Logic and Language: Exercise (Week 6)

Orestis Melkonian [6176208], Konstantinos Kogkalidis [6230067]

1 Syntax

1.1

First, we define the rules of *rightward extraction* $\widehat{\alpha}^r_\diamond$, $\widehat{\sigma}^r_\diamond$:

$$\dfrac{f : A \otimes (B \otimes \Diamond C) \to D}{\widehat{\alpha}^r_\diamond f : (A \otimes B) \otimes \Diamond C \to D} \qquad\qquad \dfrac{f : (A \otimes \Diamond C) \otimes B \to D}{\widehat{\sigma}^r_\diamond f : (A \otimes B) \otimes \Diamond C \to D}$$

We can now proceed with the derivation of

$$n \otimes ((n\backslash n)/(s/\Diamond\Box np)) \otimes ((np/n) \otimes n) \otimes ((np\backslash s)/np)) \to n$$

as follows:

$$\dfrac{\dfrac{\dfrac{\dfrac{}{n \vdash n}1_n \quad \dfrac{}{n \vdash n}1_n}{n\backslash n \vdash n\backslash n}\backslash \qquad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{np \vdash np}1_{np} \quad \dfrac{}{n \vdash n}1_n}{np\backslash n \vdash np\backslash n}\backslash}{(np\backslash n) \otimes n \vdash np}\vartriangleright^{-1} \quad \dfrac{}{s \vdash s}1_s}{np\backslash s \vdash ((np\backslash n) \otimes n)\backslash s}\backslash \quad \dfrac{\dfrac{\dfrac{}{np \vdash np}1_{np}}{\Box np \vdash \Box np}\Box}{\Diamond\Box np \vdash np}\triangledown^{-1}}{(np\backslash s)/np \vdash (((np\backslash n) \otimes n)\backslash s)/\Diamond\Box np}/}{(np\backslash s)/np \otimes \Diamond\Box np \vdash ((np\backslash n) \otimes n)\backslash s}\vartriangleright^{-1}}{((np\backslash n) \otimes n) \otimes ((np\backslash s)/np \otimes \Diamond\Box np) \vdash s}\vartriangleleft^{-1}}{(((np\backslash n) \otimes n) \otimes ((np\backslash s)/np)) \otimes \Diamond\Box np \vdash s}\widehat{\alpha}^r_\diamond}{((np\backslash n) \otimes n) \otimes ((np\backslash s)/np) \vdash s/\Diamond\Box np}\vartriangleright}{(n\backslash n)/(s/\Diamond\Box np) \vdash (n\backslash n)/(((np\backslash n) \otimes n) \otimes (np\backslash s)/np)}/}{(n\backslash n)/(s/\Diamond\Box np) \otimes (((np/n) \otimes (np\backslash s)/np) \vdash n\backslash n}\vartriangleright^{-1}}{n \otimes ((n\backslash n)/(s/\Diamond\Box np) \otimes (((np/n) \otimes n) \otimes (np\backslash s)/np)) \vdash n}\vartriangleleft^{-1}$$

2 Interpretation

2.1

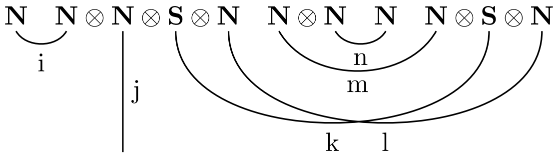
2.2

By working our way from the leaves of the proof tree, we get the following generalized Kronecker delta:

$$\textbf{island}_i \otimes \textbf{that}_{j,k,l,m} \otimes \textbf{the}_{n,o} \otimes \textbf{hurricane}_p \otimes \textbf{destroyed}_{q,r,s} \xrightarrow{\delta^{i,k,l,m,n,o}_{j,t,r,s,q,p}} \mathbf{v}^{obj}_r \in \mathrm{N}$$

$$\mathbf{v}^{obj}_r = \textbf{island}_i \otimes \textbf{that}_{i,j,k,l} \otimes \textbf{the}_{m,n} \otimes \textbf{hurricane}_n \otimes \textbf{destroyed}_{m,k,l} \quad \text{(relabeled)}$$

We give the matching diagram in the figure below:



2.3

In order to calculate the semantic value for the relative clause body 'the hurricane destroyed', we first apply **the** to **hurricane** (which leaves **hurricane** unchanged) and then proceed by the tensor application...

2.4

The interpreted type for the relative pronoun is:

$$\lceil (n\backslash n)/(s/\Diamond\Box np)\rceil = \lceil n\backslash n\rceil \otimes \lceil s/\Diamond\Box np\rceil = \lceil n\rceil \otimes \lceil n\rceil \otimes \lceil s\rceil \otimes \lceil \Diamond\Box np\rceil = N \otimes N \otimes S \otimes N$$

We can now give the following Frobenius recipe for **that**:

$$I \cong I \otimes I \xrightarrow{\eta_N \otimes \eta_N} N \otimes N \otimes N \otimes N \cong N \otimes N \otimes N \otimes I \otimes N \xrightarrow{1_N \otimes \mu_N \otimes \zeta_S \otimes 1_N} N \otimes N \otimes S \otimes N$$

In order to obtain the final interpretation, we do the following (dictated from the above recipe):

1. Reduce the rank of the transitive verb by summing over the S component, thus obtaining the following matrix:

$$\textbf{collapsed\_destroyed} = \begin{pmatrix} (\ 9 & 2 & 3\ ) \\ (\ 1 & -5 & 2\ ) \\ (\ -1 & -8 & 1\ ) \end{pmatrix}$$

2. Apply **collapsed_destroyed** to **the_hurricane** in object position:

$$\textbf{the\_hurricane\_destroyed} = \begin{pmatrix} 17 & -9 & 4 \end{pmatrix}$$

3. Multiply **the_hurricane_destroyed** element-wise with **island**:

$$\textbf{island\_that\_the\_hurricane\_destroyed} = \begin{pmatrix} -85 & -36 & 0 \end{pmatrix}$$

Note The matrix operations were computed in Python, as shown below:

```python
import numpy as np
hurricane = np.array([3,-5,5])
island = np.array([-5,4,0])
destroyed = np.array([[[4,-3,1],[5,5,2]],[[-1,-2,2],[2,-3,0]], [[-1,-4,2],[0,-4,-1]]])
the = np.array([[1, 0,0], [0,1,0], [0,0,1]])
the_hurricane = np.matmul(the, hurricane)  # == hurricane
# the_hurricane_destroyed = np.tensordot(the_hurricane, destroyed, axes=1)
collapsed_destroyed = destroyed[:,0,:] + destroyed[:,1,:]
the_hurricane_destroyed = np.matmul(hurricane, collapsed_destroyed)
island_that_the_hurricane_destroyed = island * hurricane_destroyed
```