Logic and Language: Exercise (Week 3)

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1.1

1. Unfolding the combinator f/g, we get the following derivation:

$$\dfrac{\dfrac{\dfrac{1_{A/B} : A/B \longrightarrow A/B}{\triangleright^{-1} 1_{A/B} : A/B \otimes B \longrightarrow A} \quad f : A \longrightarrow A'}{\dfrac{f \circ (\triangleright^{-1} 1_{A/B}) : A/B \otimes B \longrightarrow A'}{\triangleright(f \circ (\triangleright^{-1} 1_{A/B})) : A/B \longrightarrow A'/B}} \qquad \dfrac{\dfrac{g : B \longrightarrow B' \quad \dfrac{\dfrac{1_{A/B'} : A/B \longrightarrow A/B'}{\triangleright^{-1} 1_{A/B'} : (A/B') \otimes B' \longrightarrow A}}{\triangleleft\triangleright^{-1} 1_{A/B'} : B' \longrightarrow (A/B')\backslash A}}{\dfrac{(\triangleleft\triangleright^{-1} 1_{A/B'}) \circ g : B \longrightarrow (A/B')\backslash A}{\dfrac{\triangleleft^{-1}((\triangleleft\triangleright^{-1} 1_{A/B'}) \circ g) : (A/B') \otimes B \longrightarrow A}{\triangleright\triangleleft^{-1}((\triangleleft\triangleright^{-1} 1_{A/B'}) \circ g) : A/B' \longrightarrow A/B}}}}{(\triangleright(f \circ (\triangleright^{-1} 1_{A/B}))) \circ (\triangleright\triangleleft^{-1}((\triangleleft\triangleright^{-1} 1_{A/B'}) \circ g)) : A/B' \longrightarrow A'/B}$$

2. Applying the arrow reversal transformation $(\cdot)^\dagger$ to f/g, we get the reverse combinator:

$$\begin{aligned}
(f/g)^\dagger &= ((\triangleright(f \circ (\overset{-1}{\triangleright} 1_{A/B}))) \circ (\triangleright \overset{-1}{\triangleleft}((\triangleleft \overset{-1}{\triangleright} 1_{A/B'}) \circ g)))^\dagger \\
&= (\triangleright \overset{-1}{\triangleleft}((\triangleleft \overset{-1}{\triangleright} 1_{A/B'}) \circ g))^\dagger \circ (\triangleright(f \circ (\overset{-1}{\triangleright} 1_{A/B})))^\dagger \\
&= (\blacktriangleleft(\overset{-1}{\triangleleft}((\triangleleft \overset{-1}{\triangleright} 1_{A/B'}) \circ g))^\dagger) \circ (\blacktriangleleft(f \circ (\overset{-1}{\triangleright} 1_{A/B}))^\dagger) \\
&= (\blacktriangleleft \overset{-1}{\blacktriangleright}((\triangleleft \overset{-1}{\triangleright} 1_{A/B'}) \circ g)^\dagger) \circ (\blacktriangleleft((\overset{-1}{\triangleright} 1_{A/B})^\dagger \circ f^\dagger)) \\
&= (\blacktriangleleft \overset{-1}{\blacktriangleright}(g^\dagger \circ (\triangleleft \overset{-1}{\triangleright} 1_{A/B'})^\dagger)^\dagger) \circ (\blacktriangleleft((\overset{-1}{\blacktriangleleft} 1_{(A/B)^\dagger}) \circ f^\dagger)) \\
&= (\blacktriangleleft \overset{-1}{\blacktriangleright}(g^\dagger \circ (\blacktriangleright(\overset{-1}{\triangleright} 1_{A/B'})^\dagger))) \circ (\blacktriangleleft((\overset{-1}{\blacktriangleleft} 1_{B \oslash A}) \circ f^\dagger)) \\
&= (\blacktriangleleft \overset{-1}{\blacktriangleright}(g^\dagger \circ (\blacktriangleright \overset{-1}{\blacktriangleleft} 1_{(A/B')^\dagger}))) \circ (\blacktriangleleft((\overset{-1}{\blacktriangleleft} 1_{B \oslash A}) \circ f^\dagger)) \\
&= (\blacktriangleleft \overset{-1}{\blacktriangleright}(g^\dagger \circ (\blacktriangleright \overset{-1}{\blacktriangleleft} 1_{B' \oslash A}))) \circ (\blacktriangleleft((\overset{-1}{\blacktriangleleft} 1_{B \oslash A}) \circ f^\dagger))
\end{aligned}$$

Unfolding the combinator f/g, we get the following derivation:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{1_{B'\obslash A} : B' \obslash A \longrightarrow B' \obslash A}{\blacktriangleleft^{-1} 1_{B'\obslash A} : A \longrightarrow B' \oplus (B' \obslash A)}}{\blacktriangleright \blacktriangleleft^{-1} 1_{B'\obslash A} : A \oslash (B' \obslash A) \longrightarrow B'} \quad g^\dagger : B' \longrightarrow B}{g^\dagger \circ (\blacktriangleright \blacktriangleleft^{-1} 1_{B'\obslash A}) : A \oslash (B' \obslash A) \longrightarrow B}}{\blacktriangleright^{-1}(g^\dagger \circ (\blacktriangleright \blacktriangleleft^{-1} 1_{B'\obslash A})) : A \longrightarrow B \oplus (B' \obslash A)}}{\blacktriangleleft \blacktriangleright^{-1}(g^\dagger \circ (\blacktriangleright \blacktriangleleft^{-1} 1_{B'\obslash A})) : B \obslash A \longrightarrow B' \obslash A} \quad \dfrac{\dfrac{f^\dagger : A' \longrightarrow A \quad \dfrac{1_{B\obslash A} : B \obslash A \longrightarrow B \obslash A}{\blacktriangleleft^{-1} 1_{B\obslash A} : A \longrightarrow B \oplus (B \obslash A)}}{(\blacktriangleleft^{-1} 1_{B\obslash A}) \circ f^\dagger : A' \longrightarrow B \oplus (B \obslash A)}}{\blacktriangleleft((\blacktriangleleft^{-1} 1_{B\obslash A}) \circ f^\dagger) : B \obslash A' \longrightarrow B \obslash A}}{(\blacktriangleleft \blacktriangleright^{-1}(g^\dagger \circ (\blacktriangleright \blacktriangleleft^{-1} 1_{B'\obslash A}))) \circ (\blacktriangleleft((\blacktriangleleft^{-1} 1_{B\obslash A}) \circ f^\dagger)) : B \obslash A' \longrightarrow B' \obslash A}$$

1.2

1. $(a \oplus b)/c \longrightarrow a/(c \oslash b)$
2. $(b \oplus c) \otimes a \longrightarrow b \oplus (c \otimes a)$