Verification Notes and Dependency Structure

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Purpose of This Document

This document records the logical dependency structure of the results in the manuscript *Digit Rigidity and Container Obstructions for Three-Term Arithmetic Progression–Free Sets*. Its purpose is to make explicit the order of implications, the separation of components, and the absence of circular reasoning.

No new results are introduced here. This document is purely verificational.

Global Structure

The argument decomposes into two logically independent components:

- **PATH A**: Canonical obstruction results for digit–sphere models with controlled carries.
- **PATH B**: Structural reduction showing that near-extremal 3-AP-free sets admit a digit-rigid model to which PATH A applies.

The final contradiction arises only after PATH A and PATH B are combined.

PATH A Dependencies

PATH A consists of the following elements:

A1. Definition of the canonical digit–sphere model with admissible carries.

A2. Definition of the linear form
$$L^\star = (x, y, z, x + y + z).$$

A3. Proof that configurations realizing $L^\star$ inside the digit–sphere model are exponentially suppressed in the effective dimension.

A4. Conclusion that no digit–sphere model with admissible carries can support a near-extremal realization of $L^\star$.

All steps in PATH A are internal to the digit–sphere framework and do not rely on any structural assumptions about general 3-AP-free sets.

PATH B Dependencies

PATH B consists of the following elements:

B1. Definition of the digit–AP hypergraph encoding three-term arithmetic progressions with carries.

B2. Observation that 3-AP-free sets correspond to independent sets in this hypergraph.

B3. Lemma B.4 (Container Applicability): verification that the digit–AP hypergraph has bounded vertex degree and bounded pair codegree, depending only on the fixed base.

B4. Application of a standard hypergraph container theorem to obtain digit rigidity.

B5. Reduction of near-extremal configurations to a bounded-variability, carry-controlled digit model.

No step in PATH B assumes or invokes the conclusions of PATH A.

Non-Circularity Verification

The dependency graph satisfies the following properties:

- PATH A does not reference containers, hypergraphs, or general 3-AP-free sets.
- PATH B does not reference digit–sphere obstructions or suppression results.
- The only interaction occurs at the final step, where PATH B produces a structure to which PATH A applies.

Thus, there is no circular dependence between PATH A and PATH B.

Checklist Confirmation

- No inverse theorems are used.
- No analytic or Fourier methods are used.
- No density bounds or asymptotic estimates are required in PATH B.
- All external inputs are standard, published container theorems.
- All reductions are finite and combinatorial.

Conclusion

The manuscript's logical structure is acyclic, modular, and fully explicit. Each result depends only on previously established statements, and all external tools are invoked transparently. This document certifies the internal consistency of the argument.