

D^3 as a 2-MCFL

Orestis Melkonian, Konstantinos Kogkalidis

January 25, 2018

Universiteit Utrecht

1.3 Challenge

[This is not a hand-in exercise. If you can solve it by Dec 5, there will be a present for you!]

Let D^n be the language over an n -symbol alphabet, lexicographically ordered $a_1 < \dots < a_n$, where words satisfy the following conditions:

1. each word contains an equal number of the n alphabet symbols
2. for every prefix p of a word, the number of a_i in $p \geq$ the number of a_{i+1} ($1 \leq i \leq n-1$)

D^n generalizes the familiar language of balanced brackets, in which case you have an alphabet of size 2, say $\{a, b\}$, with ‘opening bracket’ a preceding ‘closing bracket’ b in the lexicographic ordering.

The conjecture (Makoto Kanazawa, p.c.) is that for $n \geq 2$, D^n is the language of a non-wellnested $(n-1)$ -MCFG.

Give a 2-MCFG for D^3 , i.e. words over a 3-letter alphabet $\{a, b, c\}$ (with the usual lexicographic order) satisfying conditions (1) and (2) above. Give the ACG encoding of your MCFG for D^3 .

Reference M. Moortgat (2014), A note on multidimensional Dyck languages.

Some examples

DYCK WORDS

- `abc`
- `aabbcc`
- `abcabcabcabc`

Some examples

DYCK WORDS

- abc
- aabbcc
- abcabcabacbc

NON-DYCK WORDS

- aabb

Some examples

DYCK WORDS

- abc
- aabbcc
- abcabcabcabc

NON-DYCK WORDS

- aabb
- aabbbcc

Some examples

DYCK WORDS

- abc
- aabbcc
- abcabcabcabc

NON-DYCK WORDS

- aabb
- aabbbcc
- abcacb

Some examples

DYCK WORDS

- *abc*
- *aabbcc*
- *abcabcabcabc*

NON-DYCK WORDS

- *aabb*
- *aabbbcc*
- *abcacb*

ababacbcabcc

First-match policy

Some examples

DYCK WORDS

- abc
- aabbcc
- abcabcabcabc

NON-DYCK WORDS

- aabb
- aabbbcc
- abcacb

a b a b a c b c a b c c



First-match policy

Some examples

DYCK WORDS

- *abc*
- *aabbcc*
- *abcabcabcabc*

NON-DYCK WORDS

- *aabb*
- *aabbbcc*
- *abcacb*

ababacbcabcc



First-match policy

Some examples

DYCK WORDS

- *abc*
- *aabbcc*
- *abcabcabcabc*

NON-DYCK WORDS

- *aabb*
- *aabbbcc*
- *abcacb*



First-match policy

Some examples

DYCK WORDS

- *abc*
- *aabbcc*
- *abcabcabcabc*

NON-DYCK WORDS

- *aabb*
- *aabbbcc*
- *abcacb*



First-match policy

G_0 : Grammar of triple insertions

$$S(xy) \leftarrow W(x, y). \quad (1)$$

$$W(\epsilon, xy\mathbf{abc}) \leftarrow W(x, y). \quad (2)$$

$$W(\epsilon, x\mathbf{a}y\mathbf{bc}) \leftarrow W(x, y). \quad (3)$$

.....

$$W(\mathbf{ab}x\mathbf{c}y, \epsilon) \leftarrow W(x, y). \quad (60)$$

$$W(\mathbf{abc}xy, \epsilon) \leftarrow W(x, y). \quad (61)$$

$$W(\epsilon, \mathbf{abc}). \quad (62)$$

$$W(\mathbf{a}, \mathbf{bc}). \quad (63)$$

$$W(\mathbf{ab}, \mathbf{c}). \quad (64)$$

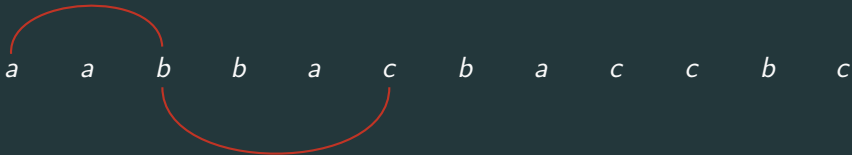
$$W(\mathbf{abc}, \epsilon). \quad (65)$$

G_0 : Grammar of triple insertions

a a b b a c b a c c b c

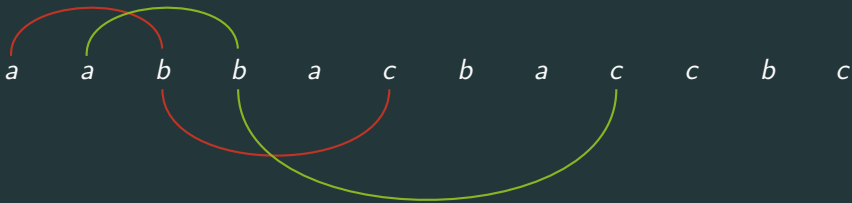
Straddling counter-example

G_0 : Grammar of triple insertions



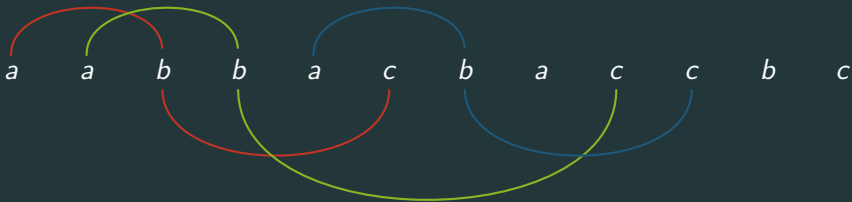
Straddling counter-example

G_0 : Grammar of triple insertions



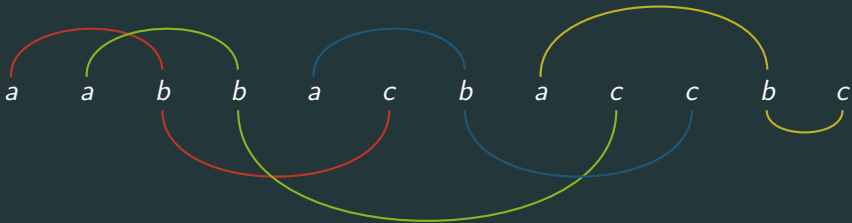
Straddling counter-example

G_0 : Grammar of triple insertions



Straddling counter-example

G_0 : Grammar of triple insertions



Straddling counter-example

Meta-grammars: Introduction

NOTATION

$\mathcal{O}_m[\textit{conclusion} \leftarrow \textit{premises} \mid \{\textit{partial orderings of inserted elements}\}]$.

Meta-grammars: Introduction

NOTATION

$\mathcal{O}_m[\textit{conclusion} \leftarrow \textit{premises} \mid \{\textit{partial orderings of inserted elements}\}]$.

META-GRAMMAR G_1

$\mathcal{O}_2[W \leftarrow \epsilon \mid \{a < b < c\}]$.	} TRIPLE INSERTION
$\mathcal{O}_2[W \leftarrow W_{xy} \mid \{x < y, a < b < c\}]$.	

Meta-grammars: Introduction

NOTATION

$\mathcal{O}_m[\textit{conclusion} \leftarrow \textit{premises} \mid \{\textit{partial orderings of inserted elements}\}].$

META-GRAMMAR G_1

$\mathcal{O}_2[W \leftarrow \epsilon \mid \{a < b < c\}].$	} TRIPLE INSERTION
$\mathcal{O}_2[W \leftarrow W_{xy} \mid \{x < y, a < b < c\}].$	
$+$	
$\mathcal{O}_2[W \leftarrow W_{xy}, W_{zw} \mid \{x < y, z < w\}].$	} INTERLEAVING WORDS

G₂: Adding states

$$\left. \begin{array}{l} \mathcal{O}_2[A^+ \leftarrow \epsilon \mid \{a\}]. \\ \mathcal{O}_2[B^+ \leftarrow \epsilon \mid \{b\}]. \\ \mathcal{O}_2[C^+ \leftarrow \epsilon \mid \{c\}]. \end{array} \right\} \text{BASE CASES}$$

$$\left. \begin{array}{l} \mathcal{O}_2[C^- \leftarrow A^+, B^+ \mid \{x < y < z < w\}]. \\ \mathcal{O}_2[B^- \leftarrow A^+, C^+ \mid \{x < y < z < w\}]. \\ \mathcal{O}_2[A^- \leftarrow B^+, C^+ \mid \{x < y < z < w\}]. \\ \mathcal{O}_2[A^+ \leftarrow C^-, B^- \mid \{x < y < z < w\}]. \\ \mathcal{O}_2[B^+ \leftarrow C^-, A^- \mid \{x < y < z < w\}]. \\ \mathcal{O}_2[C^+ \leftarrow B^-, A^- \mid \{x < y < z < w\}]. \\ \forall K \in \mathcal{S} \setminus W : \mathcal{O}_2[K \leftarrow K_{xy}, W_{zw} \mid \{x < y, z < w\}]. \end{array} \right\} \text{COMBINATIONS}$$

$$\left. \begin{array}{l} \mathcal{O}_2[W \leftarrow A^+, A^- \mid \{x < y < z < w\}]. \\ \mathcal{O}_2[W \leftarrow C^-, C^+ \mid \{x < y < z < w\}]. \end{array} \right\} \text{CLOSURES}$$

