

# Basic R: Matrices

*Ozy Meghani*

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## Matrix problems

### 1. Suppose

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$$

(a) Check that  $A^3 = \mathbf{0}$

(b) Replace the third column of A by the sum of the second and third columns

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First, produce A

```
A <- matrix(c(1,1,3,5,2,6,-2,-1,-3), nrow = 3, byrow = TRUE)
```

A

```
##      [,1] [,2] [,3]
## [1,]    1    1    3
## [2,]    5    2    6
## [3,]   -2   -1   -3
```

```
A%%A%%A
```

```
##      [,1] [,2] [,3]
## [1,]    0    0    0
## [2,]    0    0    0
## [3,]    0    0    0
```

Then, add the columns 2 and 3 and assign the sum to the third column

```
A[,3] <- A[,2] + A[,3]
```

A

```
##      [,1] [,2] [,3]
## [1,]    1    1    4
## [2,]    5    2    8
## [3,]   -2   -1   -4
```

### 2. Create the following matrix B with 15 rows

$$B = \begin{bmatrix} 10 & -10 & 10 \\ 10 & -10 & 10 \\ \dots & \dots & \dots \\ 10 & -10 & 10 \end{bmatrix}$$

Calculate the 3x3 matrix  $B^T B$ . You can make this calculation with the function `crossprod()`. See the documentaion.

```
B <- matrix(c(10,-10,10), nrow= 15, ncol= 3, byrow = T)
t(B) %*% B
```

```
##      [,1] [,2] [,3]
## [1,] 1500 -1500 1500
## [2,] -1500 1500 -1500
## [3,] 1500 -1500 1500
```

3. Create a 6 x 6 matrix `matE` with every element equal to 0. check what the functions `row()` and `col()` return when applied to `matE`.

Now, create the 6 x 6 matrix:

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

---

Here is `matE`, a 6x6 matrix of 0's followed by `row(matE)` and `col(matE)`

```
matE <- matrix(rep(0,36), nrow = 6, byrow = TRUE)
```

```
# Note what the functions row() and col() do  
row(matE)
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6]  
## [1,]    1    1    1    1    1    1  
## [2,]    2    2    2    2    2    2  
## [3,]    3    3    3    3    3    3  
## [4,]    4    4    4    4    4    4  
## [5,]    5    5    5    5    5    5  
## [6,]    6    6    6    6    6    6
```

```
col(matE)
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6]  
## [1,]    1    2    3    4    5    6  
## [2,]    1    2    3    4    5    6  
## [3,]    1    2    3    4    5    6  
## [4,]    1    2    3    4    5    6  
## [5,]    1    2    3    4    5    6  
## [6,]    1    2    3    4    5    6
```

```
# With a little experimentation you would see  
# that the specified pattern is in the |1|'s  
row(matE)-col(matE)
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6]  
## [1,]    0   -1   -2   -3   -4   -5  
## [2,]    1    0   -1   -2   -3   -4  
## [3,]    2    1    0   -1   -2   -3  
## [4,]    3    2    1    0   -1   -2  
## [5,]    4    3    2    1    0   -1  
## [6,]    5    4    3    2    1    0
```

```
# so you use the locations of the 1's to modify matE
matE[abs(row(matE)-col(matE))==1] <- 1
matE
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6]
## [1,]    0    1    0    0    0    0
## [2,]    1    0    1    0    0    0
## [3,]    0    1    0    1    0    0
## [4,]    0    0    1    0    1    0
## [5,]    0    0    0    1    0    1
## [6,]    0    0    0    0    1    0
```

---

4. Look at the help for the function `outer()`. Now, create the following patterned matrix:

$$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 7 & 8 \end{bmatrix}$$

```
a <- 0:4
A <- outer(a,a,"+")
A
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    0    1    2    3    4
## [2,]    1    2    3    4    5
## [3,]    2    3    4    5    6
## [4,]    3    4    5    6    7
## [5,]    4    5    6    7    8
```

5. Create the following patterned matrices. Your solutions should be generalizable to enable creating larger matrices with the same structure.

(a)

$$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 & 0 \\ 2 & 3 & 4 & 0 & 1 \\ 3 & 4 & 0 & 1 & 2 \\ 4 & 0 & 1 & 2 & 3 \end{bmatrix}$$

```
outer(a, a, "+")%5
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    0    1    2    3    4
## [2,]    1    2    3    4    0
## [3,]    2    3    4    0    1
## [4,]    3    4    0    1    2
## [5,]    4    0    1    2    3
```

(b)

$$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 8 & 9 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 9 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{bmatrix}$$

```
outer(0:9, 0:9, "+")%10
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10]
## [1,]    0    1    2    3    4    5    6    7    8    9
## [2,]    1    2    3    4    5    6    7    8    9    0
## [3,]    2    3    4    5    6    7    8    9    0    1
## [4,]    3    4    5    6    7    8    9    0    1    2
## [5,]    4    5    6    7    8    9    0    1    2    3
## [6,]    5    6    7    8    9    0    1    2    3    4
## [7,]    6    7    8    9    0    1    2    3    4    5
## [8,]    7    8    9    0    1    2    3    4    5    6
## [9,]    8    9    0    1    2    3    4    5    6    7
## [10,]   9    0    1    2    3    4    5    6    7    8
```

