Statistical Process Control

Statistical QA Approaches

- Statistical process control (SPC)
  - Monitors production process to prevent poor quality
- Acceptance sampling
  - Inspects random sample of product to determine if a lot is acceptable
- Design of Experiments

Statistical Quality Assurance

- Purpose: Assure that processes are performing in an acceptable manner
- Methodology: Monitor process output using statistical techniques
  - If results are acceptable, no further action is required
  - Unacceptable results call for corrective action

Acceptance Sampling:

Quality assurance that relies primarily on inspection *before* and *after production*

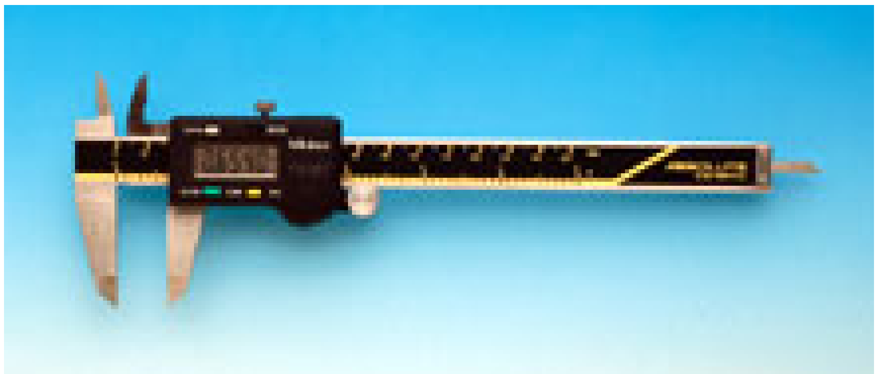
Statistical Process Control (SPC):

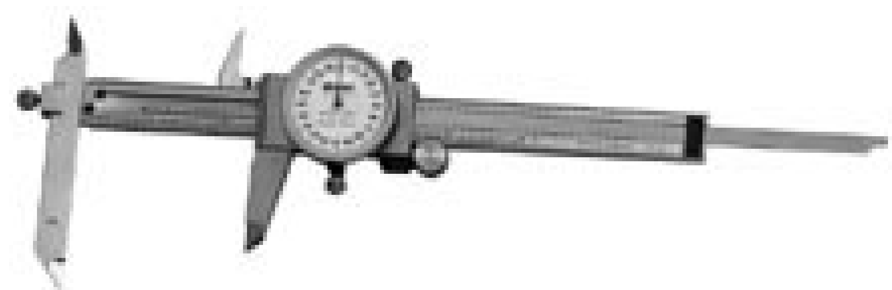
Quality control efforts that occur *during production*

What is SPC?

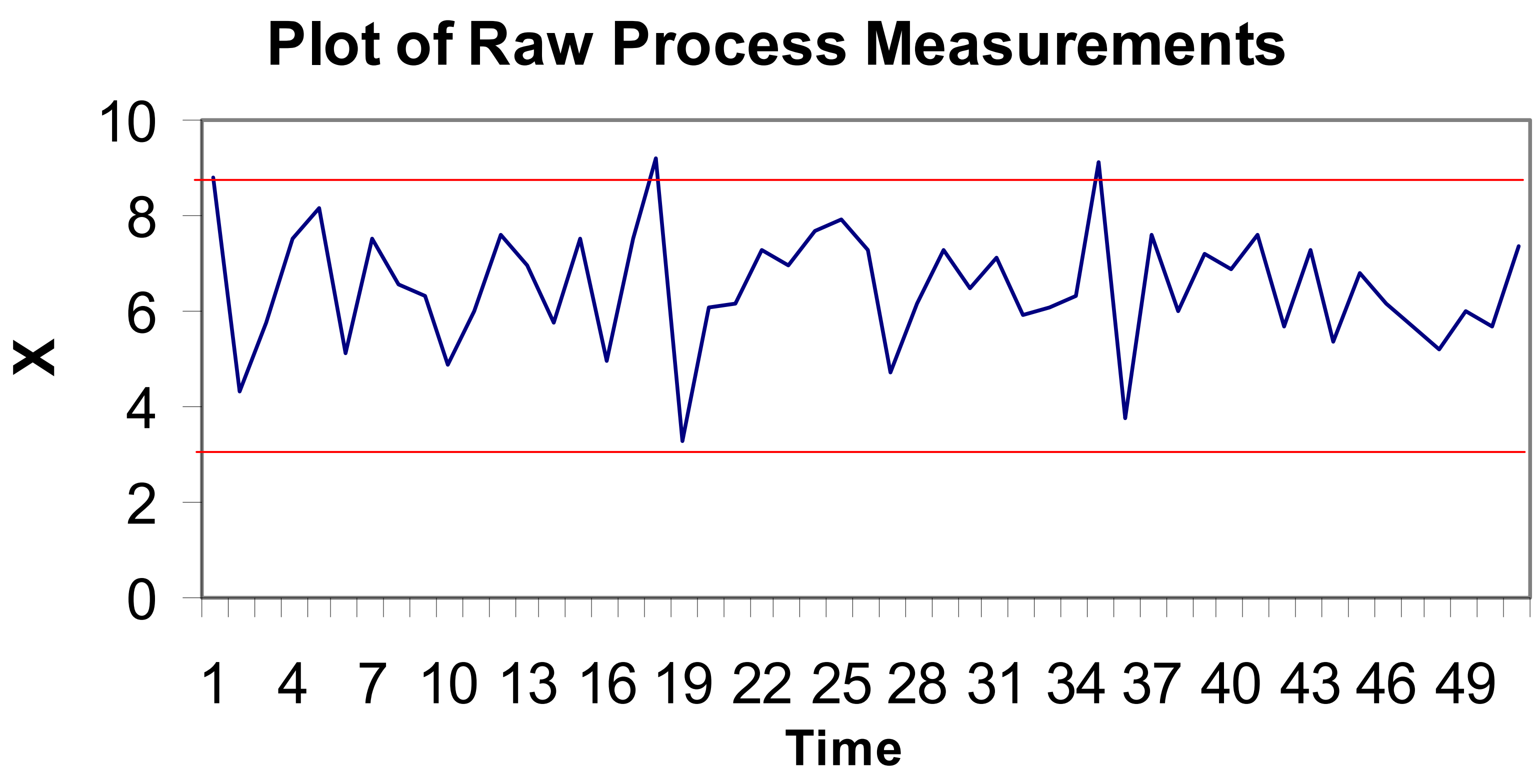
- A simple, yet powerful, collection of tools for graphically analyzing process data
- Has one primary purpose: to tell you when you have a problem.
- Invented by Walter Shewhart at AT&T to minimize process tampering
- Important because unnecessary process changes increase instability and **increase** the error rate
- SPC will identify when a problem (or special cause variation) occurs

To control, you have to measure!

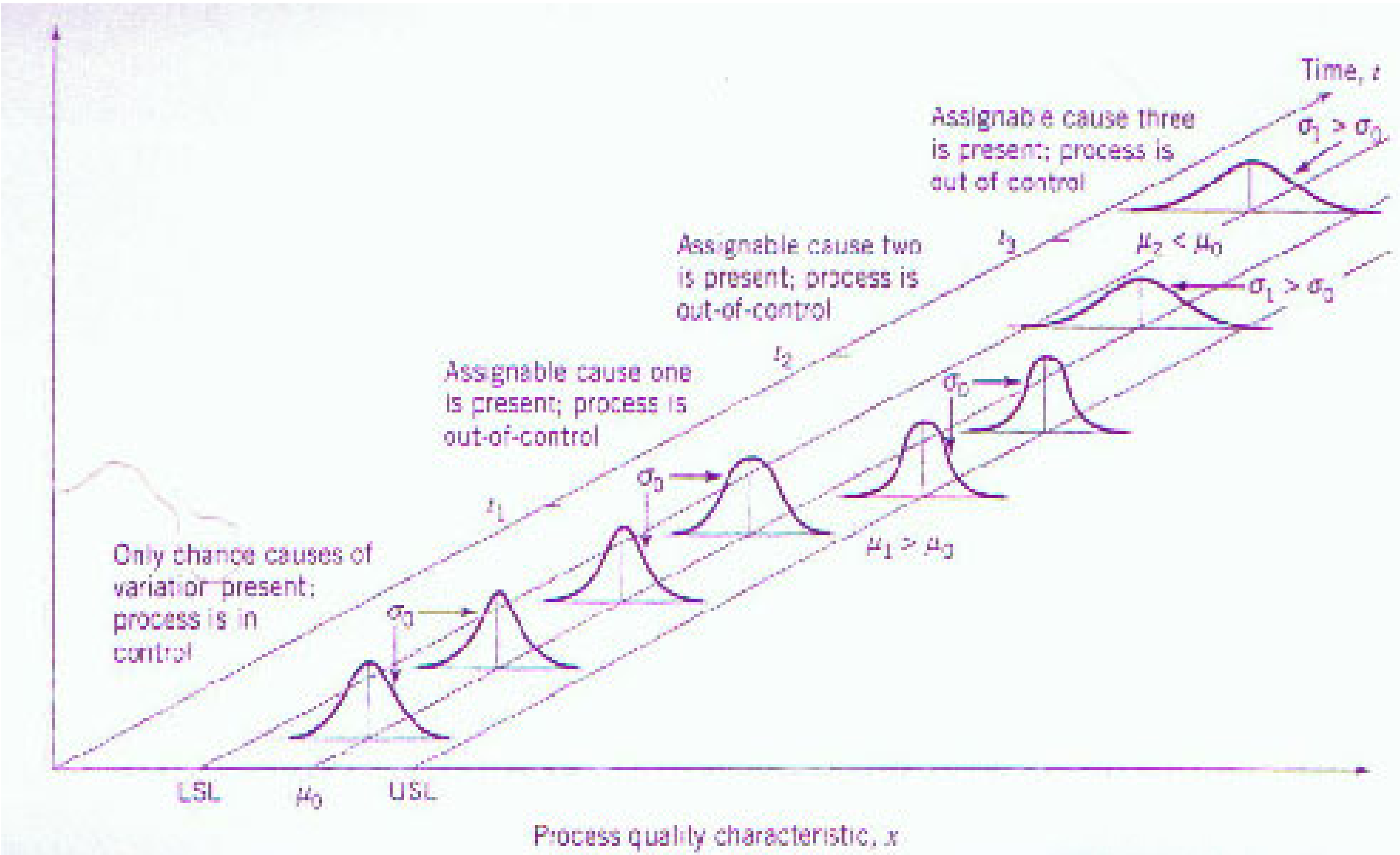




Production Data *always* has *some* Variability



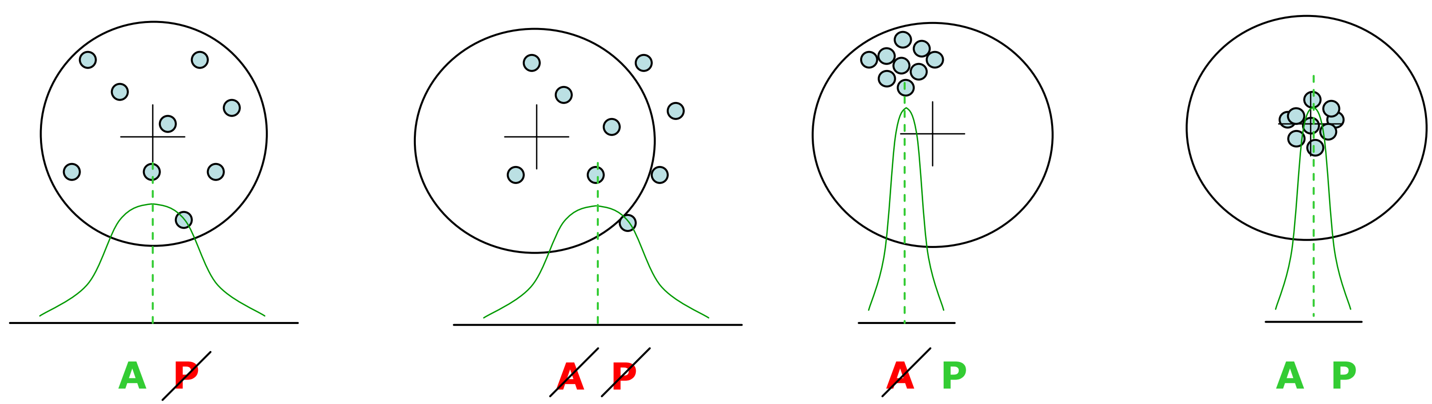
Chance and Assignable Causes of Quality Variation



Accuracy and Precision

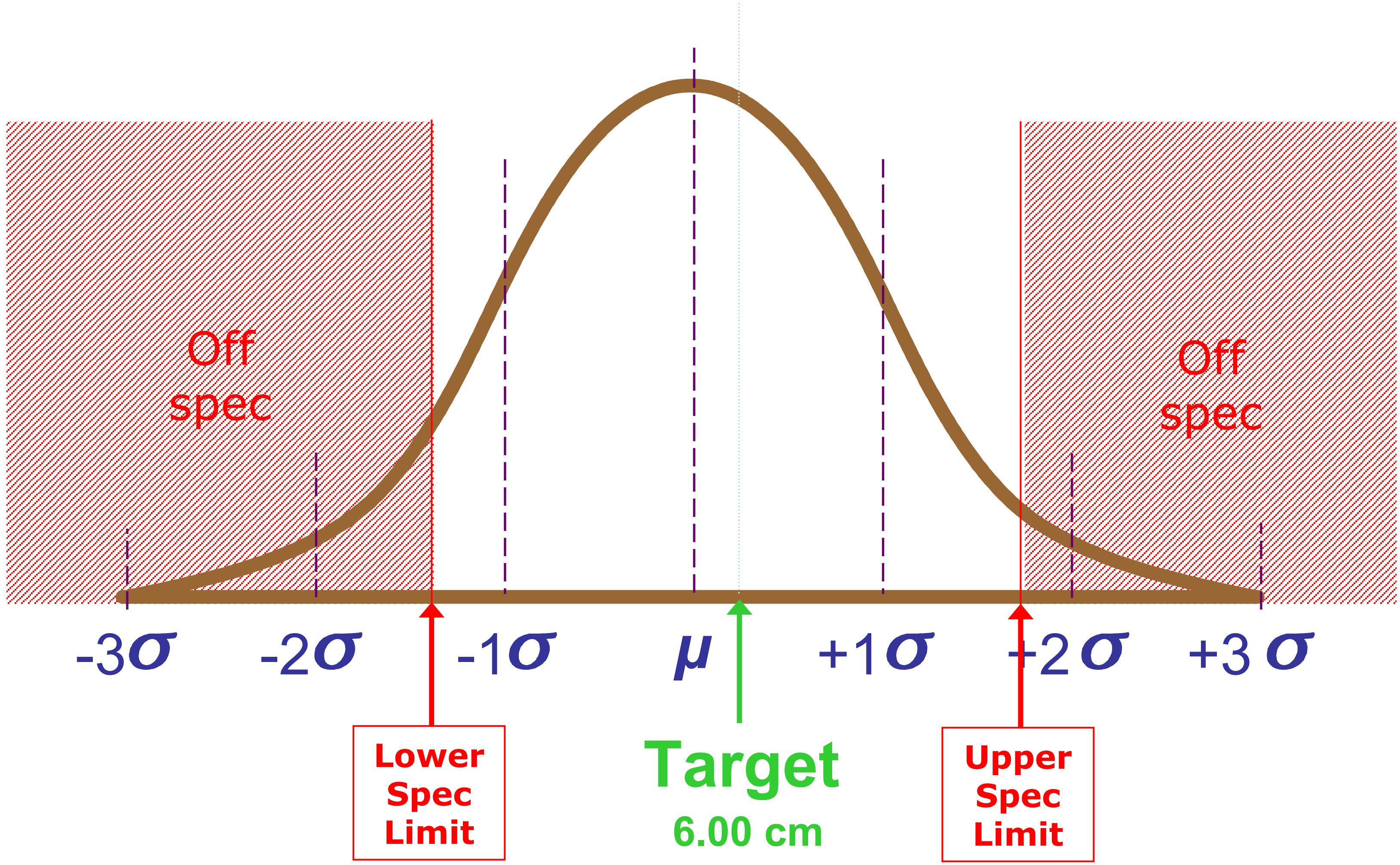
Examples of quality characteristics: Painted surface, thickness, hardness, and resistance to fading or chipping, viscosity, sweetness, electrical resistance, frequency, …

- We can control only those characteristics that can be *counted*, *evaluated* or *measured*



Engineering characteristics may show problems with **accuracy** or with **precision**

Normal Distribution—Shaft Diameter
(What is this plot of data telling us?)

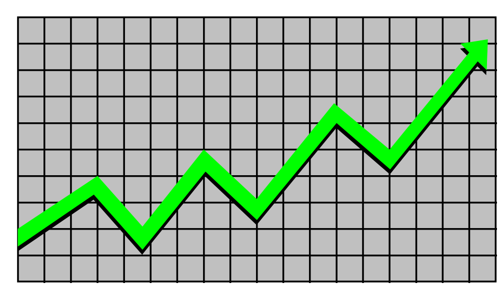


Causes of Variation

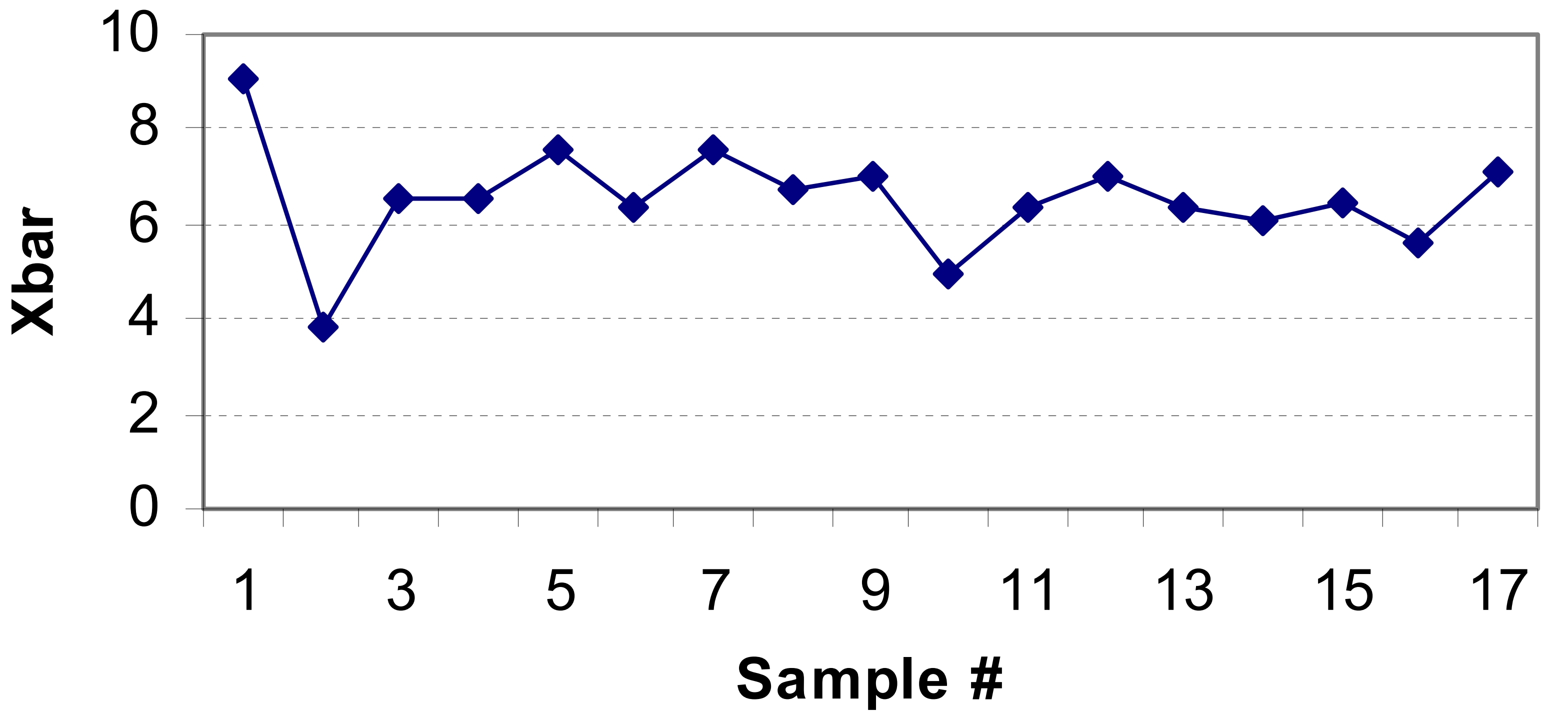
- Common Causes
  - Variation inherent in a process
  - Can be eliminated only through improvements in the system
- Assignable Causes
  - Variation due to identifiable factors
  - Can be modified through operator or management action

Assignable Causes are controlled by SPC

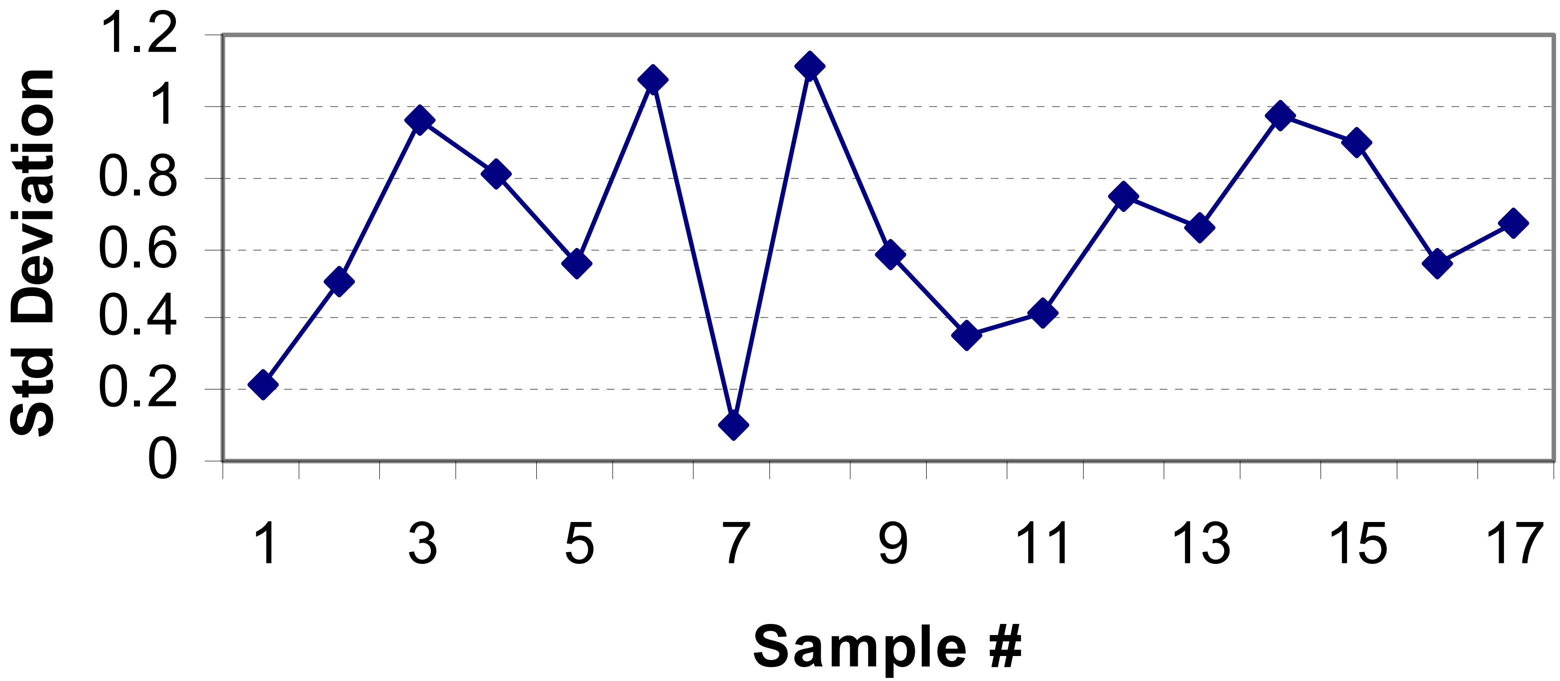
- Take periodic samples from process
- Plot sample points on a *control chart*
- Determine if process is within limits
- Prevent quality problems



Plot of Sample Averages



Plot of Sample Standard Deviation



Control Charts

- A key tool in SPC
- Graph establishing process control limits
- Charts for variables
  - –Mean (X-bar), Range (R), EWMA, CUSUM
- Charts for attributes
  - –p, np and c

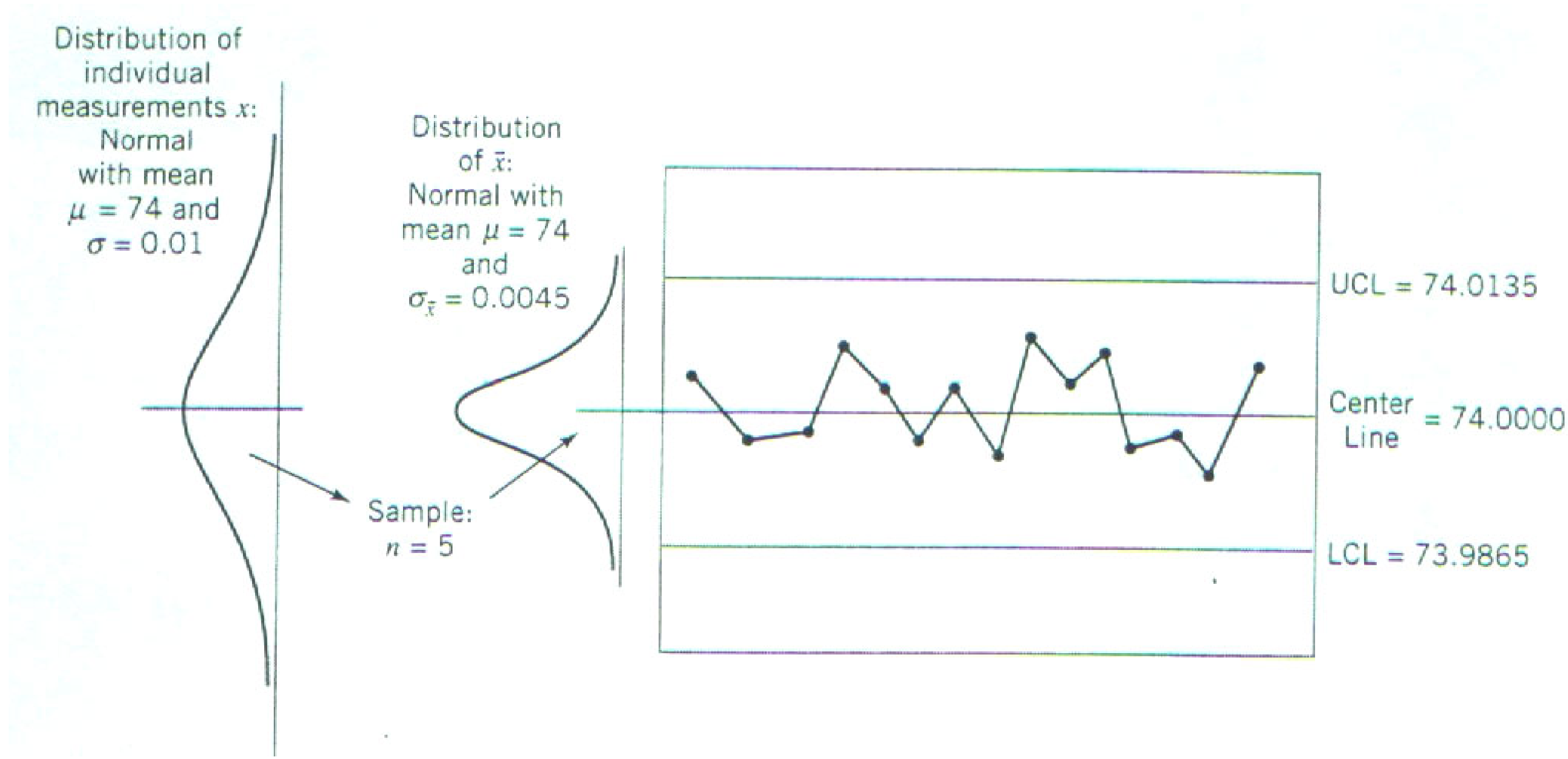
The Shewhart Control Chart

- A *time-ordered plot* of sample statistics
- When chart is within control limits
  - Only random or common causes present
  - We leave the process alone
- Plot of <u>each point</u> is the test of hypothesis:

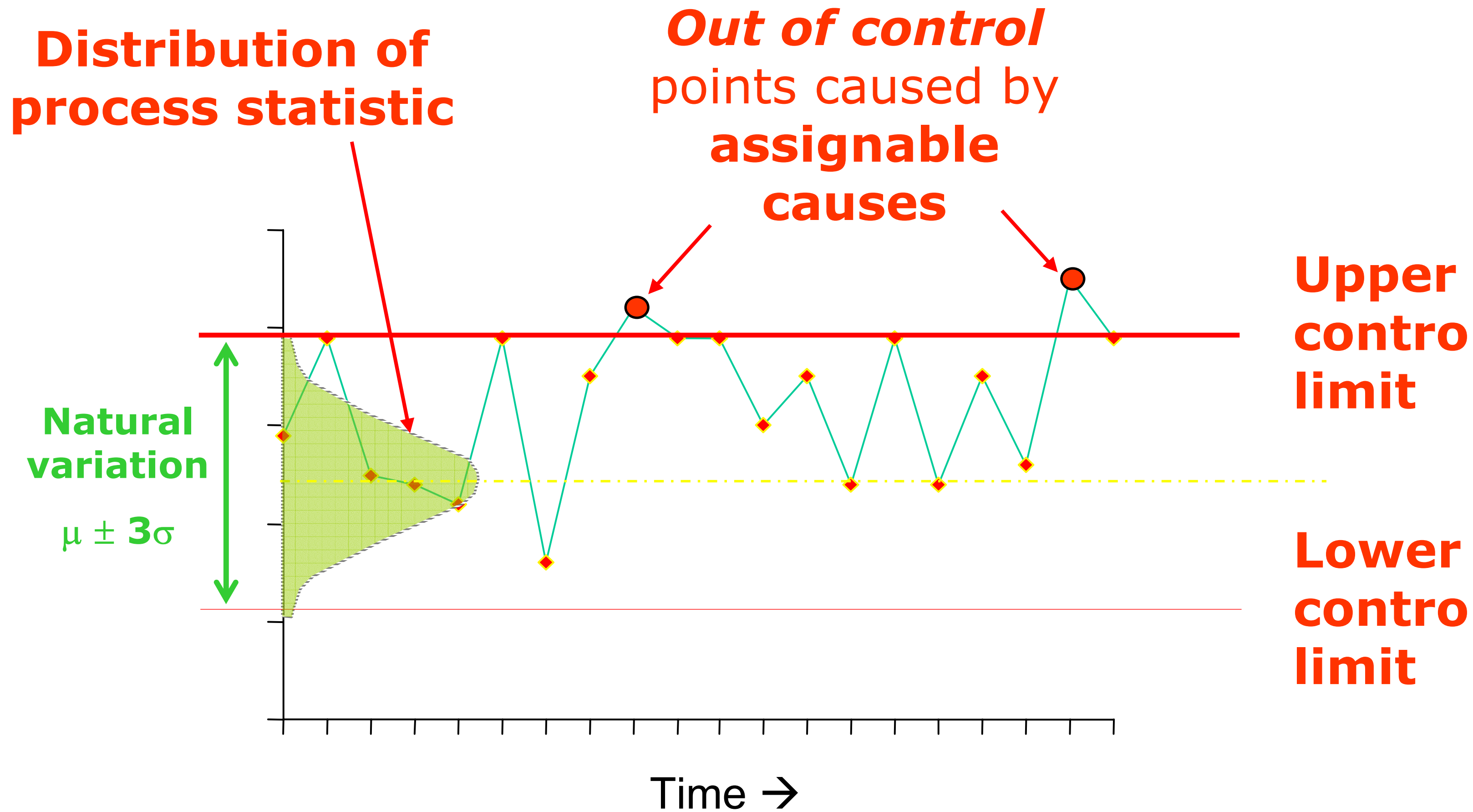
  H_0: Process is “in control” vs.

  H_1: Process is out of control and requires investigation

Relationship between the process and the control chart



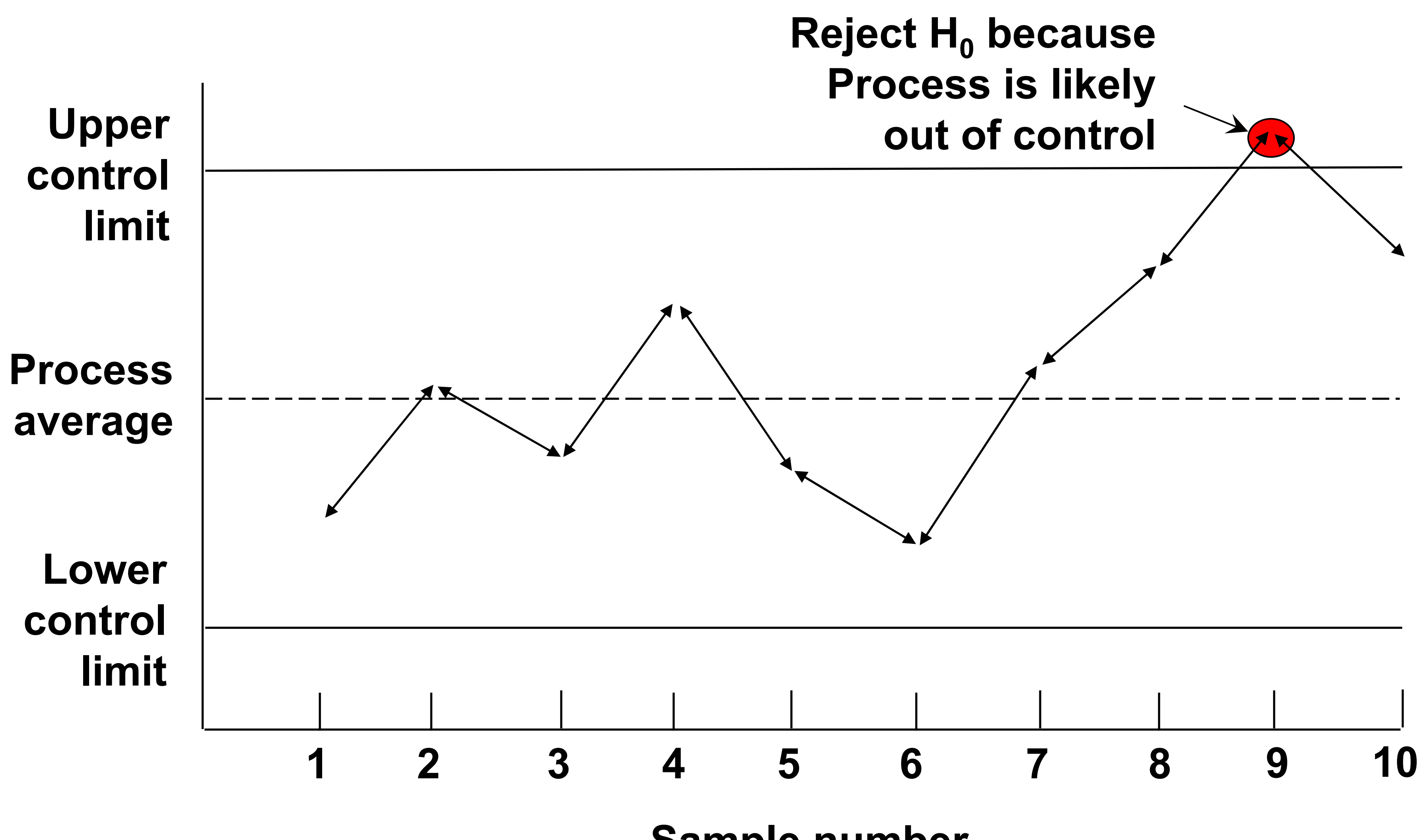
How Does the Chart Work?



A Process Is “In Control” If

- No sample points outside limits
- Most points near process average
- About equal number of points above & below centerline
- Points appear randomly distributed
- A process “in control” is supposed to be under the influence of random causes only

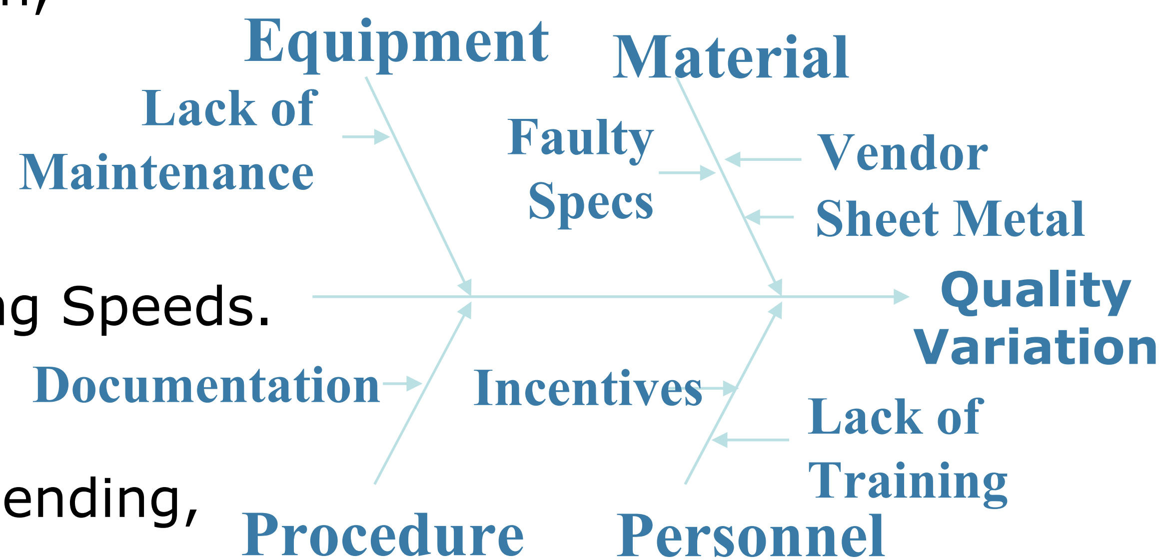
The Signal from a Control Chart



Potential Reasons for Variation

- **The Operator:**
  Training, Supervision,
  Technique.

- **The Method:**
  Procedures, Set-up,
  Temperature, Cutting Speeds.



- **The Material:**
  Moisture content, Blending,
  Contamination.

- **The Machine:**
  Set-up, Machine condition, Inherent Precision

- **Management: Poor process management; poor systems**

Charts may signal incorrectly!

Charts repeatedly apply hypothesis testing!

Type I error with charts:

Concluding that a process is not in control when it actually is

Type II error with charts:

Concluding that a process is in control when it is not

Two Types of Process Data

Variables — ***"Things we measure"***

- **Length**
- **Weight**
- **Time**
- **Blood pressure**
- **Volume**
- **Temperature**
- **Diameter**
- **Tensile Strength**
- **Strength of Solution**

Attributes — ***"Things we count"***

- **Number or percent of defective items in a lot.**
- **Number of defects per item.**
- **Types of defects.**
- **Value assigned to defects (minor = 1, major = 5, critical = 10)**

Types of Control Charts

- Basic Types
  - Most typical three
    - X-Bar and R
    - p chart
    - c chart
  - Depend Upon Data Type
    - Variables
    - Attribute
- Advances Types: CUSUM, EWMA, Multivariate
- Recall that plotting points on a control chart is the repeated application of *Hypothesis Testing*

Types of Shewhart Control Charts

Control Charts for Variables Data

$\bar{X}$ and R charts: for sample averages and ranges.

$\bar{X}$ and s charts: for sample means and standard deviations.

Md and R charts: for sample medians and ranges.

$\bar{X}$ charts: for individual measures; uses moving ranges.

Control Charts for Attributes Data

p charts: proportion of units nonconforming.

np charts: number of units nonconforming.

c charts: number of nonconformities.

u charts: number of nonconformities per unit.

Control Charts For Variables

- Mean chart (X-Bar Chart) ← for accuracy

  Uses average of a sample:

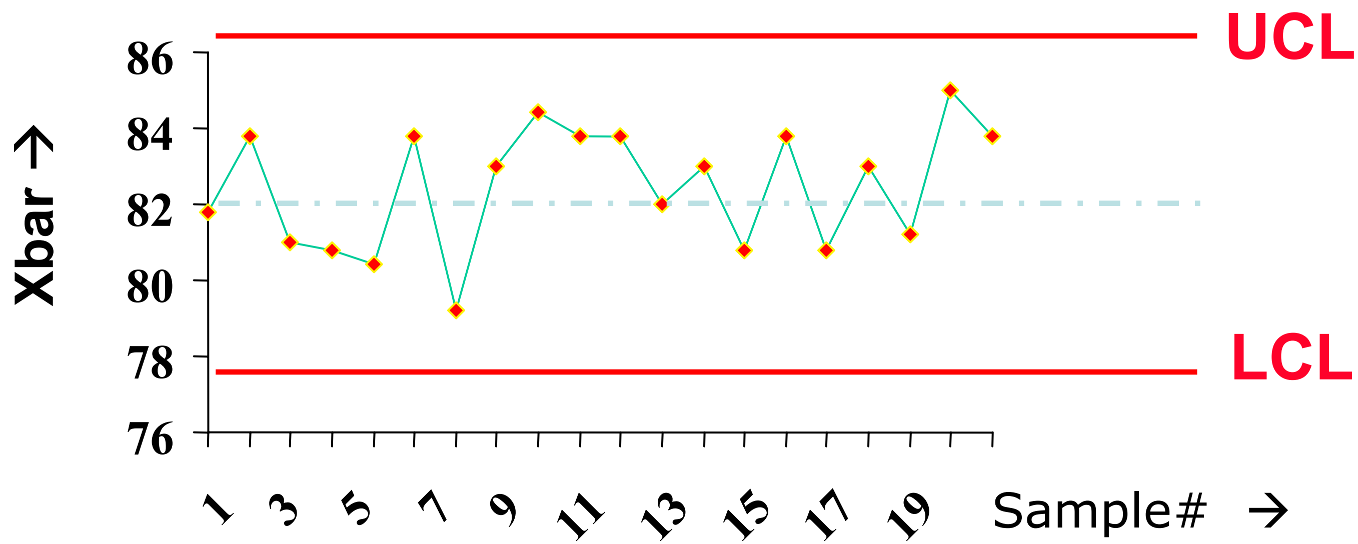
  $$\text{X-Bar} = (x_1+x_2+x_3+x_4+x_5)/5$$

- Range chart (R-Chart) ← for precision

  Uses amount of dispersion in a sample

  $$\mathbf{R = max\,(x_i) - min(x_i)}$$

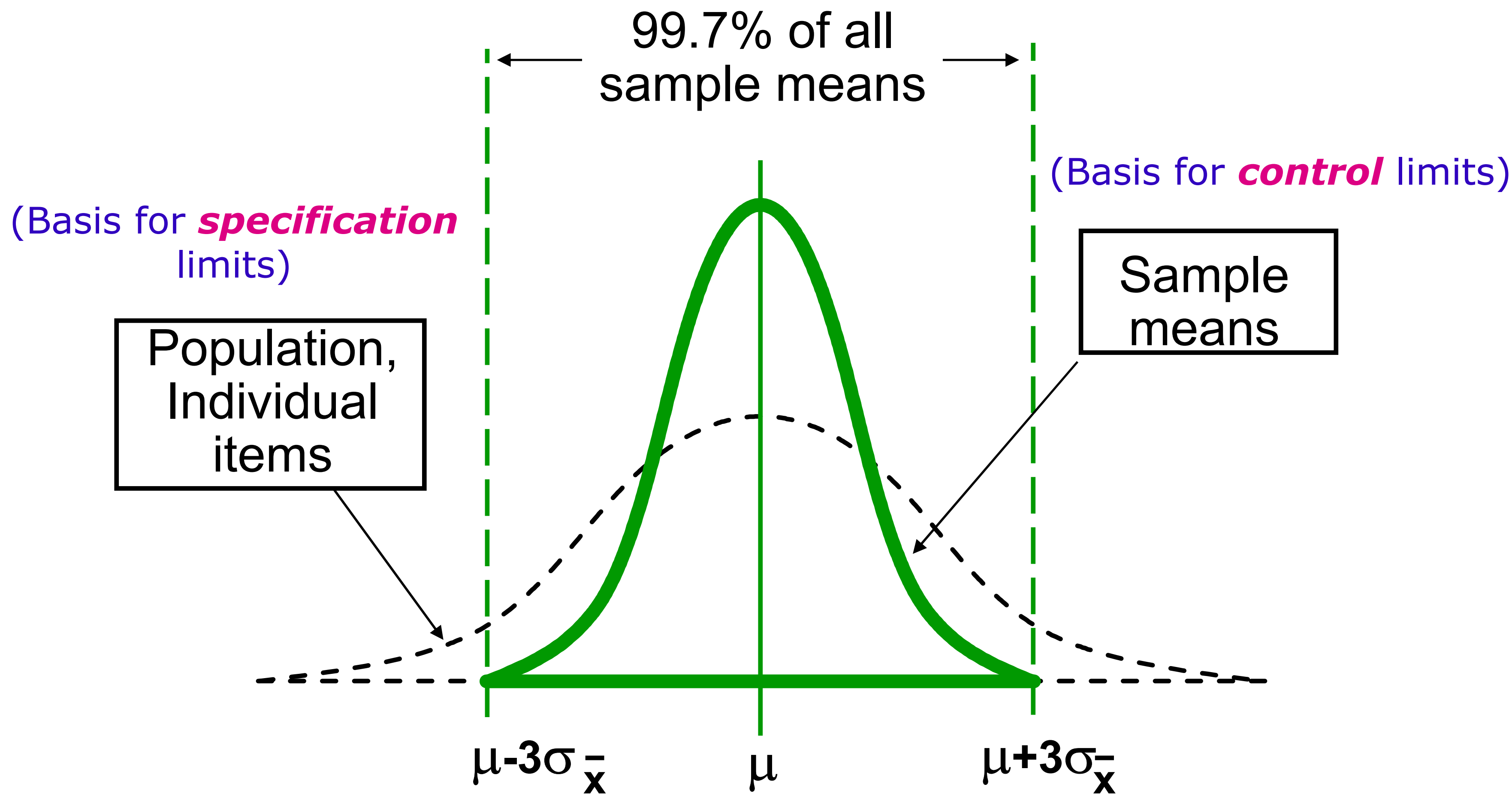
Xbar Chart helps control *Accuracy*



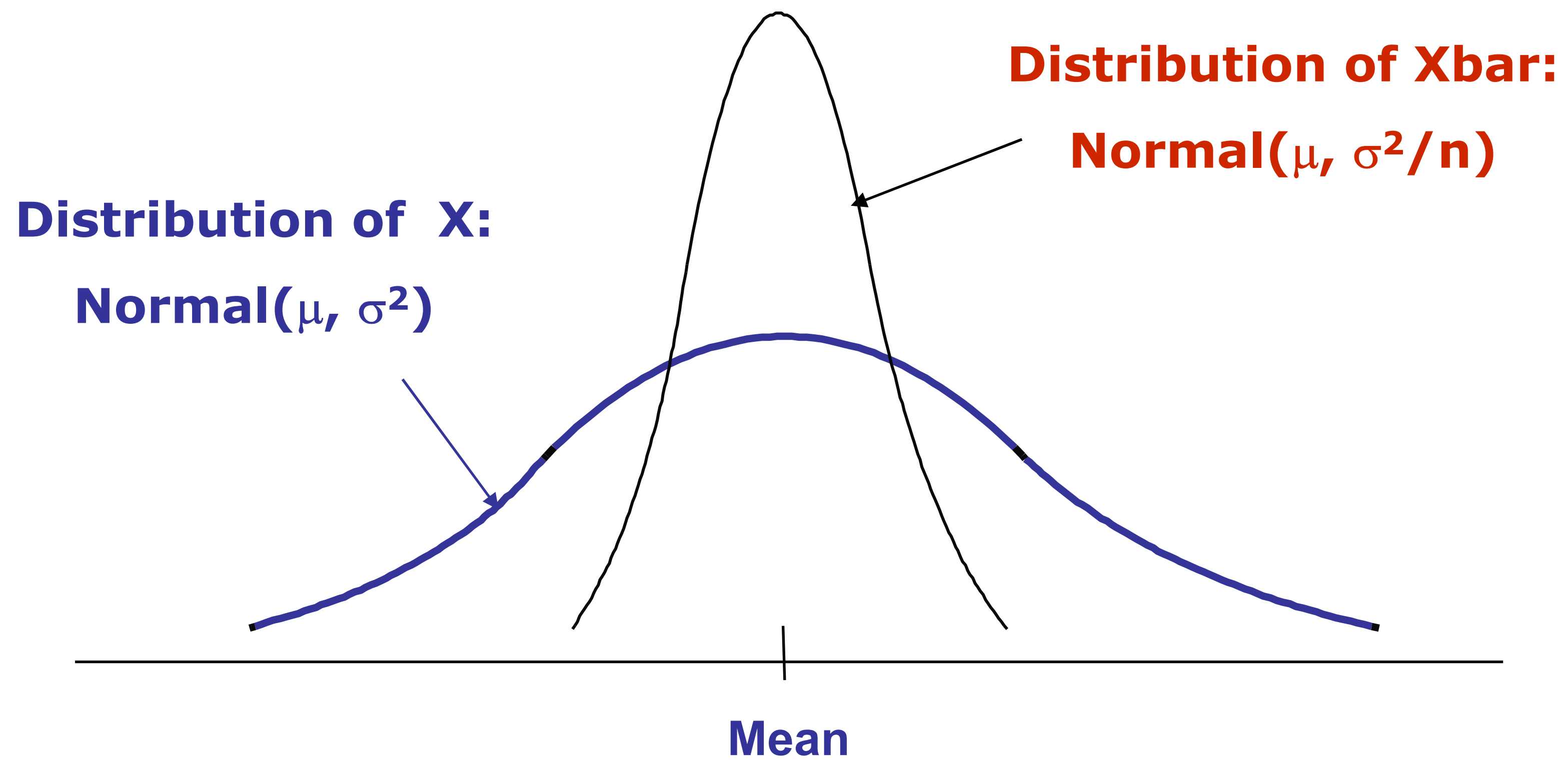
- Average Xbar = 82.5 kg
- Standard Deviation of X bar = σ_{xbar} = 1.6 kg
- **Control Limits** = Average Xbar $\pm$ 3 σ_{xbar}
  = 82.5 $\pm$ 3 $\times$ 1.6 = [77.7, 87.3]

Here, the process is "in control" (i.e., the mean is stable)

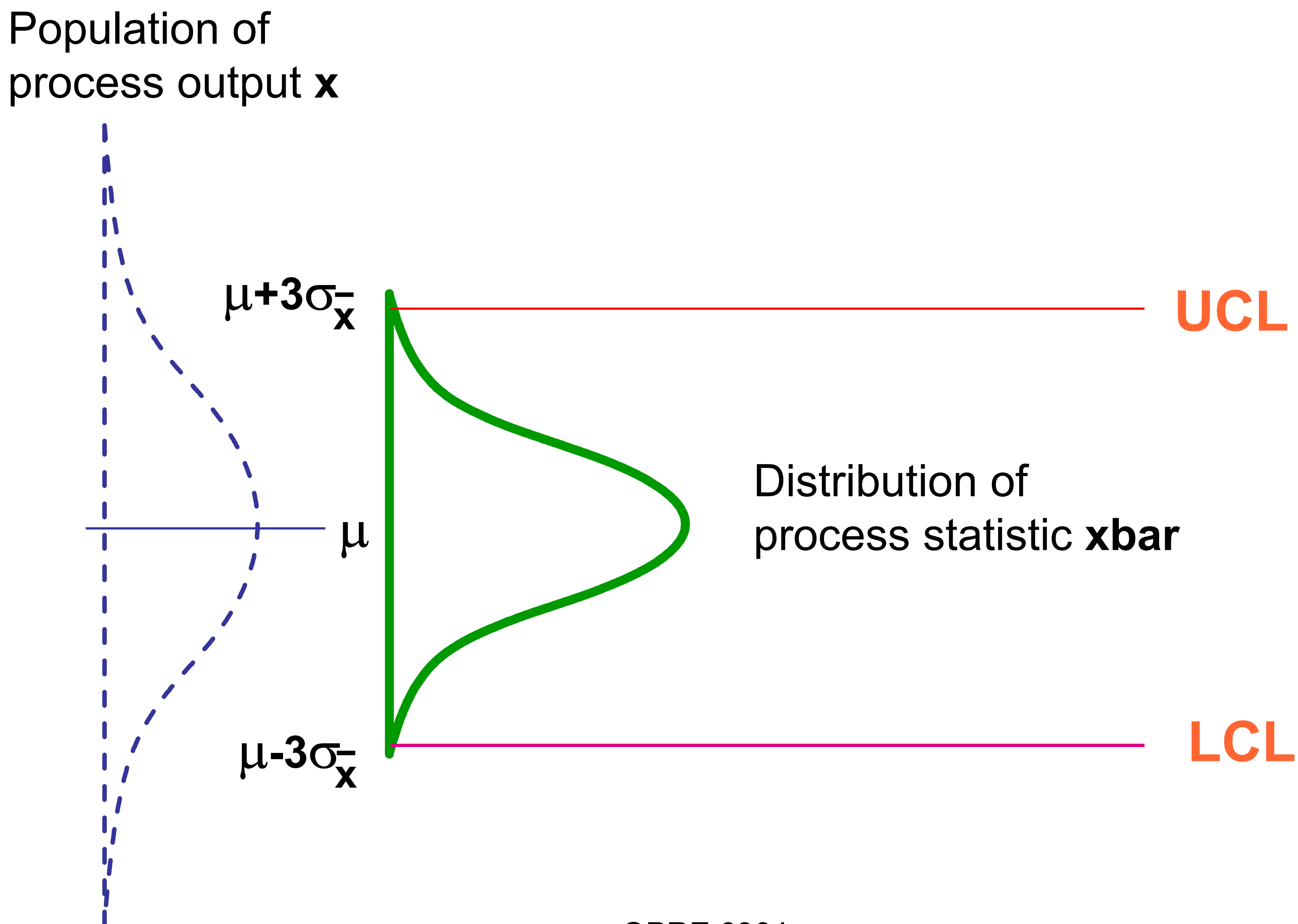
Central Limit Theorem



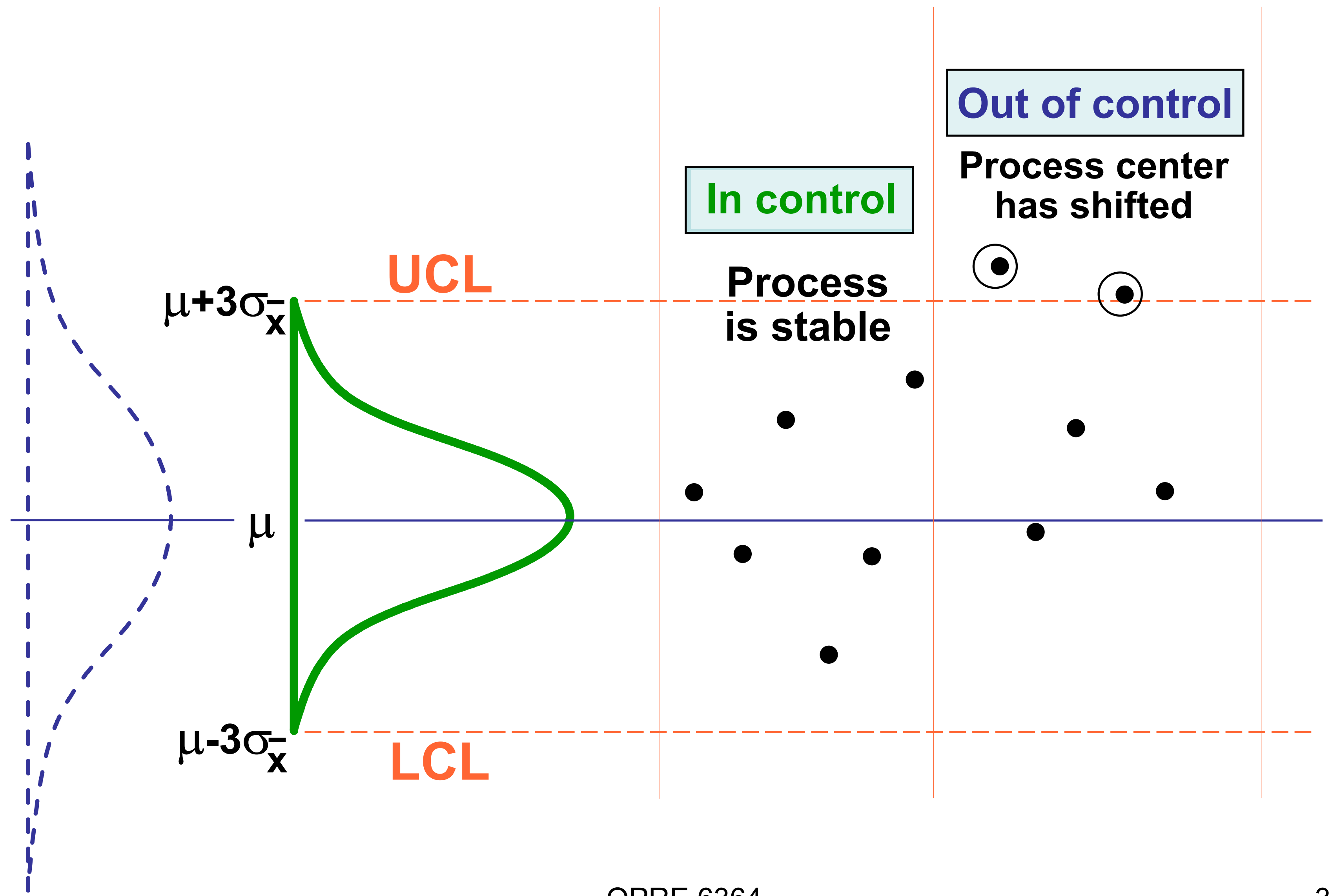
Distribution of Xbar--a Process Statistic



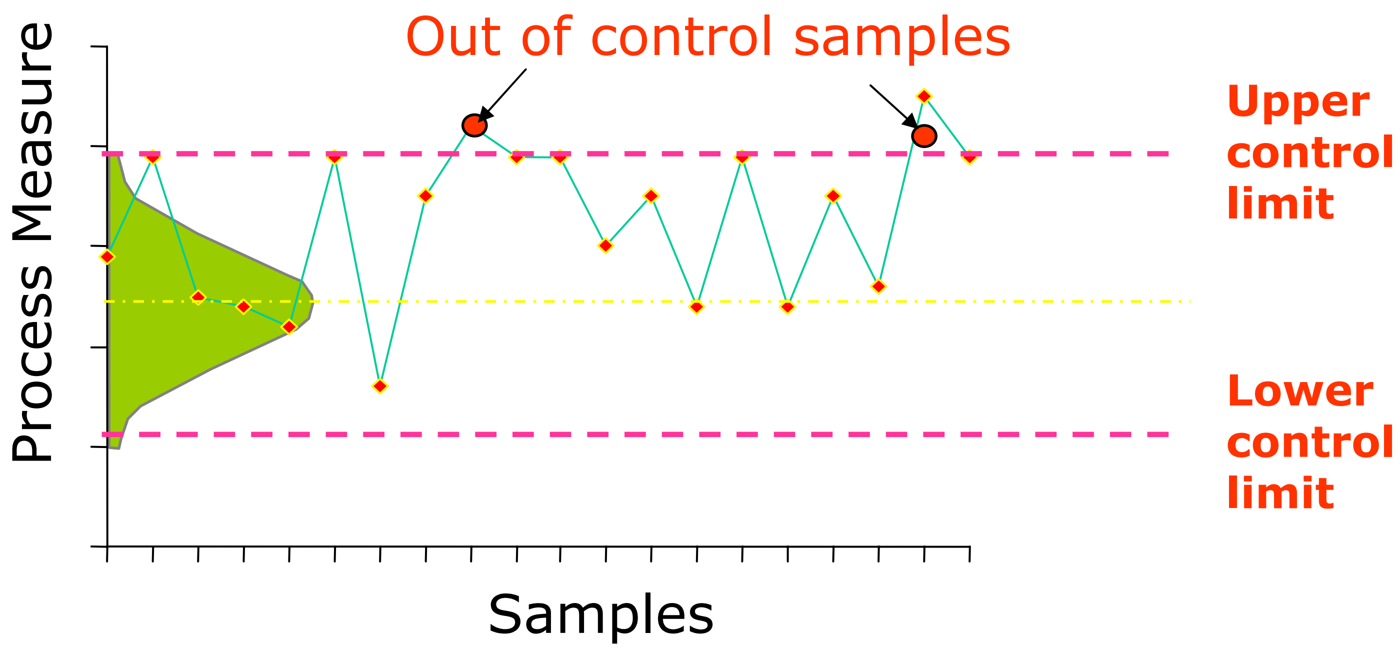
SPC Control Limits



Process Control by Control Limits



Routine use of the Process Control Chart



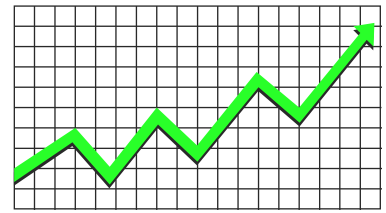
- **Data/Information**: Monitor process variability over time
- **Control Limits**: **Average $\pm$ z $\times$ Normal Variability**
- **Decision Rule**:
  - Ignore variability when points are within limits
  - Investigate variation when outside as "abnormal"
- **Errors**: **Type I** - False alarm (unnecessary investigation)

  Type II - Missed signal (to identify and correct)

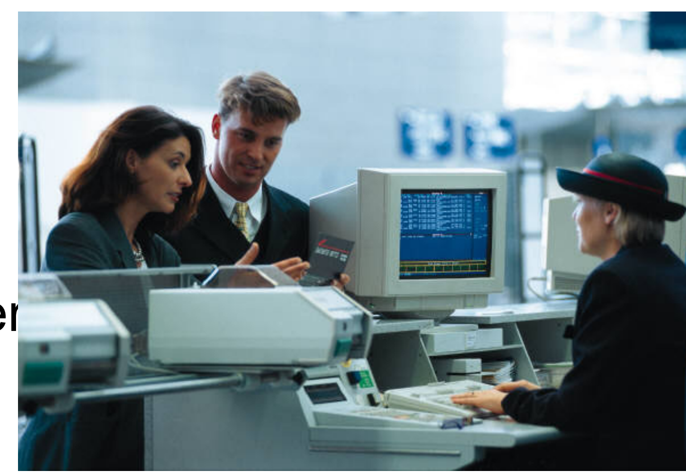
SPC Applied To Services

- Nature of defect is different in services
- Service defect is a failure to meet customer requirements
- Monitor times, customer satisfaction

Service SPC Examples



- Hospitals
  - Timeliness, responsiveness, accuracy
- Grocery Stores
  - Check-out time, stocking, cleanliness
- Airlines
  - Luggage handling, waiting times, courtesy
- Fast food restaurants
  - Waiting times, food quality, cleanliness
- Banks
  - Daily balance errors, # of customer served, transactions completed, courtesy



Control Charts

- Basic Types
  - Most typical three
    - X-Bar and R
    - p chart
    - c chart
  - Depend Upon Data Type
    - Variables
    - Attribute
- All are Applications of *Hypothesis Testing*

Variations and Control

Random or Common Variation:

Natural or inherent variations in the output of process are created by **countless minor factors, too many to investigate economically**

Assignable or Special Variation:

A variation whose **cause can be identified**

$\Rightarrow$ Assignable variations push the charts beyond control limits

$\Rightarrow$ Their causes **must be investigated, detected and removed**

Assignable cause examples: Tool wear, equipment that needs adjustment, defective materials, human factors (carelessness, fatigue, noise and other distractions, failure to follow correct procedures), failure of pumps, heaters, etc.

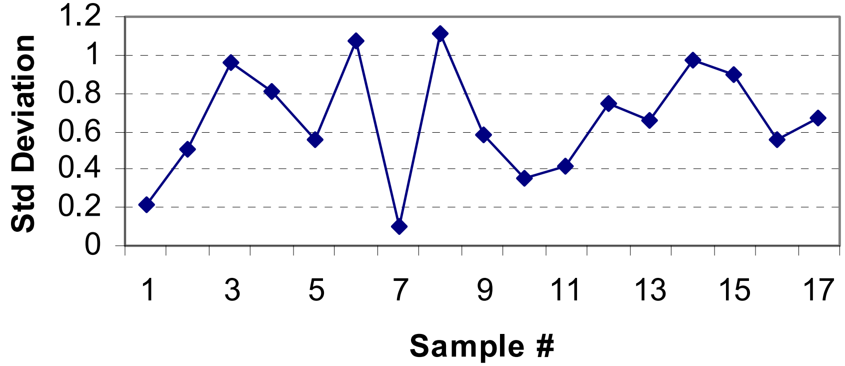
Special Causes of Variation

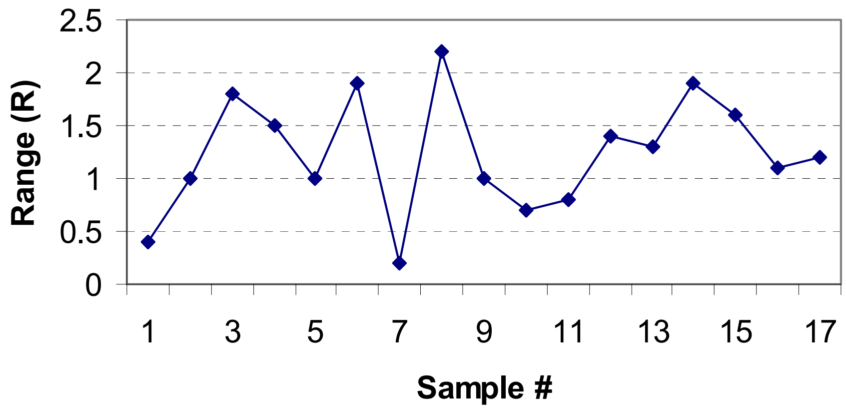
- Also called assignable cause of variation
- When an assignable cause is active, the chart goes beyond control limits
- In SPC, when some unusual or external cause occurs, the cause is identified and data point removed to calculate true control limits
- Attempting to improve a process (containing special cause variation) without removing the special cause only increases the instability and variation of the process

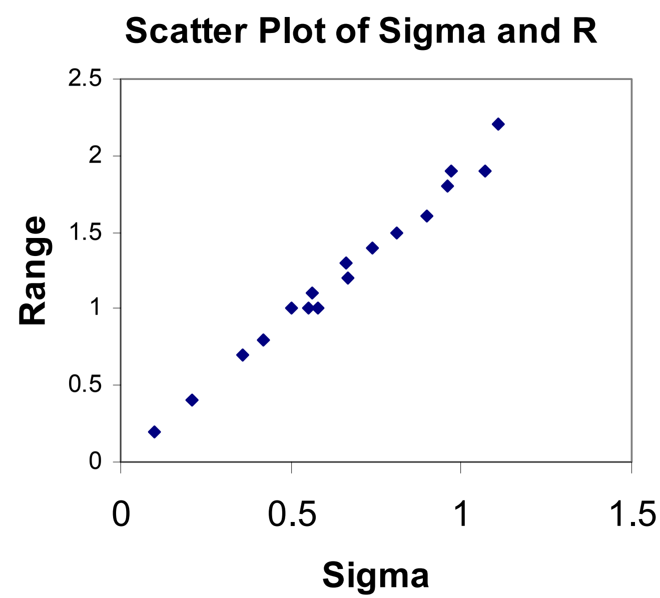
Common Causes of Variation

- Also called random causes of variation
- When only common causes are active, the chart remains stable and within control limits
- In SPC, when only random causes are active, no single cause is at fault. Any process improvement effort now must consider all sources of variation, generally the factors inherent in the technology of the process
- A process with only common cause of variation is stable and predictable and it forms the basis for measuring process capability

We can use Range in place of Std Deviation to control Precision







Correlation(s, R) = 0.9934

Control Charts for Variables

- Mean chart (X-Bar Chart)
  - –Uses average of a sample
- Range chart (R-Chart)
  - –Uses amount of dispersion in a sample

Construction of Control Chart

- Control limits must be based only on historic process data that are “in-control”
- We draw tentative limit lines and check if any points fall outside the limits
- If some points fall outside, non-random causes are present; discard those data points and re-calculate control limits
- Repeat calculation of limits if necessary

Three Sigma Control Limits

- The use of 3-sigma limits generally gives good results in practice (ARL = $1/(\alpha/2)$.
- If the distribution of the quality characteristic is reasonably well approximated by the normal distribution, then the use of 3-sigma limits is applicable.
- These limits are often referred to as <u>action limits</u>.

Warning Limits on Control Charts

- Warning limits (if used) are typically set at 2 standard deviations from the mean.
- If one or more points fall between the warning limits and the control limits, or close to the warning limits the process may not be operating properly.
- Good thing: Warning limits often increase the *sensitivity* of the control chart.
- Bad thing: Warning limits could result in an increased risk of false alarms.

Calculation of Xbar Chart Control Limits

$$\text{Def}: \quad \text{Xbar} = \bar{x} = (\mathrm{x}_1 + x_2 + x_3 + x_4 + x_5)/5$$

$$\text{Range } R = Max\ x_i - Min\ x_i$$

A quick method for finding control limits is to use

average sample range $\overline{R}$ as a measure of process variability.

Upper control limit, $\text{UCL} = \bar{\bar{x}} + z\sigma_{xbar} = \bar{\bar{x}} + A_2\overline{R}$

Lower control limit, $\text{LCL} = \bar{\bar{x}} - z\sigma_{xbar} = \bar{\bar{x}} - A_2\overline{R}$

where $\overline{R}$ = Average of sample ranges and A_2 is found from a table.

Process Control Chart Factors

Sample (Subgroup) Size (n)	Control Limit Factor for Averages (Mean Charts) (A_2)	UCL Factor for Ranges (Range Charts) (D_4)	LCL Factor for Ranges (Range Charts) (D_3)	Factor for Estimating Sigma ($\sigma = \bar{R}/d_2$) (d_2)
2	1.880	3.267	0	1.128
3	1.023	2.575	0	1.693
4	0.729	2.282	0	2.059
5	0.577	2.115	0	2.326
6	0.483	2.004	0	2.534
7	0.419	1.924	0.076	2.704
8	0.373	1.864	0.136	2.847
9	0.337	1.816	0.184	2.970
10	0.308	1.777	0.223	3.078

Process Data Example:

Select 25 small samples (in this case, n = 4)

Find $\bar{X}$ and R of each sample.

The $\bar{X}$ chart is used to control the process mean.

The R chart is used to control process variation.

	Sample Number					
	1	2	3	4	25	
X Values	4	7	6	7		
	6	3	9	6		
	5	8	8	6		
	5	6	9	5		
Sum	20	24	32	24	28	Total
$\bar{X}$	5	6	8	6	7	150
R	2	5	3	2	3	75

$\overline{X}$ and R Charts Factors

n	A_2	D_3	D_4	d_2
2	1.880	3.267	0	1.128
3	1.023	2.575	0	1.693
4	0.729	2.282	0	2.059

	Sample Number					
	1	2	3	4	25	
Values	4	7	6	7		
	6	3	9	6		
	5	8	8	6		
	5	6	9	5		
Sum	20	24	32	24	28	Total
$\overline{X}$	5	6	8	6	7	150
R	2	5	3	2	3	75

$\overline{X}$ and R Limits

n	A_2	D_3	D_4	d_2
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	Sample Number					
	1	2	3	4	25	
Values	4	7	6	7		
	6	3	9	6		
	5	8	8	6		
	5	6	9	5		
Sum	20	24	32	24	28	Total
$\overline{X}$	5	6	8	6	7	150
R	2	5	3	2	3	75

$\overline{\overline{X}} = 150 / 25 = 6$

$\overline{R} = 75 / 25 = 3$

$A_2\overline{R} = 0.729(3) = 2.2$

$UCL_{\overline{X}} = \overline{\overline{X}} + A_2\overline{R} = 6 + 2.2 = 8.2$

$LCL_{\overline{X}} = \overline{\overline{X}} - A_2\overline{R} = 6 - 2.2 = 3.8$

$UCL_R = D_4\overline{R} = 2.282(3) = 6.8$

$LCL_R = D_3\overline{R} = 0(3) = 0$

$\overline{X}$ and R Chart Plots

n	A_2	D_3	D_4	d_2
2	1.880	3.267	0	1.128
3	1.023	2.575	0	1.693
4	0.729	2.282	0	2.059

	Sample Number 1	2	3	4	...	25	Total
Values	4	7	6	7			
	6	3	9	6			
	5	8	8	6			
	5	6	9	5			
Sum	20	24	32	24		28	
$\bar{X}$	5	6	8	6		7	150
R	2	5	3	2		3	75

$\overline{\overline{X}} = 150 / 25 = 6$

$\overline{R} = 75 / 25 = 3$

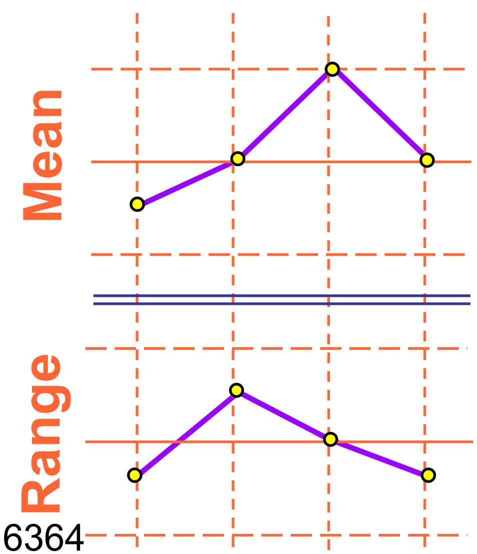
$A_2\overline{R} = 0.729(3) = 2.2$

$UCL_{\bar{X}} = \overline{\overline{X}} + A_2\overline{R} = 6 + 2.2 = 8.2$

$LCL_{\bar{X}} = \overline{\overline{X}} - A_2\overline{R} = 6 - 2.2 = 3.8$

$UCL_R = D_4\overline{R} = 2.282(3) = 6.8$

$LCL_R = D_3\overline{R} = 0(3) = 0$



$UCL_{\bar{X}} = 8.2$

$\overline{\overline{X}} = 6.0$

$LCL_{\bar{X}} = 3.8$

$UCL_R = 6.8$

$\overline{R} = 3.0$

$LCL_R = 0$

Example: **Xbar chart Control Limits by σ_{xbar}**

A quality control manager took five samples (S1, S2, S3, S4, S5), each with four observations, of the diameter of shafts manufactured on a lathe machine. The manager computed the mean of each sample and then computed the grand mean. All values are in cm. Use this information to obtain 3-sigma (i.e., z=3) control limits for means of future times. It is known from previous experience that the **standard deviation** σ_x of the process is 0.02 cm.

Observation	S1	S2	S3	S4	S5
1	12.11	12.15	12.09	12.12	12.09
2	12.10	12.12	12.09	12.10	12.14
3	12.11	12.10	12.11	12.08	12.13
4	12.08	12.11	12.15	12.10	12.12
Xbar	12.10	12.12	12.11	12.10	12.12

Example of Control Limits Calculations using $\sigma_{\mathbf{xbar}}$

$$\bar{\bar{x}} = \frac{12.10 + 12.12 + 12.11 + 12.10 + 12.12}{5} = 12.11$$

and $\sigma = 0.02$ (given). Note that sample size $n = 4$.

$$\text{Hence} \quad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{0.02}{\sqrt{4}} = 0.01$$

Upper control limit:

$$\text{UCL} = \bar{\bar{x}} + z\sigma_{\bar{x}} = 12.11 + 3 \times 0.01 = 12.14$$

Lower control limit:

$$\text{LCL} = \bar{\bar{x}} - z\sigma_{\bar{x}} = 12.11 - 3 \times 0.01 = 12.08$$

$$\text{where} \quad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

= Standard deviation of distribution of sample means $\bar{x}$

σ = Process standard deviation and n = Sample size.

Control Limit Factors

Sales Size	Factor for Xbar limits	Factor for R LCL	Factor for R UCL
n	A_2	D_3	D_4
2	1.88	0	3.27
3	1.02	0	2.57
4	0.73	0	2.28
5	0.58	0	2.11
6	0.48	0	2.00
7	0.42	0.08	1.92
8	0.37	0.14	1.86
9	0.43	0.18	1.82
10	0.31	0.22	1.78
11	0.29	0.26	1.74
12	0.27	0.28	1.72
13	0.25	0.31	1.69
14	0.24	0.33	1.67
15	0.22	0.35	1.65
16	0.21	0.36	1.64
17	0.20	0.38	1.62
18	0.19	0.36	1.61
19	0.19	0.40	1.60
20	0.18	0.41	1.59

Xbar Control Limits by **Rbar**

Observation	S1	S2	S3	S4	S5
1	12.11	12.15	12.09	12.12	12.09
2	12.10	12.12	12.09	12.10	12.14
3	12.11	12.10	12.11	12.08	12.13
4	12.08	12.11	12.15	12.10	12.12
Range *R*	0.03	0.05	0.06	0.04	0.05

$$\overline{R} = \text{Average of sample ranges}$$

$$\overline{R} = \frac{0.03 + 0.05 + 0.06 + 0.04 + 0.05}{5} = 0.046$$

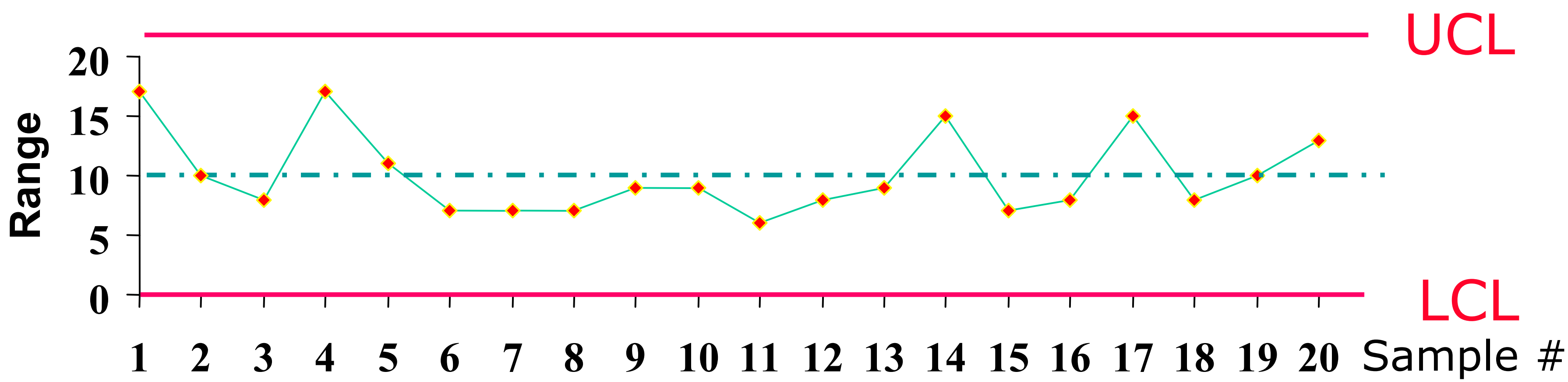
Sample size $n = 4$, therefore $A_2 = 0.73$ from table

Hence, Upper/Lower Control Limits are

$$\text{UCL} = \overline{\overline{x}} + A_2\overline{R} = 12.11 + 0.73 \times 0.046 = 12.14$$

$$\text{LCL} = \overline{\overline{x}} - A_2\overline{R} = 12.11 - 0.73 \times 0.046 = 12.08$$

Range (R) Chart helps control *Precision*



- Average Range R = 10.1 kg
- Standard Deviation of Range = 3.5 kg
- Control Limits: $10.1 \pm 3 \times 3.5 = [20.6, 0]$
  - Process here is “in control” (i.e., precision is stable)

Range Control Chart Control Limits

The control chart used to monitor process dispersion or precision is the R - chart.

Upper control limit, $\mathrm{UCL}_{\mathrm{R}} = \mathrm{D}_4 \overline{R}$

Lower control limit, $\mathrm{LCL}_{\mathrm{R}} = \mathrm{D}_3 \overline{R}$

where D_3 and D_4 are obtained from the Control Limit Factors table.

Example: R chart Limits

Observation	S1	S2	S3	S4	S5
1	12.11	12.15	12.09	12.12	12.09
2	12.10	12.12	12.09	12.10	12.14
3	12.11	12.10	12.11	12.08	12.13
4	12.08	12.11	12.15	12.10	12.12
Xbar	12.10	12.12	12.11	12.10	12.12
Range *R*	0.03	0.05	0.06	0.04	0.05

$\overline{R}$ = Average of sample ranges

$$= \frac{0.03 + 0.05 + 0.06 + 0.04 + 0.05}{5} = 0.046$$

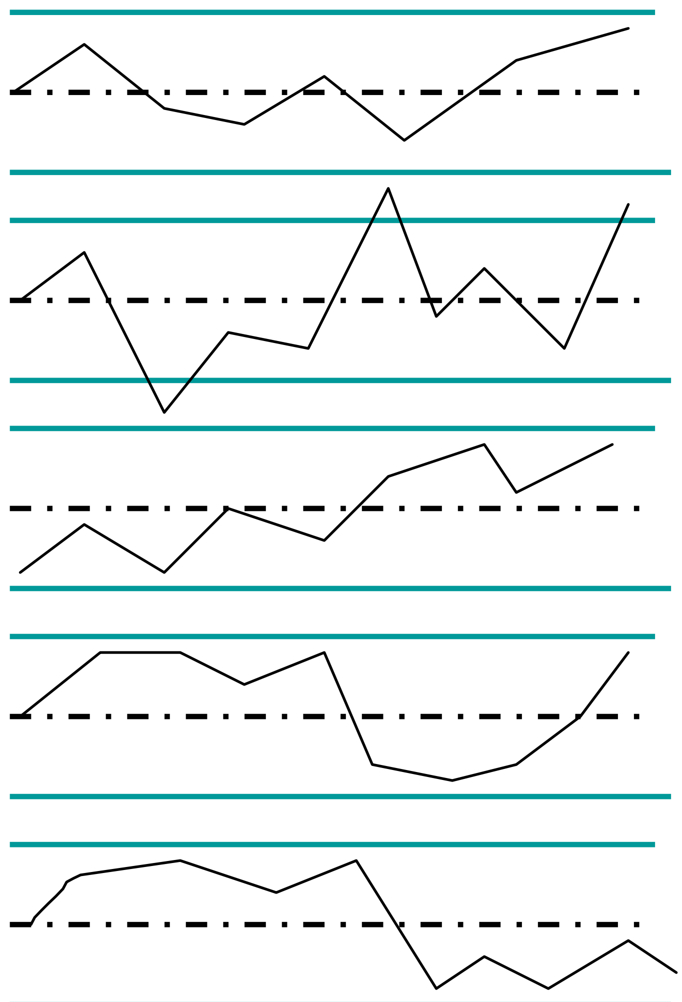
$n = 4$. Therefore $D_3 = 0.00$ and $D_4 = 2.28$ from table.

Hence, *Upper / Lower Control Limits* are

$$\text{UCL}_R = D_4\overline{R} = 2.280 \times .046 = 0.105$$

$$\text{LCL}_R = D_3\overline{R} = 0.000 \times .046 = 0.00$$

Performance Variation Patterns



Stable

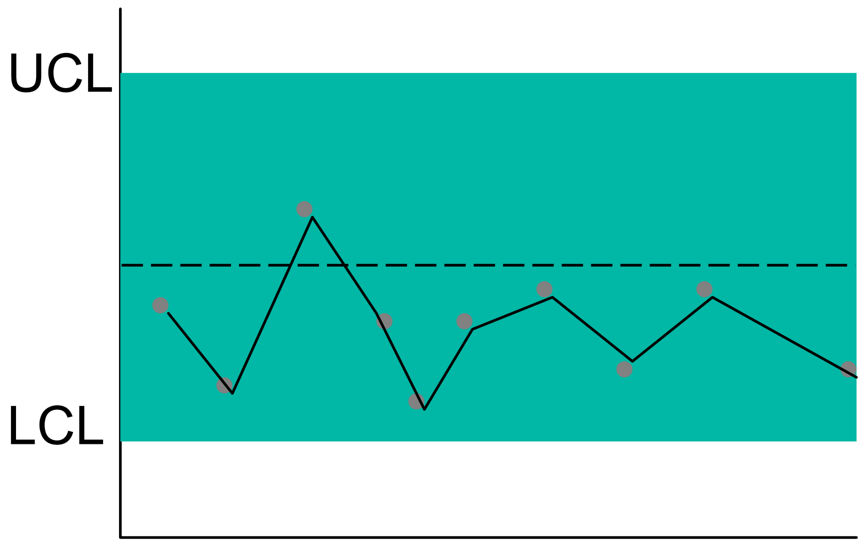
Unstable

Trend

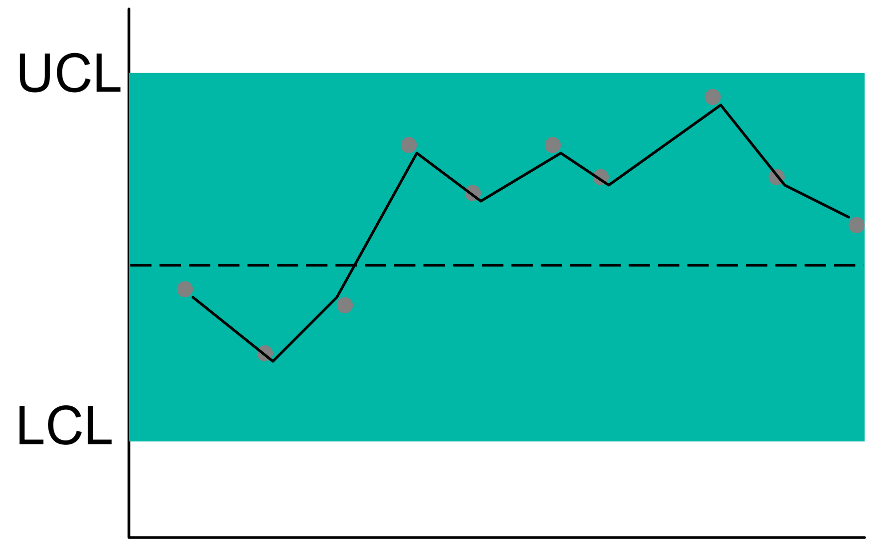
Cyclical

Shift

Abnormal Control Chart Patterns

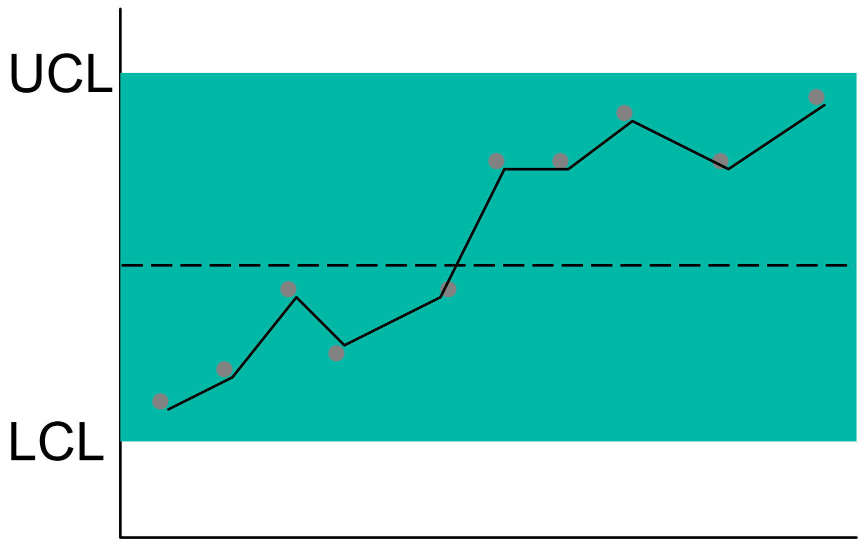


Sample observations consistently below the center line

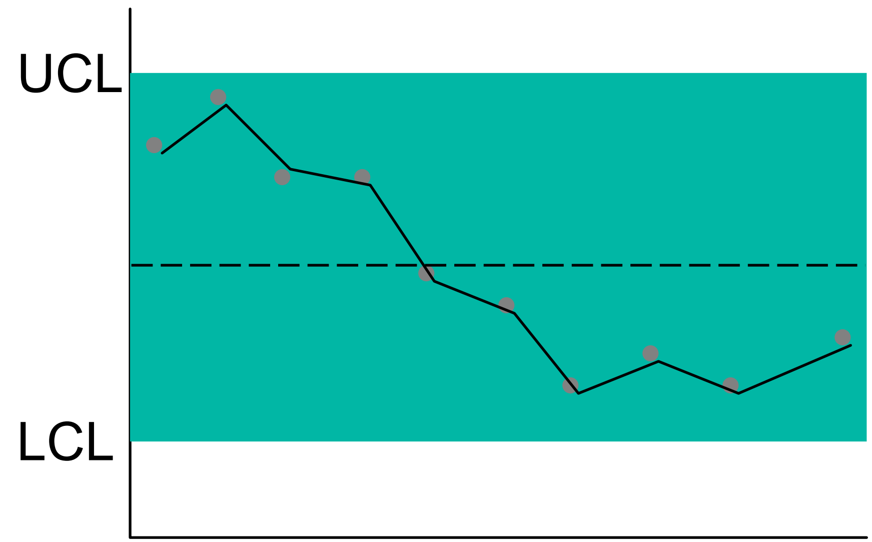


Sample observations consistently above the center line

Abnormal Control Chart Patterns

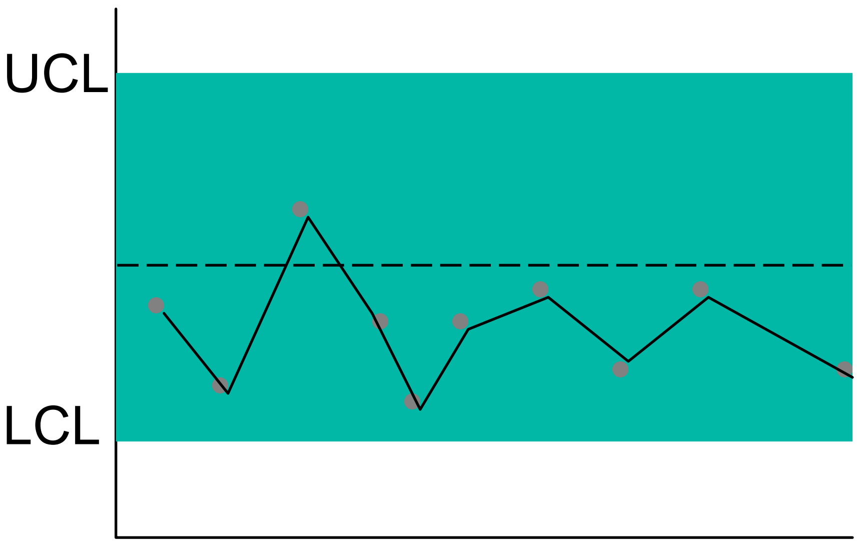


Sample observations consistently increasing

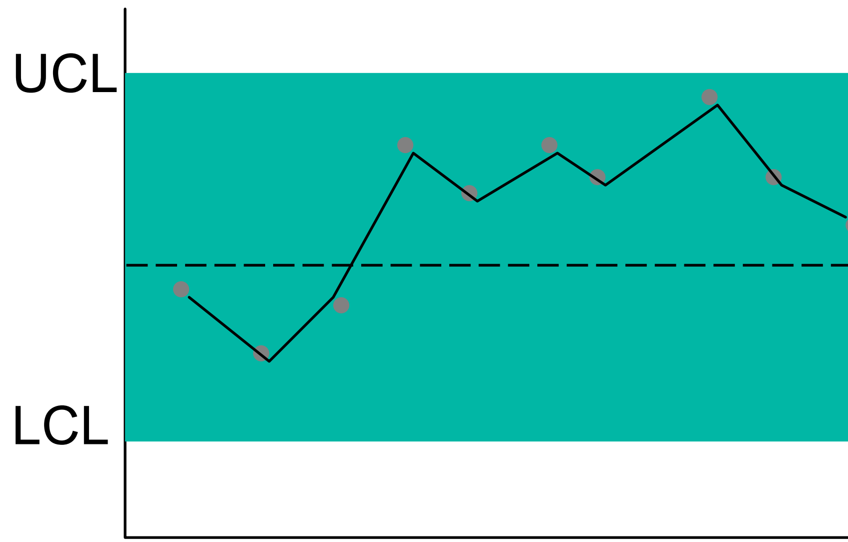


Sample observations consistently decreasing

Abnormal Control Chart Patterns

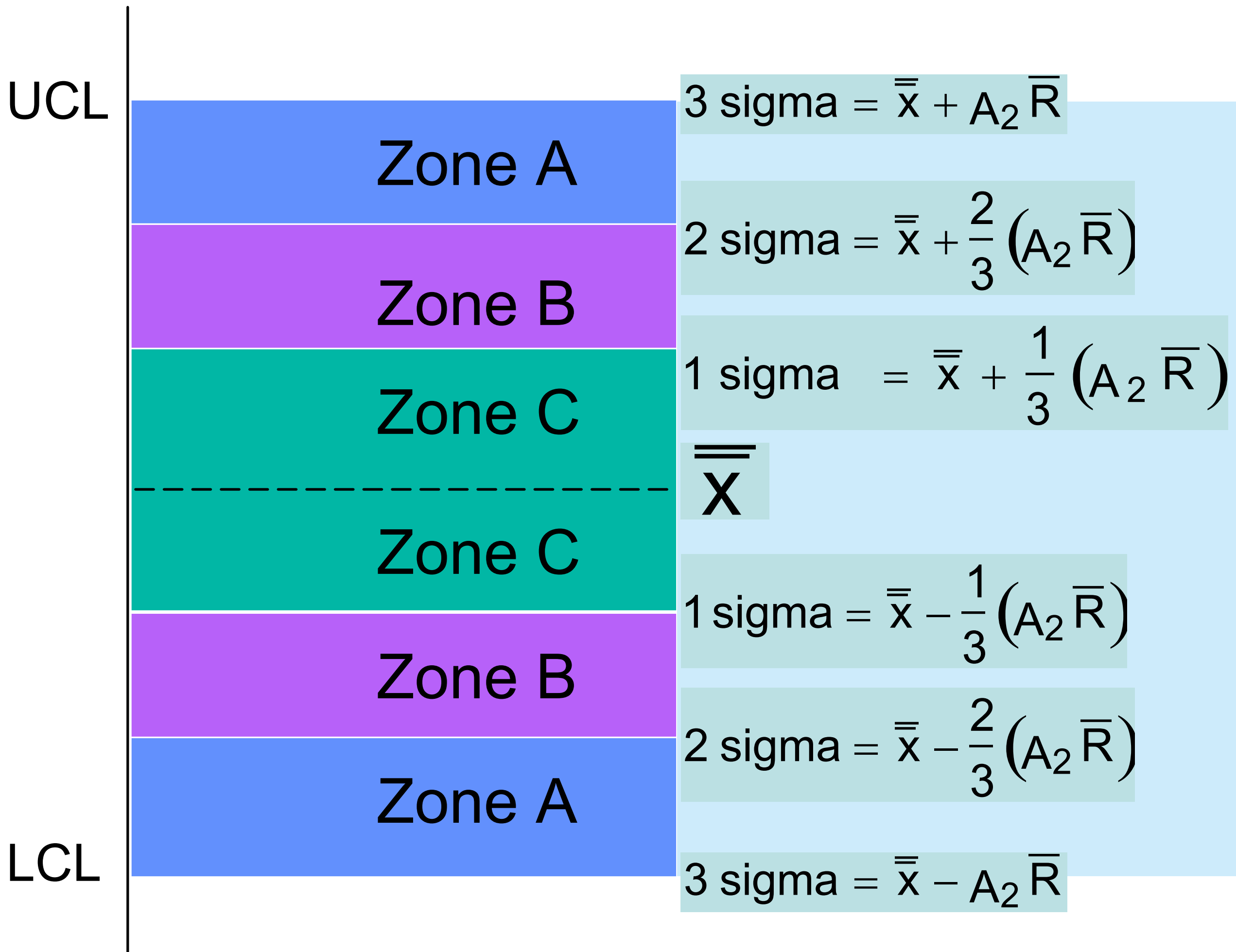


Sample observations consistently below the center line



Sample observations consistently above the center line

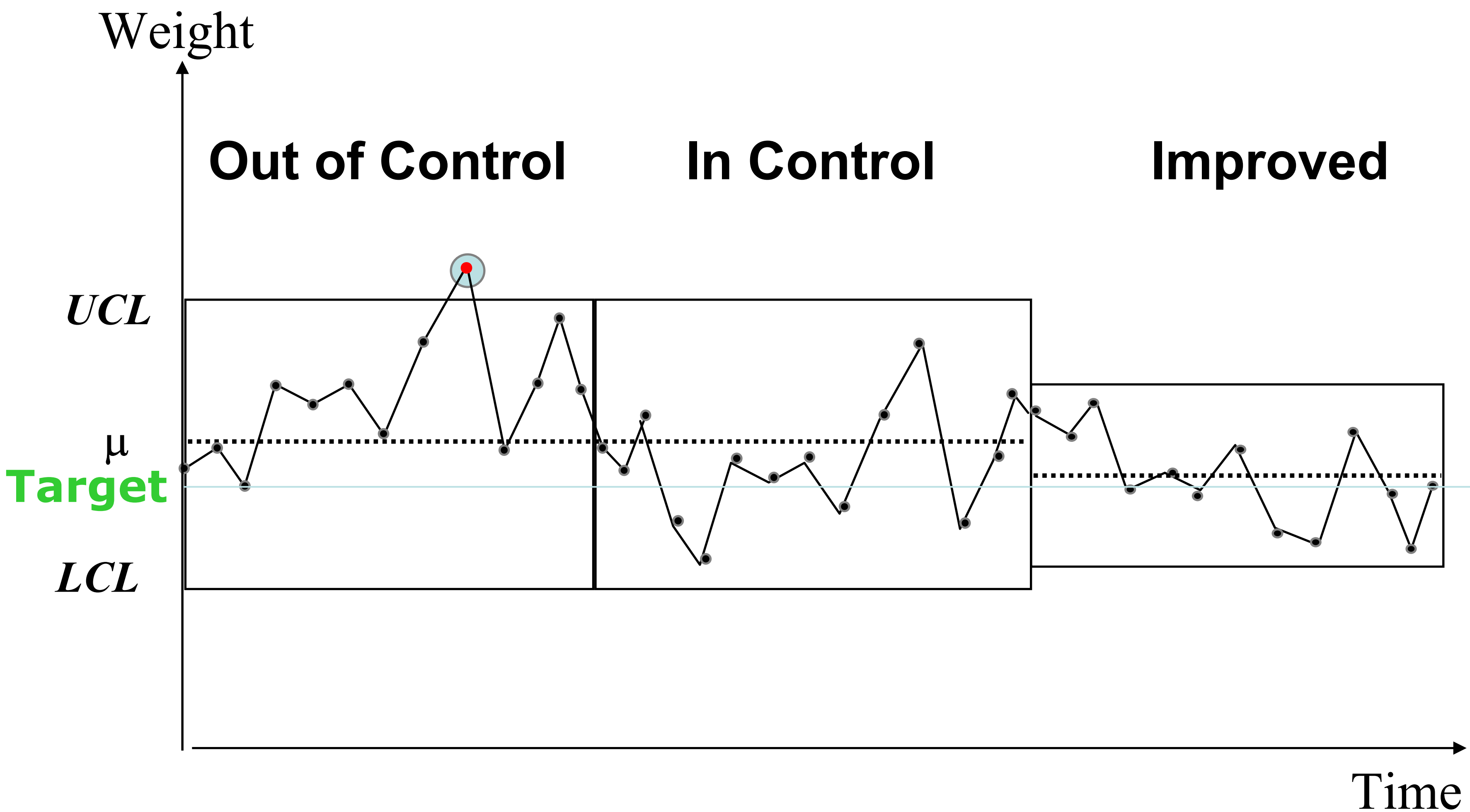
Zones For Non-Random Pattern Tests



Abnormal Control Chart Patterns

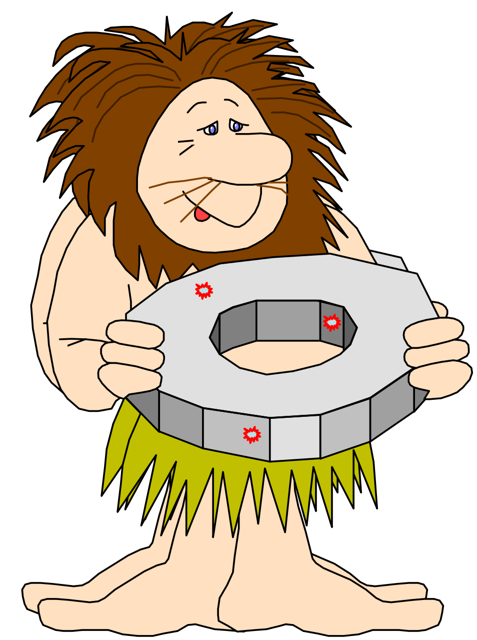
1. 8 consecutive points on one side of the center line.
2. 8 consecutive points up or down across zones.
3. 14 points alternating up or down.
4. 2 out of 3 consecutive points in zone A but still inside the control limits.
5. 4 out of 5 consecutive points in zone A or B.

From Control to Improvement



Defect Control For Attributes

- p Charts
  - –Calculate percent defectives in sample
- c Charts
  - –Count number of defects in item



Use of p-Charts

- When observations can be placed into two categories.
  - Good or bad
  - Pass or fail
  - Operate or don't operate
- When the data consists of multiple samples of several observations each

Control Limits for p–Chart

Control chart for attributes, used to monitor the proportion of defectives in a process.

Upper control limit, $\mathrm{UCL_p} = p + z\,\sigma_p$

Lower control limit, $\mathrm{LCL_p} = p - z\,\sigma_p$

where from Binomial distribution,

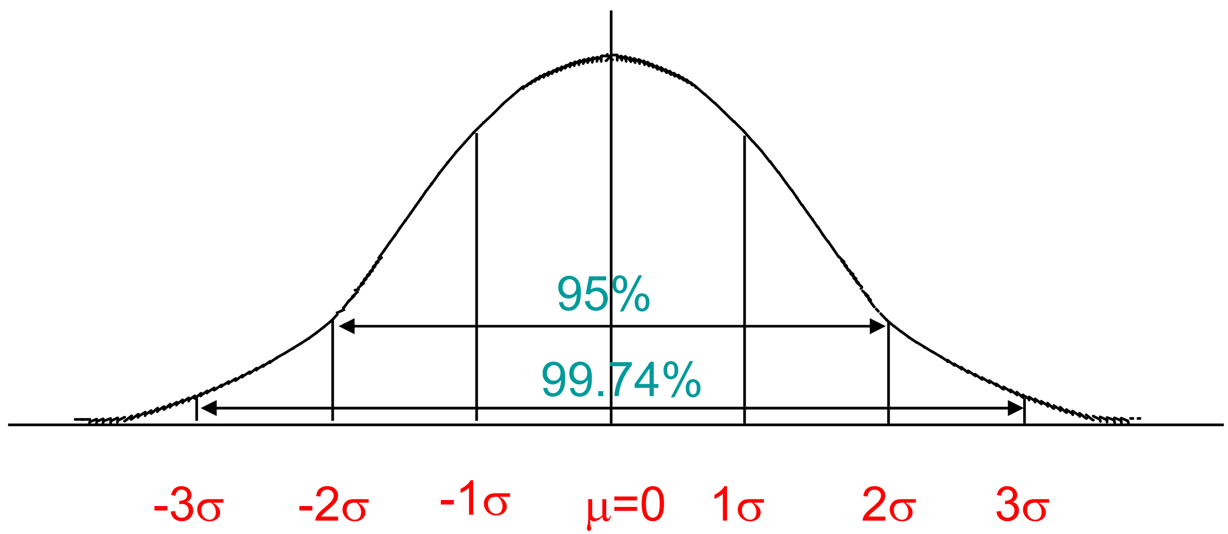
$$\sigma_{\mathrm{p}} = \sqrt{\frac{p(1-p)}{n}}$$

and p is the nominal fraction of defectives in the process.

If p is unknown, it can be estimated as $\overline{p}$ from history. The estimate, $\overline{p}$, replaces p.

Sometimes LCL is negative due to approximate formula. Use $\mathrm{LCL} = 0$.

The Normal Distribution still applies



p Chart Data

		Sample number					
		1	2	3	4	25	Total
Sample size	n	50	50	50	50	50	1250
Number of defective items found in sample	#def	2	4	0	3	2	50
Fraction defective in sample	p	.04	.08	0	.06	.04	1.00

p Chart Calculations

$$\bar{p} = \frac{\sum \#def}{\sum n} = \bar{p}$$

$$3\sigma_P = 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$= 3\sqrt{\frac{.04(.96)}{50}}$$

$$= 0.083$$

$$UCL_P = \bar{p} + 3\sigma_P$$

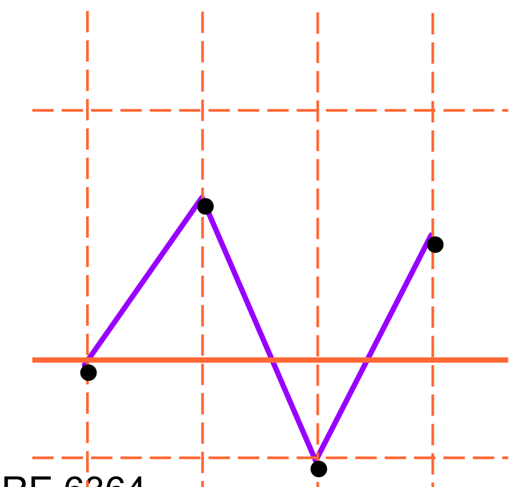
$$= .04 + .083 = .123$$

$$UCL_P = \bar{p} - 3\sigma_P$$

$$= .04 - .083 = 0$$

can't be negative

	Sample number					
	1	2	3	4	25	Total
n	50	50	50	50	50	1250
#def	2	4	0	3	2	50
p	.04	.08	0	.06	.04	1.00



$UCL_P = 0.123$

$\bar{p} = 0.04$

$LCL_P = 0$

Example:
p chart data:

A QC manager counted the **number of defective nuts produced** by an automatic machine in 12 samples. Using the data shown, construct a control chart that will describe 99.74 % of the chance variation in the process when the process in control. Each sample contained 200 nuts.

Sample #	**Number of Defectives**
1	10
2	9
3	8
4	11
5	12
6	8
7	13
8	11
9	9
10	10
11	8
12	11
Total	120

p Chart Solution

$$\overline{p} = \frac{120}{12 \times 200} = 0.05$$

$$\sigma_{\mathrm{p}} = \sqrt{\frac{\overline{p}(1-\overline{p})}{n}} = \sqrt{\frac{0.05(1-0.05)}{200}} = 0.015$$

$$\mathrm{z} = 3$$

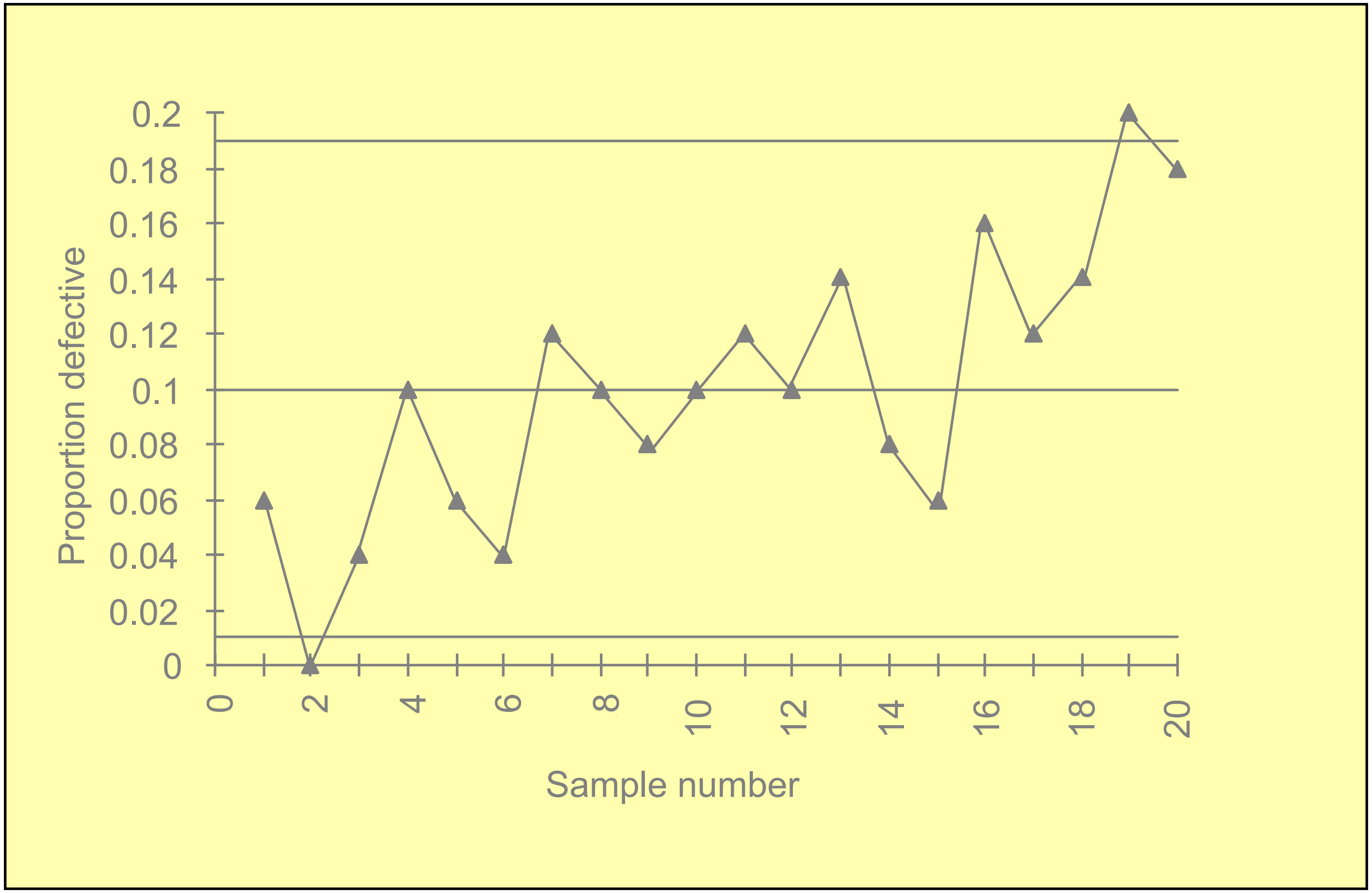
Upper control limit,

$$\mathrm{UCL}_{\mathrm{p}} = \overline{p} + z\,\sigma_{\overline{p}} = 0.05 + 3 \times 0.15 = 0.095$$

Lower control limit,

$$\mathrm{LCL}_{\mathrm{p}} = \overline{p} - z\,\sigma_{\overline{p}} = 0.05 - 3 \times 0.15 = 0.005$$

Example of p-Chart



Number of Defects/Unit: c-Charts

Use only when the number of occurrences per unit of measure can be counted; non-occurrences cannot be counted.

- Scratches, chips, dents, or errors per item
- Cracks or faults per unit of distance
- Breaks or Tears per unit of area
- Bacteria or pollutants per unit of volume
- Calls, complaints, failures per unit of time

c-Chart Controls Defects/Unit

Discrete Quality Measurement:

D = Number of "*defects*" (errors) *per unit of work*

Examples of Defects:

Number of typos/page, errors/thousand transactions, equipment breakdowns/shift, bags lost/thousand flown, power outages/year, customer complaints/month, defects/car...

If n = No. of opportunities for defects to occur, and
p = Probability of a defect/error occurrence in each
then D ~ Binomial (n, p) with mean np, variance np(1-p)
$\cong$ Poisson (np) with mean = variance = np , if
n is large (≥ 20) and p is small (≤ 0.05)

With ***c*** = np = *average number of defects per unit*,

Control limits = $c \pm 3\sqrt{c}$

c-Chart Control Limits

Upper control limit, $\mathrm{UCL}_c = c + z\sqrt{c}$

Lower control limit, $\mathrm{LCL}_c = c - z\sqrt{c}$

where c is the mean and number of defects per unit, and

$\sqrt{c}$ is the standard deviation.

c actually has a Poisson distribution. But for practical reasons the normal distribution approximation to Poisson is used.

c-Chart Example: Hotel Suite Inspection--Defects Discovered/room

Day	Defects	Day	Defects	Day	Defects
1	2	10	4	19	1
2	0	11	2	20	1
3	3	12	1	21	2
4	1	13	2	22	1
5	2	14	3	23	0
6	3	15	1	24	3
7	1	16	3	25	0
8	0	17	2	26	1
9	0	18	0		
				Total	39

Recall c-Chart Limits

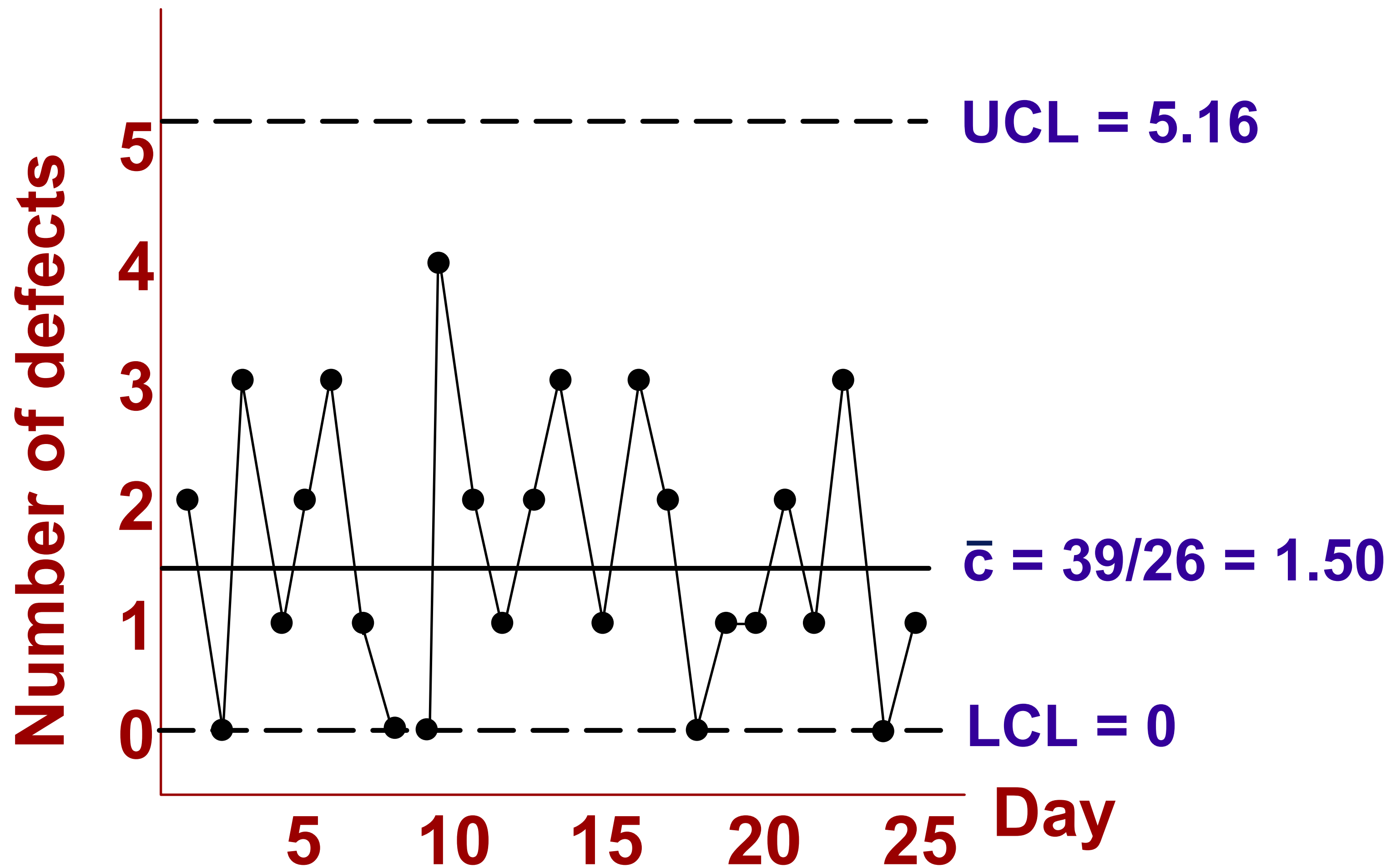
$$\text{Process average} = \bar{c} = \frac{\text{Total \# defects}}{\text{\# samples}}$$

$$\text{Sample standard deviation} = \sigma_c = \sqrt{\bar{c}}$$

$$\text{UCL} = \bar{c} + z\sigma_c$$

$$\text{LCL} = \bar{c} - z\sigma_c$$

c Chart for Hotel Suite Inspection



Example of **c Chart**

A bank manager receives a certain **number of complaints each day** about the bank's service. Complaints for 14 days are given in the table shown. Construct a control chart using three-sigma limits.

Day	Number of complaints
1	3
2	6
3	4
4	5
5	4
6	0
7	2
8	5
9	6
10	0
11	3
12	1
13	0
14	3
Total	42

c Chart Solution

$$c = \frac{42}{14} = 3$$

$$\sqrt{c} = 1.73$$

Upper control limit, $UCL_c = c + z\sqrt{c} = 3 + 3 \times 1.73 = 8.2$

Lower control limit, $LCL_c = c - z\sqrt{c} = 3 - 3 \times 1.73 = 0.0$

where c is the mean and number of defects per unit.

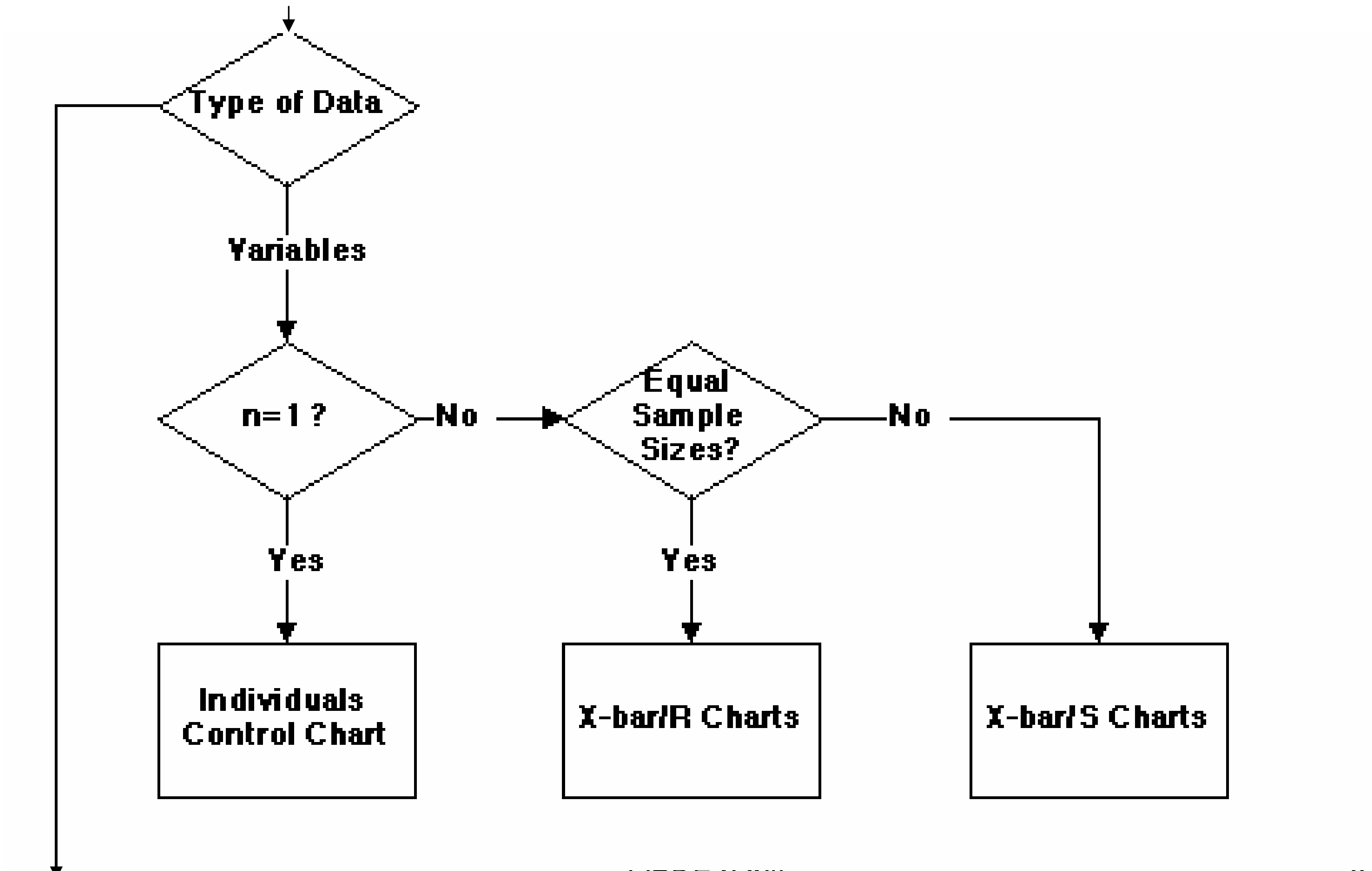
$\sqrt{c}$ is the standard deviation.

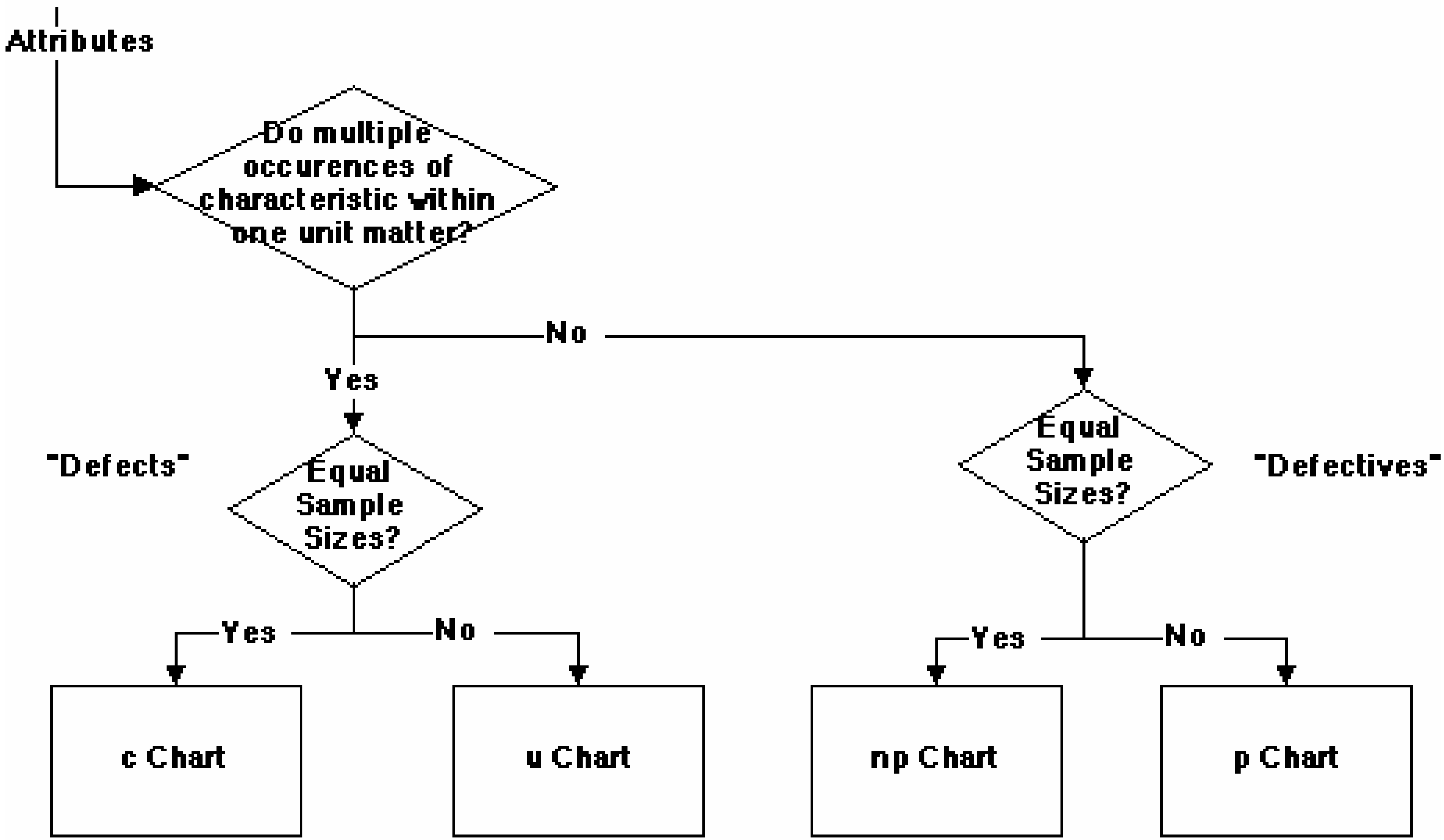
For practical reasons, normal distribution approximation to Poisson is used.

Control Charts Summary

- X-bar and R charts
  - Variables data
  - Application of ***normal distribution*** (by Central Limit Theorem)
- p charts
  - Attributes data (defects per n observations)
  - Application of ***binomial distribution***
- c charts
  - Attributes data (defects per inspection)
  - Application of ***Poisson distribution***

Which Chart to Use?

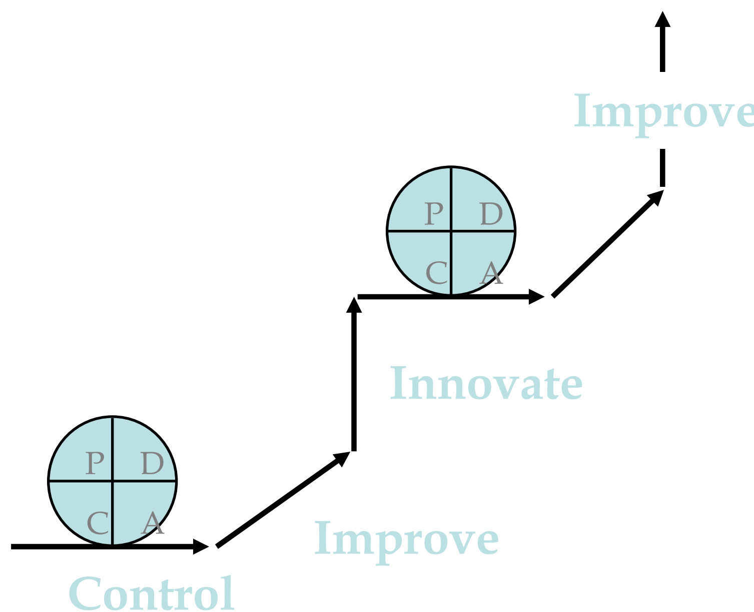



Attributes
Do multiple occurences of characteristic within one unit matter?
No
Yes
"Defects"
Equal Sample Sizes?
Equal Sample Sizes?
"Defectives"
Yes
No
Yes
No
c Chart
u Chart
np Chart
p Chart

Summary of SPC

- Statistical process control provides simple, yet powerful, for managing process while avoiding process tampering
- A process 'in control' (i.e.; exhibiting no special cause variation) is ripe for the next stage--breakthrough process improvement
- A process still burdened with special cause variation is still in the problem-solving stage

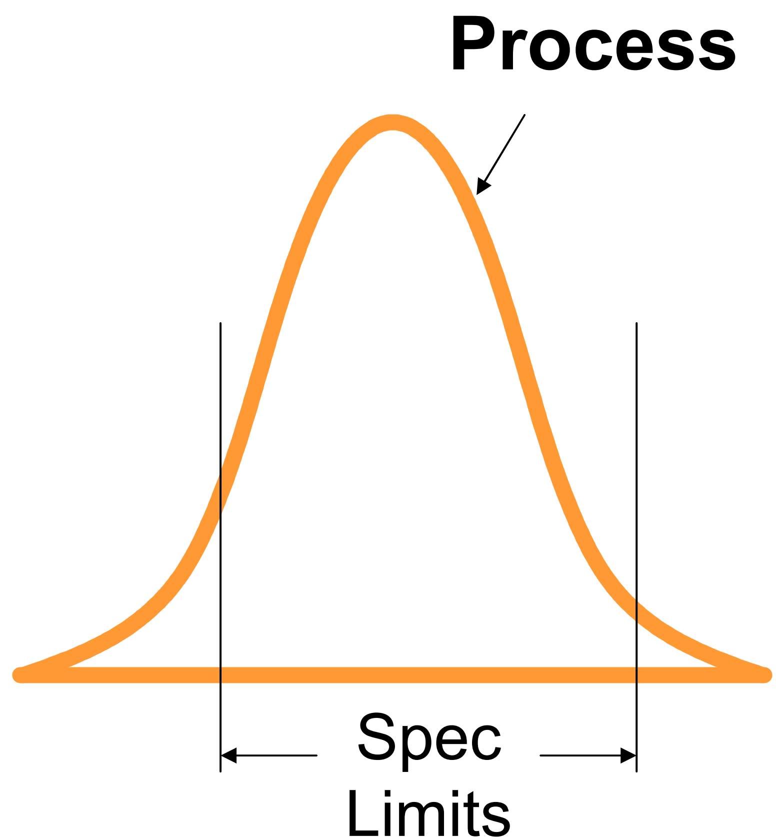
Process Improvement



- Measurement
  - External and Internal
- Analysis
  - Analyze Variation
- Control
  - Adjust Process
- Improvement
  - Reduce Variation
- Innovation
  - Redesign Product/Process

Process capability:

The inherent variability of process output relative to the variation allowed by the design or customer specification



Process Capability Analysis

- Differs *Fundamentally* from Control Charting
  - ✓ Focuses on improvement, not control
  - ✓ <u>Variables</u>, not attributes, data involved
  - ✓ Capability studies address range of ***individual*** outputs
  - ✓ Control charting addresses range of ***sample*** measures
- Assumes Normal Distribution
  - ✓ Remember the Empirical Rule?
  - ✓ Inherent capability (6$\boldsymbol{s_x}$) is compared to ***specifications***
- Requires process first to be ***in Control***

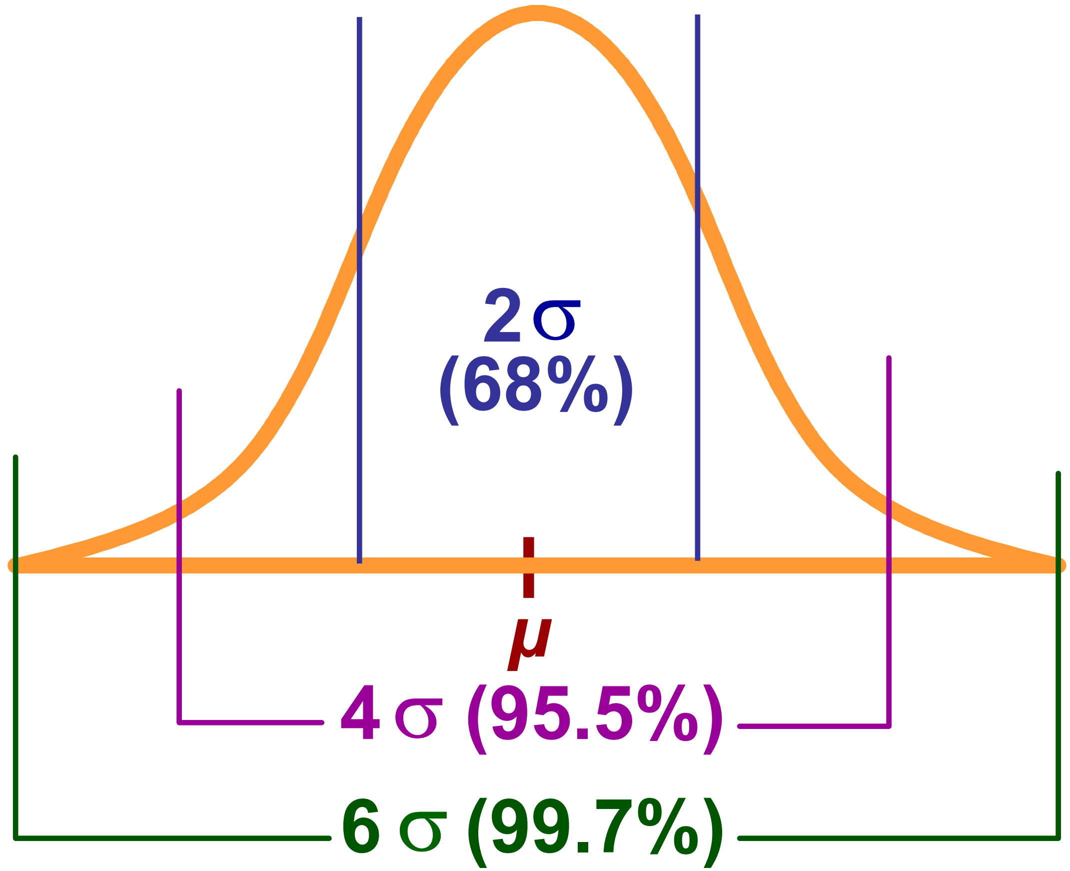
Why measure Process Capability?

Process variability can greatly impact customer satisfaction

Three common terms for variability:

1. **Tolerances:** Specifications for range of acceptable values established by engineering design or customer requirements
2. **Process variability:** Natural or inherent variability in a process
3. **Control limits:** Statistical limits that reflect the inherent variation of sample statistics

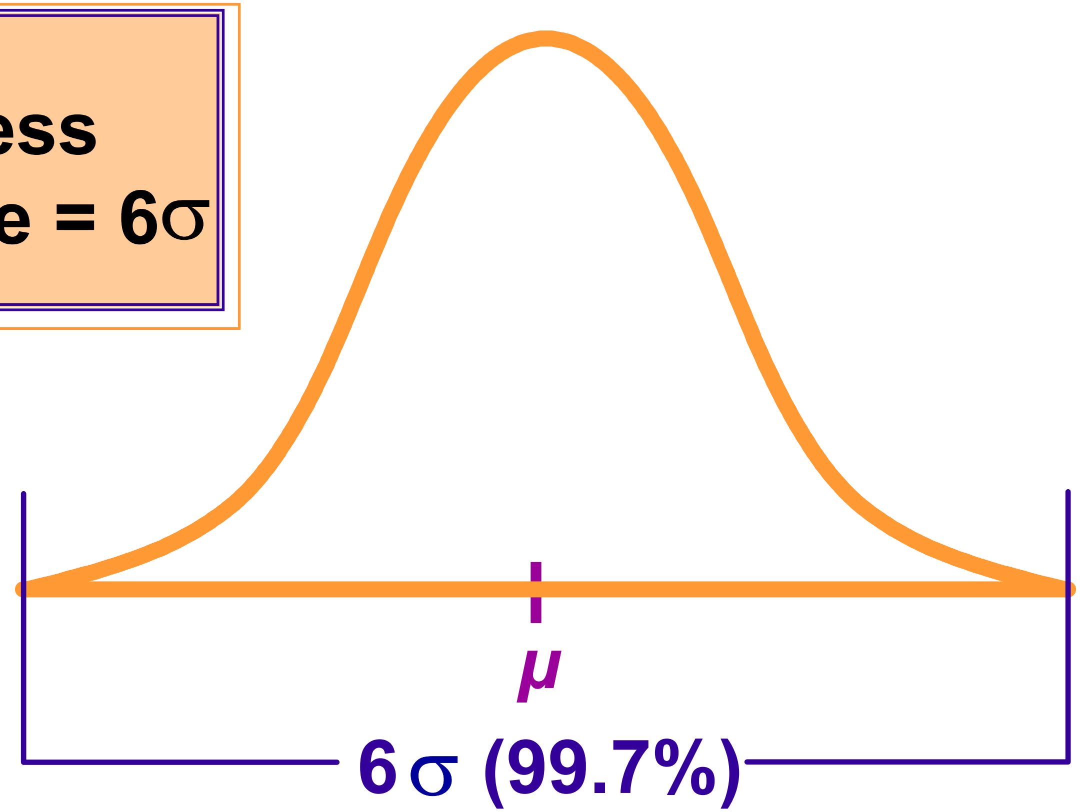
Process Capability is based on Normal Curve



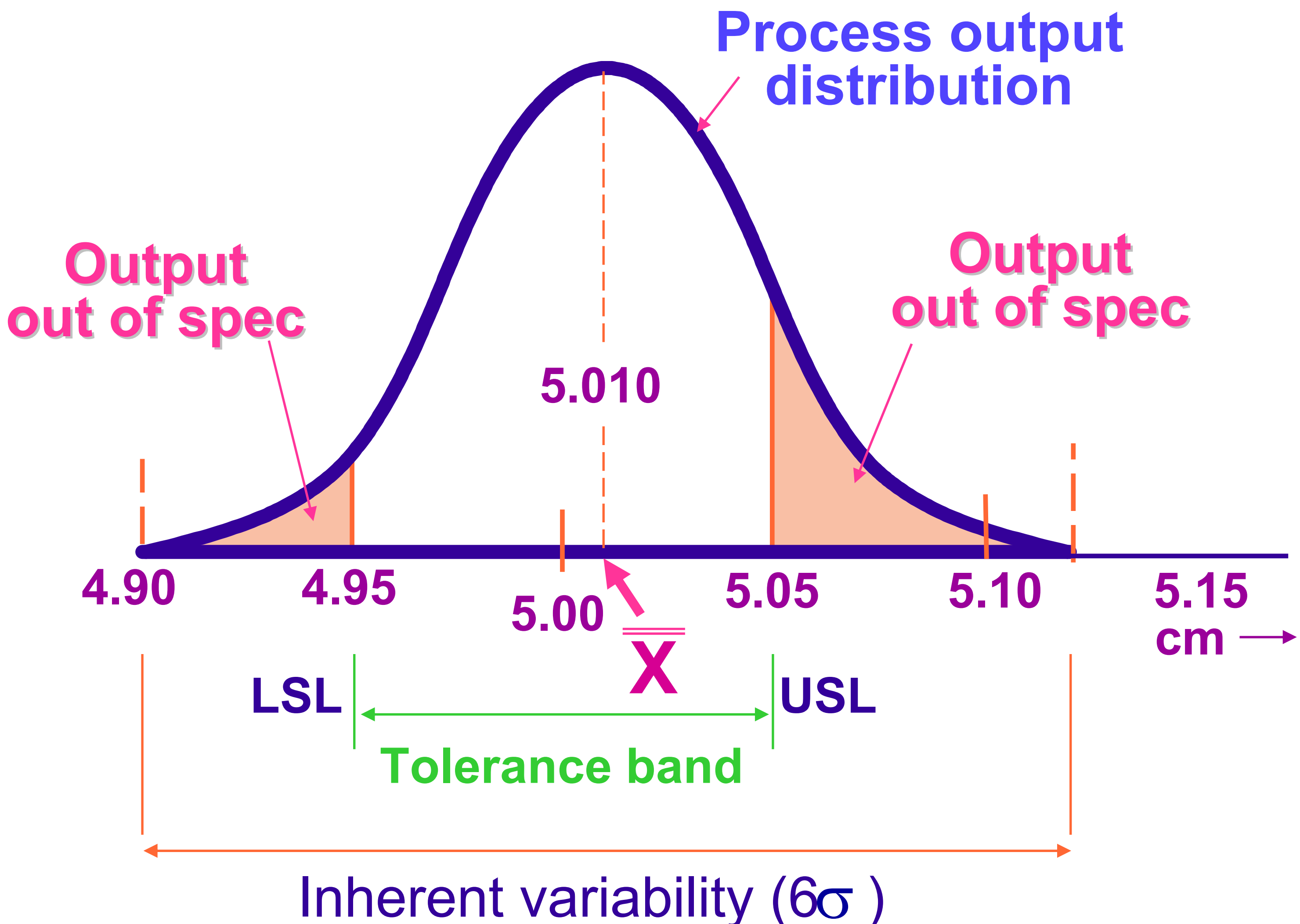
The Range of Process Output

The range in which "all" output can be produced.

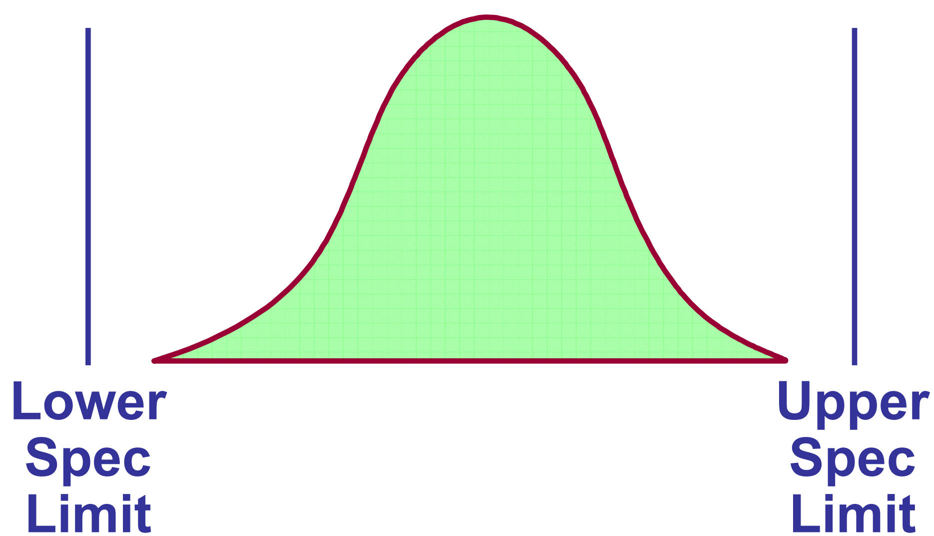
Process range = 6σ



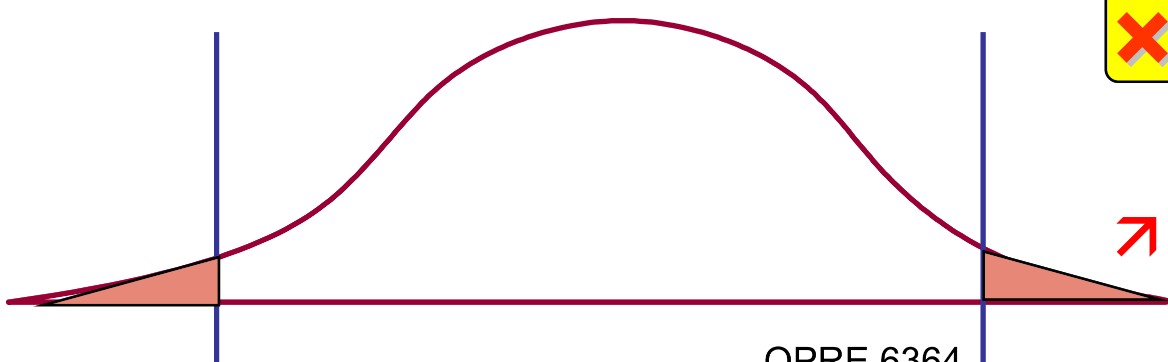
Process Capability Concept



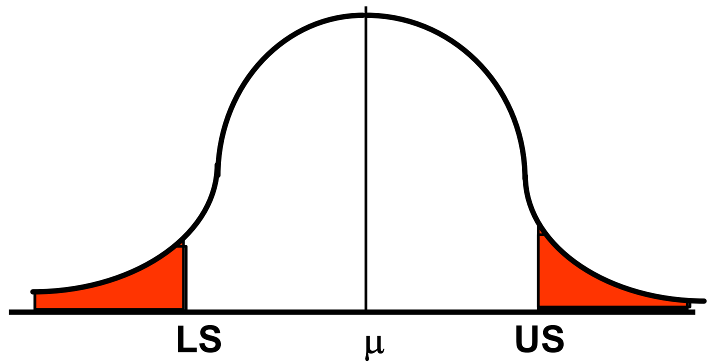
Two Process Capabilities



- This process is **CAPABLE** of producing all good output.
- Control the process.



- This process is **NOT CAPABLE**.
- INSPECT - Sort out the defectives



Capability Analysis

Capability analysis determines whether the inherent variability of the process output falls within the acceptable range of the variability allowed by the design specifications for the process output.

The range of possible solutions:

1. **Redesign** the process so that it can achieve the desired output
2. Use an **alternate process** that can achieve the desired output
3. Retain the current process but attempt to eliminate unacceptable output using **100 percent inspection**
4. Examine the **specification** to see whether they are necessary or could be **relaxed** without adversely affecting customer satisfaction.

Process Capability Ratio C_p

$$\text{Process capability ratio} = C_p$$

$$C_p = \frac{\text{Specification width}}{\text{Process width}}$$

$$C_p = \frac{USL - LSL}{6\sigma}$$

Motorola Corporation uses Six Sigma management.

For Motorola, $C_p = 2$

Process Capability Index C_{pk}

Index C_{pk} compares the spread and location of the process, relative to the specifications.

$$C_{pk} = \text{the smaller of: } \begin{cases} \dfrac{\text{Upper Spec Limit} - \bar{\bar{X}}}{3\sigma} \\ \text{OR} \\ \dfrac{\bar{\bar{X}} - \text{Lower Spec Limit}}{3\sigma} \end{cases}$$

Alternate Form

$$C_{pk} = \frac{Z_{min}}{3}$$

Where Z_{min} is the smaller of:

$$\begin{cases} \dfrac{\text{Upper Spec Limit} - \bar{\bar{X}}}{\sigma} \\ \text{OR} \\ \dfrac{\bar{\bar{X}} - \text{Lower Spec Limit}}{\sigma} \end{cases}$$

Process Capability Ratio C_{pk}

Normal distribution => 99.73% of output falls in ($\mu \pm 3\sigma$) when the process is centered. If the process is <u>not centered</u>, we use

$$C_{pk} = \text{Min}\,[(US - \mu) / 3\sigma,\ (\mu - LS) / 3\sigma]$$

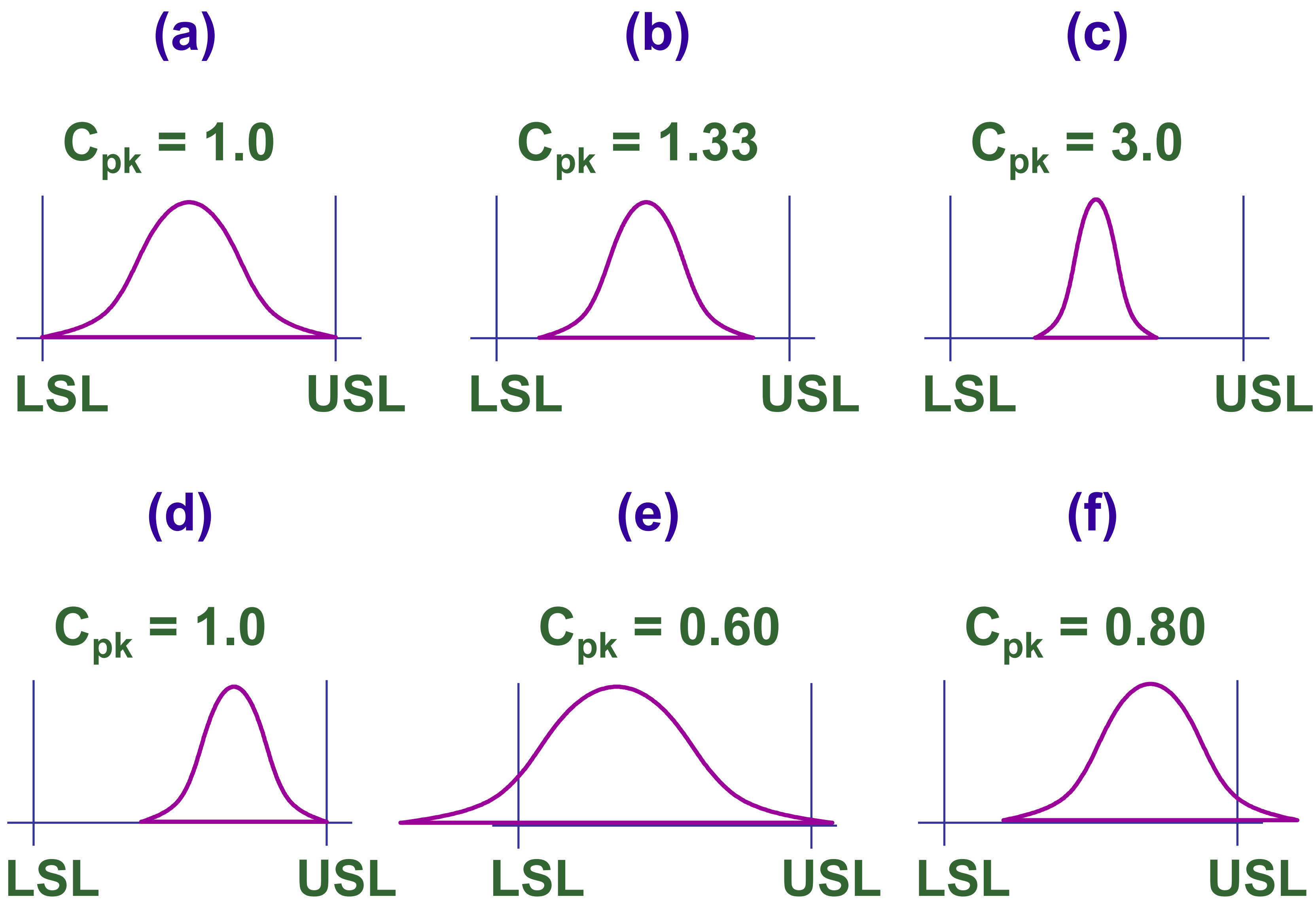
<u>Example</u>. MBPF: $C_{pk} = \text{Min}[0.1894, 0.5952] = 0.1984$

With centered process $(US - \mu) = (\mu - LS)$. Then

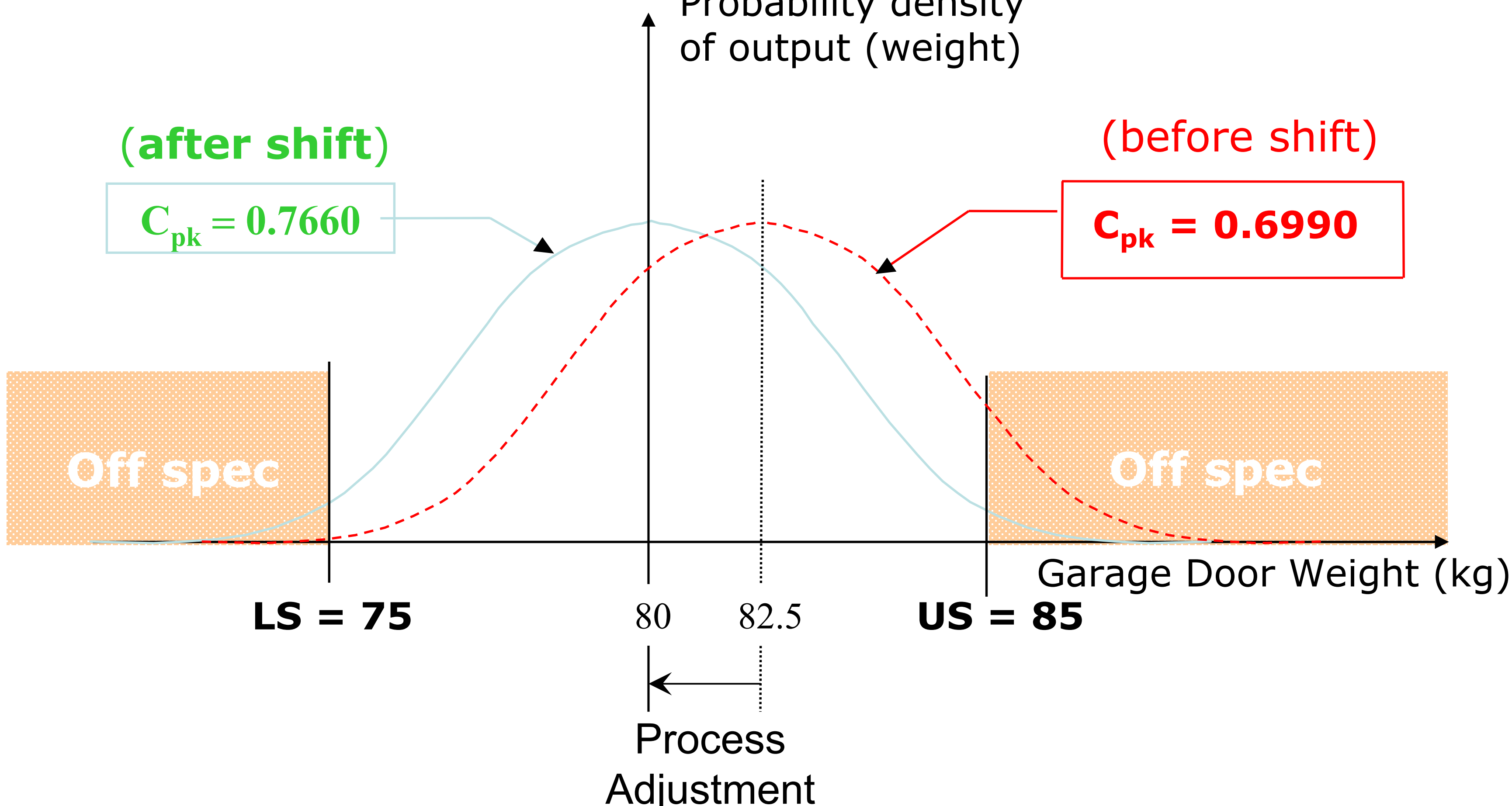
$$C_{pk} = C_p = (US - LS)/ 6\sigma = \frac{\text{Voice of the Customer}}{\text{Voice of the Process}} = 0.3968$$

C_p = 0.86	1	1.1	1.3	1.47	1.63	2.0
Defects/m = 10K	3K	1K	100	10	1ppm	2 ppm

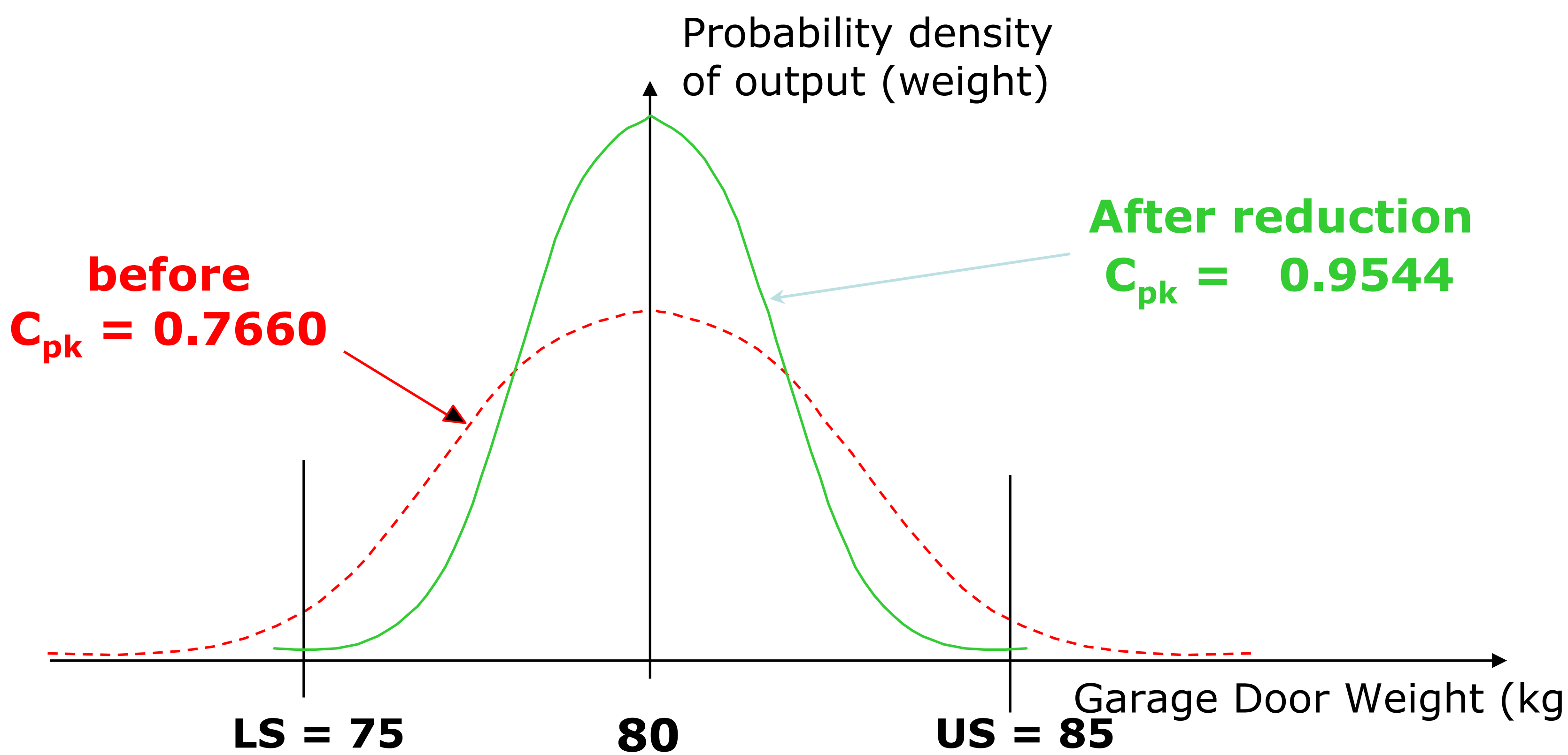
Process Capability: C_{pk} examples



Capability Improvement by *Mean Shift*



Capability Improvement by *Variance Reduction*



Process Control and Capability: Review

- Every process displays some variability—normal or abnormal
- Control charts can identify abnormal variability
- Control charts may give false (or missed) alarms by mistaking normal (abnormal) for abnormal (normal) variability
- On-line control leads to early detection and correction
- A process “in control” indicates only its internal stability
- Improving process capability involves changing the mean and/or reducing normal variability requiring a long term investment

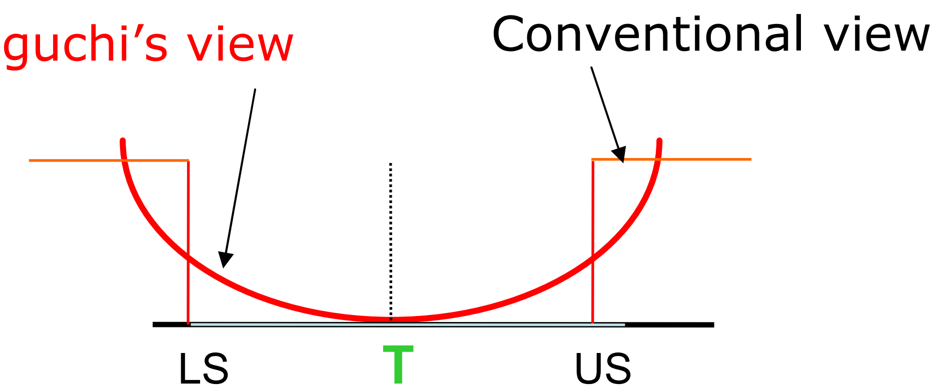
Design for Capable Processing

- Simplify
  - Fewer parts, steps
  - Modular design
- Standardize
  - Less variety
  - Standard, proven parts, and procedures
- Mistake-proof
  - Clear specs
  - Ease of assembly, disassembly, servicing

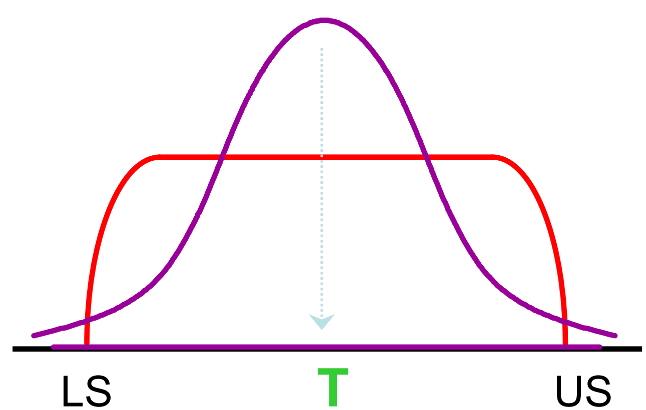
Taguchi Quality Philosophy

Loss = $k(P - T)^2$

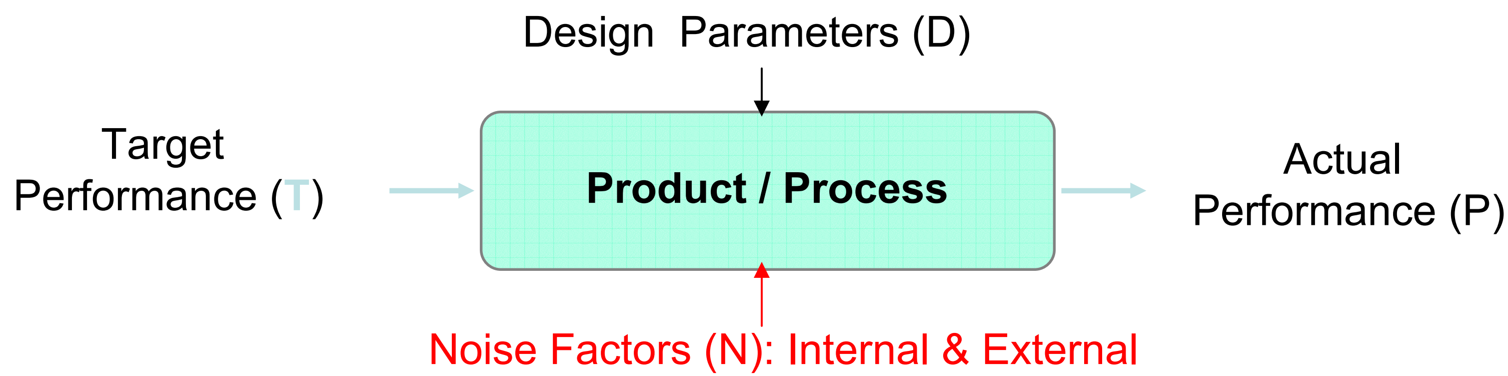
not 0 if within specs
and 1 if outside



On Target
is more important than
Within Specs



Robust Design



- Identify Product/Process Design Parameters that
  - Have significant / little influence on Performance
  - Minimize performance variation due to Noise factors
  - Minimize the processing cost
- Methodology: Design of Experiments (DOE)
- Examples - Chocolate mix, Ina Tile Co., Sony TV

The Design Process

- Goal
  - Develop high quality, low cost products, fast
- Importance
  - 80% product cost, 70% quality, 65% success
- Conventional
  - Technology-driven, Isolated, Sequential, Iterative
- Difficulties
  - Revisions, cost overruns, delays, returns, recalls
- Solution
  - Customer-driven (QFD), jointly planned, producible

Concurrent Design

- Objective
  - Interfunctional coordination to satisfy customer
  - Involve manufacturing, suppliers, R&D
- Prerequisites
  - Break down barriers
  - Cross functional training
  - Communication, teamwork, group decisions
- Result
  - Fewer revisions, miscommunication, delays
- Difficulties
  - Time consuming, complex, organizational

References

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Control chart case study

http://www.qualityamerica.com/knowledgecente/knowctrSPC_Articles.htm

SPC articles