Logic and Language: Exercise (Week 6)

Orestis Melkonian [6176208], Konstantinos Kogkalidis [6230067]

1 Syntax

1.1

First, we define the rules of *rightward extraction* $\widehat{\alpha}^r_\diamond$, $\widehat{\sigma}^r_\diamond$:

$$\frac{f : A \otimes (B \otimes \Diamond C) \to D}{\widehat{\alpha}^r_\diamond f : (A \otimes B) \otimes \Diamond C \to D} \qquad\qquad \frac{f : (A \otimes \Diamond C) \otimes B \to D}{\widehat{\sigma}^r_\diamond f : (A \otimes B) \otimes \Diamond C \to D}$$

We can now proceed with the derivation of

$$n \otimes ((n\backslash n)/(s/\Diamond\Box np)) \otimes ((np/n) \otimes n) \otimes ((np\backslash s)/np)) \to n$$

as follows:

$$
\begin{prooftree}
\AxiomC{}
\RightLabel{1_n}
\UnaryInfC{$n \vdash n$}
\AxiomC{}
\RightLabel{1_n}
\UnaryInfC{$n \vdash n$}
\RightLabel{$\backslash$}
\BinaryInfC{$n\backslash n \vdash n\backslash n$}
\AxiomC{}
\RightLabel{1_{np}}
\UnaryInfC{$np \vdash np$}
\AxiomC{}
\RightLabel{1_n}
\UnaryInfC{$n \vdash n$}
\RightLabel{$/$}
\BinaryInfC{$np/n \vdash np/n$}
\RightLabel{$\triangleright^{-1}$}
\UnaryInfC{$(np/n) \otimes n \vdash np$}
\AxiomC{}
\RightLabel{1_s}
\UnaryInfC{$s \vdash s$}
\RightLabel{$\backslash$}
\BinaryInfC{$np\backslash s \vdash ((np/n) \otimes n)\backslash s$}
\AxiomC{}
\RightLabel{1_{np}}
\UnaryInfC{$np \vdash np$}
\RightLabel{$\Box$}
\UnaryInfC{$\Box np \vdash \Box np$}
\RightLabel{$\triangledown^{-1}$}
\UnaryInfC{$\Diamond\Box np \vdash np$}
\RightLabel{$/$}
\BinaryInfC{$(np\backslash s)/np \vdash (((np/n) \otimes n)\backslash s)/\Diamond\Box np$}
\RightLabel{$\triangleright^{-1}$}
\UnaryInfC{$(np\backslash s)/np \otimes \Diamond\Box np \vdash ((np/n) \otimes n)\backslash s$}
\RightLabel{$\triangleleft^{-1}$}
\UnaryInfC{$((np/n) \otimes n) \otimes ((np\backslash s)/np \otimes \Diamond\Box np) \vdash s$}
\RightLabel{$\widehat{\alpha}^r_\diamond$}
\UnaryInfC{$(((np/n) \otimes n) \otimes ((np\backslash s)/np)) \otimes \Diamond\Box np \vdash s$}
\RightLabel{$\triangleright$}
\UnaryInfC{$((np/n) \otimes n) \otimes ((np\backslash s)/np) \vdash s/\Diamond\Box np$}
\RightLabel{$/$}
\BinaryInfC{$(n\backslash n)/(s/\Diamond\Box np) \vdash (n\backslash n)/(((np/n) \otimes n) \otimes (np\backslash s)/np)$}
\RightLabel{$\triangleright^{-1}$}
\UnaryInfC{$(n\backslash n)/(s/\Diamond\Box np) \otimes (((np/n) \otimes (np\backslash s)/np) \vdash n\backslash n$}
\RightLabel{$\triangleleft^{-1}$}
\UnaryInfC{$n \otimes ((n\backslash n)/(s/\Diamond\Box np) \otimes (((np/n) \otimes n) \otimes (np\backslash s)/np)) \vdash n$}
\end{prooftree}
$$

2 Interpretation

2.1

We start by assigning a temporary variable at each rule application in the proof tree:

