CodeForces Round 1037 Div. 3E

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Trying to build the original a felt like the most promising path. Given p_i and s_i, a_i would definitely need to be a multiple of both. This me think of the LCM of p_i and s_i. We may theorize that if there is exist some valid a, then one such a must consist of $a_i = \text{lcm}(p_i, s_i)$.

Let's assume there exists some valid a given p and s. $p_1 = a_1$ must be true as well as $s_n = a_n$, meaning that if p and s are good, p_1 is a multiple of s_1 and s_n is a multiple of p_n. Then, for 1 and n, the condition $a_i = \text{lcm}(p_i, s_i)$ would be satisfied. Now, take some arbitrary a_i where $1 < i < n$. We know that

$$\gcd(p_{i-1}, a_i) = p_i$$

and

$$\gcd(s_{i+1}, a_i) = s_i$$

If we were to change a_i into $\text{lcm}(p_i, s_i)$, $p_1 \dots p_{i-1}$ wouldn't change and $s_{i+1} \dots s_n$ would also be the same. Furthermore, $\gcd(p_{i-1}, a_i) = p_i$ would imply $\gcd(p_{i-1}, \text{lcm}(p_i, s_i)) = p_i$, as a_i being a multiple of p_i and s_i would mean it is a multiple of $\text{lcm}(p_i, s_i)$. Thus, $\gcd(p_{i-1}, a_i)$ must be a multiple of $\gcd(p_{i-1}, \text{lcm}(p_i, s_i))$, and since $\gcd(p_{i-1}, a_i) = p_i$, $\gcd(p_{i-1}, \text{lcm}(p_i, s_i)) = p_i$ as $\text{lcm}(p_i, s_i)$ is still a multiple of p_i. Similar logic would hold for s_i. Since the rest of a, remains the same, if p_i and s_i remain the same so do $p_{i+1} \dots p_n$ and $s_1 \dots s_{i-1}$. We could repeat this process until $a_i = \text{lcm}(p_i, s_i)$ for all i.

Now that we have a greedy method to find a valid a, we can just create a based on our rule and then check if the resulting p' and s' match the inputs. Since we would need to use lcm and gcd $O(n)$ times, our solution is $O(n \log a_i)$.