Logic and Language: Exercise (Week 5)

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1 LG: continuation semantics

1.1

For each term, we compute the ILL type by first observing the polarity of the sequent and then using the table to interpret complex types.

$$\begin{aligned}
&\lceil \mathbf{some} \rceil \\
=&\lceil np \otimes n \rceil^{\perp} \qquad \{\textit{Negative Hypothesis}\} \\
=&(\lceil np \rceil^{\perp} \otimes \lceil n \rceil)^{\perp} \qquad \{\ \lceil A \otimes B \rceil \textit{ with } A \textit{ and } B \textit{ positive}\}
\end{aligned}$$

$$\begin{aligned}
&\lceil \mathbf{popular} \rceil \\
=&\lceil n \otimes n \rceil^{\perp} \qquad \{\textit{Negative Hypothesis}\} \\
=&(\lceil n \rceil^{\perp} \otimes \lceil n \rceil)^{\perp} \qquad \{\ \lceil A \otimes B \rceil \textit{ with } A \textit{ and } B \textit{ positive}\}
\end{aligned}$$

$$\begin{aligned}
&\lceil \mathbf{saint} \rceil \\
=&\lceil n \rceil \qquad \{\textit{Positive Hypothesis}\}
\end{aligned}$$

$$\begin{aligned}
&\lceil \mathbf{arrived} \rceil \\
=&\lceil np \otimes s \rceil^{\perp} \qquad \{\textit{Negative Hypothesis}\} \\
=&(\lceil np \rceil \otimes \lceil s \rceil^{\perp})^{\perp} \qquad \{\lceil A \otimes B \rceil \textit{ with } A \textit{ positive and } B \textit{ negative}\}
\end{aligned}$$

$$\begin{aligned}
&\alpha \\
=&\lceil s \rceil^{\perp} \qquad \{\textit{Negative Hypohesis}\}
\end{aligned}$$

$$\begin{aligned}
&z \\
=&\lceil np \rceil \qquad \{\textit{Positive Hypothesis}\}
\end{aligned}$$

$$\begin{aligned}
&y \\
=&\lceil n \rceil \qquad \{\textit{Positive Hypothesis}\}
\end{aligned}$$

The ILL types then are:

TERM	TYPE
$\lceil \textbf{some} \rceil$	$(\lceil np \rceil^{\perp} \otimes \lceil n \rceil)^{\perp}$
$\lceil \textbf{popular} \rceil$	$(\lceil n \rceil^{\perp} \otimes \lceil n \rceil)^{\perp}$
$\lceil \textbf{saint} \rceil$	$\lceil n \rceil$
$\lceil \textbf{arrived} \rceil$	$(\lceil np \rceil \otimes \lceil s \rceil^{\perp})^{\perp}$
α	$\lceil s \rceil^{\perp}$
z	$\lceil np \rceil$
y	$\lceil n \rceil$

1.2

SOURCE TYPE	CONSTANT	$\lceil . \rceil^{\ell}$
n/n	popular	$\lambda \langle c, y \rangle.(c\ (\lambda z.\ \wedge (y\ z)\ (\text{POPULAR}\ z)))$

1.3

1. We compute the interpretation below:

$$\lceil \ddagger \rceil = \lambda a_0.(\lceil \textbf{arrived} \rceil\ \langle \lambda \beta_0.(\lceil \textbf{some} \rceil\ \langle \beta_0, \lambda \gamma_0.(\lceil \textbf{popular} \rceil\ \langle \gamma_0, \lambda a_1.(\lceil \textbf{saint} \rceil\ a_1) \rangle) \rangle), a_0 \rangle)$$

