Logic and Language: Exercise (Week 5)

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1

2 Pregroups

2.1

$$(4) \qquad \qquad \underbrace{\frac{1^l \to 1^l 1}{1^l \to 1}}_{1^l \to 1} \stackrel{(1)}{\to} \qquad \qquad \underbrace{\frac{1}{1 \to 1^l 1}}_{1 \to 1^l} \stackrel{(2)}{\to} \qquad \qquad \underbrace{\frac{1}{1 \to 1^l 1}}_{1 \to 1^l} \stackrel{(1)}{\to} \qquad \qquad \underbrace{}$$

$$(5) (\rightarrow): \frac{\underline{A^{rl} \to A^{rl}1} \ (1) \ \underline{1 \to A^{r}A} \ (2)}{\underline{A^{rl} \to A^{rl}(A^{r}A)} \ (1)} \xrightarrow{A^{rl}A^{r} \to 1} \ (2)}{\underline{A^{rl} \to (A^{rl}A^{r})A} \ (1)} \xrightarrow{\underline{A^{rl}A^{r} \to 1} \ \to} \ \underline{1A \to A} \ (1)}$$

$$(\leftarrow): \frac{\overline{A \to A1} \ (1) \quad \overline{1 \to A^r A^{rl}} \ (2)}{\underline{A \to A(A^r A^{rl})} \ (1) \qquad \overline{AA^r \to 1} \ (3)} \\ \underline{A \to (AA^r)A^{rl}} \ (1) \qquad \overline{AA^r \to 1} \ \to \qquad \overline{1A^{rl} \to A^{rl}} \ (1) \\ \underline{A \to A^{rl}} \ (1)$$

 $(6) (\rightarrow)$:

$$\frac{\overline{(AB)^{l} \to (1(AB))^{l}} \to \overline{1 \to (B^{l}A^{l})^{r}(B^{l}A^{l})}}{\frac{(AB)^{l} \to ((B^{l}A^{l})^{r}(B^{l}A^{l})(AB))^{l}}{(AB)^{l} \to ((B^{l}A^{l})^{r}(B^{l}A^{l})(AB))^{l}}} (1)}{\frac{(AB)^{l} \to ((B^{l}A^{l})^{r}(B^{l}(A^{l}(AB)))^{l}}{(AB)^{l} \to ((B^{l}A^{l})^{r}(B^{l}(A^{l}AB)))}} (1)}{\frac{(AB)^{l} \to ((B^{l}A^{l})^{r}(B^{l}(A^{l}AB)))}{(AB)^{l} \to ((B^{l}A^{l})^{r}(B^{l}(B)))^{l}} \to \overline{1B \to B}} (1)}{\underline{(AB)^{l} \to ((B^{l}A^{l})^{r}(B^{l}(B)))^{l}}} \to \overline{B^{l}B \to 1}} (2)}{\underline{(AB)^{l} \to ((B^{l}A^{l})^{r}(B^{l}B))^{l}}} \to \overline{(B^{l}A^{l})^{r}(B^{l}A^{l})^{r}(B^{l}B))^{l}}} (5)}$$