Logic and Language: Exercise (Week 6)

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1 Syntax

1.1

First, we define the rules of *rightward extraction* $\widehat{\alpha}_\diamond^r$, $\widehat{\sigma}_\diamond^r$:

$$\frac{f : A \otimes (B \otimes \Diamond C) \to D}{\widehat{\alpha}_\diamond^r f : (A \otimes B) \otimes \Diamond C \to D} \qquad \frac{f : (A \otimes \Diamond C) \otimes B \to D}{\widehat{\sigma}_\diamond^r f : (A \otimes B) \otimes \Diamond C \to D}$$

We can now proceed with the derivation of

$$n \otimes ((n\backslash n)/(s/\Diamond\Box np)) \otimes ((np/n) \otimes n) \otimes ((np\backslash s)/np)) \to n$$

as follows:

$$
\begin{array}{c}
\cfrac{
  \cfrac{
    \cfrac{
      \cfrac{\overline{np \vdash np}\,^{1_{np}} \quad \overline{n \vdash n}\,^{1_n}}{\cfrac{np\backslash n \vdash np\backslash n}{(np\backslash n) \otimes n \vdash np}\,^{\triangleright^{-1}}}\,^{\backslash} \quad \overline{s \vdash s}\,^{1_s}
    }{np\backslash s \vdash ((np\backslash n) \otimes n)\backslash s}\,^{\backslash} \quad
    \cfrac{\overline{np \vdash np}\,^{1_{np}}}{\cfrac{\Box np \vdash \Box np}{\Diamond\Box np \vdash np}\,^{\nabla^{-1}}}\,^{\Box}
  }{
    \cfrac{(np\backslash s)/np \vdash (((np\backslash n) \otimes n)\backslash s)/\Diamond\Box np}{
    \cfrac{(np\backslash s)/np \otimes \Diamond\Box np \vdash ((np\backslash n) \otimes n)\backslash s}{
    \cfrac{((np\backslash n) \otimes n) \otimes ((np\backslash s)/np \otimes \Diamond\Box np) \vdash s}{
    \cfrac{(((np\backslash n) \otimes n) \otimes ((np\backslash s)/np)) \otimes \Diamond\Box np \vdash s}{
    \cfrac{((np\backslash n) \otimes n) \otimes ((np\backslash s)/np) \vdash s/\Diamond\Box np}{}\,^{\triangleright}
    }\,^{\widehat{\alpha}_\diamond^r}
    }\,^{\triangleleft^{-1}}
    }\,^{\triangleright^{-1}}
    }\,^{/}
  }
}{}
\end{array}
$$

$$
\cfrac{\overline{n \vdash n}\,^{1_n} \quad \overline{n \vdash n}\,^{1_n}}{n\backslash n \vdash n\backslash n}\,^{\backslash} \qquad ((np\backslash n) \otimes n) \otimes ((np\backslash s)/np) \vdash s/\Diamond\Box np
$$

$$
\cfrac{(n\backslash n)/(s/\Diamond\Box np) \vdash (n\backslash n)/(((np\backslash n) \otimes n) \otimes (np\backslash s)/np)}{
\cfrac{(n\backslash n)/(s/\Diamond\Box np) \otimes (((np/n) \otimes (np\backslash s)/np) \vdash n\backslash n}{
n \otimes ((n\backslash n)/(s/\Diamond\Box np) \otimes (((np/n) \otimes n) \otimes (np\backslash s)/np)) \vdash n}\,^{\triangleleft^{-1}}
}\,^{\triangleright^{-1}}
$$

2 Interpretation

2.1

2.2

By working our way from the leaves of the proof tree, we get the following generalized Kronecker delta:

$$\mathbf{island}_i \otimes \mathbf{that}_{j,k,l,m} \otimes \mathbf{the}_{n,o} \otimes \mathbf{hurricane}_p \otimes \mathbf{destroyed}_{q,r,s} \xrightarrow{\delta^{i,k,l,m,n,o}_{j,t,r,s,q,p}} \mathbf{v}_r^{obj} \in \mathrm{N}$$

$$\mathbf{v}_r^{obj} = \mathbf{island}_i \otimes \mathbf{that}_{i,j,k,l} \otimes \mathbf{the}_{m,n} \otimes \mathbf{hurricane}_n \otimes \mathbf{destroyed}_{m,k,l} \quad \text{(relabeled)}$$

We give the matching diagram in the figure below:

$$\mathbf{N} \quad \mathbf{N} \otimes \mathbf{N} \otimes \mathbf{S} \otimes \mathbf{N} \quad \mathbf{N} \otimes \mathbf{N} \quad \mathbf{N} \quad \mathbf{N} \otimes \mathbf{S} \otimes \mathbf{N}$$

i   j   n   m   k   l