CodeForces Educational Round 178 E

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August 3, 2025

Since Bob gets to see Alice's move before he plays, it seems as if he has an advantage. What if we give him the n card? Then, he can only lose if Alice has the 1 card. However, if Alice plays any card besides the 1 card, Bob takes that card, and if Alice plays the 1, Bob can just try to play any other card. As such, if Bob has the n card, if he has more than one card he is guaranteed to win. Otherwise, if he only has the n card, he loses.

Now we consider the case of Alice having the n card. If she has the 1 card as well, then Bob definitely loses since all his cards get eaten by Alice's n card. If Bob has the 1 card, however, Alice can't play the n card anymore since Bob will take it, and we know what happens when Bob has the n card and also another card.

Thus, Alice has to play her next highest card. If Bob has a stronger card than that, then Bob will win since he will just play his strongest card if Alice doesn't play the n card, and his 1 card otherwise. If Alice's second strongest card is stronger than Bob's strongest card, then Alice wins as she plays that card every time and wins.

This is equivalent to checking who owns the $n-1$ card, because if Alice has the $n-1$ card it is her second strongest card and definitely stronger than all of Bob's cards (remember in this case Alice has the n card), and if Bob owns the $n-1$ card he overpowers any non-n card that Alice plays. The only edge case is when the $n-1$ card and the 1 card are the same, i.e when $n=2$. In such a case, Alice trivially wins since she plays her 1 and takes Bob's n. We solve the problem in $O(n)$.