I. $\mathbb{D} = ?$, $\mathbb{W} = ?$

$\mathbb{D} = \{\text{"Bitstrings"}\}$

$\mathbb{W} = 2\mathbb{N} = \{0, 2, 4, 6, \dots\}$

II. $f(x) = x^2 + 1 \qquad g(x) = x + 2$

$$f \circ g = f(x+2) = (x+2)^2 + 1 = x^2 + 4x + 5$$

$$g \circ f = g(x^2+1) = (x^2+1) + 2 = x^2 + 3$$

$$f + g = x^2 + 1 + x + 2 = x^2 + x + 3$$

$$f \cdot g = (x^2+1)(x+2) = x^3 + 2x^2 + x + 2$$

III. a) $\prod_{i=0}^{10} i$

$$\underline{0}\cdot 1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6\cdot 7\cdot 8\cdot 9\cdot 10 = 0$$

b) $\prod_{i=5}^{8} i = 5\cdot 6\cdot 7\cdot 8 = 1680$

c) $\prod_{i=1}^{100} (-1)^i = (-1)^1\cdot(-1)^2\cdot(-1)^3 = (-1)^{\sum_{i=1}^{100} i}$

$$= (-1)^{\frac{100\cdot 101}{2}} = (-1)^{5050} = 1$$

d) $\prod_{j=1}^{10} 2 = 2^{10}$

IV. a) $\sum_{j=0}^{8} 3\cdot 2^j = 3\cdot 2^0 + 3\cdot 2^1 + 3\cdot 2^2\cdot$

b) $\sum_{k=1}^{8} 2^k = \frac{2^9-1}{2-1} - \frac{2^1-1}{2-1} = 2^9 - 2 = 510$

c) $\sum_{l=2}^{8} (-3)^l = (-3)^2 + (-3)^3 + (-3)^4 + (-3)^5 + (-3)^6 + (-3)^7 + (-3)^8$

$= 4923$

d) $\sum_{i=0}^{8} 2\cdot(-3)^i = 2\frac{(-3)^9-1}{-3-1} = 9842$

V.

$$\sum_{k=99}^{200} k^3 = \sum_{k=0}^{200} k^3 - \sum_{k=0}^{98} k^3 = \frac{200^2(200+1)^2}{4} - \frac{98^2(98+1)^2}{4}$$

$$= 404010000 - 23532201$$

$$= \underline{\underline{380477799}}$$

III. Man hat nacheinander

a) $$\prod_{i=0}^{10} i = 0 \cdot 1 \cdot 2 \cdot 3 \cdot \cdots 10 \;=\; 0$$

b) $$\prod_{i=5}^{8} i = 5 \cdot 6 \cdot 7 \cdot 8 \;=\; 1680$$

c) $$\prod_{i=1}^{100} (-1)^i = (-1)^{\sum_{i=1}^{100} i} \;=\; (-1)^{\frac{100\cdot 101}{2}} \;=\; (-1)^{50\cdot 101} \;=\; (-1)^{5050} \;=\; 1$$

d) $$\prod_{i=1}^{10} 2 = 2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2 \;=\; 2^{10}$$

IV. Bekanntlich gilt (endliche geometrische Reihe): $\sum_{k=0}^{n-1} q^k = \frac{q^n - 1}{q-1}$

a) $$\sum_{j=0}^{8} 3\cdot 2^j = 3\sum_{j=0}^{8} 2^j \;=\; 3\frac{2^9-1}{2-1} \;=\; 1533$$

b) $$\sum_{k=1}^{8} 2^k = \frac{2^9-1}{2-1} - \frac{2^1-1}{2-1} \;=\; 2^9 - 2 \;=\; 510$$

c) $$\sum_{l=2}^{8} (-3)^l = \left((-3)^2 + (-3)^3 + ... + (-3)^8\right) \;=\; (-3)^2\left(1 + (-3) + (-3)^2 + ... + (-3)^6\right)$$
$$= (-3)^2 \frac{(-3)^7 - 1}{-3-1} \;=\; 9\frac{1+(3)^7}{4} \;=\; 4923$$

d) $$\sum_{i=0}^{8} 2\cdot(-3)^i = 2\frac{(-3)^9-1}{-3-1} \;=\; 9842$$

V.

$$\sum_{k=99}^{200} k^3 = \sum_{k=0}^{200} k^3 - \sum_{k=0}^{98} k^3$$
$$= \frac{200^2(200+1)^2}{4} - \frac{98^2(98+1)^2}{4} \;=\; 404010000 - 23532201 \;=\; 380477799$$