$$
\begin{prooftree}
\AxiomC{}
\RightLabel{1_n}
\UnaryInfC{$n \vdash n$}
\AxiomC{}
\RightLabel{1_n}
\UnaryInfC{$n \vdash n$}
\RightLabel{$\backslash$}
\BinaryInfC{$\color{red}{m} : n\backslash n \vdash n\backslash n$}
\AxiomC{}
\RightLabel{1_{np}}
\UnaryInfC{$np \vdash np$}
\AxiomC{}
\RightLabel{1_n}
\UnaryInfC{$n \vdash n$}
\RightLabel{$/$}
\BinaryInfC{$\color{red}{f} : np/n \vdash np/n$}
\RightLabel{$\triangleright^{-1}$}
\UnaryInfC{$\color{red}{g} : (np/n) \otimes n \vdash np$}
\AxiomC{}
\RightLabel{1_s}
\UnaryInfC{$s \vdash s$}
\RightLabel{$\backslash$}
\BinaryInfC{$\color{red}{h} : np\backslash s \vdash ((np/n) \otimes n)\backslash s$}
\AxiomC{}
\RightLabel{1_{np}}
\UnaryInfC{$np \vdash np$}
\RightLabel{$\Box$}
\UnaryInfC{$\color{red}{b} : \Box np \vdash \Box np$}
\RightLabel{$\triangledown^{-1}$}
\UnaryInfC{$\color{red}{d} : \Diamond\Box np \vdash np$}
\RightLabel{$/$}
\BinaryInfC{$\color{red}{i} : (np\backslash s)/np \vdash (((np/n) \otimes n)\backslash s)/\Diamond\Box np$}
\RightLabel{$\triangleright^{-1}$}
\UnaryInfC{$\color{red}{j} : (np\backslash s)/np \otimes \Diamond\Box np \vdash ((np/n) \otimes n)\backslash s$}
\RightLabel{$\triangleleft^{-1}$}
\UnaryInfC{$\color{red}{k} : ((np/n) \otimes n) \otimes ((np\backslash s)/np \otimes \Diamond\Box np) \vdash s$}
\RightLabel{$\widehat{\alpha}^r_\diamond$}
\UnaryInfC{$\color{red}{l} : (((np/n) \otimes n) \otimes ((np\backslash s)/np)) \otimes \Diamond\Box np \vdash s$}
\RightLabel{$\triangleright$}
\UnaryInfC{$\color{red}{n} : ((np/n) \otimes n) \otimes ((np\backslash s)/np) \vdash s/\Diamond\Box np$}
\RightLabel{$/$}
\BinaryInfC{$\color{red}{o} : (n\backslash n)/(s/\Diamond\Box np) \vdash (n\backslash n)/(((np/n) \otimes n) \otimes (np\backslash s)/np)$}
\RightLabel{$\triangleright^{-1}$}
\UnaryInfC{$\color{red}{p} : (n\backslash n)/(s/\Diamond\Box np) \otimes (((np/n) \otimes (np\backslash s)/np) \vdash n\backslash n$}
\RightLabel{$\triangleleft^{-1}$}
\UnaryInfC{$\color{red}{q} : n \otimes ((n\backslash n)/(s/\Diamond\Box np) \otimes (((np/n) \otimes n) \otimes (np\backslash s)/np)) \vdash n$}
\end{prooftree}
$$

We now work our way top-down through the proof-tree, writing the interpretation of each formula using the rules of 3.1.

$$
\begin{aligned}
f &: (1_{np} \otimes \eta_n \otimes 1_n) \circ (1_{np\otimes n} \otimes 1_n \otimes 1_n) \circ (1_{np\otimes n} \otimes \epsilon_n) && (1)\\
g \equiv \triangleright^{-1} f &: (f \otimes 1_n) \circ (1_{np} \otimes \epsilon_n) && (2)\\
h &: (1_{np} \otimes \eta_{(np/n)\otimes n} \otimes 1_s) \circ (1_{np} \otimes g \otimes 1_{((np/n)\otimes n)\otimes s}) \circ (\epsilon_{np} \otimes 1_{((np/n)\otimes n)\otimes s}) && (3)\\
i &: (h \otimes \eta_{np} \otimes 1_{np}) \circ (1_{((np/n)\otimes n)\backslash s)\otimes s)} \otimes d \otimes 1_{np}) \otimes (1_{((np/n)\otimes n)\backslash s)\otimes s} \otimes \epsilon_s) && (4)\\
j \equiv \triangleright^{-1} i &: (i \otimes 1_{\Diamond\Box np}) \circ (1_{((np/n)\otimes n)\backslash s} \otimes \epsilon_{\Diamond\Box np}) && (5)\\
k \equiv \triangleleft^{-1} i &: (1_{(np/n)\otimes n} \otimes i) \circ (\epsilon_{(np/n)\otimes n} \otimes 1_s) && (6)\\
l \equiv \widehat{\alpha}^r_\diamond k &: \alpha \circ k && (7)\\
m &: (1_n \otimes \eta_n \otimes 1_n) \circ (1_n \otimes 1_n \otimes 1_{n\otimes n}) \circ (\epsilon_n \otimes 1_{n\otimes n}) && (8)\\
n \equiv \triangleright l &: (1_{(np/n)\otimes n} \otimes \eta_{(np\backslash s)/np}) \circ (l \otimes 1_{(np\backslash s)/np}) && (9)\\
o &: (m \otimes \eta_{(((np/n)\otimes n)\otimes (np\backslash s)/np)} \otimes 1_{(s/\Diamond\Box np)}) \circ (1_{(n\backslash n)\otimes(((np/n)\otimes n)\otimes(np\backslash s)/np)} \otimes n \otimes 1_{s/\Diamond\Box np}) \circ (1_{(n\backslash n)\otimes(((np/n)\otimes} && (10)
\end{aligned}
$$

