TINSTAAFL — theorem

Optical Technologies.

there's no such thing as a free lunch

SAYING

said to emphasize that you cannot get something for nothing:

[http://dictionary.cambridge.org]

… in order to be graded after the full lecture,
you <u>must</u> visit the exercises!

next exercises: 2017-05-23

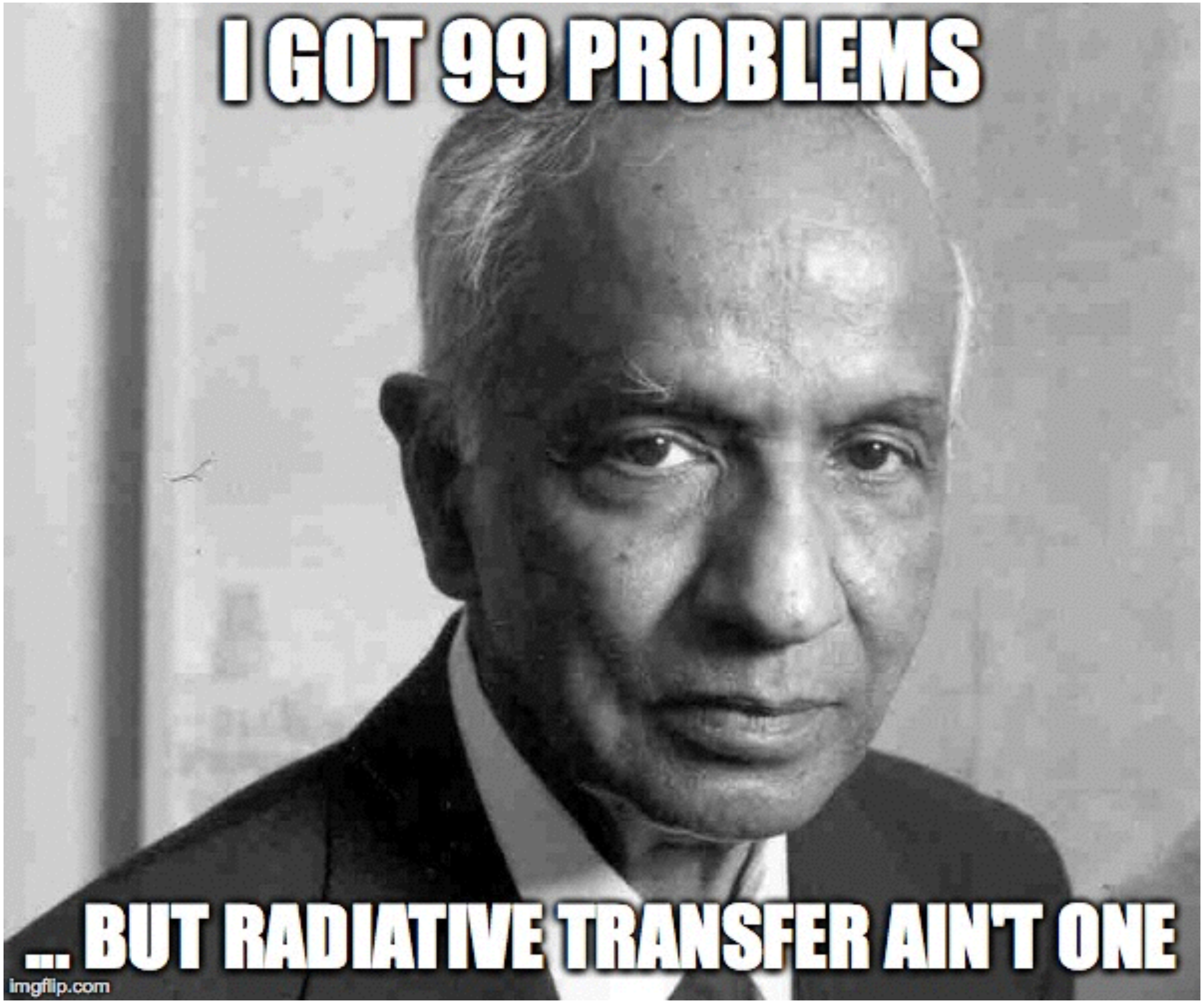
Problem <u>hand-out</u> date: 2017-06-28

Solution <u>hand-in</u> date: 2017-07-12

2 students per problem OK!

report-volume: 10 pages!

Recap — L3:



[Subrahmanyan “Chandra“ Chandrasekhar]

Lecture focus on tissue optics: outline

Computational tissue optics

Light transport in turbid media (Optical absorption)

Monte Carlo (MC) method

Pseudo random numbers (PRNs)

- Lecture 4:
  Computational tissue optics
  (meet long-term goal)

postpone acoustics to 2017-06-28

Top-down approach:

- to understand light transport in <u>general</u> turbid media you need to *master the MC method*
- ~~to master MC method you need to *sample PRNs*~~
- ~~to sample PRNs you need to actually *code*~~

Bottom-up approach:

- ~~Lecture 1:~~
  ~~*PRN generation / quality control*~~
- ~~Lecture 2:~~
  ~~*MC sampling strategies*~~
- ~~Lecture 3:~~
  ~~*MC simulation in turbid media*~~
- ~~Exercise 1:~~
  ~~*extensive Python code examples*~~

Lecture 4

Light transport in turbid media II

Outline

- Recap — L3

Lecture 4: Light transport in turbid media II

4.1 - Implicit assumptions of the RTE

4.2 - The statistical minutiae of photon trails

4.3 - Modeling the irradiation source

4.4 - Measuring the volumetric energy density

4.5 - Exemplary use-cases

4.1 Implicit assumptions of the RTE

Radiative transfer theory:

- RTE is purely phenomenological - it’s aim is to describe, not to explain!
- based on implicit assumptions:

  (1) radiation energy carried by particles (photons). Photons carry energy quanta (see L3-3.2.1)

  (2) photons propagate in quasi-homogeneous media. Trails proceed along the straight rays of geometrical optics (see L3-3.2.1)

  (3) photons interact with matter and exchange energy and momentum through quasi-elastic scattering at successive „events“: (see L3-3.2.2) (see L3-3.2.4)

    (3.1) absorption: governed by Beer-Lambert law

    (3.2) redirection: governed by scattering phase function

4.2 The statistical minutiae of photon trails

Exact results for *isotropic random walks* in d-dimensions:

- PDF for walker position:

$$P_N(\vec{R}) = \Big(\frac{d}{2\pi N\langle s^2\rangle}\Big)^{d/2} \exp\Big\{-d\frac{R^2}{2N\langle s^2\rangle}\Big\}$$

Note:

- PDF tends towards Gaussian distribution in the limit $N \to \infty$
- width depends on 2nd moment of displacements $\langle s^2\rangle = \int s^2 p(\vec{s})\, \mathrm{d}\vec{s}$
- average squared displacement (cf. lecture 2-2.2) $\langle R^2\rangle = \langle s^2\rangle N$

- PDF for walker displacement from the origin:

$$P_N(R) = S_d R^{d-1} P_N(\vec{R})$$

Reasons why the above formulae might fail:

- non-uniform *step length* distributions
- strong correlations between *step directions*
- interaction between different walkers

4.2 The statistical minutiae of photon trails

- amend photon packet data structure by observable

Code Listing 1: photon packet displacement

```
    def d(self):
        """displacement

        Implements displacement of the photon packet relativ to start

        Returns:
            d (float) end-to-end displacement of photon trail
        """
        x,y,z = self.x
        x0, y0, z0 = self.x0
        return np.sqrt((x-x0)**2 + (y-y0)**2 + (z-z0)**2)
```

(module: `photonPacket3D.py`)

- sampling of displacements at final photon position

Code Listing 2: loop to sample displacents

```
while(nP):
    random.seed(nP)
    myPhoton = PhotonPacket(lenSamp.generate, dirSamp.generate)
    while(myPhoton.exists()):
        myPhoton.propagate()
        myPhoton.absorb(a)
    print nP, myPhoton.d(), myPhoton.nSteps
    nP -= 1
```

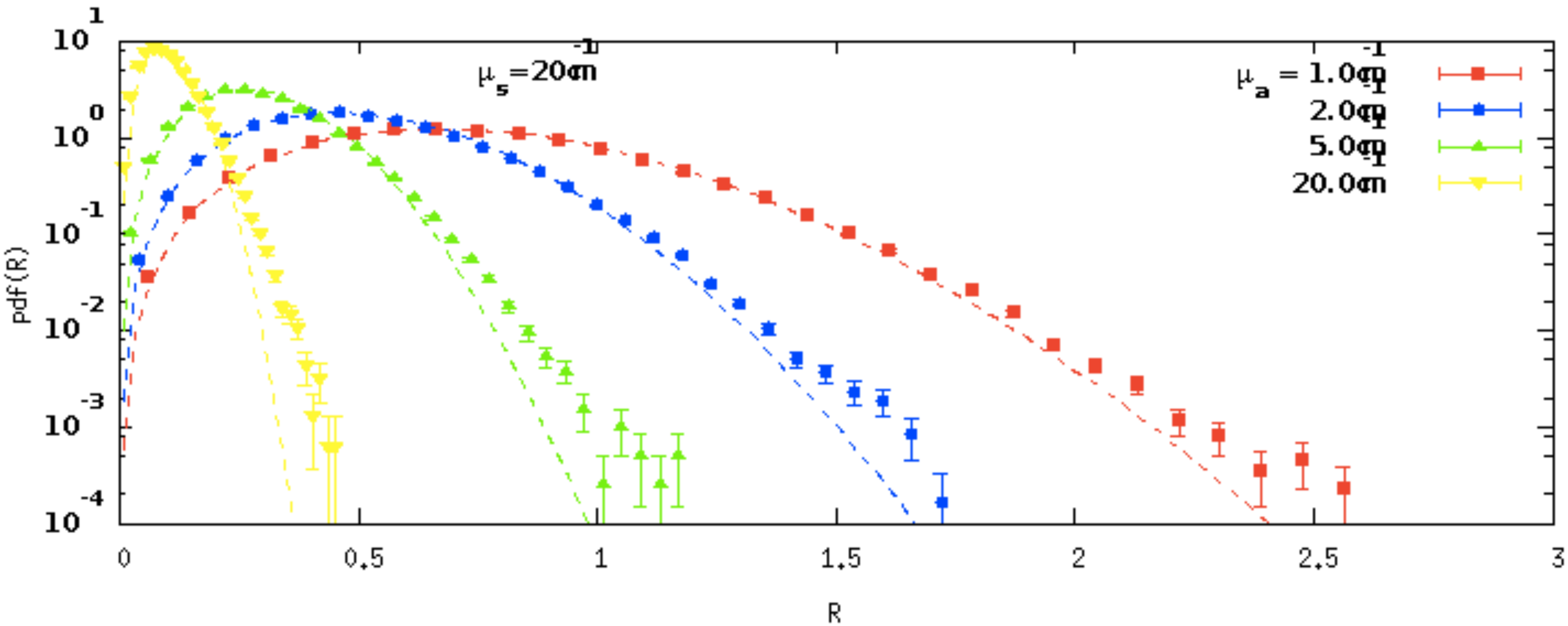
(script: `main_displacement.py`)

End-to-end distance of photon packet trails:

- *naive* expectation: consistent with (pure) 3D random walk statistics

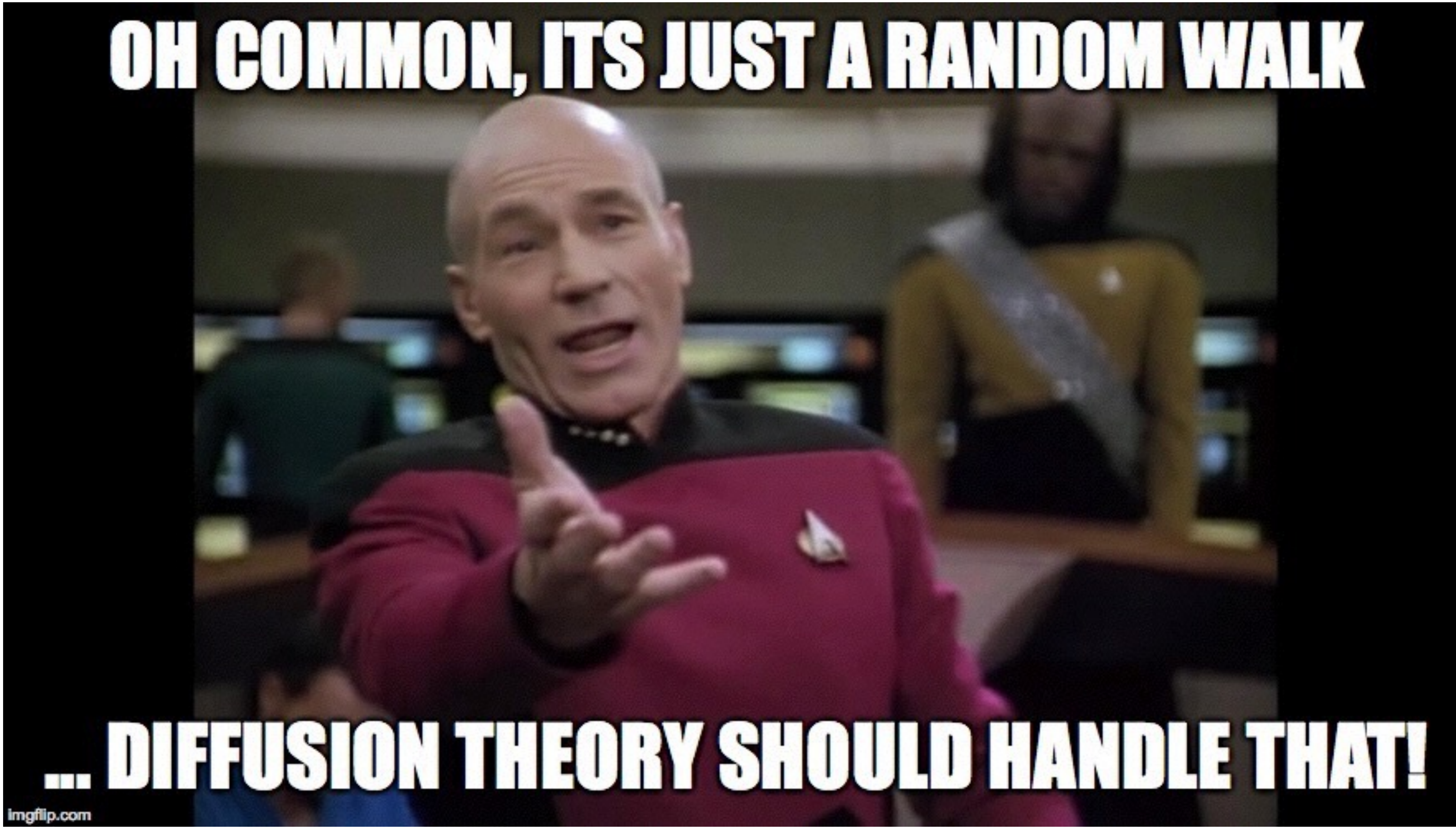
$$P(R) = 4\pi R^2 \left(\frac{3}{2\pi N\langle s^2\rangle}\right)^{3/2} \exp\left\{-\frac{3}{2}\frac{R^2}{N\langle s^2\rangle}\right\} \qquad \langle s^2\rangle = 2/\mu_t^2, \quad N \to \infty$$

- *less naive*: should hold in diffusion approximation limit $\mu_a \ll \mu_s$ of RTE (see lecture 3-3.1.2)
- observation 1: *istotropic* scattering



indeed gets worse as $\mu_a \to \mu_s$ … but why?

Picard is upset!

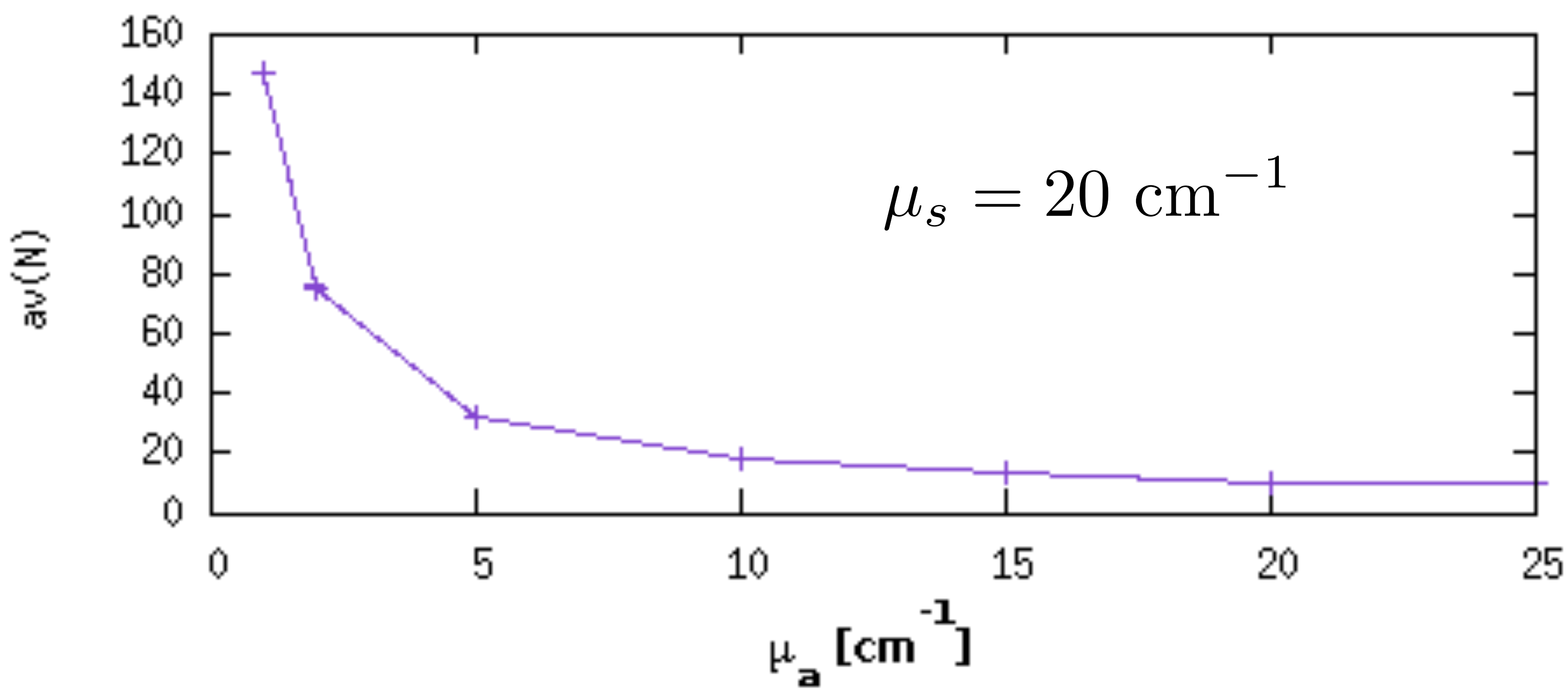


4.2 The statistical minutiae of photon trails

Optical Technologies.

