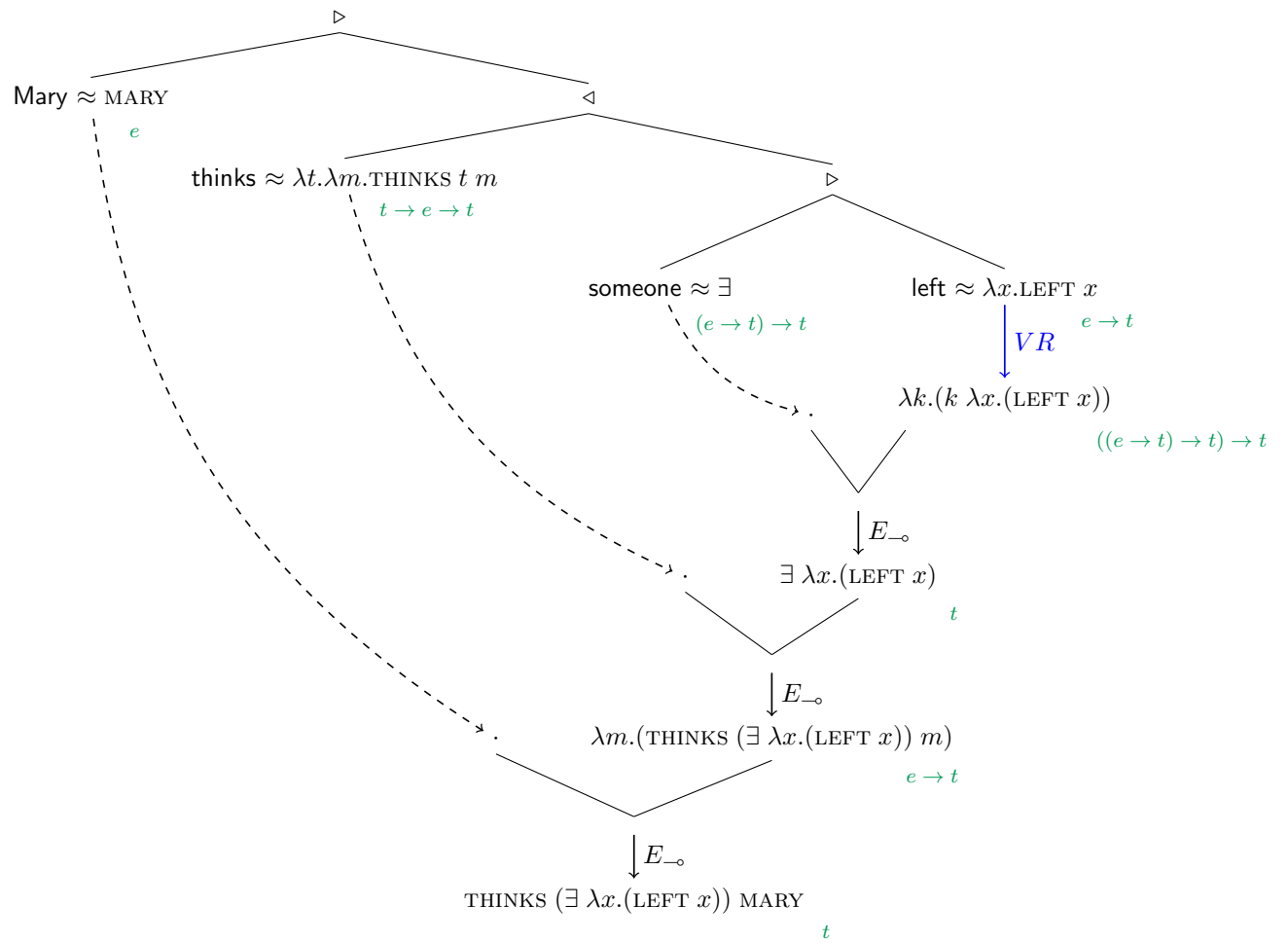# Logic and Language: Exercise (Week 2)

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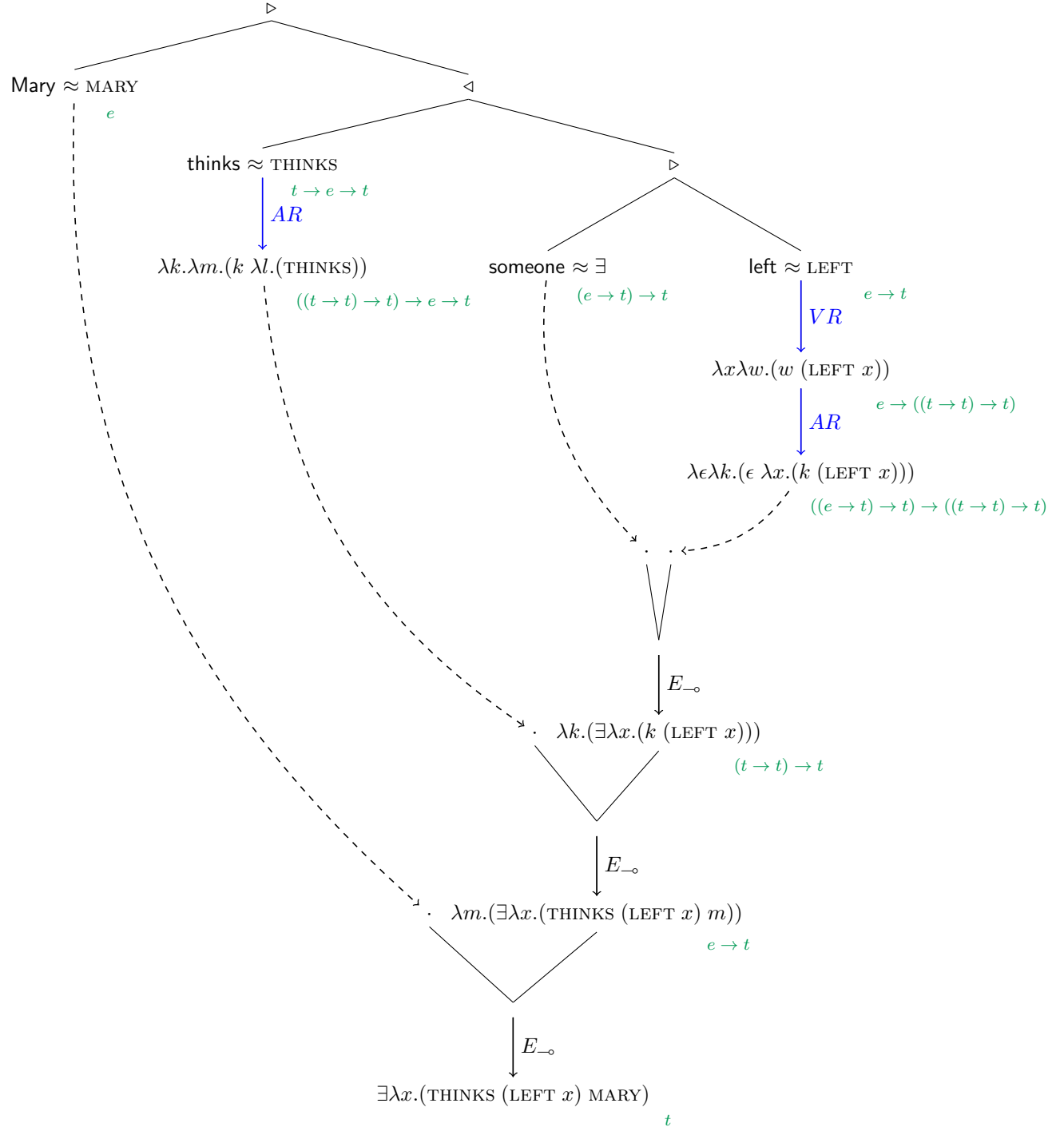
1 Hendriks

1.1 Local Interpretation

Clarification: Basic term translations implicitly apply η-expansions, in order to transform the term in the desired format.

1.2   Non-Local Interpretation

$\triangleright$

Mary $\approx$ MARY
  e

  thinks $\approx$ THINKS
    $t \to e \to t$
    $\downarrow AR$
  $\lambda k.\lambda m.(k\ \lambda l.(\text{THINKS}))$
    $((t \to t) \to t) \to e \to t$

  $\triangleleft$

  someone $\approx \exists$
    $(e \to t) \to t$

  left $\approx$ LEFT
    $e \to t$
    $\downarrow VR$
  $\lambda x\lambda w.(w\ (\text{LEFT}\ x))$
    $e \to ((t \to t) \to t)$
    $\downarrow AR$
  $\lambda\epsilon\lambda k.(\epsilon\ \lambda x.(k\ (\text{LEFT}\ x)))$
    $((e \to t) \to t) \to ((t \to t) \to t)$

  $\triangleright$

  $\downarrow E_{\multimap}$

  $\lambda k.(\exists\lambda x.(k\ (\text{LEFT}\ x)))$
    $(t \to t) \to t$

  $\downarrow E_{\multimap}$

  $\lambda m.(\exists\lambda x.(\text{THINKS}\ (\text{LEFT}\ x)\ m))$
    $e \to t$

  $\downarrow E_{\multimap}$

  $\exists\lambda x.(\text{THINKS}\ (\text{LEFT}\ x)\ \text{MARY})$
    t

2

2 Barker

2.1 Left-to-right incremental

$(\text{Mary} \rhd (\text{thinks} \lhd (\text{someone} \rhd \text{left})))^{\rightsquigarrow}\ (\lambda x.x)$

$\equiv\quad \lambda k.(\text{Mary}^{\rightsquigarrow}\ \lambda n.((\text{thinks} \lhd (\text{someone} \rhd \text{left}))^{\rightsquigarrow}\ \lambda m.(k\ (m\ n))))\ (\lambda x.x)$

$\rightarrow_\beta^*\quad \text{Mary}^{\rightsquigarrow}\ \lambda n.((\text{thinks} \lhd (\text{someone} \rhd \text{left}))^{\rightsquigarrow}\ \lambda m.(m\ n))$

$\equiv\quad \lambda k.(k\ \text{MARY})\ \lambda n.((\text{thinks} \lhd (\text{someone} \rhd \text{left}))^{\rightsquigarrow}\ \lambda m.(k\ (m\ n)))$

$\rightarrow_\beta^*\quad (\text{thinks} \lhd (\text{someone} \rhd \text{left}))^{\rightsquigarrow}\ \lambda m.(m\ \text{MARY})$

$\equiv\quad \lambda k.((\text{thinks}^{\rightsquigarrow}\ \lambda m.((\text{someone} \rhd \text{left})^{\rightsquigarrow}\ \lambda n.(k\ (m\ n))))\ \lambda m.(m\ \text{MARY})$

$\rightarrow_\beta^*\quad \text{thinks}^{\rightsquigarrow}\ \lambda m.((\text{someone} \rhd \text{left})^{\rightsquigarrow}\ \lambda n.(m\ n\ \text{MARY}))$

$\equiv\quad \lambda k.(k\ \text{THINKS})\ \lambda m.((\text{someone} \rhd \text{left})^{\rightsquigarrow}\ \lambda n.(m\ n\ \text{MARY}))$

$\rightarrow_\beta^*\quad (\text{someone} \rhd \text{left})^{\rightsquigarrow}\ \lambda n.(\text{THINKS}\ n\ \text{MARY})$

$\equiv\quad \lambda k.(\text{someone}^{\rightsquigarrow}\ \lambda n.(\text{left}^{\rightsquigarrow}\ \lambda m.(k\ (m\ n))))\ \lambda n.(\text{THINKS}\ n\ \text{MARY})$

$\rightarrow_\beta^*\quad \text{someone}^{\rightsquigarrow}\ \lambda n.(\text{left}^{\rightsquigarrow}\ \lambda m.(\text{THINKS}\ (m\ n)\ \text{MARY}))$

$\equiv\quad \exists \lambda n.(\lambda k.(k\ \text{LEFT})\ \lambda m.(\text{THINKS}\ (m\ n)\ \text{MARY}))$

$\rightarrow_\beta^*\quad \exists \lambda n.(\text{THINKS}\ (\text{LEFT}\ n)\ \text{MARY})$

2.2 Right-to-left incremental

$(\text{Mary} \rhd (\text{thinks} \lhd (\text{someone} \rhd \text{left})))^{\leftsquigarrow}\ (\lambda x.x)$

$\equiv\quad \lambda k.((\text{thinks} \lhd (\text{someone} \rhd \text{left}))^{\leftsquigarrow}\ \lambda m.(\text{Mary}^{\leftsquigarrow}\ \lambda n.(k\ (m\ n))))\ (\lambda x.x)$

$\rightarrow_\beta^*\quad (\text{thinks} \lhd (\text{someone} \rhd \text{left}))^{\leftsquigarrow}\ \lambda m.(\text{Mary}^{\leftsquigarrow}\ \lambda n.(m\ n))$

$\equiv\quad (\text{thinks} \lhd (\text{someone} \rhd \text{left}))^{\leftsquigarrow}\ \lambda m.(\lambda k.(k\ \text{MARY})\ \lambda n.(m\ n))$

$\rightarrow_\beta^*\quad (\text{thinks} \lhd (\text{someone} \rhd \text{left}))^{\leftsquigarrow}\ \lambda m.(m\ \text{MARY})$

$\equiv\quad \lambda k.((\text{someone} \rhd \text{left})^{\leftsquigarrow}\ \lambda n.((\text{thinks})^{\leftsquigarrow}\ \lambda m.(k\ (m\ n))))\ \lambda m.(m\ \text{MARY})$

$\equiv\quad \lambda k.((\text{someone} \rhd \text{left})^{\leftsquigarrow}\ \lambda n.(\lambda k.(k\ \text{THINKS})\ \lambda m.(k\ (m\ n))))\ \lambda m.(m\ \text{MARY})$

$\rightarrow_\beta^*\quad \lambda k.((\text{someone} \rhd \text{left})^{\leftsquigarrow}\ \lambda n.(k\ (\text{THINKS}\ n)))\ \lambda m.(m\ \text{MARY})$

$\rightarrow_\beta^*\quad (\text{someone} \rhd \text{left})^{\leftsquigarrow}\ \lambda n.(\text{THINKS}\ n\ \text{MARY})$

$\equiv\quad \lambda k.(\text{left}^{\leftsquigarrow}\ \lambda m.(\text{someone}^{\leftsquigarrow}\ \lambda n.(k\ (m\ n))))\ \lambda n.(\text{THINKS}\ n\ \text{MARY})$

$\equiv\quad \lambda k.(\lambda k.(k\ \text{LEFT})\ \lambda m.(\exists \lambda n.(k\ (m\ n))))\ \lambda n.(\text{THINKS}\ n\ \text{MARY})$

$\rightarrow_\beta^*\quad \lambda k.(k\ \text{LEFT})\ \lambda m.(\exists \lambda n.(\text{THINKS}\ (m\ n)\ \text{MARY}))$

$\rightarrow_\beta^*\quad \exists \lambda n.((\text{THINKS}\ (\text{LEFT}\ n)\ \text{MARY}))$

2.3   Observations

Whereas the local derivation was the easiest one to produce with Hendrik's model, Barker's seems unable to create it. Contrary to Hendriks', Barker's model is deterministic and leaves no choice to the parser about the rules applied or the continuations created. In that sense, it is not surprising that only one of the interpretations could be associated with the given sentence. As to why it is this particular intepretation that we obtain with both the left-to-right and right-to-left approaches, two points must be considered.

Obviously, there will come a point in which we will have to translate $\mathsf{someone} \triangleright \mathsf{left}$, which will produce a lambda abstraction that will accept a continuation k. Now, k will necessarily include the word thinks, as can be seen by the translation rules for $\mathsf{thinks} \triangleleft (\mathsf{someone} \triangleright \mathsf{left})$ (i.e. $(M \triangleleft N)^{\rightsquigarrow}$ and $(M \triangleleft N)^{\leftsquigarrow}$). In both cases, we examine whether a continuation containing THINKS as a sub-expression will be passed as an argument to $\mathsf{someone} \triangleright \mathsf{left}$, notated as N inside the rules. In the right-to-left case, it is immediate that $N^{\leftsquigarrow}$ is passed an argument that contains THINKS, namely $M^{\leftsquigarrow} \mapsto \lambda k.(k \text{ THINKS})$. In the left-to-right case, we have to consider an additional translation step, namely the application $(M^{\rightsquigarrow} \; \lambda m.(N^{\rightsquigarrow} \ldots m \ldots))$. Expanding $M^{\rightsquigarrow}$ to $\lambda k.(k \text{ THINKS})$ and applying a β-reduction leads to $(N^{\rightsquigarrow} \; \ldots \text{THINKS} \ldots)$, which again passes a continuation k containing THINKS to N.

Moreover, looking carefully at the corresponding translation rule for $\mathsf{someone} \triangleright \mathsf{left}$, we notice that the given continuation k, which contains THINKS as a sub-expression, is always positioned in an inner scope than that of $\mathsf{someone}$.

Hence, as the non-local interpretation positions THINKS in an outer scope than that of SOMEONE, it cannot be derived from Barker's model.

3 Plotkin

CONSTANT	SOURCE TYPE	TARGET VALUE	TARGET TERM
	A	$\lceil A \rceil$	type: $\overline{A} = (\lceil A \rceil \multimap \bot) \multimap \bot$
Mary	np	e	$\lambda k.(k \text{ MARY})$
someone	np	e	$\exists$
left	$np \backslash s$	$e \multimap (t \multimap \bot) \multimap \bot$	$\lambda k'.(k' \ \lambda x.\lambda k.(k \ (\text{LEFT } x)))$
thinks	$(np \backslash s)/s$	$t \multimap ((e \multimap (t \multimap \bot) \multimap \bot) \multimap \bot) \multimap \bot$	$\lambda k.(k \ \lambda t.\lambda k'.(k' \ \lambda x.\lambda c.(c \ (\text{THINKS } t \ x))))$

First, we compute the inner interpretations:

$$\overline{\text{someone} \triangleright \text{left}} \quad \equiv \lambda k.(\underline{\lambda k'.(k' \ \lambda x.\lambda k''.(k'' \ (\text{LEFT } x)))} \ \underline{\lambda m.(\exists \lambda n.(m \ n \ k)))}$$

$$\rightarrow^*_\beta \lambda k.(\exists \lambda n.(k \ (\text{LEFT } n))) \tag{1}$$

$$\overline{\text{thinks} \triangleleft (\text{someone} \triangleright \text{left})} \quad \equiv \lambda k.(\overline{\text{thinks}} \ \lambda m.(\overline{\text{someone} \triangleright \text{left}} \ \lambda n.(m \ n \ k)))$$

$$\equiv \lambda k''.(\underline{\lambda k.(k \ \lambda t.\lambda k'.(k' \ \lambda x.\lambda c.(c \ (\text{THINKS } t \ x))))} \ \underline{\lambda m.(\overline{\text{someone} \triangleright \text{left}} \ \lambda n.(m \ n \ k''))})$$

$$\rightarrow^*_\beta \lambda k''.(\overline{\text{someone} \triangleright \text{left}} \ \lambda n.(k'' \ \lambda x.\lambda c.(c \ (\text{THINKS } n \ x))))$$

$$\overset{(1)}{\equiv} \lambda k''.(\underline{\lambda k.(\exists \lambda n.(k \ (\text{LEFT } n)))} \ \underline{\lambda n.(k'' \ \lambda x.\lambda c.(c \ (\text{THINKS } n \ x))))}$$

$$\rightarrow^*_\beta \lambda k''.(\exists \lambda n.(k'' \ \lambda x.\lambda c.(c \ (\text{THINKS } (\text{LEFT } n) \ x))))$$

$$\rightarrow_\alpha \lambda k.(\exists \lambda n.(k \ \lambda x.\lambda c.(c \ (\text{THINKS } (\text{LEFT } n) \ x)))) \tag{2}$$

We can now compute the interpretation by giving the empty context (ϵ) as the initial continuation:

$$\overline{\text{Mary} \triangleright (\text{thinks} \triangleleft (\text{someone} \triangleright \text{left}))} \ \epsilon$$

$$\equiv \quad \lambda k.(\overline{\text{thinks} \triangleleft (\text{someone} \triangleright \text{left})} \ \lambda m.(\lambda k.(k \text{ MARY}) \ \lambda n.(m \ n \ k))) \ (\lambda x.x)$$

$$\rightarrow^*_\beta \quad \overline{\text{thinks} \triangleleft (\text{someone} \triangleright \text{left})} \ \lambda m.(\lambda k.(k \text{ MARY}) \ \lambda n.(m \ n \ (\lambda x.x)))$$

$$\overset{(2)}{\equiv} \quad \lambda k.(\exists \lambda n.(k \ \lambda x.\lambda c.(c \ (\text{THINKS } (\text{LEFT } n) \ x)))) \ \lambda m.(\underline{\lambda k.(k \text{ MARY})} \ \underline{\lambda n.(m \ n \ (\lambda x.x))})$$

$$\rightarrow^*_\beta \quad \lambda k.(\exists \lambda n.(k \ \lambda x.\lambda c.(c \ (\text{THINKS } (\text{LEFT } n) \ x)))) \ \underline{\lambda m.(m \text{ MARY } (\lambda x.x))}$$

$$\rightarrow^*_\beta \quad \exists \lambda n.(\text{THINKS } (\text{LEFT } n) \text{ MARY})$$

3.1 Observations

Plotkin's model surpasses Barker's due to its ability to handle quantifiers. However it still falls short of producing the local interpretation of the given sentence, largely due to the same reasons. As before, Plotkin's model is strict in terms of how much control it assigns to the parser; only a single, deterministic parse is possible for a sentence. Similarly, the sub-phrase someone $\triangleright$ left will at some point be evaluated and await for a continuation k as an argument, which will be positioned at an inner scope than that of SOMEONE (notated as $\overline{N}$ in the $N^A \triangleright M^{A \backslash B}$ rule).

Now, k cannot contain LEFT, as it is already positioned in an outer scope (notated as $\overline{M}$ in the rule). Moreover, if no other word were to be contained in k, the term would be ill-typed as it requires a constant to the right. Therefore, either THINK or MARY would have to be inserted, thus enforcing the global interpretation.