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Time series forecasting models: A comparative study of some Models with application to inflation data

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Abstract

This study examined and compared six basic time series forecasting models (Exponential model, Double Exponential model, Holt-Winter models, Time Series linear regression model, the ad-hoc Bootstrapping model and the Self Adjusting model) with application to twenty-four Months Nigeria's CPI inflation sample data, from January 2009 to December 2010 inflation data. With the aids of five different standard forecasting accuracy measures (MSE, MAE, RMSE, SSE, and MAPE), results from the out-of-sample forecasts shows that the double exponential model with a smoothening constant of 0.68 is the best forecasting model for the Nigeria inflation rate data among the other ad-hoc model considered.

Keywords

Forecast, Error, Exponential, Smoothening, Constant, Inflation

1. Introduction

Time series data forecast has been a research issue for a very long time now [*1*, *2*, *3*]. Plenty of methods from different research fields have been proposed for forecasting. Once the data for Time series to be forecasted have been captured, the analyst's next step is to select a model for forecasting future values. Various statistical and graphic techniques may be useful to the analyst in the selection processes. But the choice of methods and models to use on any time series data depends on factors such as simplicity, accuracy and stability on the time series data. Therefore, the need for quick, reliable, simple and medium term forecast of various time series data cannot be over-emphasized. Models such as exponential models, Holt-Winter models, linear regression etc provide a quick simple and comprehensive solution to forecasting time series data.

Many research works like [*4*], [*5*], [*6*] etc have compared few ad-hoc model with different forecasting accuracy but not as extensive as it has been done in this study. As a majority of recent research works concentrate on the classical Box-Jenkins model, on the excuse that the ad-hoc model lacks of objective statistical identification and diagnostic system for evaluating the "goodness" of models. But contrarily, carefully chosen ad-hoc model do almost as well as the Box-Jenkins classical model as documented in two large-scale empirical studies by Makridakis in [*7*] and [*8*]

However, George W. Wilson [*9*], defines inflation as "too much money chasing too few goods". A more thorough and acceptable definition would be that inflation is —a persistent increase in the level of consumer prices or a persistent decline in the purchasing power of money, caused by an increase in available currency and credit beyond the proportion of available goods and services, The American Heritage Dictionary, [*10*]. Whatever the definition however the results are the same, inflation means that over time a consumer or company can purchase less with their money. Over time the cost of goods or services will raise meaning that the purchasing power of the consumer or company will be decreased.

There are several ways of measuring inflation, and one of the ways is by using the Consumer Price Index (CPI) which looks at the price changes of a common consumer goods and services (i.e. food, clothing, gasoline). The Central Bank of Nigeria (CBN) measures the CPI Indicator by calculating and publishing the monthly CPI values. The CPI assumes that the same quantity of product is bought by the consumer year over year, this ignores the fact that as prices increase consumer preference may change, for example if the price of a bag of Rice increase dramatically the consumer may begin to buy bag of "Garri" instead meaning that their food costs might not be as high.

The formulation of exponential smoothening forecast model arose in the 1950's from the original work of Brown [*11*], [*12*] and Holt [*13*], who were working on creating forecasting model for inventory control system. The main idea is to construct forecasts of future values as weighted average of past observation with the more recent observation carrying more weight in determining forecasts than observations in the more distant past. The weight diminishes with time but they do so in exponential way as,

$$\beta_j = \beta^j \tag{1}$$

where $-1 < \beta < 1$ and $j = 1, 2, 3 \dots\dots\dots$ represent the specific period in the past.

The Holt-Winter method is a member of the smoothening method family [*14*], where a second parameter is introduced to indicate the period of the seasonal component. It is observed that the Holt-Winter and the double exponential parameter estimates are recursively obtained while the linear trend regression model have their parameter obtained via least square method.