End-to-end distance of photon packet trails:

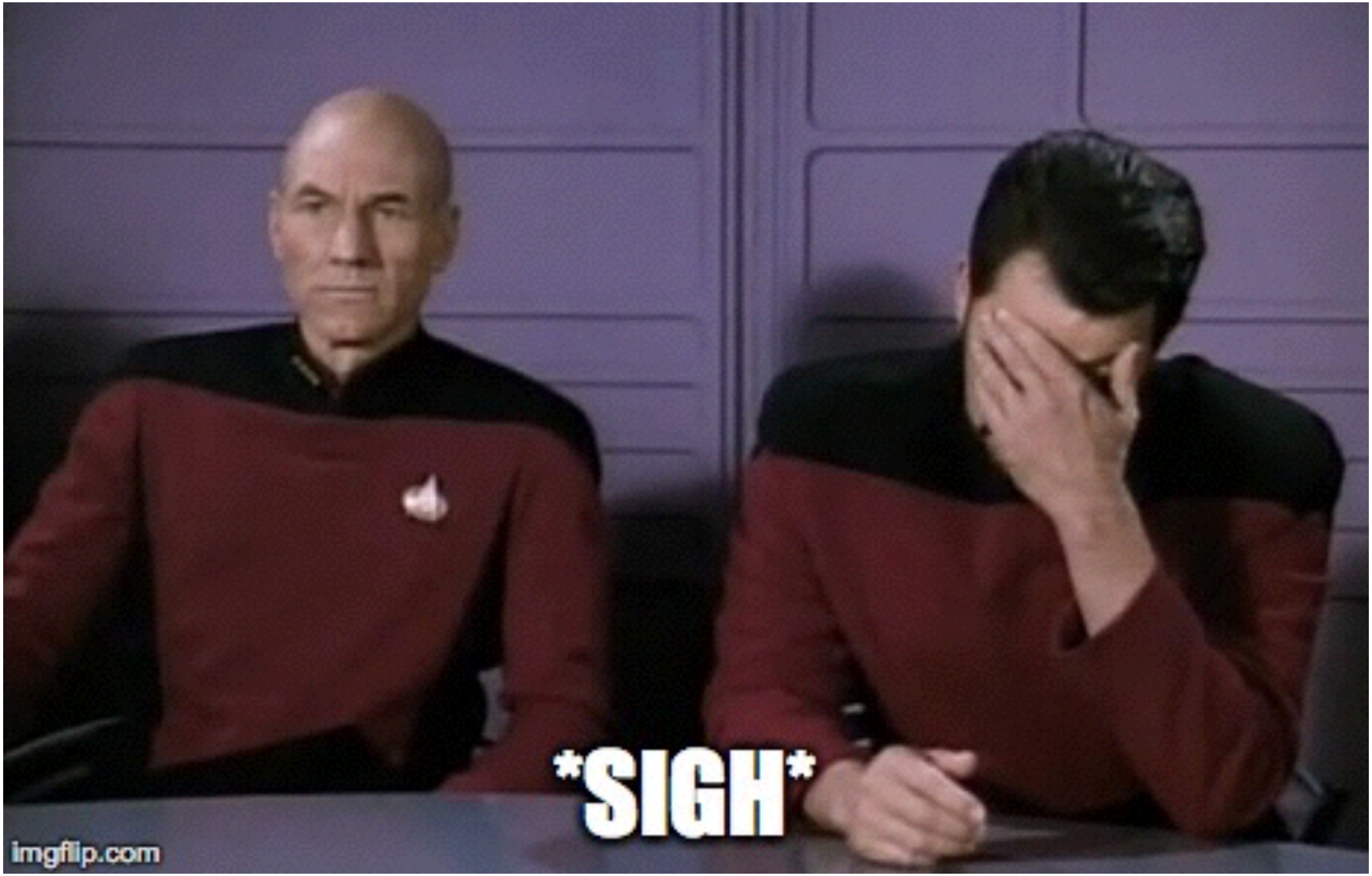
- diffusion theory holds … but only in the asymptotic regime $N \to \infty$
- here: absorption keeps trails rather short



- asymptotic probability function for walker position is not valid, yet!
- … walks are simply too short!

$$P_N(\vec{R}) = \cancel{\left(\frac{d}{2\pi N\langle s^2\rangle}\right)^{d/2}\exp\left\{-d\frac{R^2}{2N\langle s^2\rangle}\right\}}$$

Riker knew!



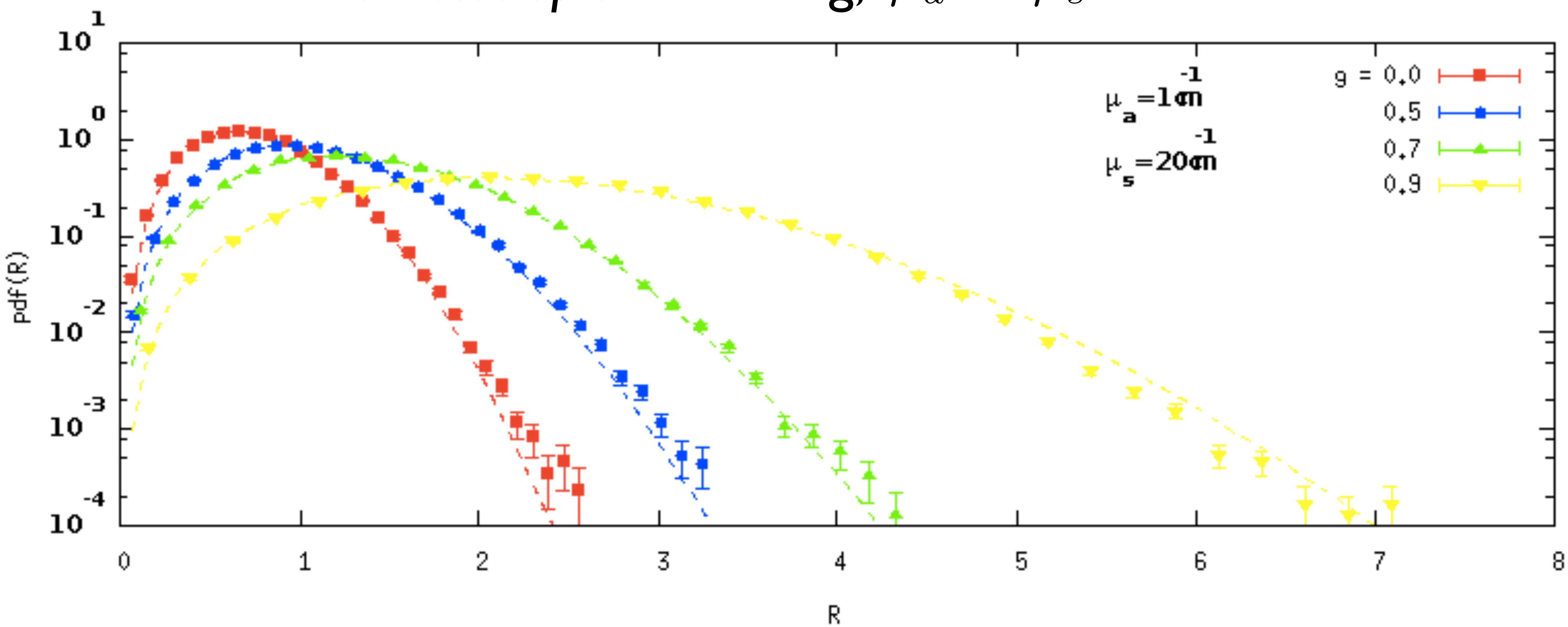
4.2 The statistical minutiae of photon trails

End-to-end distance of photon packet trails:

- *less naive* expectation: consistent with custom 3D random walk statistics

$$P(R) = 4\pi R^2\left(\frac{3}{2\pi\langle R^2\rangle}\right)^{3/2}\exp\left\{-\frac{3}{2}\frac{R^2}{\langle R^2\rangle}\right\} \qquad \langle s^2\rangle \equiv \langle R^2\rangle/N$$

- *less naive*: should hold in diffusion approximation limit $\mu_a \ll \mu_s$ of RTE (see lecture 3-3.1.2)
- observation 2: *non-istotropic* scattering, $\mu_a \ll \mu_s$



only changes scale as $g \to 1$ within diff. limit … thats ok!

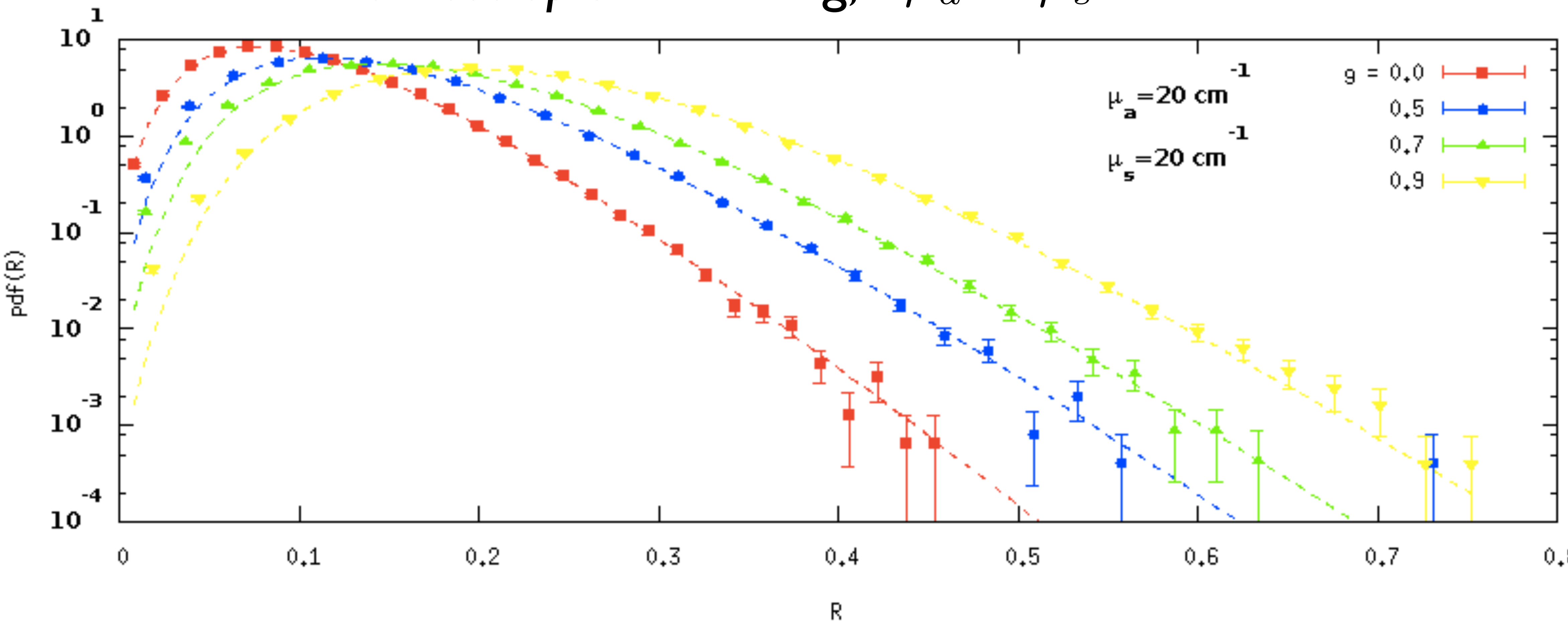
End-to-end distance of photon packet trails:

- in the full RTE regime, effective scaling form describes data well

$$P(R) \sim (R-\delta R)^{\alpha} \exp\{-\beta R\}$$

e.g., at $g = 0.5$: $\delta R = -0.012$, $\alpha = 4.5$, $\beta = 36.3$

- observation 3: *non-isotropic* scattering, $\mu_a \approx \mu_s$

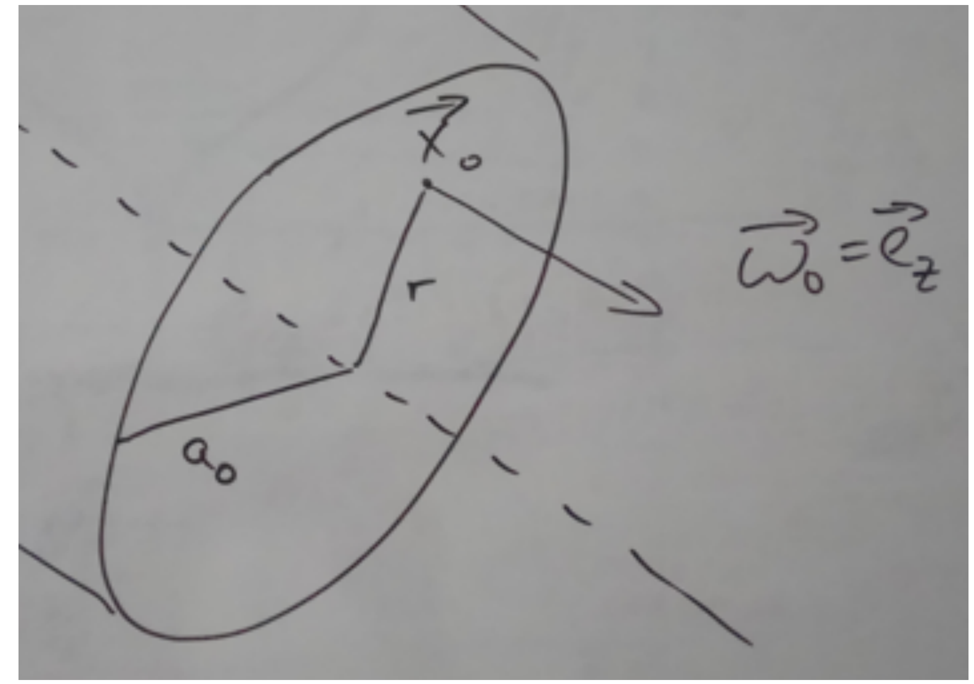


Note: long displacements exhibit increased probability!

4.3 Modeling the irradiation source

4.3.1 Simple beam

- simple approach to model spatially extended irradiation source
- model:
  - beam incident along plain normal of tissue
  - circular beam with top-hat beam profile



- method:
  - beam centered at $(r, z) = (0, 0)$ cm
  - beam radius a_0
  - sample initial position via inversion method

    radial coordinate: $p_r(X) = 2\frac{X}{a_0^2} \to r = a_0\sqrt{u_1}$

    azimuthal angle: $p_\phi(X) = \frac{1}{2\pi} \to \phi = 2\pi u_2$

    $u_1, u_2 \in [0, 1)$
  - fixed directional cosines:

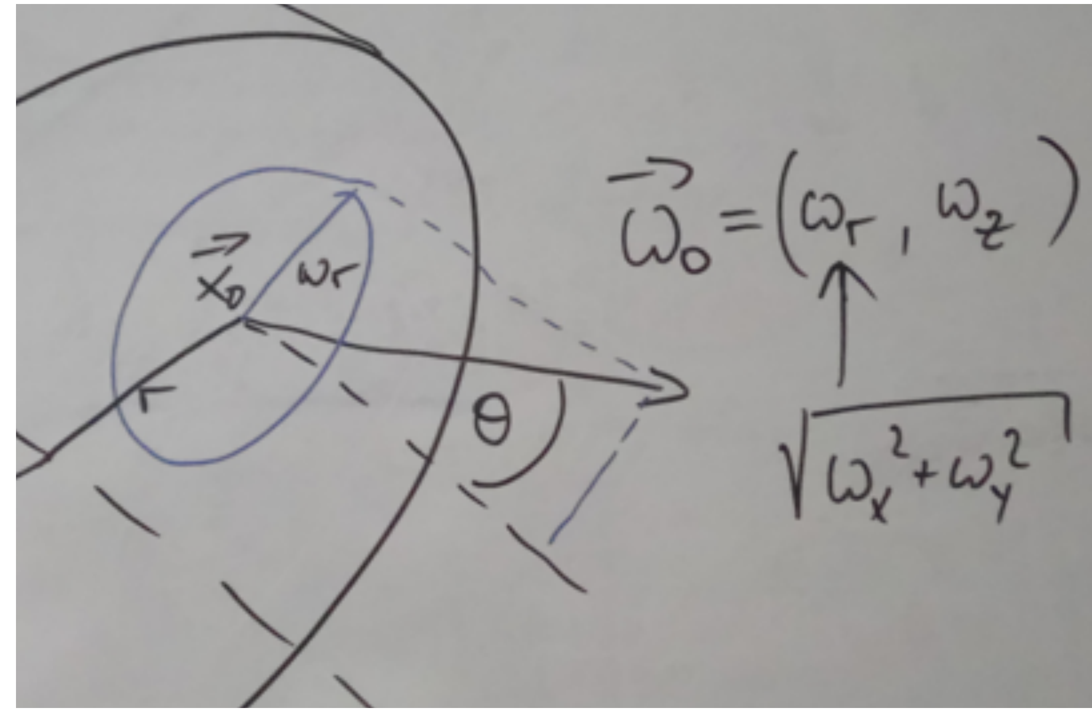
    $(\omega_x, \omega_y, \omega_z) = (0, 0, 1)$ (normally incident)
- mapping:
  - see module: `irradiationSource.py`

4.3 Modeling the irradiation source

Optical Technologies.

4.3.1 Realistic source fiber

- easy approach to mimic source fiber with finite numerical aperture
- model:
  - circular fiber surface
  - limited numerical aperture
  - refractive index mismatch



- method:
  - beam centered at $(r, z) = (0, 0)\ \mathrm{cm}$
  - beam radius a_0, numerical aperture NA
  - refractive index of fiber and medium: n_f, n_M
  - sample initial position as before
  - directional cosines:

$$\theta = \sin^{-1}\left(NA\,\frac{n_f}{n_m}\right) u_1 \qquad \omega_z = \cos(\theta)$$

$$\phi = 2\pi u_2 \qquad \omega_x = (1-\omega_z^2)^{1/2}\cos(\phi) \qquad u_1, u_2 \in [0, 1)$$

$$\omega_y = (1-\omega_z^2)^{1/2}\sin(\phi)$$

[Mourant *et al.*, Opt. Lett., 21 (1996) 546]

- mapping:
  - see module: `irradiationSource.py`

4.4 Measuring the volumetric energy density

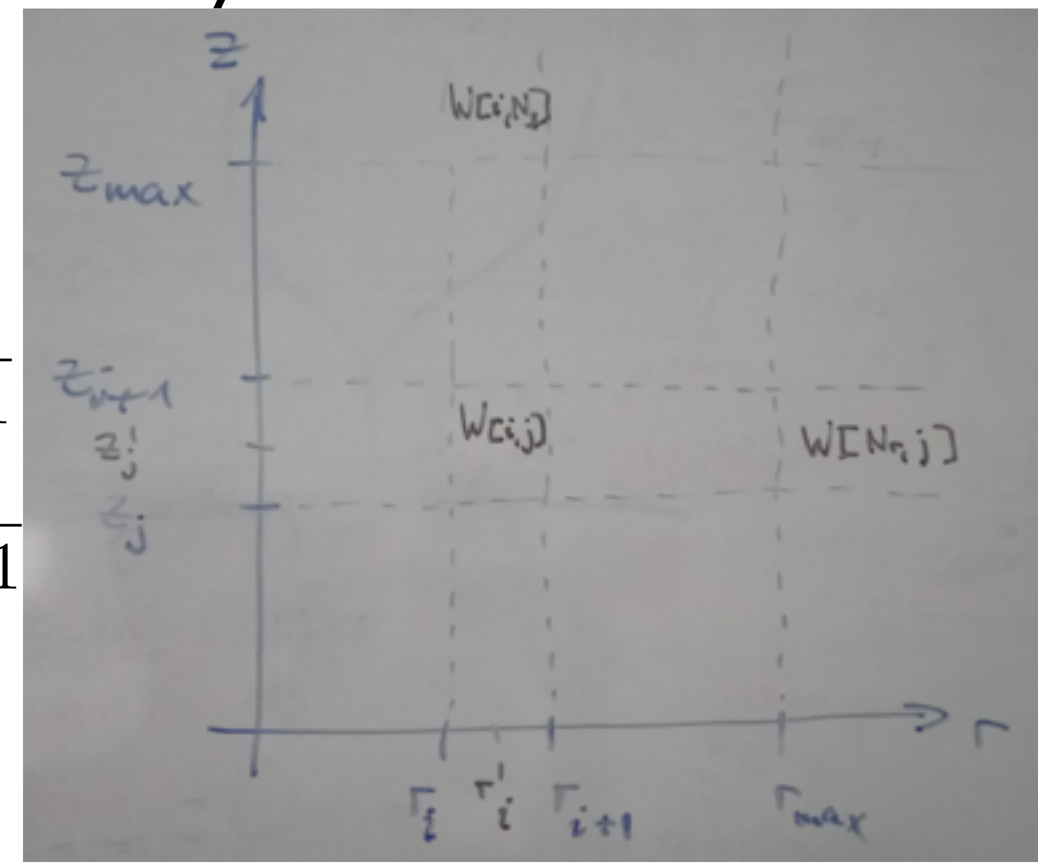
- model:
  - energy decrements of individual photons must be accumulated locally
  - observable: steady state volumetric energy density
- method:
  - use discrete mesh to represent domain
  - here: only radially symmetric beams