G_3 : $G_2 +$ Universal triple insertion

$$G_3 = G_2 + \forall K \in \mathcal{S} \setminus W :$$

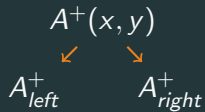
$$\mathcal{O}_2[[K \leftarrow K_{xy} \mid \{x < y, a < b < c\}]].$$



YOU SHALL NOT PARSE!

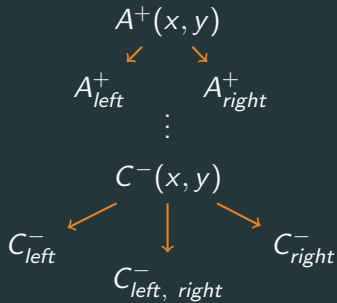
Refining states

EXAMPLE



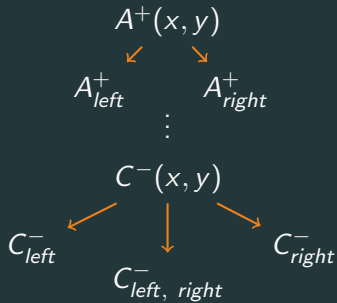
Refining states

EXAMPLE



Refining states

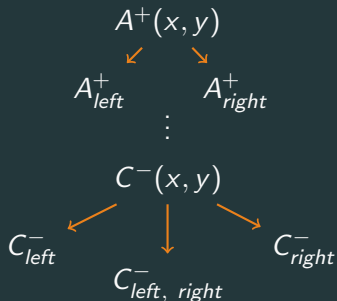
EXAMPLE



WHY?

Refining states

EXAMPLE

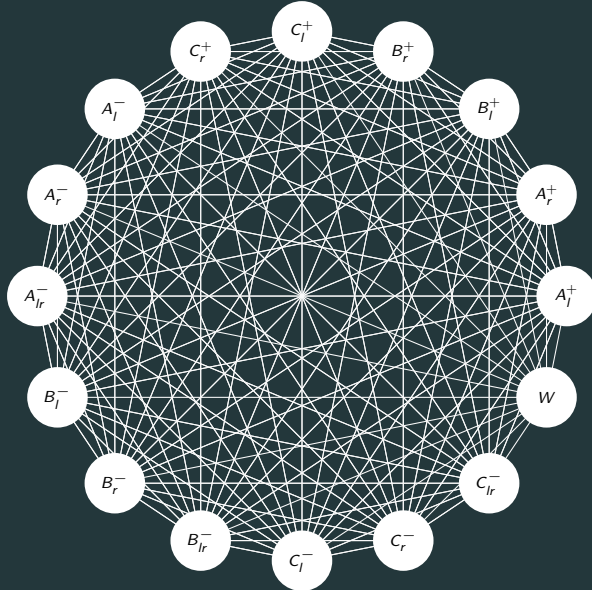


WHY?

NEW ORDERS IN INTERACTIONS

$$C^-(xz, wy) \leftarrow A^+_{left}(x, y), B^+(z, w).$$

Refining states: Interactions



G₄: Automatic Rule Inference

STATE DESCRIPTORS \mathcal{D}

$$W \mapsto (\epsilon, \epsilon)$$

$$A_l^+ \mapsto (a, \epsilon)$$

$$A_r^+ \mapsto (\epsilon, a)$$

$$B_l^+ \mapsto (b, \epsilon)$$

$$B_r^+ \mapsto (\epsilon, b)$$

$$C_l^+ \mapsto (c, \epsilon)$$

$$C_r^+ \mapsto (\epsilon, c)$$

$$A_l^- \mapsto (bc, \epsilon)$$

$$A_r^- \mapsto (\epsilon, bc)$$

$$A_{lr}^- \mapsto (b, c)$$

$$B_l^- \mapsto (ac, \epsilon)$$

$$B_r^- \mapsto (\epsilon, ac)$$

$$B_{lr}^- \mapsto (a, c)$$

$$C_l^- \mapsto (ab, \epsilon)$$

$$C_r^- \mapsto (\epsilon, ab)$$

$$C_{lr}^- \mapsto (a, b)$$

Automatic Rule Inference: Example

$$\text{CASE OF } A_{lr}^-(x_b, y_c) + B_{lr}^-(z_a, w_c)$$

PERMUTATION

\vdots

(zxw, y)