(c)

$$\begin{bmatrix} 0 & 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 \\ 1 & 0 & 8 & 7 & 6 & 5 & 4 & 3 & 2 \\ 2 & 1 & 0 & 8 & 7 & 6 & 5 & 4 & 3 \\ 3 & 2 & 1 & 0 & 8 & 7 & 6 & 5 & 4 \\ 4 & 3 & 2 & 1 & 0 & 8 & 7 & 6 & 5 \\ 5 & 4 & 3 & 2 & 1 & 0 & 8 & 7 & 6 \\ 6 & 5 & 4 & 3 & 2 & 1 & 0 & 8 & 7 \\ 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 & 8 \\ 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 \end{bmatrix}$$

```
outer(0:8, 0:8, "-")%9
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9]
## [1,]    0    8    7    6    5    4    3    2    1
## [2,]    1    0    8    7    6    5    4    3    2
## [3,]    2    1    0    8    7    6    5    4    3
## [4,]    3    2    1    0    8    7    6    5    4
## [5,]    4    3    2    1    0    8    7    6    5
## [6,]    5    4    3    2    1    0    8    7    6
## [7,]    6    5    4    3    2    1    0    8    7
## [8,]    7    6    5    4    3    2    1    0    8
## [9,]    8    7    6    5    4    3    2    1    0
```

6. Solve the following system of linear equations by setting up and solving the matrix equation  $\mathbf{Ax} = \mathbf{y}$ .

$$\begin{aligned} x_1 + 2x_2 + 3x_3 + 4x_4 + 5x_5 &= 7 \\ 2x_1 + x_2 + 2x_3 + 3x_4 + 4x_5 &= -1 \\ 3x_1 + 2x_2 + x_3 + 2x_4 + 3x_5 &= -3 \\ 4x_1 + 3x_2 + 2x_3 + x_4 + 2x_5 &= 5 \\ 5x_1 + 4x_2 + 3x_3 + 2x_4 + x_5 &= 17 \end{aligned}$$

```

y <- c(7, -1, -3, 5, 17)

A <- matrix(0, nrow = 5, ncol = 5)
A <- abs(col(A)-row(A)) +1

x <- solve(A, y)

```

**7. Create a 6 x 10 matrix of random integers chosen from 1,2,...,10 by executing the following two lines of code:**

```

set.seed(75)
aMat <- matrix(sample(10, size=60, replace=TRUE), nr=6)

```

Use the matrix you have created to answer these questions:

(a) Find the number of entries in each row which are greater than 4.

```

set.seed(75)
aMat <- matrix(sample(10, size=60, replace=TRUE), nr=6)

apply(aMat, 1, function(x){sum(x>4)})

## [1] 4 7 6 2 6 7

```

(b) Which rows contain exactly two occurrences of the number seven?

```

which(apply(aMat, 1, function(x){sum(x == 7) == 2}))

## [1] 5

```

(c) Find those pairs of columns whose total (over both columns) is greater than 75. The answer should be a matrix with two columns; so, for example, the row (1,2) in the output matrix means that the sum of columns 1 and 2 in the original matrix is greater than 75. Repeating a column is permitted; so, for example, the final output matrix could contain the rows (1,2), (2,1), and (2,2).

```

aMatcolsums <- colSums(aMat)
which(outer(aMatcolsums, aMatcolsums, "+") > 75, arr.ind = TRUE)

##      row col
## [1,]   2   2
## [2,]   6   2
## [3,]   8   2
## [4,]   2   6
## [5,]   8   6
## [6,]   2   8
## [7,]   6   8
## [8,]   8   8

```

What if repetitions are not permitted? Then only (1,2) from (1,2),(2,1) and (2,2) would be permitted.

**8. Calculate**

$$(a) \sum_{i=1}^{20} \sum_{j=1}^5 \frac{i^4}{(3+j)}$$


---

```
sum((1:20)^4 * sum(1/(3+(1:5))))
```

```
## [1] 639215.3
```

```
# or
```

```
sum(outer((1:20)^4, (3+(1:5)), "/"))
```

```
## [1] 639215.3
```

---

$$(b) \sum_{i=1}^{20} \sum_{j=1}^5 \frac{i^4}{(3+ij)}$$

```
sum((1:20)^4/(3 + outer(1:20, 1:5, "*")))
```

```
## [1] 89912.02
```

$$(c) \sum_{i=1}^{10} \sum_{j=1}^i \frac{i^4}{(3+ij)}$$

```
sum( outer(1:10, 1:10, function(i,j){(i>=j)*i^4/(3 + i*j)}))
```

```
## [1] 6944.743
```