D^3 AS A 2-MCFL

Orestis Melkonian, Konstantinos Kogkalidis

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Utrecht University

Introduction/Definition

Some examples

Dyck words

- abc
- aabbcc
- abcabcabacbc

Some examples

Dyck words

- abc
- aabbcc
- abcabcabacbc

Non-dyck words

- aabb

Some examples

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- abcabcabacbc

Non-dyck words

- aabb
- aabbbcc

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- abcacb

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- aabb
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$$a\,b\,a\,b\,a\,c\,b\,c\,a\,b\,c\,c$$

First-match policy

Some examples

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a b a b a c b c a b c c

First-match policy

Some examples

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- abc
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Non-dyck words

- aabb
- aabbbcc
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$ababacbcabcc$

First-match policy

Some examples

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Non-dyck words

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$a\,b\,a\,b\,a\,c\,b\,c\,a\,b\,c\,c$

First-match policy

Motivation

G_0: Grammar of triple insertions

$$\mathsf{S}(xy) \leftarrow \mathsf{W}(x, y). \tag{1}$$

$$\mathsf{W}(\epsilon, xy\mathbf{abc}) \leftarrow \mathsf{W}(x, y). \tag{2}$$

$$\mathsf{W}(\epsilon, x\mathbf{a}y\mathbf{bc}) \leftarrow \mathsf{W}(x, y). \tag{3}$$

$$\ldots\ldots$$

$$\mathsf{W}(\mathbf{ab}x\mathbf{c}y, \epsilon) \leftarrow \mathsf{W}(x, y). \tag{60}$$

$$\mathsf{W}(\mathbf{abc}xy, \epsilon) \leftarrow \mathsf{W}(x, y). \tag{61}$$

$$\mathsf{W}(\epsilon, \mathbf{abc}). \tag{62}$$

$$\mathsf{W}(\mathbf{a}, \mathbf{bc}). \tag{63}$$

$$\mathsf{W}(\mathbf{ab}, \mathbf{c}). \tag{64}$$

$$\mathsf{W}(\mathbf{abc}, \epsilon). \tag{65}$$

G_0: Grammar of triple insertions

$a \quad a \quad b \quad b \quad a \quad c \quad b \quad a \quad c \quad c \quad b \quad c$

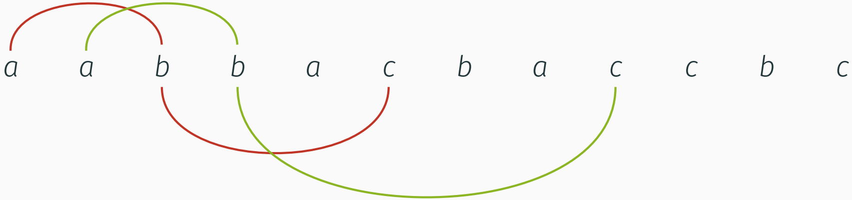
Straddling counter-example

G_0: Grammar of triple insertions

$$a \quad a \quad b \quad b \quad a \quad c \quad b \quad a \quad c \quad c \quad b \quad c$$

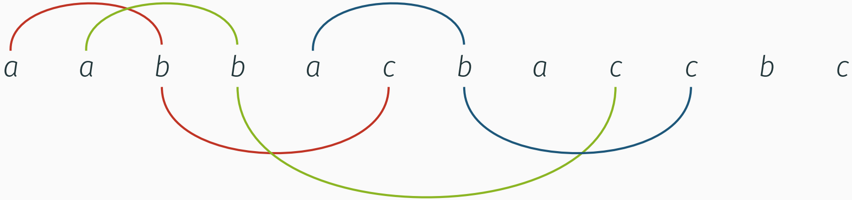
Straddling counter-example

G_0: Grammar of triple insertions



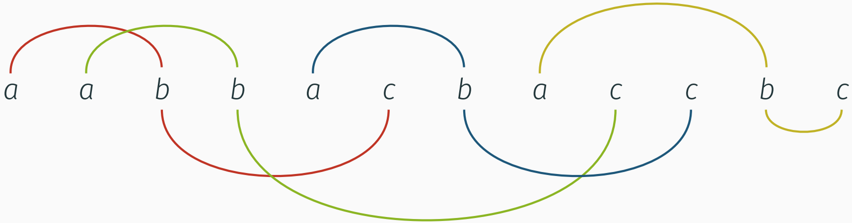
Straddling counter-example

G_0: Grammar of triple insertions



Straddling counter-example

G_0: Grammar of triple insertions



Straddling counter-example

Notation

$$\mathcal{O}_m[\![conclusion \leftarrow premises \mid \{partial\ orderings\ of\ inserted\ elements\}$$

Meta-grammars: Introduction

Notation

$$\mathcal{O}_m[\![conclusion \leftarrow premises \mid \{partial\ orderings\ of\ inserted\ elements\}]\!]$$

Meta-grammar G_1

$$\left.\begin{array}{l} \mathcal{O}_2[\![W \leftarrow \epsilon \mid \{a < b < c\}]\!]. \\ \mathcal{O}_2[\![W \leftarrow W_{xy} \mid \{x < y,\ a < b < c\}]\!]. \end{array}\right\} \begin{array}{c} \text{TRIPLE} \\ \text{INSERTION} \end{array}$$

Notation

$$\mathcal{O}_m[\![\textit{conclusion} \leftarrow \textit{premises} \mid \{\textit{partial orderings of inserted elements}\}]\!]$$

Meta-grammar G_1

$$\left.\begin{array}{l}\mathcal{O}_2[\![W \leftarrow \epsilon \mid \{a < b < c\}]\!].\\ \mathcal{O}_2[\![W \leftarrow W_{xy} \mid \{x < y,\ a < b < c\}]\!].\end{array}\right\} \text{TRIPLE INSERTION}$$

$+$

$$\mathcal{O}_2[\![W \leftarrow W_{xy}, W_{zw} \mid \{x < y,\ z < w\}]\!].\ \Big\} \text{INTERLEAVING WORDS}$$

G_2: Adding states

$$\left.\begin{array}{l}\mathcal{O}_2[\![A^+ \leftarrow \epsilon \mid \{a\}]\!].\\ \mathcal{O}_2[\![B^+ \leftarrow \epsilon \mid \{b\}]\!].\\ \mathcal{O}_2[\![C^+ \leftarrow \epsilon \mid \{c\}]\!].\end{array}\right\}\ \text{BASE CASES}$$

$$\left.\begin{array}{l}\mathcal{O}_2[\![C^- \leftarrow A^+, B^+ \mid \{x < y < z < w\}]\!].\\ \mathcal{O}_2[\![B^- \leftarrow A^+, C^+ \mid \{x < y < z < w\}]\!].\\ \mathcal{O}_2[\![A^- \leftarrow B^+, C^+ \mid \{x < y < z < w\}]\!].\\ \mathcal{O}_2[\![A^+ \leftarrow C^-, B^- \mid \{x < y < z < w\}]\!].\\ \mathcal{O}_2[\![B^+ \leftarrow C^-, A^- \mid \{x < y < z < w\}]\!].\\ \mathcal{O}_2[\![C^+ \leftarrow B^-, A^- \mid \{x < y < z < w\}]\!].\\ \forall\ \mathrm{K} \in \mathcal{S} \setminus \mathrm{W} :\ \mathcal{O}_2[\![\mathrm{K} \leftarrow \mathrm{K}_{xy}, \mathrm{W}_{zw} \mid \{x < y,\ z < w\}]\!].\end{array}\right\}\ \text{COMBINATIONS}$$

$$\left.\begin{array}{l}\mathcal{O}_2[\![W \leftarrow A^+, A^- \mid \{x < y < z < w\}]\!].\\ \mathcal{O}_2[\![W \leftarrow C^-, C^+ \mid \{x < y < z < w\}]\!].\end{array}\right\}\ \text{CLOSURES}$$

G_3: G_2 + Universal triple insertion

$$G_3 = G_2 + \forall\, K \in \mathcal{S} \setminus W : \quad \mathcal{O}_2[\![K \leftarrow K_{xy} \mid \{x < y,\ a < b < c\}]\!].$$

Demo

Refining states

Example

$$
\begin{array}{ccc}
 & A^{+}(x, y) & \\
\swarrow & & \searrow \\
A^{+}_{left} & & A^{+}_{right}
\end{array}
$$

Refining states

Example

$$\begin{tikzcd}
& A^{+}(x, y) \arrow[dl] \arrow[dr] & \\
A^{+}_{left} & & A^{+}_{right}
\end{tikzcd}$$

$$\vdots$$

$$\begin{tikzcd}
& C^{-}(x, y) \arrow[dl] \arrow[d] \arrow[dr] & \\
C^{-}_{left} & C^{-}_{left,\ right} & C^{-}_{right}
\end{tikzcd}$$

Refining states

Example

$$
\begin{array}{ccc}
 & A^{+}(x,y) & \\
\swarrow & & \searrow \\
A^{+}_{left} & & A^{+}_{right} \\
 & \vdots & \\
 & C^{-}(x,y) & \\
\swarrow & \downarrow & \searrow \\
C^{-}_{left} & C^{-}_{left,\ right} & C^{-}_{right}
\end{array}
$$

Why?

Refining states

Example

$$\begin{tikzcd}[column sep=small]
 & A^{+}(x,y) \arrow[dl] \arrow[dr] & \\
A^{+}_{left} & \vdots & A^{+}_{right}
\end{tikzcd}$$

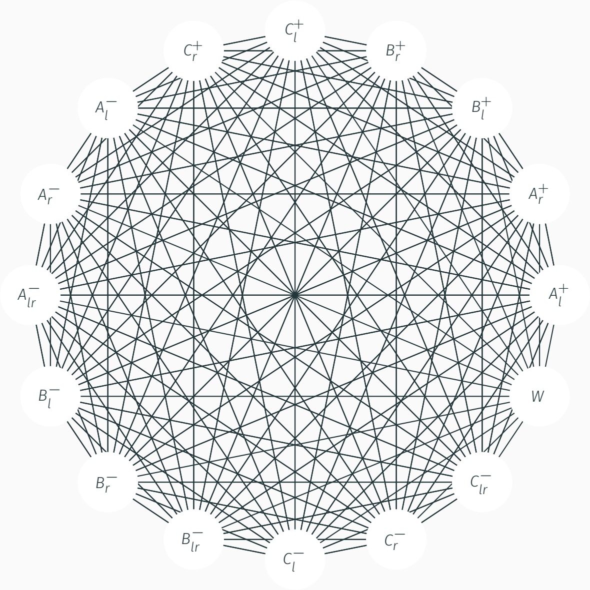
$$\begin{tikzcd}[column sep=small]
 & C^{-}(x,y) \arrow[dl] \arrow[d] \arrow[dr] & \\
C^{-}_{left} & C^{-}_{left,\ right} & C^{-}_{right}
\end{tikzcd}$$

Why?

New orders in interactions

$$C^{-}(xz,\ wy) \leftarrow A^{+}_{left}(x,\ y),\ B^{+}(z,\ w).$$

Refining states: Interactions



G_4: Automatic Rule Inference

State descriptors $\mathcal{D}$

$$
\begin{aligned}
\mathrm{W} &\mapsto (\epsilon, \epsilon) & \mathrm{A}_r^- &\mapsto (\epsilon, bc) \\
\mathrm{A}_l^+ &\mapsto (a, \epsilon) & \mathrm{A}_{lr}^- &\mapsto (b, c) \\
\mathrm{A}_r^+ &\mapsto (\epsilon, a) & \mathrm{B}_l^- &\mapsto (ac, \epsilon) \\
\mathrm{B}_l^+ &\mapsto (b, \epsilon) & \mathrm{B}_r^- &\mapsto (\epsilon, ac) \\
\mathrm{B}_r^+ &\mapsto (\epsilon, b) & \mathrm{B}_{lr}^- &\mapsto (a, c) \\
\mathrm{C}_l^+ &\mapsto (c, \epsilon) & \mathrm{C}_l^- &\mapsto (ab, \epsilon) \\
\mathrm{C}_r^+ &\mapsto (\epsilon, c) & \mathrm{C}_r^- &\mapsto (\epsilon, ab) \\
\mathrm{A}_l^- &\mapsto (bc, \epsilon) & \mathrm{C}_{lr}^- &\mapsto (a, b)
\end{aligned}
$$

Automatic Rule Inference: Example

$$\textsc{Case of } A^{-}_{lr}(x_b,\ y_c) + B^{-}_{lr}(z_a,\ w_c)$$

<u>PERMUTATION</u>

$$\vdots$$

$$(zxw,\ y)$$

$$\vdots$$

$$(xzw,\ y)$$

$$\vdots$$

Automatic Rule Inference: Example

Case of $A^{-}_{lr}(x_b,\ y_c) + B^{-}_{lr}(z_a,\ w_c)$

Permutation		Descriptor
$\vdots$		$\vdots$
$(zxw,\ y)$	$\longrightarrow$	$(abc,\ c)$
$\vdots$		$\vdots$
$(xzw,\ y)$	$\longrightarrow$	$(bac,\ c)$
$\vdots$		$\vdots$

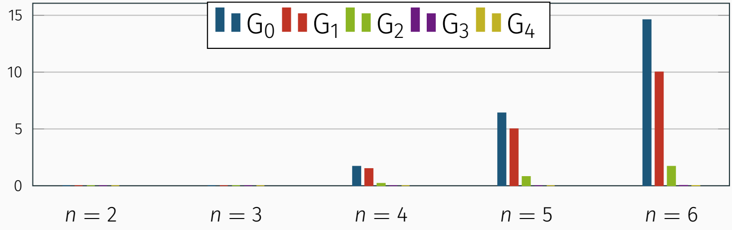
Automatic Rule Inference: Example

Case of $A^{-}_{lr}(x_b,\ y_c) + B^{-}_{lr}(z_a,\ w_c)$

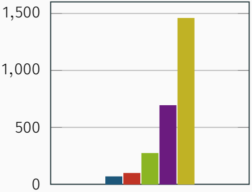
$$
\begin{array}{ccccc}
\underline{\text{PERMUTATION}} & & \underline{\text{DESCRIPTOR}} & & \underline{\text{ELIMINATED}} \\
 & & & & \vdots \\
\vdots & & \vdots & & \\
 & & & \nearrow & (c,\ \epsilon) \mapsfrom C^{+}_{l} \\
(zxw,\ y) & \longrightarrow & (abc,\ c) & & \\
 & & & \searrow & (\epsilon,\ c) \mapsfrom C^{+}_{r} \\
\vdots & & \vdots & & \vdots \\
(xzw,\ y) & \longrightarrow & (bac,\ c) & \longrightarrow & (bac,\ c) \notin \mathcal{D} \\
\vdots & & \vdots & & \vdots
\end{array}
$$

Results

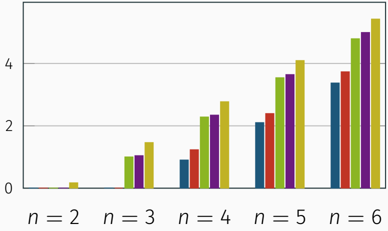
Counter-examples (%)



Rule size



Computation Time (in log(sec))



Correspondences: Young Tableaux

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1

3

5

1	2
3	4
5	6

Correspondences: Promotion on Young Tableaux

1	2
3	4
5	6

	1
2	3
4	5

Correspondences: Promotion on Young Tableaux

1	2
3	4
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1	
2	3
4	5

1	2
3	4
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1	3
2	
4	5

Correspondences: Promotion on Young Tableaux

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Growth rules