2. The adjucted $\cdot^{\ell}$ translations are the following:

$$\begin{aligned}
\lceil \textbf{some} \rceil^{\ell} &= \lambda \langle x, k \rangle.(\exists \lambda z.(\wedge\ (k\ \lambda \theta.(\theta\ z))\ (x\ z))) \\
\lceil \textbf{popular} \rceil^{\ell} &= \lambda \langle c, k \rangle.(c\ (\lambda z.(\wedge\ (\text{POPULAR}\ z)\ (k\ \lambda \theta.(\theta\ z))))) \\
\lceil \textbf{saint} \rceil^{\ell} &= \lambda c.(c\ \text{SAINT}) \\
\lceil \textbf{arrived} \rceil^{\ell} &= \lambda \langle k, c \rangle.(k\ \lambda z.(c\ (\text{ARRIVED}\ z)))
\end{aligned}$$

We can now corroborate the α-equivalence of the two $\cdot^{\ell}$ translations:

$$\begin{aligned}
\lceil \ddagger \rceil^{\ell} = \lambda a_0.(&\underline{\lambda \langle k, c \rangle.(k\ \lambda z.(c\ (\text{ARRIVED}\ z)))} \\
&\langle \lambda \beta_0.(\lambda \langle x, k \rangle.(\exists \lambda z.(\wedge\ (k\ \lambda \theta.(\theta\ z))\ (x\ z)))\ \langle \beta_0, \lambda \gamma_0.(\\
&\quad \lambda \langle c, k \rangle.(c\ (\lambda z.(\wedge\ (\text{POP}\ z)\ (k\ \lambda \theta.(\theta\ z)))))\ \langle \gamma_0, \lambda a_1.(\lambda c.(c\ \text{SAINT})\ a_1) \rangle) \rangle), a_0 \rangle) \\
\to_{\beta}^{*}\ & \lambda a_0.(\underline{\lambda \langle x, k \rangle.(\exists \lambda z.(\wedge\ (k\ \lambda \theta.(\theta\ z))\ (x\ z)))}\ \langle \lambda z.(a_0\ (\text{ARRIVED}\ z)), \lambda \gamma_0.(\\
&\quad \lambda \langle c, k \rangle.(c\ (\lambda z.(\wedge\ (\text{POPULAR}\ z)\ (k\ \lambda \theta.(\theta\ z)))))\ \langle \gamma_0, \lambda a_1.(\lambda c.(c\ \text{SAINT})\ a_1) \rangle) \rangle) \\
\to_{\beta}^{*}\ & \lambda a_0.(\exists \lambda z.(\wedge\ (\underline{\lambda \langle c, k \rangle.(c\ (\lambda z.(\wedge\ (\text{POPULAR}\ z)\ (k\ \lambda \theta.(\theta\ z)))))} \\
&\quad \langle \lambda \theta.(\theta\ z), \lambda a_1.(\lambda c.(c\ \text{SAINT})\ a_1) \rangle)\ (a_0\ (\text{ARRIVED}\ z))) \\
\to_{\beta}^{*}\ & \lambda a_0.(\exists \lambda z.(\wedge\ (\underline{\lambda \theta.(\theta\ z)}\ (\lambda z.(\wedge\ (\text{POPULAR}\ z)\ (\underline{\lambda c.(c\ \text{SAINT})}\ (\lambda \theta.(\theta\ z))))))\ (a_0\ (\text{ARRIVED}\ z))) \\
\to_{\beta}^{*}\ & \lambda a_0.(\exists \lambda z.(\wedge\ (\wedge\ (\text{POPULAR}\ z)\ (\text{SAINT}\ z))\ (a_0\ (\text{ARRIVED}\ z))) \\
\to_{\alpha}\ & \lambda c.(\exists \lambda z.(\wedge\ (\wedge\ (\text{POPULAR}\ z)\ (\text{SAINT}\ z))\ (c\ (\text{ARRIVED}\ z))) \\
\to_{\alpha}\ & \lambda c.(\exists \lambda x.(\wedge\ (\wedge\ (\text{POPULAR}\ x)\ (\text{SAINT}\ x))\ (c\ (\text{ARRIVED}\ x))) \\
=\ & \lceil \dagger \rceil^{\ell}
\end{aligned}$$

2 Pregroups

2.1

(4)

$(\to)$

$$\dfrac{\dfrac{}{1^l \to 1^l 1}\,(1) \qquad \dfrac{}{1^l 1 \to 1}\,(2)}{1^l \to 1}\,\to$$

