

Logic and Language: Exercise (Week 5)

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1 LG: continuation semantics

1.1

For each term, we compute the ILL type by first observing the polarity of the sequent and then using the table to interpret complex types.

$$\begin{aligned} \llbracket \text{some} \rrbracket &= \llbracket np/n \rrbracket^\perp && \{ \text{Negative Hypothesis} \} \\ &= (\llbracket np \rrbracket^\perp \otimes \llbracket n \rrbracket)^\perp && \{ \llbracket A/B \rrbracket \text{ with } A \text{ and } B \text{ positive} \} \\ &= (np^\perp \otimes n)^\perp && \{ np \text{ and } n \text{ positive} \} \end{aligned}$$

$$\begin{aligned} \llbracket \text{popular} \rrbracket &= \llbracket n/n \rrbracket^\perp && \{ \text{Negative Hypothesis} \} \\ &= (\llbracket n \rrbracket^\perp \otimes \llbracket n \rrbracket)^\perp && \{ \llbracket A/B \rrbracket \text{ with } A \text{ and } B \text{ positive} \} \\ &= (n^\perp \otimes n)^\perp && \{ n \text{ positive} \} \end{aligned}$$

$$\begin{aligned} \llbracket \text{saint} \rrbracket &= \llbracket n \rrbracket && \{ \text{Positive Hypothesis} \} \\ &= n && \{ n \text{ positive} \} \end{aligned}$$

$$\begin{aligned} \llbracket \text{arrived} \rrbracket &= \llbracket np \backslash s \rrbracket^\perp && \{ \text{Negative Hypothesis} \} \\ &= (\llbracket np \rrbracket \otimes \llbracket s \rrbracket)^\perp && \{ \llbracket B \backslash A \rrbracket \text{ with } B \text{ positive and } A \text{ negative} \} \\ &= (np \otimes s^\perp)^\perp && \{ np \text{ positive, } s \text{ negative} \} \end{aligned}$$

$$\begin{aligned} \alpha &= \llbracket s \rrbracket \\ &= s^\perp && \{ s \text{ negative} \} \end{aligned}$$

$$\begin{aligned} z &= \llbracket np \rrbracket \\ &= np && \{ np \text{ positive} \} \end{aligned}$$

$$\begin{aligned} y &= \llbracket n \rrbracket \\ &= n && \{ n \text{ positive} \} \end{aligned}$$

The ILL types then are:

TERM	TYPE
$\llbracket \text{some} \rrbracket$	$(np^\perp \otimes n)^\perp$
$\llbracket \text{popular} \rrbracket$	$(n^\perp \otimes n)^\perp$
$\llbracket \text{saint} \rrbracket$	n
$\llbracket \text{arrived} \rrbracket$	$(np \otimes s^\perp)^\perp$
α	s^\perp
z	np
y	n

1.2

SOURCE TYPE	CONSTANT	$\llbracket \cdot \rrbracket^\ell$
n/n	popular	$\lambda \langle c, y \rangle. (c (\lambda z. \wedge (y z) (\text{POPULAR } z)))$

1.3

1. We compute the interpretation below:

$$\llbracket \dagger \rrbracket = \lambda a_0. (\llbracket \text{arrived} \rrbracket \langle \lambda \beta_0. (\llbracket \text{some} \rrbracket \langle \beta_0, \lambda \gamma_0. (\llbracket \text{popular} \rrbracket \langle \gamma_0, \lambda a_1. (\llbracket \text{saint} \rrbracket a_1) \rangle \rangle \rangle), a_0 \rangle)$$

2. The adjucted \cdot^ℓ translations are the following:

$$\begin{aligned} \llbracket \text{some} \rrbracket^\ell &= \lambda \langle x, k \rangle. (\exists \lambda z. (\wedge (k \lambda \theta. (\theta z)) (x z))) \\ \llbracket \text{popular} \rrbracket^\ell &= \lambda \langle c, k \rangle. (c (\lambda z. (\wedge (\text{POPULAR } z) (k \lambda \theta. (\theta z)))))) \\ \llbracket \text{saint} \rrbracket^\ell &= \lambda c. (c \text{ SAINT}) \\ \llbracket \text{arrived} \rrbracket^\ell &= \lambda \langle k, c \rangle. (k \lambda z. (c (\text{ARRIVED } z))) \end{aligned}$$