$$r \in [0, r_{\max}] \to \{r_i = i\Delta r\}_{i=0}^{N_r-1};\ \Delta r = \frac{r_{\max}}{N_r - 1}$$

$$z \in [0, z_{\max}] \to \{z_i = i\Delta z\}_{i=0}^{N_z-1};\ \Delta z = \frac{z_{\max}}{N_z - 1}$$

$$W(r, z) \to \{W[i,j]\}_{i=0\ldots N_r}^{j=0\ldots N_z}$$

extra bins used for sanity checks!



binning:

$$r \in [r_i, r_{i+1}) \to i,\ r \geq r_{\max} \to N_r$$

$$z \in [z_j, z_{j+1}) \to j,\ z \geq z_{\max} \to N_z$$

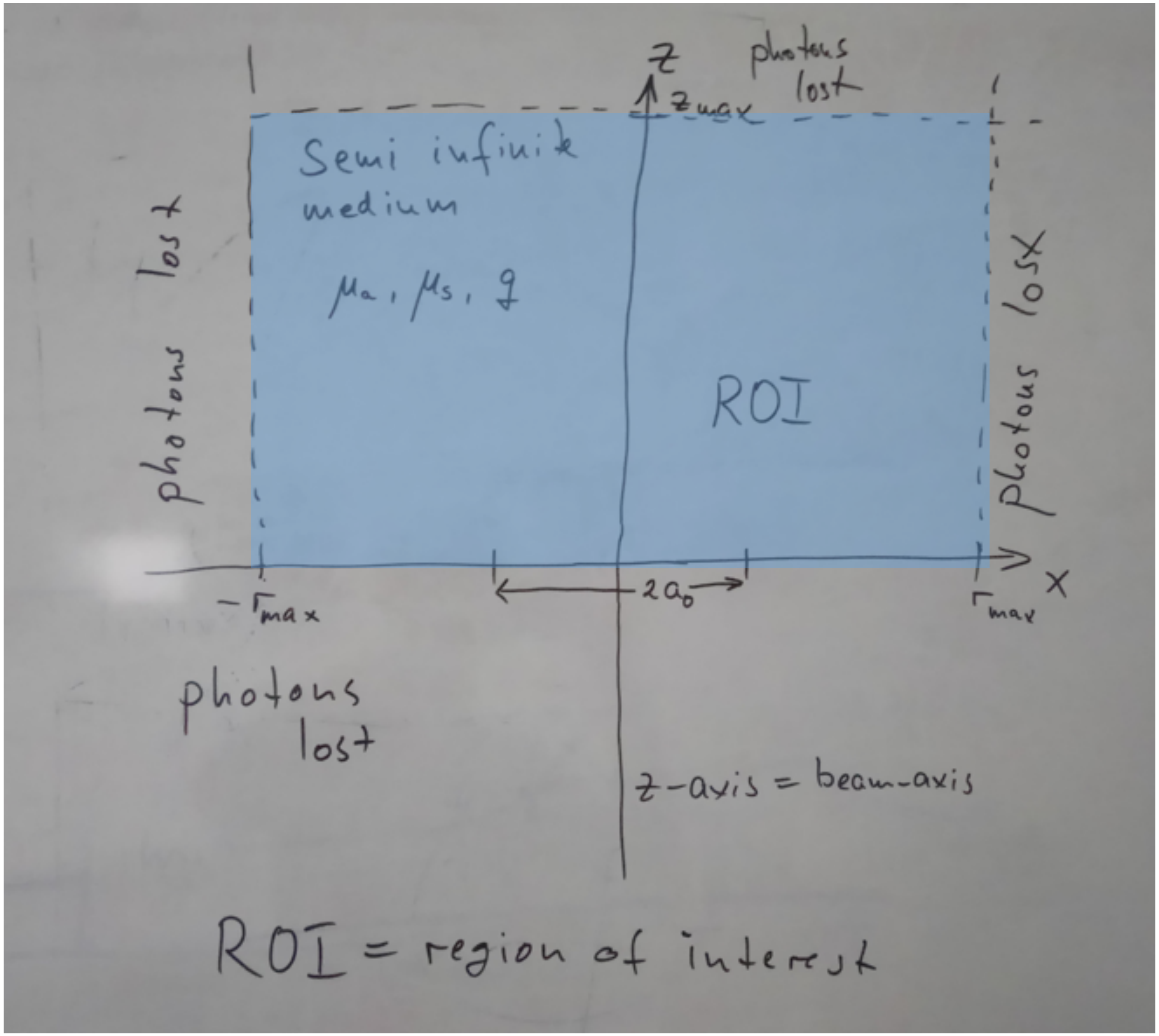
$$\Delta W(r, z) \to W[i, j]$$

normalization:

$$W[i,j] \leftarrow W[i,j]/(2\pi N \Delta z (i + 0.5)\Delta r^2)\ [\mathrm{cm}^{-3}]$$

- mapping: see module `volumetricEnergyDensity.py`

Geometry for the numerical experiments:



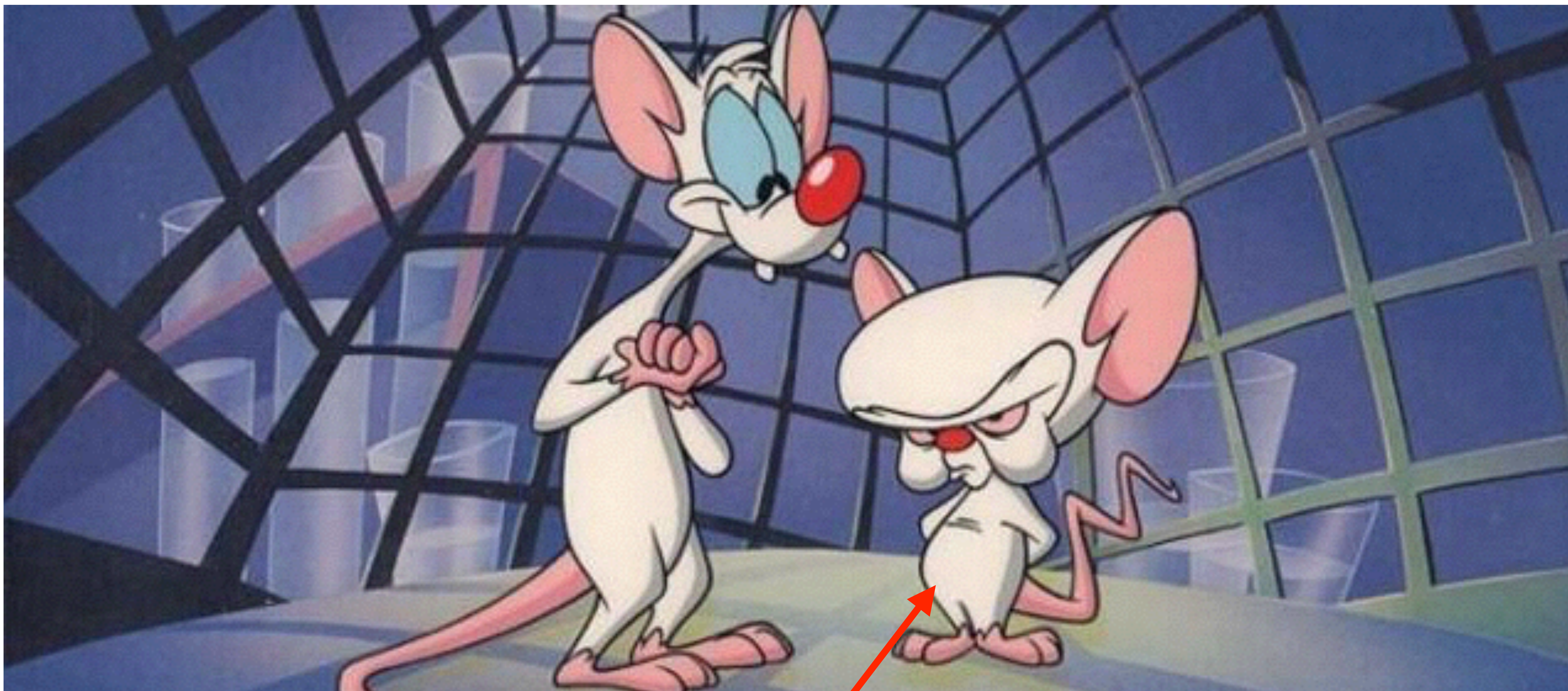
4.5 Exemplary use-cases

Isotropic tissue - murine liver

4.5 Isotropic tissue - murine liver

Numerical experiments:

- considering optical properties of tissue determined *in vitro*
- source: murine (albino) liver tissue



