Project 1: Maximum Flow and Linear Programming

Advanced Algorithms 2011

University of Copenhagen

26th April 2011

1 Introduction

This is the first mandatory assignment for the course Advanced Algorithms 2011. Assignments must be completed in groups of 2 or 3 students. Permission for individual assignments is only granted in special cases. The assignment is handed out the 26th of April 2011 and is due on the 4th of May 2011, at 22:00. To pass the assignment the group must have completed most of the questions satisfactory. To be allowed to resubmit the group must have made a reasonable attempt at solving most of the questions. The assignment will be corrected as quickly as possible, but not later than the 12th of May 2011. Eventual resubmissions are due on the 19th of May 2011. Completed assignments must be uploaded to Absalon (this page). All descriptions and arguments should be kept concise while still containing relevant points. The assignment consists of three parts, where the first two are mostly programming assignments.

2 Software and programming languages

There are no requirements on your choice of programming language, but you should select one which has bindings for a Linear Programming solver (LP-solver). The problems you are asked to solve are relatively small, which means that the ease of use of the LP-solver is much more important than its efficiency. We recommend using either the pulp-or library for Python or lp_solve for Java.

3 Part 1

In this project you are going to investigate the flow of cars towards Copenhagen during the morning rush-hours. More specifically, you are supposed to answer the following question: How many cars per minute can get from the suburbs through the main highways into the center of the city. To simplify the problem, you receive an undirected graph $G = (V, E)$. Four of its vertices define the center of Copenhagen (26, 27, 28 and 29 in Figure 1), and seven of its vertices define the suburbs (vertex 0 through 6). Other vertices correspond to street intersections, edges correspond to street-segments and the weights of edges are the number of cars that the street-segment can carry per minute. The graph is shown in Figure 1. A Java-class implementing the graph in Figure 1 is attached as an appendix.

Question 1.1

Argue why G is not a flow network.

Question 1.2

Describe how to convert G into a flow network in which the maximum flow is the maximum number of cars that can enter the city from the seven suburbs per minute. None of the streets are one-way streets.

Question 1.3

Report the maximum number of cars that can enter the city per minute using the graph and an LP-solver of your choice.

Question 1.4

Report which street segments should have their capacity increased if more cars were to enter the center.

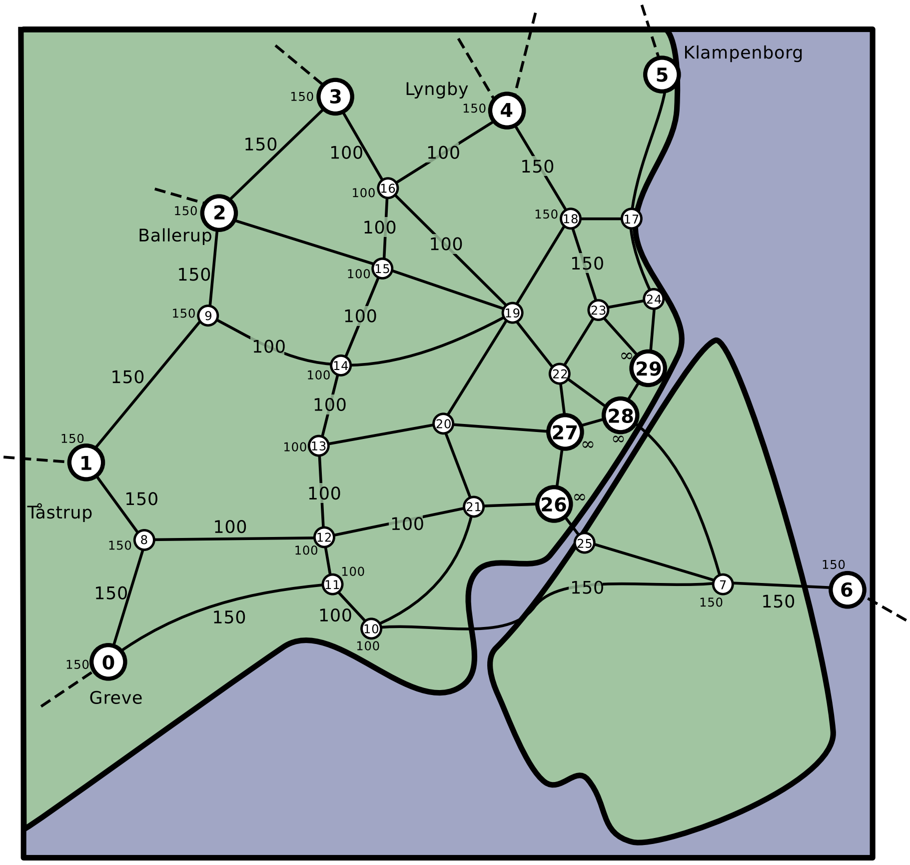

Figure 1: Graph of Copenhagen. Vertices 0-6 are the suburbs and vertices 26-29 are the city center locations. The capacity (number of cars per minute) of each street-segment is shown in large font for most of the edges. If nothing is specified, the capacity of a street is 30. For most vertices, its capacity is shown next to the vertex. If nothing is specified, the capacity of a vertex is also 30.

4 Part 2

In this part we wish to limit the rate of cars that can enter intersections. We therefore add a capacity constraints to intersections as shown in Figure 1.

Question 2.1

Describe how this modification of the problem can be solved by modifying the graph only.

Question 2.2

Describe how this problem can be solved by modifying the LP-formulation of the max-flow problem only.

Question 2.3

Name an advantage to each of the two methods.

Question 2.4

Report the maximum number of cars that can enter the city per minute using the vertex-constraints and an LP-solver of your choice.

5 Part 3

In Question 2.1 you had to handle the problem of adding capacity constraints on intersections, by modifying the graph to obtain a flow network. In this part, you have to formalise this reduction and prove that it is equivalent.

Assume $G = (V, E)$ is a network (not a flow network) with capacity constraints on intersections and let $G' = (V', E')$ be the flow network obtained by modifying G as in Question 2.1.

Question 3.1

Extend the flow properties and definitions so they take intersection-capacities into account. Specifically, show how to define capacity constraint and flow conservation properties as well as the value of a flow for such a problem.

Question 3.2

Given a flow $f : V \times V \to \mathbb{R}$ in G, define the equivalent flow $f' : V \times V \to \mathbb{R}$ in G'.

Question 3.3

Prove that f' obeys your definition of capacity constraint and flow conservation from Question 3.1.

Question 3.4

Prove that $|f'| = |f|$.