\vdots

(xzw, y)

\vdots

Automatic Rule Inference: Example

CASE OF $A_{lr}^-(x_b, y_c) + B_{lr}^-(z_a, w_c)$

PERMUTATION

DESCRIPTOR

\vdots

\vdots

$(zxw, y) \longrightarrow (abc, c)$

\vdots

\vdots

$(xzw, y) \longrightarrow (bac, c)$

\vdots

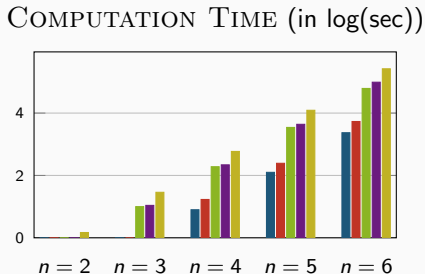
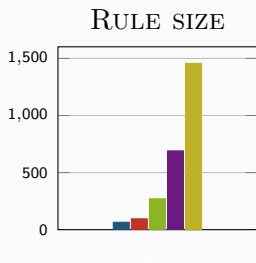
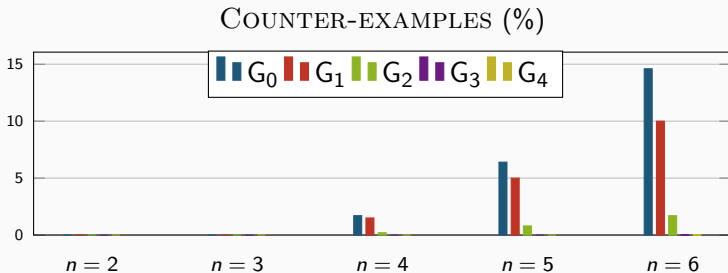
\vdots

Automatic Rule Inference: Example

CASE OF $A_{lr}^-(x_b, y_c) + B_{lr}^-(z_a, w_c)$

<u>PERMUTATION</u>	<u>DESCRIPTOR</u>	<u>ELIMINATED</u>
\vdots	\vdots	\vdots
(zxw, y)	$\longrightarrow (abc, c)$	$\begin{array}{l} \nearrow (c, \epsilon) \leftarrow C_l^+ \\ \searrow (\epsilon, c) \leftarrow C_r^+ \end{array}$
\vdots	\vdots	\vdots
(xzw, y)	$\longrightarrow (bac, c)$	$\longrightarrow (bac, c) \notin \mathcal{D}$
\vdots	\vdots	\vdots

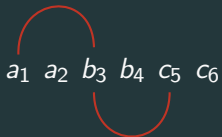
Results



Correspondences: Young Tableaux

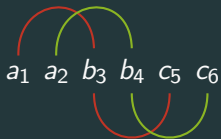
a_1 a_2 b_3 b_4 c_5 c_6

Correspondences: Young Tableaux



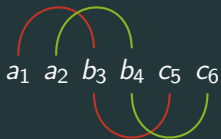
1	
3	
5	

Correspondences: Young Tableaux



1	2
3	4
5	6

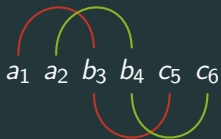
Correspondences: Promotion on Young Tableaux



1	2
3	4
5	6

	1
2	3
4	5

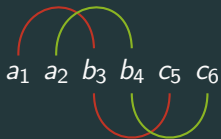
Correspondences: Promotion on Young Tableaux



1	2
3	4
5	6

1	
2	3
4	5

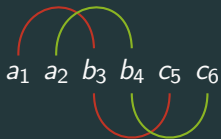
Correspondences: Promotion on Young Tableaux



1	2
3	4
5	6

1	3
2	
4	5

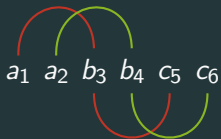
Correspondences: Promotion on Young Tableaux



1	2
3	4
5	6

1	3
2	5
4	

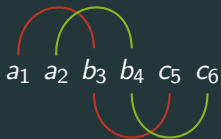
Correspondences: Promotion on Young Tableaux



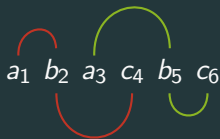
1	2
3	4
5	6

1	3
2	5
4	6

Correspondences: Promotion on Young Tableaux



1	2
3	4
5	6



1	3
2	5
4	6

Correspondences: Spider Webs

GROWTH RULES