Witt and Witt [*15*] found that accuracy is the most important evaluation criterion, and because of their clear definitions, the Mean Absolute Percentage Error (MAPE), Root Mean Square Error (RMSE), Mean absolute Error (MAE), Mean Square Error (MSE), Sum of Square Error (SSE), and the Variance (VAR) was selected for this study.

The organization of this article is as follows. Section 2 describes the data used in this study as well as details methodology of the analysis, Section 3 presents empirical results and brief discussion of the results obtained, and Section 4 concludes the study.

2. Materials and Methods

The six basic forecasting models considered in this study were compared with the application of the monthly CPI inflation food rate data with a weight of *507.03* taken from the Central Bank of Nigeria (CBN) statistical Bulletin, ranging from January *2009* to December *2010*. This sample were divided into two parts; the in-sample data set (50% of the first part of the available time series data) and with the remaining part of the time series assigned to the out-of-sample data set. Then the competing models are "ran" through the out-of-sample data while forecasting h-steps ahead each time (h is the forecast horizon of interest) while updating the smoothening parameter(s) as one moves through the out-of-sample data. In the process of generating these h-step-ahead forecasts for the competing models, the comparison of the competing forecasts with the actual values that withheld as we generate our forecast and then use the standard forecasting accuracy measure like MSE, MAE, RMSE, MAPE to choose the best (most accurate) smoothening forecasting model as indicated by the out-of-sample forecasting experiment for further use. The smoothening procedure for the single exponential model given as;

$$Z_t = a + \varepsilon_t \tag{2}$$

Where a represents the constant, ε_t stand for the residuals. To forecast the $t+1$ moment in the moment t, the series was computed recursively;

$$\hat{Z}_{t+1} = \alpha Z_t + (1-\alpha)\hat{Z}_t \qquad where\ t = 1, \overline{x+h} \tag{3}$$

Where the number of available observation shown by x, h is the time horizon for which the forecast is made and α is the smoothing factor which can take value between *0* and *1*. The value of α is usually determined by minimizing the sum of square of the forecast error;

$$\frac{1}{t}\sum_{i=0}^{t-1}(Z_t - \hat{Z}_{t+1})^2 = \frac{1}{t}\sum_{i=0}^{t-1} e_{t+1}^2 \tag{4}$$

In (*3*), the relationship is applied recursively for each observation from the series. Each new smoothed value $\hat{Z}_{t+1}$ is computed as the weighted average of the current observation Z_t and the previous smoothed observation $\hat{Z}_t$. Thus each smoothed value $\hat{Z}_{t+1}$ is the weighted average of the previous t observation. (*3*) can be written as;

$$\hat{Z}_{t+1} = \alpha \textstyle\sum_{i=1}^{t}(1-\alpha)^i\ \hat{Z}_{t+1-s} \tag{5}$$

The initial value of $\hat{Z}_1$ is usually equal to Z or the average of the initial value of the series. However, for the double exponential smoothening model, we applied two equations recursively for the Z_t namely;

$$S_t = \alpha Z_t + (1-\alpha)\, S_{t-1} \tag{6}$$

$$= \alpha D_t + (1-\alpha)\, D_{t-1} \tag{7}$$

Where S_t is the single smoothed series, D_t, the double smoothed series and the α stands for the smoothing parameter, between $0 < \alpha \leq 1$. The forecast from the double smoothing are computed as;

$$\hat{Z}_{t+h} = \left(2 + \frac{\alpha h}{1-\alpha}\right)S_t - \left(1 + \frac{\alpha h}{1-\alpha}\right)D_t = \left\{2S_t - D_t + \frac{\alpha}{1-\alpha}(S_t - D_t)h\right\} \tag{8}$$

$2S_t - D_t$ and $\frac{\alpha}{1-\alpha}(S_t - D_t)$ in equation (*8*) are the intercept and slope respectively . The initial values for S_1 and D_1 are usually set to be equal with Z_1 or with the average of the initial values of the series.

In Holt-Winter model, the forecast series is given as;

$$\hat{Z}_{t+h} = \hat{a} + \hat{b}h \quad (9)$$

Where a and b are the intercept and slope which are computed recursively;

$$a_t = \alpha Z_t + (1-\alpha)(a_{t-1} + b_{t-1}) \quad (10)$$

$$b_t = \beta(a_t - a_{t-1}) + (1+\beta)b_{t-1} \quad (11)$$

Where α and β are smoothening factor which can be found within the interval $\alpha, \beta \in [0,1]$ being determined by minimizing the sum of squares of the forecast errors. The initial value of a_1 is usually is usually Z_1 while b is set to be equal with the average of the initial value of the series or with the difference of the initial observation.

Assuming a single observation and smoothed value of December *2009* is available, this study also considered such forecast (known as Bootstrapping forecast) given as;

$$S_{t+1} = \alpha Z_{Origin} + (1-\alpha)S_t \quad (12)$$

Where Z_{Origin} value remains constant. In the linear regression model, the line period is taken as 1-month and coded as x value under each month since data is time series data. Thus, the origin set as January *2009*, $x = 0$ and February, $x = 1$.

The theoretical framework of the n-dependent self-adjusting model as described by [*16*], has a smoothing parameter E_t, and Z_i the value of the observation for period i, then;

$$E_1 = Z_1 \quad (13)$$

And the Nth period given as,

$$E_N = \frac{Z_1 + 2Z_2 + 3Z_3 + \cdots + NZ_N}{1+2+3+\cdots+N} = \frac{\sum_{t=1}^{N} tZ_t}{\sum_{t=1}^{N} t}, N \geq 3 \quad (14)$$

Therefore,

$$E_N = \frac{2}{N(N+1)} \sum_{t=1}^{N} tZ_t = \frac{2}{N(N+1)} \left[\sum_{t=1}^{N-1} tZ_t + NZ_N \right] \quad (15)$$

$$E_N = (1-\omega_N)E_{N-1} + \omega_N Z_N \quad (16)$$

where, $\sum_{t=1}^{N} t = \frac{N(N+1)}{2}$, $E_{N-1} = \frac{2}{N(N+1)} \sum_{t=1}^{N-1} tZ_t$ and $\omega_N = \frac{2}{N+1}$

ω_N is the variable weighted constant which adjusts itself throughout a particular problem depending on the number of observation N

However, the forecast for h period is given as;

$$F_{N+h} = E_n + hT_n \quad (17)$$

$$T_i = Z_i = d_i \quad (18)$$

$$T_N = \frac{d_1 + 2d_2 + 3d_3 + \cdots + Nd_N}{1+2+3+\cdots+N} = \frac{2}{N(N+1)} \sum_{t=1}^{N} td_t \quad (19)$$

Where T_n is the trend, h the forecast horizon, and E_n the smoothing parameter.

To demonstrate how well the different forecasting method performs, the following shows the measures of accuracy which have been considered in this study.

$$Variance\ of\ forecast\ (Var) = \frac{\sum_{i=2}^{t}(t-\bar{t})^2}{t} \quad (20)$$

$$Mean\ Square\ Error\ forecast\ (MSE) = \frac{\sum_{i=2}^{t} e_i^2}{t} \quad (21)$$

$$Mean\ Absolute\ Error\ forecast\ (MAE) = \frac{\sum_{i=2}^{t} |e_i|^2}{t} \quad (22)$$

$$Mean\ Absolute\ Percentage\ Error\ forecast\ (MAPE) = 100X \frac{\sum_{i=2}^{t} \frac{|e_i|}{d_i}}{t} \% \quad (23)$$

$$Root\ Mean\ Square\ Error\ forecast\ (RMSE) = \sqrt{\frac{\sum_{i=2}^{t} e_i^2}{t}} \quad (24)$$

$$Sum\ of\ Square\ Error\ forecast\ (SSE) = \sum_{i=2}^{t} e_i^2$$

Where, d_i is the the original data point at time *i,* and e_i the forecasting error

3. Results and Discussion

The computed out-of-sample forecasts and errors for the Nigeria CPI inflation data (Jan. *2010* – Dec. *2010*) was obtained from the six basic ad-hoc time series models (Simple Exponential Smoothing (SES), Double Exponential Smoothing (DES), Holt-Winter model Simple Exponential Smoothing (HWSES), Time series linear Regression model (TLR), ad-hoc Bootstraping model (BM) and the Self-Adjusting model (SAM)) are shown in table 1. The choice of the smoothing constants ($\alpha = 0.880$ for the SEM, $\alpha = 0.680$ for DES, $\alpha = 0.680$, $\gamma = 0.680$ for HWSES, $\alpha = 0.300$ for BM, $\alpha = 95.830, \beta = 1.220$ for LTR) best satisfy the condition and criteria of forecast [17]. The Statistics of errors forecasts (VAR, MSE, MAE, MAPE, RMSE and SSE) comparison (see Table 2) clearly revealed the behavior of the six different models in forecasting the Nigeria's food CPI inflation rate. As observed in Table 1, the Time series linear trend regression (LTR), Bootstrapping method (BM) and the self-Adjusting model (SAM) are inappropriate and unstable models as more and more forecasts of Time series are made. This observation is in line with the findings of [18]. However, the Simple Exponential Smoothing (SES), Double

Exponential Smoothing (DES), Holt-Winter model Simple Exponential Smoothing (HWSES) shows a better performance for the twelve months forecast, with the Double Exponential smoothing (DES) the best performing model among the six compared models.

Table 1. *Summary of Models, Forecast and Errors of Inflation data.*

Models/Techniques	Smoothening constant	Years	Data	Forecast	Error
Simple Exponential Smoothing(SES)	α = 0.880	JAN(10)	103.700	102.112	1.588
		FEB(10)	104.800	103.509	1.291
		MAR(10)	105.300	104.645	0.655
		APR(10)	106.600	105.221	1.379
		MAY(10)	105.700	106.434	-0.73
		JUN(10)	110.000	105.787	4.213
		JUL(10)	111.600	109.494	2.106
		AUG(10)	113.800	111.347	2.453
		SEP(10)	114.000	113.505	0.495
		OCT(10)	114.000	113.940	0.060
		NOV(10)	114.000	113.992	0.408
		DEC(10)	115.400	114.351	1.049
Double Exponential Smoothing(DES)	α = 0.680	JAN(10)	103.700	103.302	0.398
		FEB(10)	104.800	104.445	0.355
		MAR(10)	105.300	105.588	-0.288
		APR(10)	106.600	106.731	-0.131
		MAY(10)	105.700	107.874	-2.174
		JUN(10)	110.000	109.017	0.983
		JUL(10)	111.600	110.160	1.440
		AUG(10)	113.800	111.303	2.497
		SEP(10)	114.000	112.446	1.554
		OCT(10)	114.000	113.589	0.411
		NOV(10)	114.400	114.730	-0.330
		DEC(10)	115.400	115.870	-0.470
Holt-Winter Exponential Smoothing (HWSES)	α = 0.680 γ = 0.680	JAN(10)	103.700	102.996	0.704
		FEB(10)	104.800	104.171	0.629
		MAR(10)	105.300	105.346	-0.046
		APR(10)	106.600	106.521	0.079
		MAY(10)	105.700	107.696	-1.996
		JUN(10)	110.000	108.871	1.129
		JUL(10)	111.600	110.046	1.554
		AUG(10)	113.800	111.221	2.579
		SEP(10)	114.000	112.396	1.604
		OCT(10)	114.000	113.571	0.429
		NOV(10)	114.400	114.746	-0.346
		DEC(10)	115.400	115.921	-0.521
Bootstrapping Method (BM)	α = 0.300	JAN(10)	103.700	103.590	0.110
		FEB(10)	104.800	103.470	1.330
		MAR(10)	105.300	103.360	1.940
		APR(10)	106.600	103.260	3.340
		MAY(10)	105.700	103.180	2.520
		JUN(10)	110.000	103.100	6.900
		JUL(10)	111.600	103.030	8.570
		AUG(10)	113.800	102.970	10.830
		SEP(10)	114.000	102.910	11.090
		OCT(10)	114.000	102.860	11.140
		NOV(10)	114.400	102.810	11.590
		DEC(10)	115.400	102.770	12.630
Linear Trend Regression Model (LTR)	α = 95.830 β = 1.220	JAN(10)	103.700	97.050	6.650
		FEB(10)	104.800	98.270	6.530
		MAR(10)	105.300	99.490	5.810
		APR(10)	106.600	100.710	5.890
		MAY(10)	105.700	101.930	3.770
		JUN(10)	110.000	103.150	6.850
		JUL(10)	111.600	104.370	7.230
		AUG(10)	113.800	105.590	8.210
		SEP(10)	114.000	106.810	7.190
		OCT(10)	114.000	108.030	5.970
		NOV(10)	114.400	109.250	5.150
		DEC(10)	115.400	110.470	4.930

Models/Techniques	Smoothening constant	Years	Data	Forecast	Error
Self Adjusting Model (SAM)	No smoothing constant	JAN(10)	103.700	103.200	0.500
		FEB(10)	104.800	104.800	0.000
		MAR(10)	105.300	106.400	-1.100
		APR(10)	106.600	108.000	-1.400
		MAY(10)	105.700	109.600	-3.900
		JUN(10)	110.000	111.200	-1.200
		JUL(10)	111.600	112.800	-1.200
		AUG(10)	113.800	114.400	-0.600
		SEP(10)	114.000	116.000	-2.000
		OCT(10)	114.000	117.600	-3.600
		NOV(10)	114.400	119.200	-4.800
		DEC(10)	115.400	120.800	-5.400

(Source: Authors' computation)

Table 2. *Summary of Statistics of errors comparison.*

Data	Methods/ Models	Mean forecast Error	VAR forecast Error	MSE forecast Error	MAE forecast Error	MAPE forecast Error	RMSE forecast Error	SSE forecast Error
Monthly inflation Rate (CPI) for food	Simple Exponential Smoothing(SES)	1.247	1.645	3.064	1.369	1.249	1.750	36.769
	Double Exponential Smoothing(DES)	0.353	1.436	1.441	0.919	0.831	1.200	17.299
	Holt-Winter Exponential Smoothing (HWSES)	0.483	1.415	1.531	0.968	0.875	1.237	18.372
	Bootstrapping Method(BM)	6.835	22.011	66.860	6.835	6.064	8.176	802.320
	Time Series Linear Trend Regression Model(LTR)	6.181	1.430	39.524	6.181	5.624	6.286	474.288
	Self-Adjusting Model (SAM)	-2.058	3.653	7.585	2.141	1.917	2.754	91.030

(Source: Authors' computation)

4. Conclusions

In this study, six different time series forecasting models which include Simple exponential Smoothing model, double exponential smoothing model, Holt-Winter exponential smoothing model, ad-hoc Bootstraping methods, the Time series linear regression model and the self-Adjusting model. The various model performances were compared by calculating their VAR, MSE, MAE, MAPE, RMSE and SSE. Results and analysis shown in Table *1* and *2*, clearly indicate that double exponential Smoothing (DES) is the best model for forecasting *12*-Months Nigeria's inflation food (CPI) rate data. The Holt-Winter model could also be used alternatively in forecasting *12* months inflation rate as result in table *2*, reveals the level of close comparison of both models. Results also shown that the Simple Exponential Smoothing (SES), Bootstrapping, Linear trend Regression and the Self Adjusting Model (SAM) are unstable and inappropriate models for forecasting the Nigeria CPI inflation rate data.

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