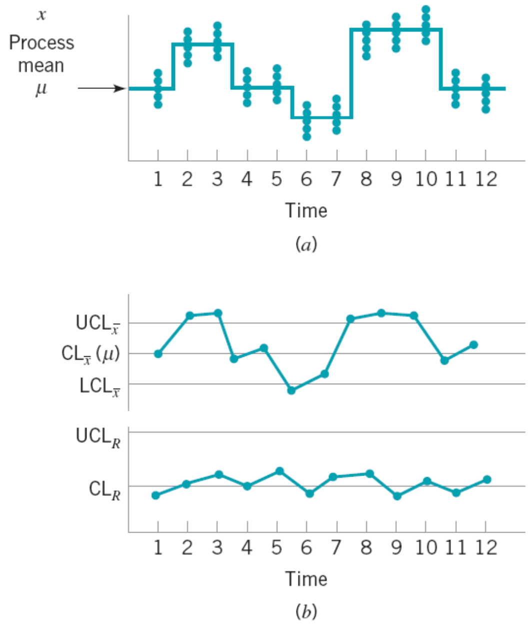


FIGURE 5.10 The snapshot approach to rational subgroups. (*a*) Behavior of the process mean. (*b*) Corresponding $\bar{x}$ and R control charts.

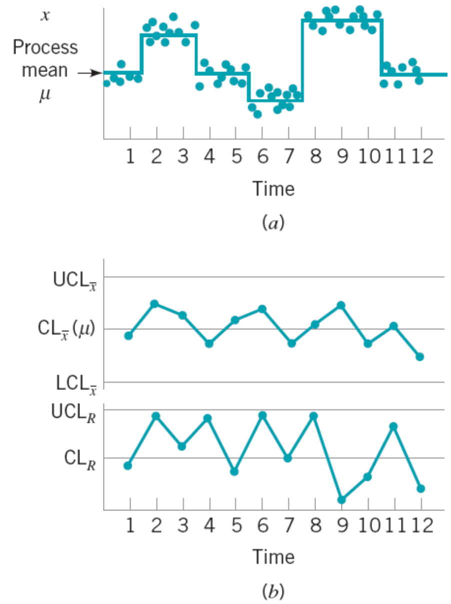


FIGURE 5.11 The random sample approach to rational subgroups. (*a*) Behavior of the process mean. (*b*) Corresponding $\bar{x}$ and R control charts.

5.3.5 Analysis of Patterns on Control Charts

A control chart may indicate an out-of-control condition either when one or more points fall beyond the upper and lower control limits or when then plotted point exhibit some nonrandom pattern. See Figure 5.12 and Figure 5.13, exhibit a cyclical behavior, yet all points fall within control limits.

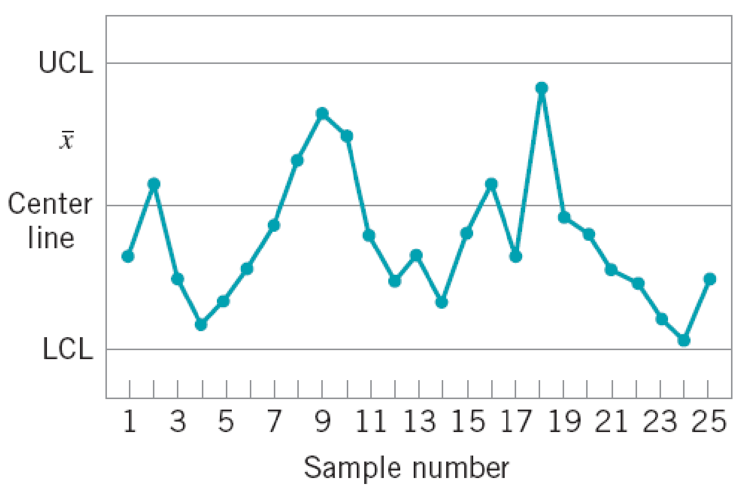


FIGURE 5.12 An $\bar{x}$ control chart.

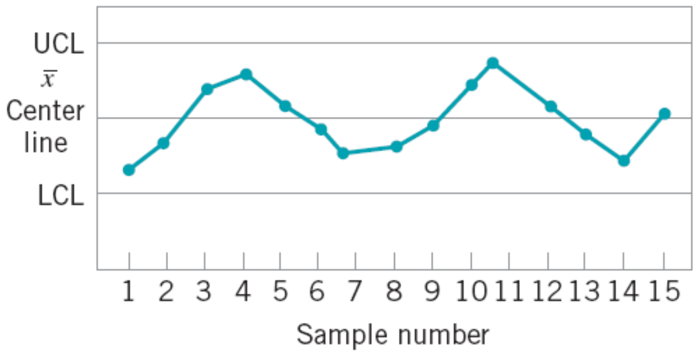


FIGURE 5.13 An $\bar{x}$ chart with a cyclic pattern.

The Western Electric Handbook (1956) suggests a set of decision rules for detecting nonrandom patterns on control charts. Specifically, it suggests concluding that the process is out of control if either

1. One point plots outside the three-sigma control limits;
2. Two out of three consecutive points plot beyond the two-sigma warning limits;
3. Four out of five consecutive points plot at a distance of one-sigma or beyond from the center line;

or

4. Eight consecutive points plot on one side of the center line.

Run: A run is sequence of observations of the same type. When we have 4 or more points in a row increase in magnitude or decrease in magnitude, this arrangement of points is called run.

■ **TABLE 5.1**

Some Sensitizing Rules for Shewhart Control Charts

Standard Action Signal:	**1.** One or more points outside of the control limits.	Western Electric Rules
	2. Two of three consecutive points outside the two-sigma warning limits but still inside the control limits.	Western Electric Rules
	3. Four of five consecutive points beyond the one-sigma limits.	Western Electric Rules
	4. A run of eight consecutive points on one side of the center line.	Western Electric Rules
	5. Six points in a row steadily increasing or decreasing.	
	6. Fifteen points in a row in zone C (both above and below the center line).	
	7. Fourteen points in a row alternating up and down.	
	8. Eight points in a row on both sides of the center line with none in zone C.	
	9. An unusual or nonrandom pattern in the data.	
	10. One or more points near a warning or control limit.	

5.3.6 Discussion of Sensitizing Rules for Control Charts

Several criteria may be applied simultaneously to a control chart to determine whether the process is out of control. The basic criteria is one or more points outside of the control limits. The supplementary criteria are sometimes used to increase the **sensitivity** of the control charts to a small process shift so that one may response more quickly to the assignable cause. Some sensitizing rules for Shewhart control charts are as follows:

1. One or more points plot outside the control limits.
2. Two out of the three consecutive points outside the 2-sigma warning limits but still inside the control limits.
3. Four of five consecutive points beyond the 1-sigma limits.
4. A run of eight consecutive points on one side of the center.
5. Six points in a row steadily increasing or decreasing.
6. 15 points in a row in zone C (both above and below the center line).
7. 14 points in a row alternating up and down.
8. 8 points in a row in both sides of the center line with none in zone C.
9. An unusual or nonrandom pattern in the data.
10. One or more points near a warning or control limit.

Among the 10 rules, first four are called the Western Electric Rules (1956)

Probability of type I error for several decision rules

Suppose that the analyst uses k decision rules and that criterion i has type I error probability α_i. Then the overall type I error or false alarm probability for the decision based on all k tests is

$$\alpha = 1 - \prod_{i=1}^{k}(1-\alpha_i)$$

5.3.7 Phase I and Phase II of Control Chart Application

- **Phase I** is a retrospective analysis of process data to construct trial control limits
  - Charts are effective at detecting large, sustained shifts in process parameters, outliers, measurement errors, data entry errors, etc.
  - Facilitates identification and removal of assignable causes

- **In phase II**, the control chart is used to monitor the process
  - Process is assumed to be reasonably stable
  - Emphasis is on process monitoring, not on bringing an unruly process into control

5.4 The Rest of the Magnificat Seven

1. Histogram or Stemplot (chapter 2)
2. Check sheet (page 199-200)
3. Pareto Chart (page 200-201)
4. Cause-and-effect diagram (page 202-203)
5. Defect concentration diagram (page 204)
6. Scatter diagram (page 204-205)
7. Control chart

5.5 Implementing SPC in a Quality Improvement program

Elements of a successful SPC program

1. Management leadership
2. A team approach
3. Education of employees at all levels
4. Emphasis on reduction variablity
5. Measuring success in quantitative (economic) terms
6. A mechanism for communicating successful results throughout the organization.

5.6 An Application of SPC

Example of applying SPC methods to improve quality and productivity in a coper plating operation at a printed board fabrication facility. The team of the company have decided to use the following tools:

1. Cause-and-effect diagram for controller downtime (page 207)

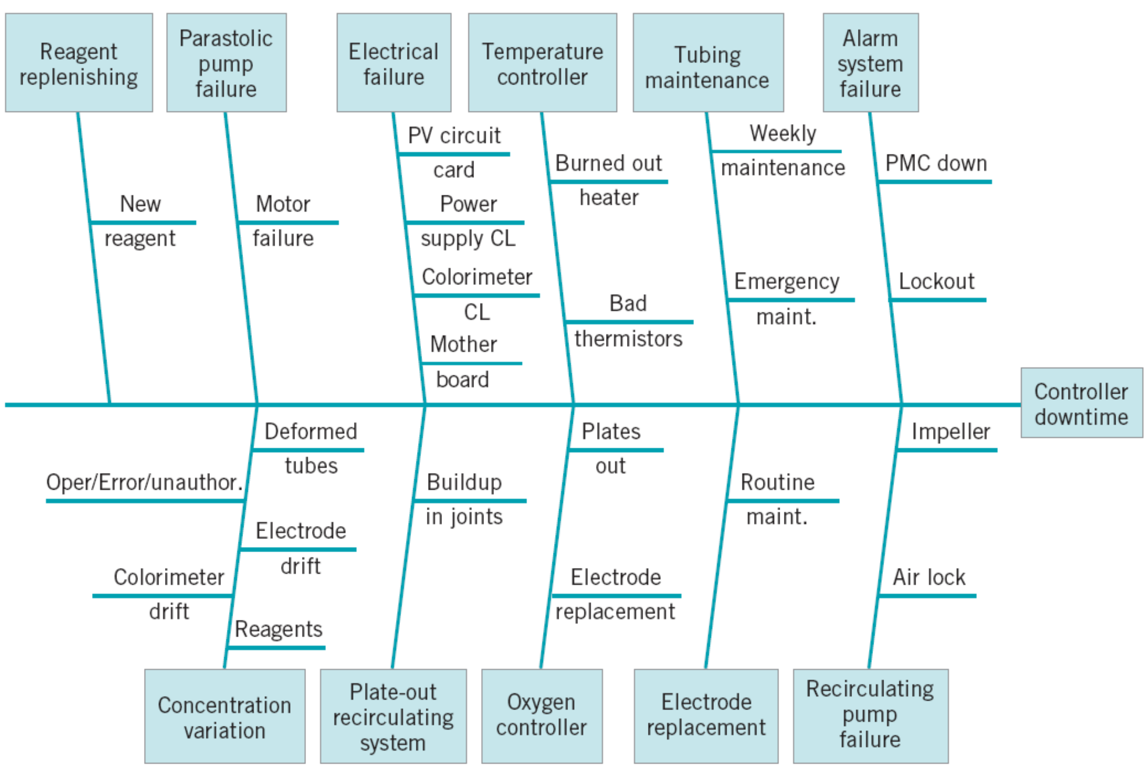


FIGURE 5.23 Cause-and-effect diagram for controller downtime.

2. Check Sheet

WEEKLY TALLY	OPERATOR ______		
WEEK ENDING ______	ERRORS	DESCRIPTION	ACTION
1. CONCENTRATION VARIATION			
a. Colorimeter drift	______		
b. Electrode failure	______		
c. Reagents	______		
d. Deformed tubes	______		
e. Oper/error/unauthorized	______		
2. ALARM SYSTEM FAILURE			
a. PMC down	______		
b. Lockout	______		
3. RECIRCULATING PUMP FAILURE			
a. Air lock	______		
b. Impeller	______		
4. REAGENT REPLENISHING			
a. New reagent	______		
5. TUBING MAINTENANCE			
a. Weekly maintenance	______		
b. Emergency maintenance	______		
6. ELECTRODE REPLACEMENT			
a. Routine maintenance	______		
7. TEMPERATURE CONTROLLER			
a. Burned out heater	______		
b. Bad thermistors	______		
8. OXYGEN CONTROLLER			
a. Plates out	______		
b. Electrode replacement	______		
9. PARASTOLIC PUMP FAILURE			
a. Motor failure	______		
10. ELECTRICAL FAILURE			
a. PV circuit card	______		
b. Power supply CL	______		
c. Colorimeter CL	______		
d. Motherboard	______		
11. PLATE-OUT RECIRCULATING			
a. Buildup at joints	______		
TOTAL COUNT			

FIGURE 5.24 Check sheet for logbook.

3. Pareto analysis of controller failures

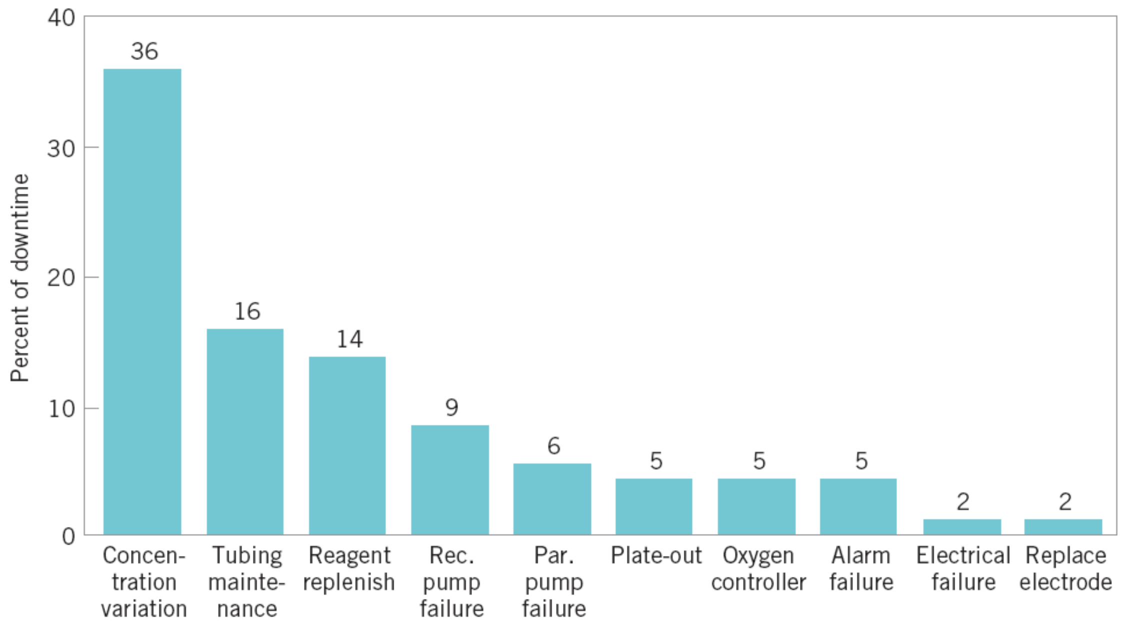


FIGURE 5.25 Pareto analysis of controller failures.

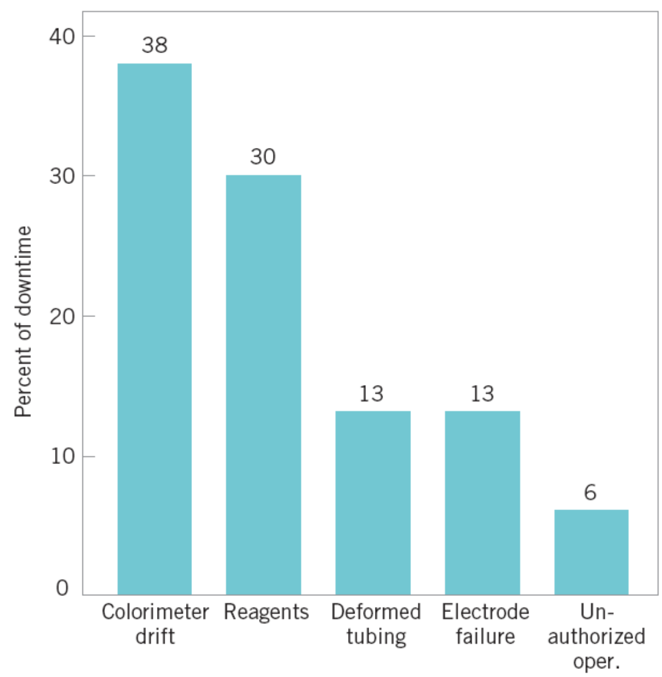


FIGURE 5.26 Pareto analysis of concentration variation.

4. X-bar (Figure 5.27) and R (Figure 5.28) Control charts

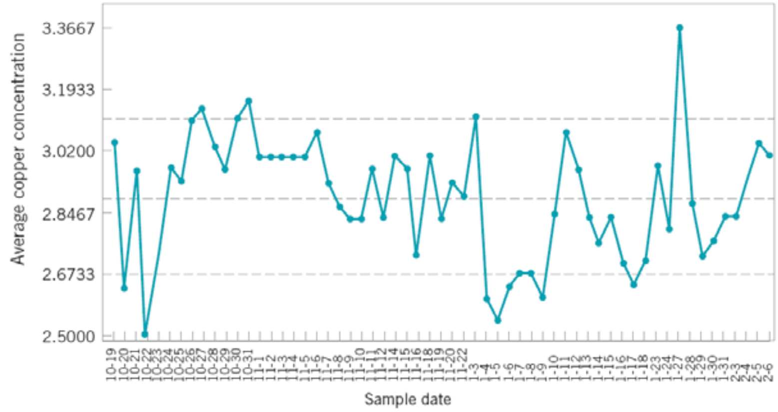


FIGURE 5.27 $\bar{x}$ chart for the average daily copper concentration.

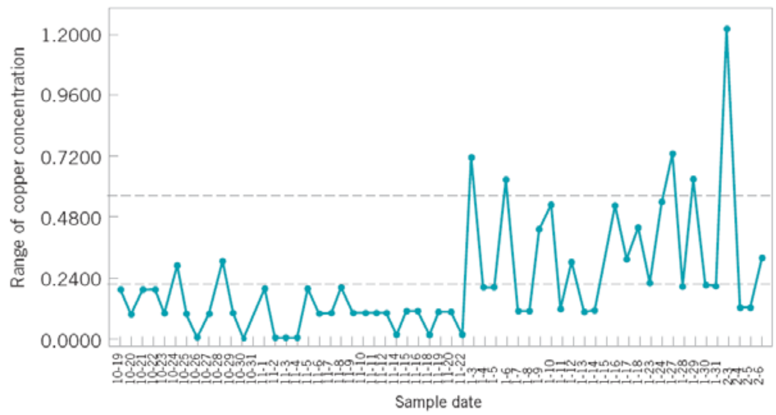


FIGURE 5.28 *R* chart for daily copper concentration.

5. Tolerance diagram in Figure 5.29 (page 211).

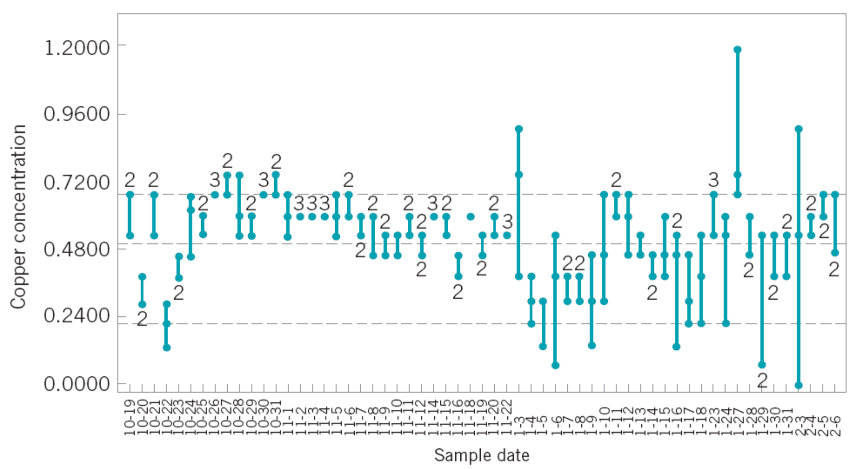


FIGURE 5.29 Tolerance diagram of daily copper concentration.

6. Factorial Design, Figure 5.30, page 212.

■ **TABLE 5.2**

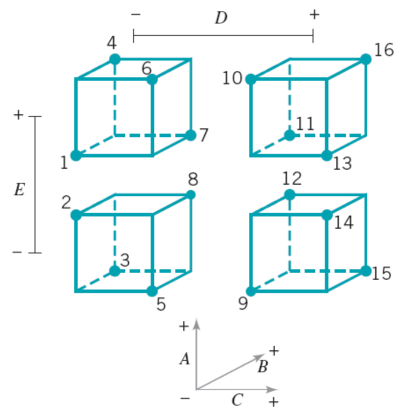
A Designed Experiment for the Plating Process

Objective: Minimize Plating Defects

Process Variables	**Low Level**	**High Level**
A = Copper concentration	−	+
B = Sodium hydroxide concentration	−	+
C = Formaldehyde concentration	−	+
D = Temperature	−	+
E = Oxygen	−	+

Experimental Design

	Variables					
Run	***A***	***B***	***C***	***D***	***E***	**Response (Defects)**
1	−	−	−	−	+	
2	+	−	−	−	−	
3	−	+	−	−	−	
4	+	+	−	−	+	
5	−	−	+	−	−	
6	+	−	+	−	+	
7	−	+	+	−	+	
8	+	+	+	−	−	
9	−	−	−	+	−	
10	+	−	−	+	+	
11	−	+	−	+	+	
12	+	+	−	+	−	
13	−	−	+	+	+	
14	+	−	+	+	−	
15	−	+	+	+	−	
16	+	+	+	+	+	



■ **FIGURE 5.30** A geometric view of the fractional factorial design for the plating process experiment.

5.7 Applications of Statistical Process Control and Quality Improvemnet Toos in Transactional and Service Business (See page 213-222)