Logic and Language: Exercise (Week 6)

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1 Syntax

1.1

First, we define the rules of *rightward extraction* $\widehat{\alpha}^r_\diamond$, $\widehat{\sigma}^r_\diamond$ as follows:

$$\frac{f : (A \otimes B) \otimes \Diamond C \to D}{\widehat{\alpha}^r_\diamond f : A \otimes (B \otimes \Diamond C) \to D} \qquad\qquad \frac{f : (A \otimes B) \otimes \Diamond C}{\widehat{\sigma}^r_\diamond f : A \otimes \Diamond C) \otimes B \to D}$$

We can now derive

$$n \otimes ((n\backslash n)/(s/\Diamond\Box np)) \otimes ((np/n) \otimes n) \otimes ((np\backslash s)/np)) \to n$$

as follows:

$$\frac{\dfrac{\dfrac{\overline{np \vdash np}\,^{1_{np}} \quad \overline{s \vdash s}\,^{1_s}}{np\backslash s \vdash np\backslash s}\,\backslash \quad \overline{np \vdash np}\,^{1_{np}}}{\dfrac{(np\backslash s)/np \vdash (np\backslash s)/np}{\dfrac{(np\backslash s)/np \otimes np \vdash np\backslash s}{\dfrac{np \vdash ((np\backslash s)/np)\backslash(np\backslash s)}{\dfrac{\Box(np \vdash \Box((np\backslash s)/np)\backslash(np\backslash s))}{\dfrac{\Diamond\Box np \vdash ((np\backslash s)/np)\backslash(np\backslash s)}{\dfrac{((np\backslash s)/np) \otimes \Diamond\Box np \vdash np\backslash s}{\dfrac{np \otimes (((np\backslash s)/np) \otimes \Diamond\Box np) \vdash s}{\dfrac{(np \otimes (np\backslash s)/np) \otimes \Diamond\Box np \vdash s}{np \otimes ((np\backslash s)/np) \vdash s/\Diamond\Box np \vdash s}\,\triangleright}\,\widehat{\alpha}^r_\diamond}\,\triangleleft^{-1}}\,\triangleleft^{-1}}\,\triangledown^{-1}}\,\Box}\,\triangleleft}\,\triangleright^{-1}}{/}}{\vdots}$$

$$\frac{\dfrac{\overline{n \vdash n}\,^{1_n} \quad \overline{n \vdash n}\,^{1_n}}{n\backslash n \vdash n\backslash n}\,\backslash \qquad np \otimes ((np\backslash s)/np) \vdash s/\Diamond\Box np \vdash s}{\dfrac{(n\backslash n)/(s/\Diamond\Box np) \vdash (n\backslash n)/(np \otimes ((np\backslash s)/np)))}{\dfrac{((n\backslash n)/(s/\Diamond\Box np)) \otimes (np \otimes ((np\backslash s)/np)) \vdash n\backslash n}{\dfrac{np \otimes ((np\backslash s)/np) \vdash ((n\backslash n)/(s\Diamond\Box np))\backslash(n\backslash n)}{np \vdash ((n\backslash n)/(s/\Diamond\Box np))\backslash(n\backslash n))/((np\backslash s)/np))}\,\triangleright}\,\triangleleft}\,\triangleright^{-1}}{/} \qquad \overline{n \vdash n}\,^{1_n}$$

$$\frac{np/n \vdash (((n\backslash n)/(s/\Diamond\Box np))\backslash(n\backslash n))/((np\backslash s)/np))/n}{\dfrac{np/n \otimes n \vdash (((n\backslash n)/(s/\Diamond\Box np))\backslash(n\backslash n))/((np\backslash s)/np))}{\dfrac{(np/n \otimes n) \otimes (np\backslash s)/np \vdash ((n\backslash n)/(s/\Diamond\Box np))\backslash(n\backslash n)}{\dfrac{((n\backslash n)/(s/\Diamond\Box np)) \otimes (((np/n) \otimes ((np\backslash s)/np)) \vdash n\backslash n}{n \otimes ((n\backslash n)/(s/\Diamond\Box np)) \otimes ((np/n) \otimes n) \otimes ((np\backslash s)/np)) \vdash n}\,\triangleleft^{-1}}\,\triangleleft^{-1}}\,\triangleright^{-1}}\,\triangleright^{-1}}{/}$$