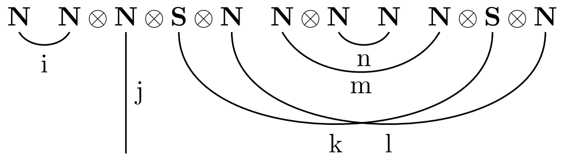
Recursively unwrapping the above interpretations, we obtain the final interpretation:

2.2

By working our way from the leaves of the proof tree, we get the following generalized Kronecker delta:

$$
\begin{aligned}
\mathbf{island}_i \otimes \mathbf{that}_{j,k,l,m} \otimes \mathbf{the}_{n,o} \otimes \mathbf{hurricane}_p \otimes \mathbf{destroyed}_{q,r,s} &\xrightarrow{\delta^{i,k,l,m,n,o}_{j,t,r,s,q,p}} \mathbf{v}^{obj}_r \in \mathrm{N}\\
\mathbf{v}^{obj}_r = &\mathbf{island}_i \otimes \mathbf{that}_{i,j,k,l} \otimes \mathbf{the}_{m,n} \otimes \mathbf{hurricane}_n \otimes \mathbf{destroyed}_{m,k,l} \quad \text{(relabeled)}
\end{aligned}
$$

We give the matching diagram in the figure below:



2.3

In order to calculate the semantic value for the relative clause body 'the hurricane destroyed', we first apply $\textbf{the}_{MN}$ to $\textbf{hurricane}_N$. The operation yields the noun-phrase $\textbf{the hurricane}_M$, represented by a row-vector equal to that of **hurricane**. The verb $\textbf{destroyed}_{MKL}$ is then applied to the resulting vector, thereupon we obtain the final result $\textbf{the hurricane destroyed}_{KL}$. Concretely, $\textbf{the hurricane destroyed}_{KL} = \textbf{destroyed}_{MKL}(\textbf{the}_{MN}\textbf{hurricane}_N)$ is a 2 by 3 matrix, the elements of which are:

$$\begin{pmatrix} 12 & -19 & 3 \\ 5 & 10 & 1 \end{pmatrix}$$

given by:

$$\textbf{the hurricane destroyed}(k,l) = \sum_{m \epsilon M} \textbf{hurricane}(m) \times \textbf{destroyed}(m,k,l), \ \forall \ k \ \epsilon \ K, \ l \ \epsilon \ L$$

The corresponding Python code is given below:

2.4

The interpreted type for the relative pronoun is:

$$\lceil (n\backslash n)/(s/\Diamond\Box np) \rceil = \lceil n\backslash n \rceil \otimes \lceil s/\Diamond\Box np \rceil = \lceil n \rceil \otimes \lceil n \rceil \otimes \lceil s \rceil \otimes \lceil \Diamond\Box np \rceil = N \otimes N \otimes S \otimes N$$

We can now give the following Frobenius recipe for **that**:

$$I \cong I \otimes I \xrightarrow{\eta_N \otimes \eta_N} N \otimes N \otimes N \otimes N \cong N \otimes N \otimes N \otimes I \otimes N \xrightarrow{1_N \otimes \mu_N \otimes \zeta_S \otimes 1_N} N \otimes N \otimes S \otimes N$$

In order to obtain the final interpretation, we do the following (dictated from the above recipe):

1. Reduce the rank of the transitive verb by summing over the S component, thus obtaining the following matrix:

$$\textbf{collapsed\_destroyed} = \begin{pmatrix} \begin{pmatrix} 9 & 2 & 3 \end{pmatrix} \\ \begin{pmatrix} 1 & -5 & 2 \end{pmatrix} \\ \begin{pmatrix} -1 & -8 & 1 \end{pmatrix} \end{pmatrix}$$

2. Apply **collapsed_destroyed** to **the_hurricane** in subject position:

$$\textbf{the\_hurricane\_destroyed} = \begin{pmatrix} 17 & -9 & 4 \end{pmatrix}$$

3. Multiply **the_hurricane_destroyed** element-wise with **island**:

$$\textbf{island\_that\_the\_hurricane\_destroyed} = \begin{pmatrix} -85 & -36 & 0 \end{pmatrix}$$

The corresponding Python code is given below: