I.

$$H_j = 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{j}. \quad j = 1,2,3$$

folgende Ungleichung $H_{2^n} \geq 1 + \frac{n}{2}, \ \forall n \in \mathbb{N}$

IV) $P(1): \ 1 + \frac{1}{2} = 1.5$

IS) $P(n): \ 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{2^n}$

$P(n+1): \ 1 + \frac{1}{2} + \ldots + \frac{1}{2^{(n+1)}}$

$$H_{2^{n+1}} = H_{2^n} + \frac{1}{2^n+1} + \frac{1}{2^n+2} + \ldots + \frac{1}{2^n+2^n}$$

$$H_{2^{n+1}} \geq H_{2^n} + 2^n\left(\frac{1}{2^{n+1}}\right)$$

$$\boxed{2^n \frac{1}{2^{n+1}} = \frac{\cancel{2^n}}{\cancel{2^n}+2^n} = \frac{1}{2}}$$

$$H_{2^{n+1}} \geq H_{2^n} + \frac{1}{2}$$

$$H_{2^{n+1}} \geq 1 + \frac{n}{2} + \frac{1}{2} = 1 + \frac{n+1}{2}$$

$\underline{\mathrm{II}}$ $\binom{2n+1}{n} > 2^{n+1}$; $n \in \mathbb{N}$; $n \geq 2$

$$\binom{2n+1}{n} = \frac{(2n+1)!}{n!\,(2n+1-n)!}$$

$$= \frac{(2n+1)!}{n! \cdot (n+1)!}$$

IV) $$\frac{(2\cdot 2+1)!}{2!\cdot(2+1)!} > 2^{2+1} = \frac{5!}{2!\cdot 3!} > 8$$

$$= \frac{120}{12} > 8 = 10 > 8$$

IS) $$\frac{(2(n+1)+1)!}{(n+1)!\,(n+2)!} > 2^{n+2}$$ $\boxed{= \binom{2n+1}{n}}$

$$= \frac{(2n+3)!}{(n+1)!\,(n+2)!} > 2^{n+2}$$

?

$$\binom{2n+3}{n+1} = \frac{(2n+3)(2n+2)\cdots(2n+3-(n+1)+1)}{1\cdot 2\cdot 3\cdots n(n+1)}$$

$$= \frac{(2n+3)(2n+2)\cdots(n+3)}{1\cdot 2\cdot 3\cdots(n+1)}$$

$$= \frac{(2n+2)(2n+3)}{(n+1)(n+2)} \cdot \binom{2n+1}{n} > 2\frac{2n+3}{n+2}2^{n+1} > 2^{n+2}$$

$$= \frac{2(n+2)-1}{n+2} = 2 - \frac{1}{n+2} > 1 \quad \forall n \geq 1$$

III.

IV) $1^2 + 3^2 = \frac{2 \cdot \cancel{3} \cdot 5}{\cancel{3}} = 10$

IS) $1^2 + 3^2 + \ldots + (2n+1)^2 = \frac{(n+1)(2n+1)(2n+3)}{3}$

für $n+1$:

$$1^2 + 3^2 + \ldots + (2n+1)^2 + (2(n+1)^2 = \frac{(n+1)(2n+1)(2n+3)}{3} + (2n+3)^2$$

$$= \frac{(n+1)(2n+1)(2n+3)}{3} + (2n+3)^2$$

$$= \frac{(n+1)(2n+1)(2n+3) + 3(2n+3)^2}{3}$$

$$= \frac{2n+3}{3}\left((n+1)(2n+1) + 3(2n+3)\right)$$

$$= \frac{2n+3}{3}\left(2n^2 + 3n + 1 + 6n + 9\right)$$

$$= \frac{2n+3}{3}\left(2n^2 + 9n + 10\right)$$

$$= \frac{(n+2)(2n+3)(2n+5)}{3}$$

Die Formel gilt für $n+1$

IV $\quad f_k \qquad 0 \; 1 \; 1 \; 2 \; 3 \; 5 \; 8 \; 13 \; 21 \; 34 \; 55$

a) $\forall n \in \mathbb{N} \; \left(f_1^2 + f_2^2 + \dots + f_n^2 = f_n f_{n+1}\right)$

1. Fib Zahl $\quad$ 2. Fib Zahl $\quad$ n-Fib Zahl

$$\sum_{k=1}^{n} f_k^2 = f_n f_{n+1}$$

IV) $P(1)$ $n=1$: $\; 1^2 = 1 \cdot 1$ ✓

IS) $P(n)$: $f_1^2 + f_2^2 + \dots + f_n^2 = f_n f_{n+1}$

$P(n+1)$: $f_1^2 + f_2^2 + \dots + f_n^2 + f_{n+1} \underline{+ f_{n+2}^2} = f_n f_{n+1} \underline{+ f_{n+2}^2}$

$$= \sum_{k=1}^{n+1} f_k^2 + \sum_{k=n+2}^{n+2} f_k^2 = f_n f_{n+1} + f_{n+2}^2$$

$\downarrow$

$? \quad + f_{n+2}^2 =$

hier abgebrochen

IS) $P(n+1)$: $f_1^2 + f_2^2 + \dots + f_n^2 + f_{n+1}^2 \quad = f_n f_{n+1} + f_{n+1}^2$

$= f_{n+1}(f_n + f_{n+1}) \;\triangleleft\; \boxed{f_n f_{n+1} + f_{n+1} f_{n+1}}$!

$= f_{n+1} f_{n+2}$

b) $\forall n \in \mathbb{N} \quad (f_1 + f_3 + \ldots + f_{2n-1} = f_{2n})$

B: $f_1 + f_3 + \ldots + f_{2n-1} + f_{2n+1} \overset{?}{=} f_{2n+2}$

$2n+1 = 2(n+1)-1$

$2n+2-1$

$2n+1$

IV) $P(1), n=1$: $f_1 = 1$, $f_{2\cdot 1} = 1$ ✓

IS) $P(n)$: $f_1 + f_3 + \ldots + f_{2n-1} = f_{2n}$

$P(n+1)$: $f_1 + f_3 + \ldots + f_{2n-1} + f_{2n+1} = f_{2n} + f_{2n+1}$;

$f_4 = \underbrace{3 + f_5 = 5}_{f_6 = 8}$

$= \underline{\underline{f_{2n+2}}}$ ✓

V. $\forall x[f(x) \rightarrow (r(x) \vee s(x))]$

$f(DI) \vee f(DO)$

$\neg r(DI) \wedge \neg s(DI)$

$\neg s(DO)$

1. $\neg r(DI) \wedge \neg s(DI)$
2. $\neg(r(DI) \vee s(DI))$
3. $f(DI) \rightarrow (r(DI) \vee s(DI))$
4. $\neg f(DI)$
5. $f(DI) \vee f(DO)$
6. $f(DO)$
7. $f(DO) \rightarrow (r(DO) \vee s(DO))$
8. $r(DO) \vee s(DO)$
9. $\neg s(DO)$
10. $r(DO)$

$\neg f(DI)$: Dienstag nicht frei

$f(DO)$: DO frei

$r(DO)$: Es regnet am DO