We can now corroborate the α -equivalence of the two \cdot^ℓ translations:

$$\begin{aligned} \llbracket \dagger \rrbracket^\ell &= \lambda a_0. (\lambda \langle k, c \rangle. (k \lambda z. (c (\text{ARRIVED } z)))) \\ &\quad \langle \lambda \beta_0. (\lambda \langle x, k \rangle. (\exists \lambda z. (\wedge (k \lambda \theta. (\theta z)) (x z))) \langle \beta_0, \lambda \gamma_0. (\\ &\quad \lambda \langle c, k \rangle. (c (\lambda z. (\wedge (\text{POP } z) (k \lambda \theta. (\theta z)))))) \langle \gamma_0, \lambda a_1. (\lambda c. (c \text{ SAINT}) a_1) \rangle \rangle \rangle), a_0 \rangle) \\ &\rightarrow_\beta^* \lambda a_0. (\lambda \langle x, k \rangle. (\exists \lambda z. (\wedge (k \lambda \theta. (\theta z)) (x z)))) \langle \lambda z. (a_0 (\text{ARRIVED } z)), \lambda \gamma_0. (\\ &\quad \lambda \langle c, k \rangle. (c (\lambda z. (\wedge (\text{POPULAR } z) (k \lambda \theta. (\theta z)))))) \langle \gamma_0, \lambda a_1. (\lambda c. (c \text{ SAINT}) a_1) \rangle \rangle \rangle) \\ &\rightarrow_\beta^* \lambda a_0. (\exists \lambda z. (\wedge (\lambda \langle c, k \rangle. (c (\lambda z. (\wedge (\text{POPULAR } z) (k \lambda \theta. (\theta z)))))) \\ &\quad \langle \lambda \theta. (\theta z), \lambda a_1. (\lambda c. (c \text{ SAINT}) a_1) \rangle \rangle) (a_0 (\text{ARRIVED } z))) \\ &\rightarrow_\beta^* \lambda a_0. (\exists \lambda z. (\wedge (\lambda \theta. (\theta z)) (\lambda z. (\wedge (\text{POPULAR } z) (\lambda c. (c \text{ SAINT}) (\lambda \theta. (\theta z)))))) (a_0 (\text{ARRIVED } z))) \\ &\rightarrow_\beta^* \lambda a_0. (\exists \lambda z. (\wedge (\wedge (\text{POPULAR } z) (\text{SAINT } z)) (a_0 (\text{ARRIVED } z)))) \\ &\rightarrow_\alpha \lambda c. (\exists \lambda z. (\wedge (\wedge (\text{POPULAR } z) (\text{SAINT } z)) (c (\text{ARRIVED } z)))) \\ &\rightarrow_\alpha \lambda c. (\exists \lambda x. (\wedge (\wedge (\text{POPULAR } x) (\text{SAINT } x)) (c (\text{ARRIVED } x)))) \\ &= \llbracket \dagger \rrbracket^\ell \end{aligned}$$

2 Pregroups

2.1

$$(4) \quad \begin{array}{ccc} & (\rightarrow) & (\leftarrow) \\ \frac{\overline{1^l \rightarrow 1^l 1} \quad (1) \quad \overline{1^l 1 \rightarrow 1} \quad (2)}{1^l \rightarrow 1} \rightarrow & & \frac{\overline{1 \rightarrow 1^l 1} \quad (2) \quad \overline{1^l 1 \rightarrow 1^l} \quad (1)}{1 \rightarrow 1^l} \rightarrow \end{array}$$

$$(5) \quad (\rightarrow): \quad \frac{\overline{A^{rl} \rightarrow A^{rl} 1} \quad (1) \quad \overline{1 \rightarrow A^r A} \quad (2)}{\overline{A^{rl} \rightarrow A^{rl}(A^r A)} \rightarrow} \rightarrow \frac{\overline{A^{rl} \rightarrow (A^{rl} A^r) A} \quad (1) \quad \overline{A^{rl} A^r \rightarrow 1} \quad (2)}{A^{rl} \rightarrow 1A} \rightarrow \frac{1A \rightarrow A \quad (1)}{A^{rl} \rightarrow A}$$

$$(\leftarrow): \quad \frac{\overline{A \rightarrow A 1} \quad (1) \quad \overline{1 \rightarrow A^r A^{rl}} \quad (2)}{\overline{A \rightarrow A(A^r A^{rl})} \rightarrow} \rightarrow \frac{\overline{A \rightarrow (AA^r) A^{rl}} \quad (1) \quad \overline{AA^r \rightarrow 1} \quad (3)}{A \rightarrow 1A^{rl}} \rightarrow \frac{1A^{rl} \rightarrow A^{rl} \quad (1)}{A \rightarrow A^{rl}}$$

$$(6) \quad (\rightarrow): \quad \frac{\overline{(AB)^l \rightarrow (1(AB))^l} \rightarrow \quad \overline{1 \rightarrow (B^l A^l)^r (B^l A^l)} \quad (3)}{\overline{(AB)^l \rightarrow (((B^l A^l)^r (B^l A^l))(AB))^l} \rightarrow} \rightarrow \frac{\overline{(AB)^l \rightarrow ((B^l A^l)^r ((B^l A^l)(AB)))^l} \quad (1)}{\overline{(AB)^l \rightarrow ((B^l A^l)^r (B^l (A^l (AB))))} \quad (1)} \rightarrow \frac{\overline{(AB)^l \rightarrow ((B^l A^l)^r (B^l (A^l A) B))} \quad (1) \quad \overline{A^l A \rightarrow 1} \quad (2)}{\overline{(AB)^l \rightarrow ((B^l A^l)^r (B^l (1B)))^l} \rightarrow} \rightarrow \frac{\overline{1B \rightarrow B} \quad (1)}{\overline{(AB)^l \rightarrow ((B^l A^l)^r (B^l B))^l} \rightarrow} \rightarrow \frac{\overline{B^l B \rightarrow 1} \quad (2)}{\overline{(AB)^l \rightarrow ((B^l A^l)^r 1)^l} \rightarrow} \rightarrow \frac{\overline{(B^l A^l)^r 1 \rightarrow (AB)^l} \quad (5)}{\overline{(AB)^l \rightarrow ((B^l A^l)^r)^l} \rightarrow}$$

2.2

We first calculate the pregroup translation of the given sequent:

$$\begin{array}{ll}
\overline{(p/((q/q)/r))/r} & \\
= \overline{(p/((q/q)/r))} r^\ell & \{\overline{A/B} = \overline{A}(\overline{B})^\ell\} \\
= p \overline{((q/q)/r)}^\ell r^\ell & \{\overline{A/B} = \overline{A}(\overline{B})^\ell\} \\
= p \overline{(q/q)}^\ell r^\ell r^\ell & \{\overline{A/B} = \overline{A}(\overline{B})^\ell\} \\
= p ((q q^\ell) r^\ell)^\ell r^\ell & \{\overline{A/B} = \overline{A}(\overline{B})^\ell\} \\
= p (q (q^\ell r^\ell))^\ell r^\ell & \{\text{rule (1) from 2.1}\} \\
= p (q^\ell r^\ell)^\ell q^\ell r^\ell & \{\text{rule (6) from 2.1}\} \\
= p r^{\ell\ell} q^{\ell\ell} q^\ell r^\ell & \{\text{rule (6) from 2.1}\}
\end{array}$$

Finally, we prove the sequent by drawing a string diagram:

$$\begin{array}{c}
p r^{\ell\ell} q^{\ell\ell} q^\ell r^\ell \\
| \quad \frown
\end{array}$$