$(\leftarrow)$

$$\dfrac{\dfrac{}{1 \to 1^l 1}\,(2) \qquad \dfrac{}{1^l 1 \to 1^l}\,(1)}{1 \to 1^l}\,\to$$

(5) $(\to)$:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A^{rl} \to A^{rl} 1}\,(1) \qquad \dfrac{}{1 \to A^r A}\,(2)}{A^{rl} \to A^{rl}(A^r A)}\,\to}{A^{rl} \to (A^{rl} A^r) A}\,(1) \qquad \dfrac{}{A^{rl} A^r \to 1}\,(2)}{A^{rl} \to 1A}\,\to \qquad \dfrac{}{1A \to A}\,(1)}{A^{rl} \to A}\,\to$$

$(\leftarrow)$:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A \to A1}\,(1) \qquad \dfrac{}{1 \to A^r A^{rl}}\,(2)}{A \to A(A^r A^{rl})}\,\to}{A \to (AA^r) A^{rl}}\,(1) \qquad \dfrac{}{AA^r \to 1}\,(3)}{A \to 1A^{rl}}\,\to \qquad \dfrac{}{1A^{rl} \to A^{rl}}\,(1)}{A \to A^{rl}}\,\to$$

(6) $(\to)$:

$$\dfrac{\dfrac{}{(AB)^l \to (1(AB))^l}\,\to \qquad \dfrac{}{1 \to (B^l A^l)^r (B^l A^l)}\,(3)}{(AB)^l \to (((B^l A^l)^r (B^l A^l))(AB))^l}\,\to$$

$$\dfrac{\dfrac{\dfrac{(AB)^l \to (((B^l A^l)^r (B^l A^l))(AB))^l}{(AB)^l \to ((B^l A^l)^r ((B^l A^l)(AB)))^l}\,(1)}{(AB)^l \to ((B^l A^l)^r (B^l (A^l (AB))))}\,(1)}{(AB)^l \to ((B^l A^l)^r (B^l ((A^l A) B)))}\,(1)$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{(AB)^l \to ((B^l A^l)^r (B^l ((A^l A) B))) \qquad \dfrac{}{A^l A \to 1}\,(2)}{(AB)^l \to ((B^l A^l)^r (B^l (1B)))^l}\,\to \qquad \dfrac{}{1B \to B}\,(1)}{(AB)^l \to ((B^l A^l)^r (B^l B))^l}\,\to \qquad \dfrac{}{B^l B \to 1}\,(2)}{(AB)^l \to ((B^l A^l)^r 1)^l}\,\textit{todo} \qquad \dfrac{}{(B^l A^l)^r 1 \to (}}{\dfrac{(AB)^l \to ((B^l A^l)^r)^l}{(AB)^l \to B^l A^l}\,(5)}$$

2.2

We first calculate the pregroup translation of the given sequent:

$$
\begin{aligned}
& \overline{(p/((q/q)/r))/r} \\
=& \overline{(p/((q/q)/r))}\ r^{\ell} \\
=& p\ \overline{((q/q)/r)}^{\ell}\ r^{\ell} \\
=& p\ (\overline{(q/q)}\ r^{\ell})^{\ell}\ r^{\ell} \\
=& p\ ((q\ q^{\ell})\ r^{\ell})^{\ell}\ r^{\ell} \\
=& p\ (q\ (q^{\ell}\ r^{\ell}))^{\ell}\ r^{\ell} && \{\textit{rule (1) from 2.1}\} \\
=& p\ (q^{\ell}\ r^{\ell})^{\ell}\ q^{\ell}\ r^{\ell} && \{\textit{rule (6) from 2.1}\} \\
=& p\ r^{\ell^{\ell}}\ q^{\ell^{\ell}}\ q^{\ell}\ r^{\ell} && \{\textit{rule (6) from 2.1}\}
\end{aligned}
$$

Finally, we prove the sequent by drawing a string diagram: