Logic and Language: Exercise (Week 6)

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1 Syntax

1.1

First, we define the rules of *rightward extraction* $\widehat{\alpha}^r_\diamond$, $\widehat{\sigma}^r_\diamond$:

$$\frac{f : A \otimes (B \otimes \Diamond C) \to D}{\widehat{\alpha}^r_\diamond f : (A \otimes B) \otimes \Diamond C \to D} \qquad \frac{f : (A \otimes \Diamond C) \otimes B \to D}{\widehat{\sigma}^r_\diamond f : (A \otimes B) \otimes \Diamond C \to D}$$

We can now proceed with the derivation of

$$n \otimes ((n\backslash n)/(s/\Diamond\Box np)) \otimes ((np/n) \otimes n) \otimes ((np\backslash s)/np)) \to n$$

as follows:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{np \vdash np}\,1_{np} \quad \dfrac{}{n \vdash n}\,1_n}{np\backslash n \vdash np\backslash n}\backslash}{(np\backslash n) \otimes n \vdash np}\triangleright^{-1} \quad \dfrac{}{s \vdash s}\,1_s}{np\backslash s \vdash ((np\backslash n) \otimes n)\backslash s}\backslash \quad \dfrac{\dfrac{\dfrac{}{np \vdash np}\,1_{np}}{\Box np \vdash \Box np}\Box}{\Diamond\Box np \vdash np}\triangledown^{-1}}{(np\backslash s)/np \vdash (((np\backslash n) \otimes n)\backslash s)/\Diamond\Box np}/}{(np\backslash s)/np \otimes \Diamond\Box np \vdash ((np\backslash n) \otimes n)\backslash s}\triangleright^{-1}}{((np\backslash n) \otimes n) \otimes ((np\backslash s)/np \otimes \Diamond\Box np) \vdash s}\triangleleft^{-1} \quad \dfrac{\dfrac{}{n \vdash n}\,1_n \quad \dfrac{}{n \vdash n}\,1_n}{n\backslash n \vdash n\backslash n}\backslash}{(((np\backslash n) \otimes n) \otimes ((np\backslash s)/np)) \otimes \Diamond\Box np \vdash s}\widehat{\alpha}^r_\diamond \quad \dfrac{}{((np\backslash n) \otimes n) \otimes ((np\backslash s)/np) \vdash s/\Diamond\Box np}\triangleright}{(n\backslash n)/(s/\Diamond\Box np) \vdash (n\backslash n)/(((np\backslash n) \otimes n) \otimes (np\backslash s)/np)}/}{(n\backslash n)/(s/\Diamond\Box np) \otimes (((np/n) \otimes (np\backslash s)/np) \vdash n\backslash n}\triangleright^{-1}}{n \otimes ((n\backslash n)/(s/\Diamond\Box np) \otimes (((np/n) \otimes n) \otimes (np\backslash s)/np)) \vdash n}\triangleleft^{-1}$$

2 Interpretation

2.1

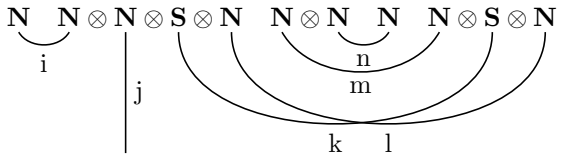
2.2

By working our way from the leaves of the proof tree, we get the following generalized Kronecker delta:

$$\mathbf{island}_i \otimes \mathbf{that}_{j,k,l,m} \otimes \mathbf{the}_{n,o} \otimes \mathbf{hurricane}_p \otimes \mathbf{destroyed}_{q,r,s} \xrightarrow{\delta^{i,k,l,m,n,o}_{j,t,r,s,q,p}} \mathbf{v}^{obj}_r \in \mathrm{N}$$

$$\mathbf{v}^{obj}_r = \mathbf{island}_i \otimes \mathbf{that}_{i,j,k,l} \otimes \mathbf{the}_{m,n} \otimes \mathbf{hurricane}_n \otimes \mathbf{destroyed}_{m,k,l} \quad \text{(relabeled)}$$

We give the matching diagram in the figure below:



2.3

In order to calculate the semantic value for the relative clause body 'the hurricane destroyed', we first apply $\mathbf{the}_{MN}$ to $\mathbf{hurricane}_N$. The operation yields the noun-phrase $\mathbf{the\ hurricane}_M$, represented by a row-vector equal to that of **hurricane**. The verb $\mathbf{destroyed}_{MKL}$ is then applied to the resulting vector, hence we obtain the final result $\mathbf{the\ hurricane\ destroyed}_{KL}$. Concretely, $\mathbf{the\ hurricane\ destroyed}_{KL} = \mathbf{destroyed}_{MKL}(\mathbf{the}_{MN}\mathbf{hurricane}_N)$ is a 2 by 3 matrix, the elements of which are:

$$\begin{pmatrix} 12 & -19 & 3 \\ 5 & 10 & 1 \end{pmatrix}$$

given by:

$$\mathbf{the\ hurricane\ destroyed}(k,l) = \sum_{n \epsilon N} \mathbf{hurricane}(n) \times \mathbf{destroyed}(n,k,l)\ \forall\ k\ \epsilon\ K,\ l\ \epsilon\ L \quad (1)$$

2.4

The interpreted type for the relative pronoun is:

$$\lceil (n\backslash n)/(s/\Diamond\Box np) \rceil = \lceil n\backslash n \rceil \otimes \lceil s/\Diamond\Box np \rceil = \lceil n \rceil \otimes \lceil n \rceil \otimes \lceil s \rceil \otimes \lceil \Diamond\Box np \rceil = N \otimes N \otimes S \otimes N$$

We can now give the following Frobenius recipe for **that**:

$$I \cong I \otimes I \xrightarrow{\eta_N \otimes \eta_N} N \otimes N \otimes N \otimes N \cong N \otimes N \otimes N \otimes I \otimes N \xrightarrow{1_N \otimes \mu_N \otimes \zeta_S \otimes 1_N} N \otimes N \otimes S \otimes N$$

In order to obtain the final interpretation, we do the following (dictated from the above recipe):

1. Reduce the rank of the transitive verb by summing over the S component, thus obtaining the following matrix:

$$\mathbf{collapsed\_destroyed} = \begin{pmatrix} \begin{pmatrix} 9 & 2 & 3 \end{pmatrix} \\ \begin{pmatrix} 1 & -5 & 2 \end{pmatrix} \\ \begin{pmatrix} -1 & -8 & 1 \end{pmatrix} \end{pmatrix}$$

2. Apply **collapsed_destroyed** to **the_hurricane** in object position:

$$\mathbf{the\_hurricane\_destroyed} = \begin{pmatrix} 17 & -9 & 4 \end{pmatrix}$$

3. Multiply **the_hurricane_destroyed** element-wise with **island**:

$$\mathbf{island\_that\_the\_hurricane\_destroyed} = \begin{pmatrix} -85 & -36 & 0 \end{pmatrix}$$

Note The matrix operations were computed in Python, as shown below: