Chapter 5 Homework

1. Determine the value of k so that $f_{X,Y}(x,y)=k(2+x+y^2)$ with support $x=1,2,3 \quad y=1,2,3$ is a joint pmf.

The joint pmf of the random variables X and Y is given in the table. Use the table for questions 2 and 3.

		X				
		0	1	2	3	4
Y	0	0.01	0.05	0.05	0.14	0.07
	1	0.06	0.02	0.11	0.07	0.09
	2	0.18	0.03	0.04	0.05	0.03

2. a) Determine $P(Y=2)$ b) Determine $f_Y(y)$ c) Determine $f_{X|y=1}(2)$ d) Determine $f_{X|y=2}(x)$

3. a) Determine $P(X=3)$ b) Determine $f_X(x)$ c) Determine $f_{Y|x=3}(1)$ d) Determine $f_{Y|x=2}(y)$

The joint pmf of the random variables X, Y and Z is given in the table. Use the table for questions 4 - 9.

Z=1

		X		
		1	2	3
Y	1	.001	.002	.003
	2	.006	.005	.004
	3	.007	.008	.009

Z=2

		X		
		1	2	3
Y	1	.07	.06	.01
	2	.08	.05	.02
	3	.09	.04	.03

Z=3

		X		
		1	2	3
Y	1	.03	.01	.05
	2	.02	.145	.04
	3	.07	.06	.08

4. a) Determine $P(Y=2)$ b) Determine $P(Z=2)$ c) Determine $P(X=2)$ (No need to determine the marginals)

5. a) Determine $P(Y=1)$ b) Determine $P(Z=1)$ c) Determine $P(X=1)$ (No need to determine the marginals)

6. a) Determine $f_{X,Y}(1,2)$ b) Determine $f_{X,Y|z=1}(1,2)$ c) Determine $f_{X,Y|z=2}(1,2)$ (No need to determine marginals)

7. a) Determine $f_{X,Y}(2,2)$ b) Determine $f_{X,Y|z=1}(2,2)$ c) Determine $f_{X,Y|z=2}(2,2)$ (No need to determine marginals)

8. a) Determine $f_{X,Z}(x,z)$ b) Determine $f_{Y|x=1,z=1}(y)$ c) Determine $f_{Y|x=1}(y)$ d) Determine $f_{Y|z=1}(y)$

9. a) Determine $f_{X,Y}(x,y)$ b) Determine $f_{Z|x=1,y=1}(z)$ c) Determine $f_{Z|x=1}(z)$ d) Determine $f_{Z|y=1}(z)$

Use $f_{X,Y,Z}(x,y,z)=\frac{1}{81}(x+xy+z)$ for $x=0,1,2 \quad y=0,1,2 \quad z=0,1,2$ to answer problems 10 – 13.

10. a) Determine $f_{X,Y}(1,2)$ b) Determine $f_{X,Y|z=1}(1,2)$ c) Determine $f_{X,Y|z=2}(1,2)$

11. a) Determine $f_{X,Y}(2,2)$ b) Determine $f_{X,Y|z=1}(2,2)$ c) Determine $f_{X,Y|z=2}(2,2)$

12. a) Determine $f_{X,Z}(x,z)$ b) Determine $f_{Y|x=1,z=1}(y)$ c) Determine $f_{Y|x=1}(y)$

d) Determine $f_{Y|z=1}(y)$ e) Determine $f_{Y|x,z}(y)$ f) Determine $f_{Y|x}(y)$

13. a) Determine $f_{X,Y}(x,y)$ b) Determine $f_{Z|x=1,y=1}(z)$ c) Determine $f_{Z|x=1}(z)$

d) Determine $f_{Z|y=1}(z)$ e) Determine $f_{Z|x,y}(z)$ f) Determine $f_{Z|x}(z)$

14. Determine k, so that $f_{X,Y}(x,y)=\begin{cases} k(x+y) & 0<x<1,\ 0<y<1 \\ 0 & otherwise \end{cases}$ is a joint pdf.

15. Determine k, so that $f_{X,Y}(x,y)=\begin{cases} kxy & 0<x<1,\ 0<y<1 \\ 0 & otherwise \end{cases}$ is a joint pdf.

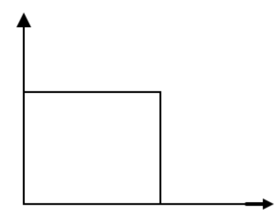
16. Determine k, so that $f_{X,Y}(x,y)=\begin{cases} k(xy^2) & 0<x<1,\ 0<y<1 \\ 0 & otherwise \end{cases}$ is a joint pdf.

17. Determine k, so that $f_{X,Y}(x,y)=\begin{cases} kx & 0<x<y<1 \\ 0 & otherwise \end{cases}$ is a joint pdf.

18. Determine k, so that $f_{X,Y}(x,y)=\begin{cases} k(x+y) & 0<x<y<1 \\ 0 & otherwise \end{cases}$ is a joint pdf.

19. Determine k, so that $f_{X,Y}(x,y)=\begin{cases} kxy & 0<x<y<1 \\ 0 & otherwise \end{cases}$ is a joint pdf.

20 The joint pdf of X and Y is $f_{X,Y}(x,y)=\begin{cases} x+y & 0<x<1,\ 0<y<1 \\ 0 & otherwise \end{cases}$ (region below)



a) Determine $f_X(x)$ b) Determine $f_Y(y)$

c) Determine $f_{X|y}(x)$ d) Determine $f_{X|y=.5}(x)$

e) Determine $f_{Y|x}(y)$ f) Determine $f_{Y|x=.25}(y)$

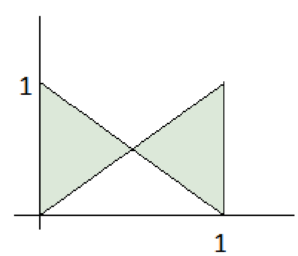
g) Determine $E[X]$ and $Var[X]$ h) Determine $E[Y]$

i) Determine $Var[Y]$ j) Determine $Cov[X,Y]$

21. The joint density of the random variables *X* and *Y* is $f_{X,Y}(x,y)=2 \text{ on } S$, where *S* is the region bounded by $x=0, x=1, y=x, y=1-x$.