In case you want to do „lab“ experiments:

cut approximately here

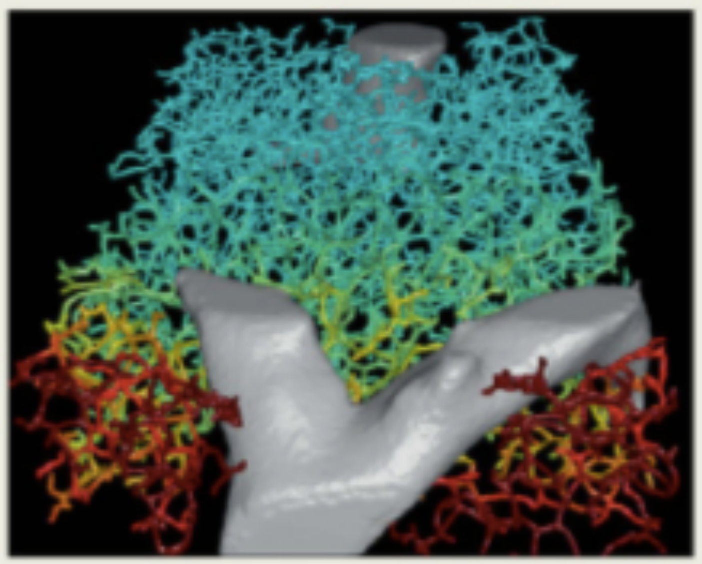
… save mice, work *in silico!*

No joke!

Optical Technologies.

Virtuelle Leber könnte Zahl der Tierversuche verringern

Forscher simulieren die Strömungsdynamik der Gallenflüssigkeit in dem Organ und sagen auf diese Weise durch Medikamente verursachte Schäden vorher



Dreidimensionales Modell des Gallennetzwerks, durch das die Galle von der Leber in den Darm fließt. Die Farben geben die Fließgeschwindigkeiten der Galle an (Blau: langsam, Rot: schnell).

Die Leber ist das zentrale Stoffwechselorgan des Körpers und maßgeblich an dessen Entgiftung beteiligt. Dies macht sie besonders anfällig für Schäden durch Medikamente. Daher sind Tierversuche zur Überprüfung der Lebertoxizität neuer Medikamente gesetzlich vorgeschrieben. Für den Abbau von Fetten und den Abtransport von Ausscheidungsprodukten bildet die Leber Gallenflüssigkeit, die durch ein fein verästeltes Kanalnetzwerk in den Darm fließt. Ein Forscherteam am Max-Planck-Institut für molekulare Zellbiologie und Genetik in Dresden hat dieses Netzwerk in Mäusen mit hochauflösenden Mikroskopen untersucht und Aufbau und Struktur der Kanäle analysiert. Dann haben die Forscher ein 3D-Modell der Gallengänge erstellt, das die Strömungseigenschaften der Gallenflüssigkeit nachstellen kann. Mit dem Modell können die Wissenschaftler Leberkrankheiten sowie Auswirkungen von Medikamenten auf die Leber erforschen, zum Beispiel die bei neuen Wirkstoffen häufig auftretende Gallestauung. Als Nächstes wollen die Wissenschaftler das Modell so verändern, dass es die Verhältnisse in der menschlichen Leber widerspiegelt. Zwar werden Tierversuche auch in absehbarer Zukunft weiter notwendig sein, das Modell könnte jedoch dazu beitragen, ihre Zahl zu verringern. (www.mpg.de/11186162)

40 MaxPlanckForschung 1 | 17

[MaxPlanckForschung Heft 1/2017, p40]

4.5 Isotropic tissue - murine liver

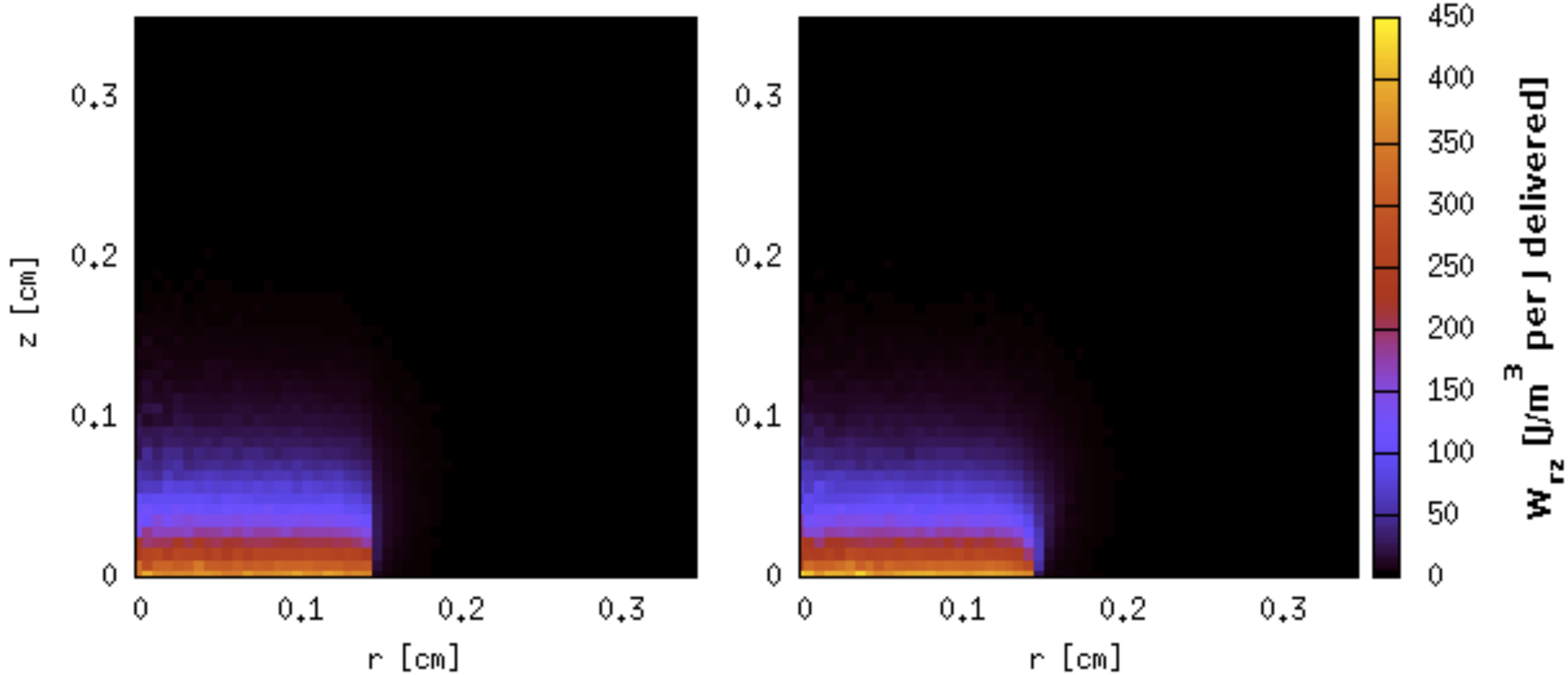
Numerical experiments:

λ [nm]	μ_a [cm^{-1}]	μ_s [cm^{-1}]	g
2100	27.2	24.5	0.80
1320	6.6	44.2	0.91
1064	5.9	60.9	0.92
800	5.7	97.0	0.94
633	6.5	143.7	0.95
488	12.2	173.5	0.93

Analysis: thin slab of tissue between glass dishes
[Parsa, Jacques, Nishioka, Appl. opt., 28 (1989) 2325]

Review of analyses & lots of tissue data:
[Cheong, Prahl, Welch, IEEE J. Quant. El., 26 (1990) 2166]

Scripts: `main_simpleBeam.py` (left)
`main_realisticFiber.py` (right)



4.5 Isotropic tissue - murine liver

Optical Technologies.

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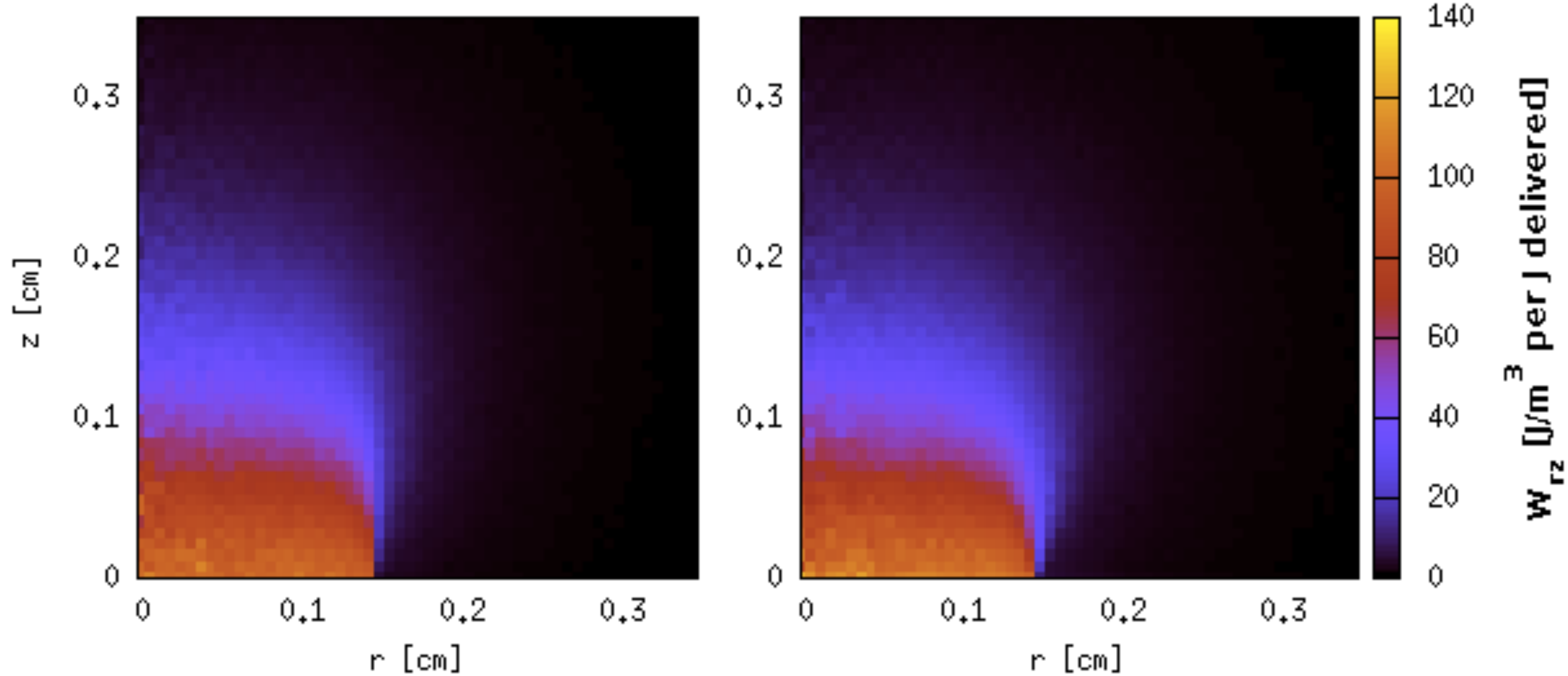

Optical Technologies.

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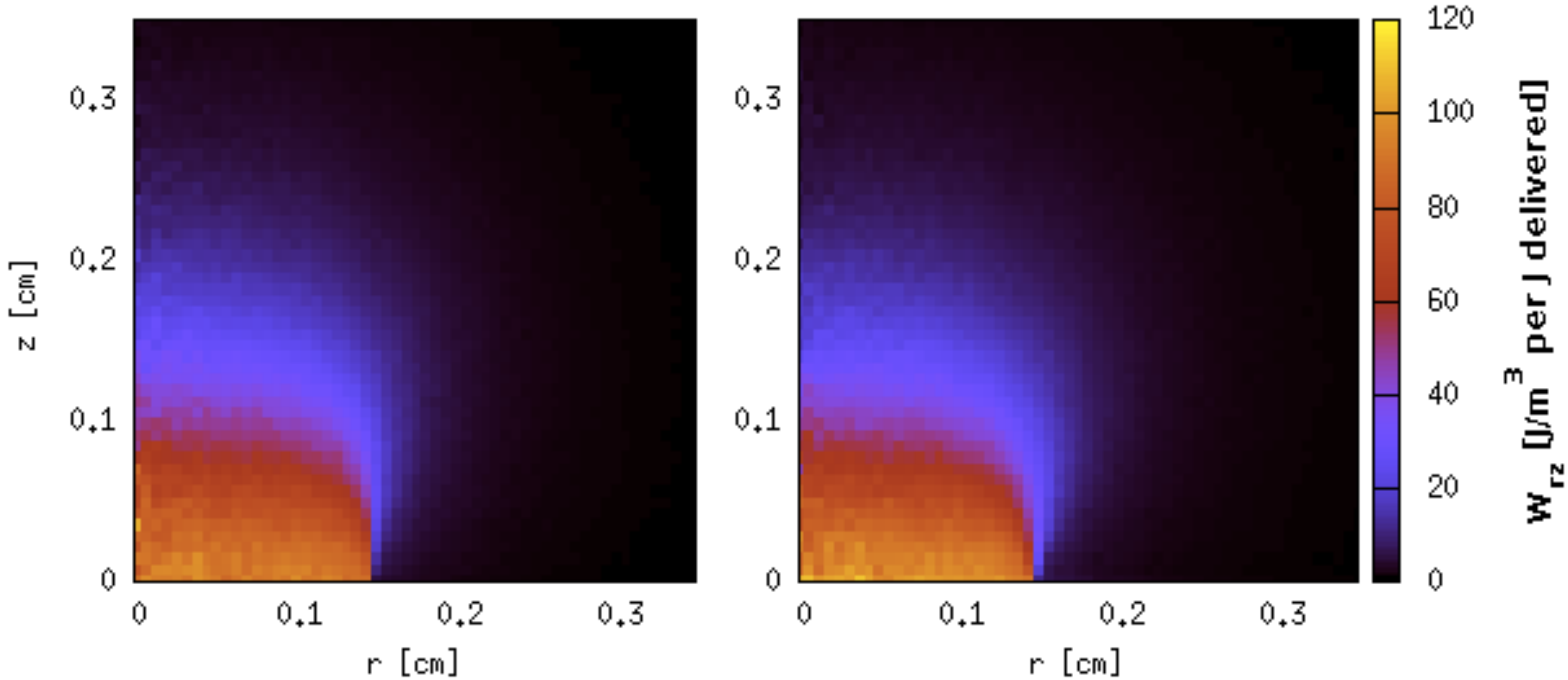

4.5 Isotropic tissue - murine liver

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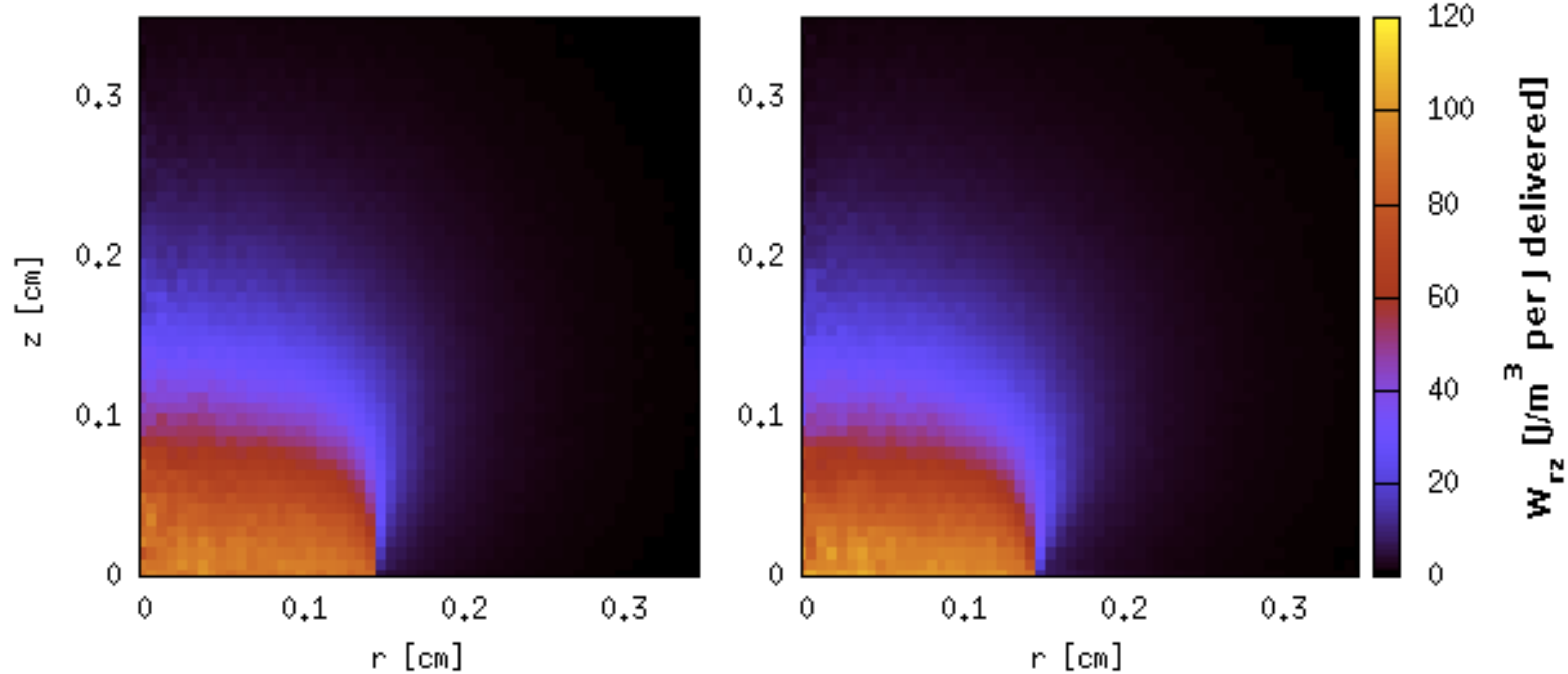

4.5 Isotropic tissue - murine liver

Optical Technologies.

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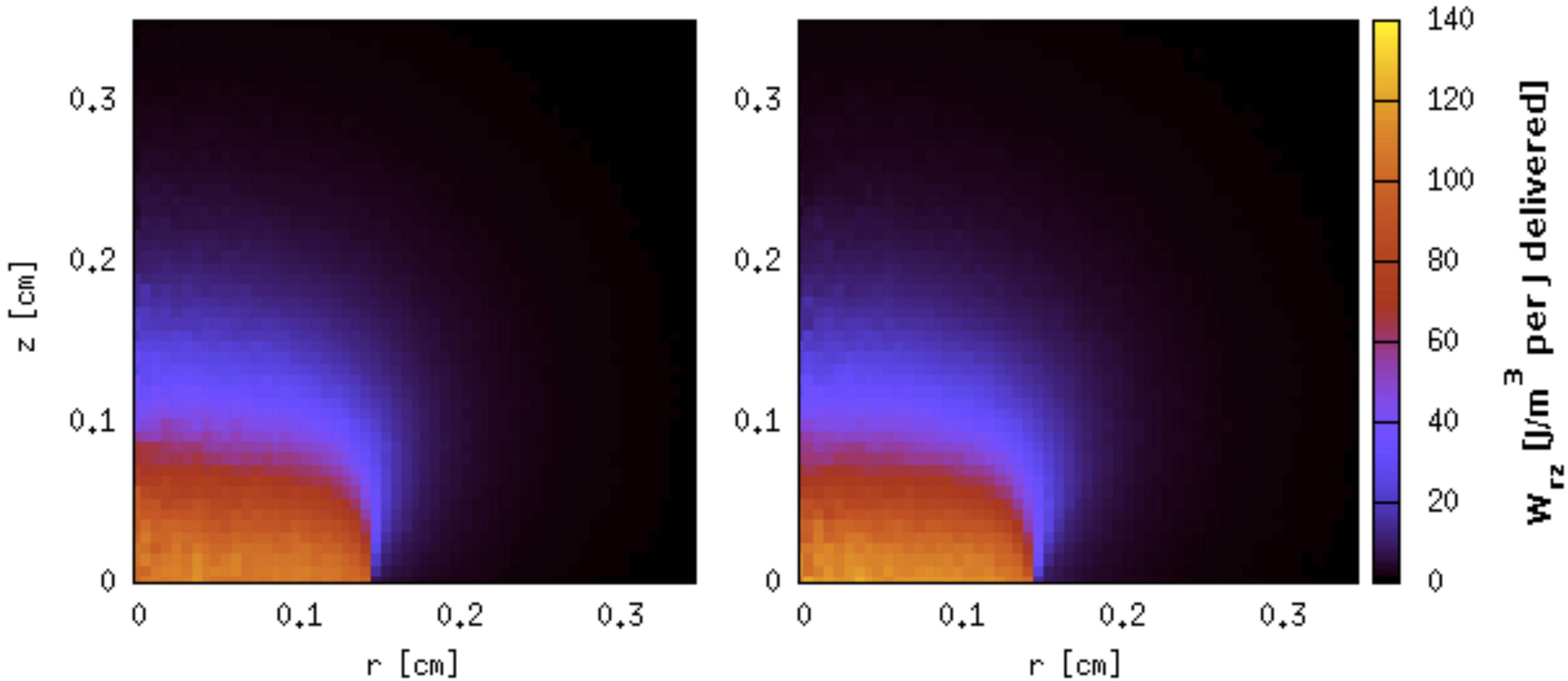

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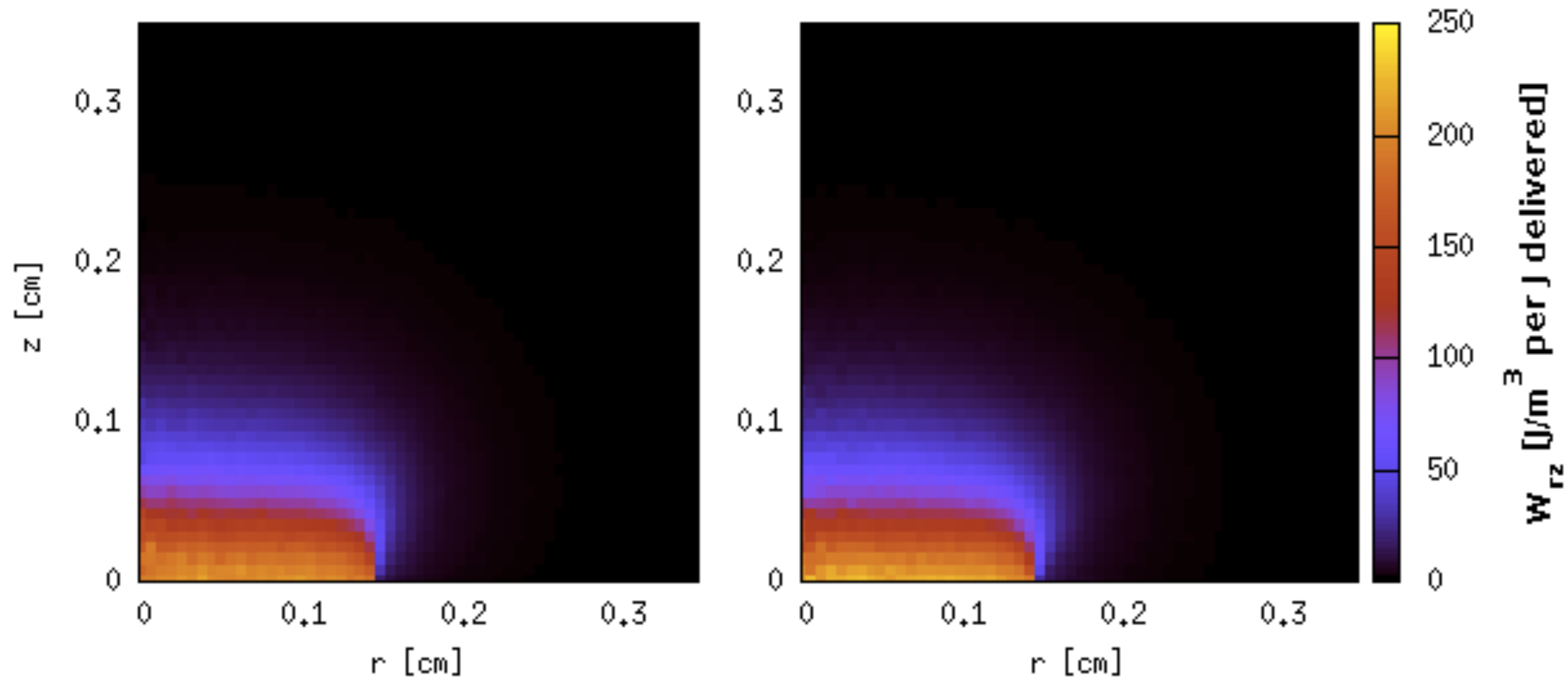

Internal fluence: (dimensionless quantity)

λ [nm]	μ_a [cm^{-1}]	μ_s [cm^{-1}]	g
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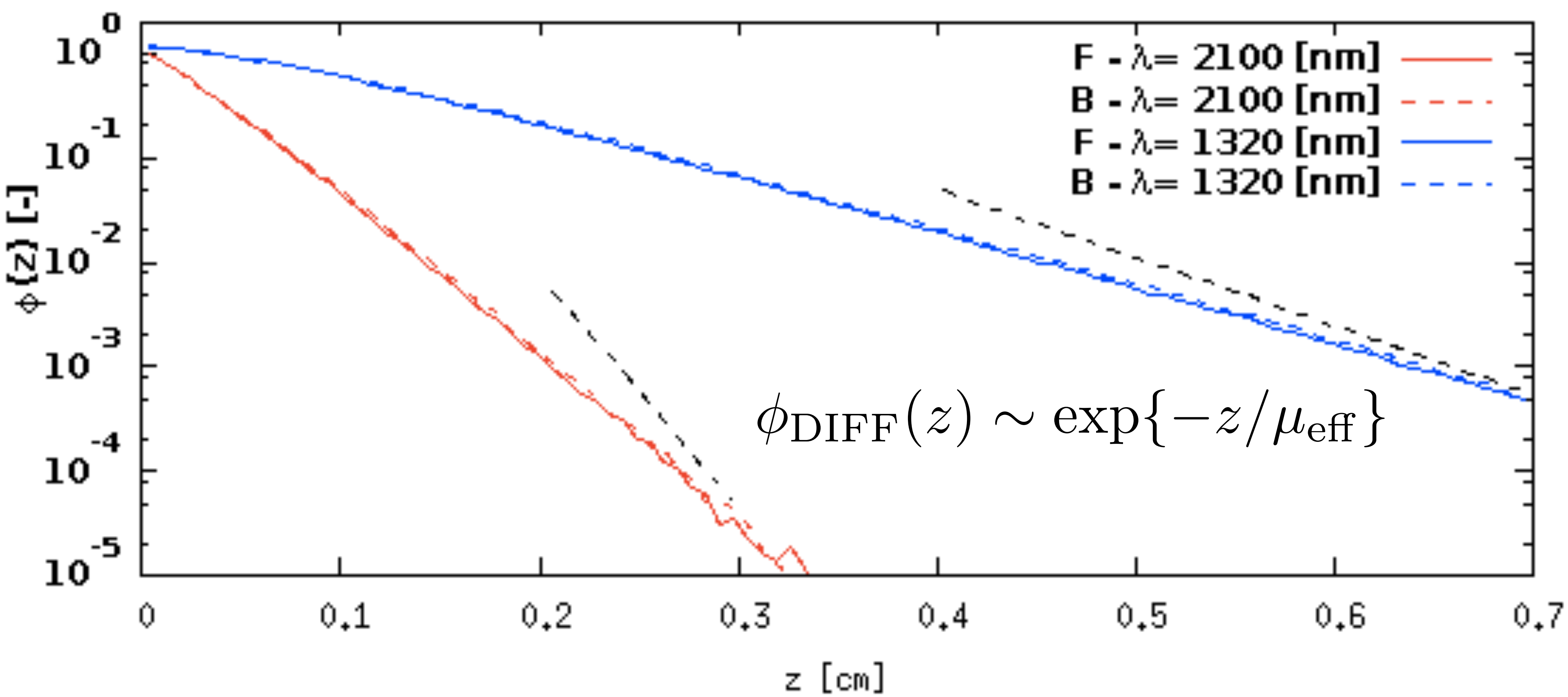
1064	5.9	60.9	0.92
800	5.7	97.0	0.94
633	6.5	143.7	0.95
488	12.2	173.5	0.93

$$\phi(z) = 2\pi \int_0^\infty \mu_a^{-1} W(r,z)\, r \; \mathrm{d}r$$

$$\downarrow$$

$$\phi_z[j] = \mu_a^{-1} \sum_{i=0}^{N_r-1} W[i,j](i+0.5)\Delta r^2$$

photon probability of being absorbed per unit depth



Notes on lecture 4:

- code not optimized for speed! Purely for illustration purpose!
- improvements: effects of refractive index mismatches
- *upcoming exercises:* non-isotropic tissue - human aorta

After lecture: example programs available at

https://github.com/omelchert/CompTissueOpt-2017.git