Appendix: Container Applicability for the Digit–AP Hypergraph

James Scott

Lemma 1 (Container Applicability)**.** *Let $q \geq 2$ be fixed. Let $L \geq 1$ and define the vertex set*

$$V = \{(i, x) : 1 \leq i \leq L,\ x \in \{0, 1, \ldots, q-1\}\}.$$

Define a 3-uniform hypergraph $\mathcal{H}$ on V whose edges are triples

$$\{(i, a_i), (i, b_i), (i, c_i)\}$$

arising from a single coordinate i, where the digits a_i, b_i, c_i form a valid digitwise contribution to a three-term arithmetic progression with admissible carry 0 or 1.

Then $\mathcal{H}$ has maximum vertex degree and maximum pair codegree bounded by constants depending only on q. Consequently, the hypergraph container theorems of Saxton–Thomason or Balogh–Morris–Samotij apply to $\mathcal{H}$.