a) Determine $f_Y(y)$ b) Determine $f_X(x)$

c) Determine $E[X]$ and $Var[X]$ d) Determine $E[Y]$ and $Var[Y]$



22. Let X and Y be continuous random variables with joint **CDF** (not the pdf)

$$F(x,y)=\frac{1}{250}\left(20xy-x^2y-xy^2\right)\text{for } 0\le x\le 5 \text{ and } 0\le y\le 5$$

a) Determine $P(3\le X)$ b) Determine $P(2\le X)$ c) Determine $P\left[(2\le X\le 3)\cap(1\le Y\le 4)\right]$

d) Determine $f_X(x)$ e) Determine $f_Y(y)$

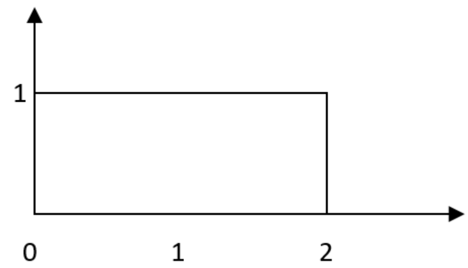
23. The joint pdf of X and Y is $f_{X,Y}(x,y) = \begin{cases} \frac{x+y}{3} & 0<x<2,\ 0<y<1 \\ 0 & otherwise \end{cases}$ (region below).

a) Determine $f_{Y|x}(y)$

b) Determine $f_{X|y}(x)$

c) Determine $E[Y \mid x]$

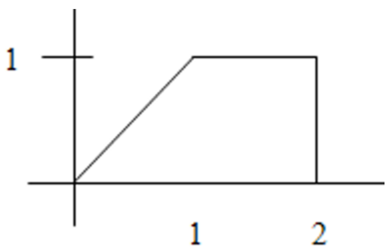
d) Determine $E[X \mid y]$



24. Suppose that the joint probability density function of the jointly continuous random variables X and Y is

$$f_{X,Y}(x,y) = \begin{cases} \frac{6}{11}x & \text{on the given region} \\ 0 & \text{otherwise} \end{cases}$$

Determine $f_Y(y)$



25. Let X and Y be continuous random variables with joint pdf $f_{X,Y}(x,y) = \begin{cases} 6x & 0<x<y<1 \\ 0 & otherwise \end{cases}$.

Given that $E[X] = \frac{1}{2}$ and $E[Y] = \frac{3}{4}$, determine $Cov[X,Y]$.

26. Given $f_{X,Y}(x,y) = \begin{cases} 12y^2 & 0 \le y \le x \le 1 \\ 0 & otherwise \end{cases}$ Determine $E[Y \mid x]$

27. Given $f_{X,Y}(x,y) = \begin{cases} 6x & 0<x<y<1 \\ 0 & otherwise \end{cases}$ and that $E[X] = \frac{1}{2}$ and $E[Y] = \frac{3}{4}$, determine $Cov[X,Y]$.

28. Let X and Y be continuous random variables with joint pdf $f_{X,Y}(x,y) = \begin{cases} 6x & 0<x<y<1 \\ 0 & otherwise \end{cases}$.

Given that $E[X] = \frac{1}{2}$ and $E[Y] = \frac{3}{4}$, determine $Cov[X,Y]$.

29. Determine $Cov[X,Y]$ for the random variables in problem 23.

30. Determine $Cov[X,Y]$ for the random variables in problem 24.

31. Given that $V[X]=1.2$, $V[Y]=2.8$ and $Cov[X,Y]=3$:

a) Determine $V[X+Y]$ b) Determine $V[X-Y]$ c) Determine $V[2X-4Y]$

d) Determine $V[Y+X]$ e) Determine $V[Y-X]$ f) Determine $V[4Y-2X]$

32. Given that $V[X]=1.2$, $V[Y]=2.8$ and $Cov[X,Y]=-3$:

a) Determine $V[X+Y]$ b) Determine $V[X-Y]$ c) Determine $V[2X-4Y]$

d) Determine $V[Y+X]$ e) Determine $V[Y-X]$ f) Determine $V[4Y-2X]$

33. Given that $V[X]=2.1$, $V[Y]=3.5$ and $Cov[X,Y]=-4$:

a) Determine $V[X+Y]$ b) Determine $V[X-Y]$ c) Determine $V[2X-4Y]$

d) Determine $V[Y+X]$ e) Determine $V[Y-X]$ f) Determine $V[4Y-2X]$

34. Given that $V[X]=2.1$, $V[Y]=1.5$ and $Cov[X,Y]=5$:

a) Determine $V[X+Y]$ b) Determine $V[X-Y]$ c) Determine $V[2X-4Y]$

d) Determine $V[Y+X]$ e) Determine $V[Y-X]$ f) Determine $V[4Y-2X]$