Logic and Language: Exercise (Week 2)

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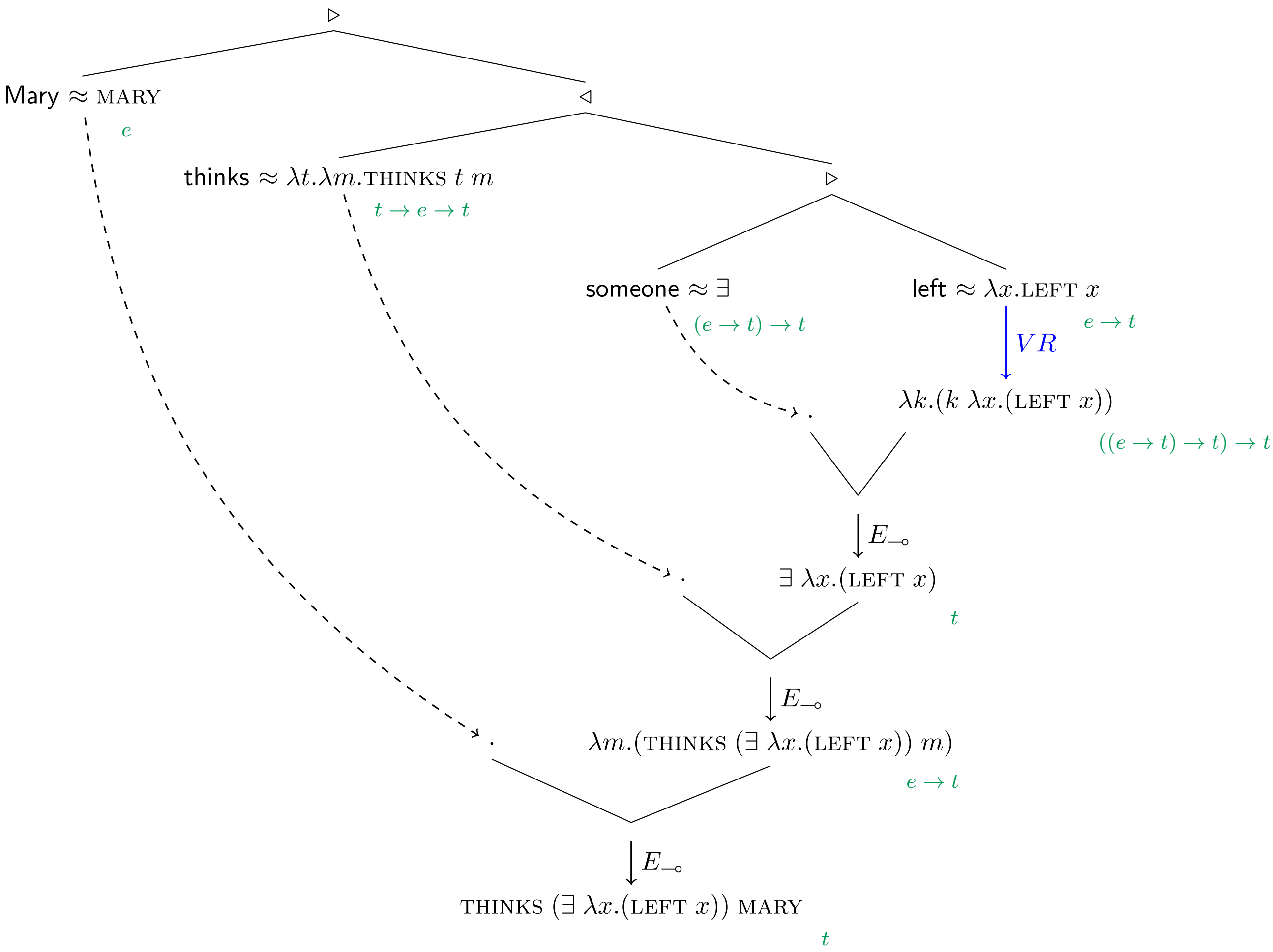
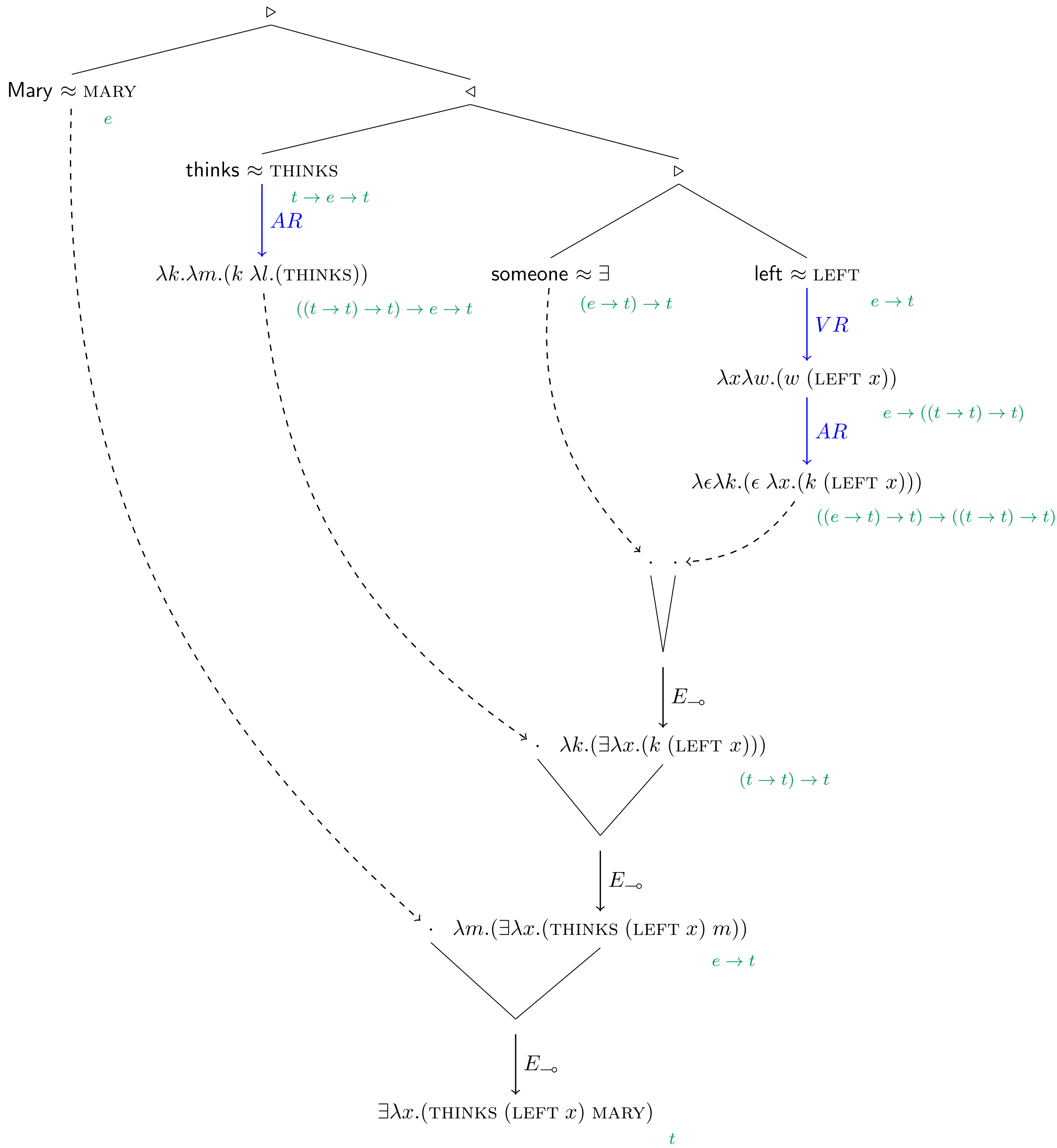
1 Hendriks

1.1 Local Interpretation



Clarification: Basic term translations implicitly apply η-expansions, in order to transform the term in the desired format.

1.2 Non-Local Interpretation



2 Barker

2.1 Left-to-right incremental

$$
\begin{aligned}
& (\mathsf{Mary} \rhd (\mathsf{thinks} \lhd (\mathsf{someone} \rhd \mathsf{left})))^{\rightsquigarrow}\ (\lambda x.x) \\
\equiv\ & \underline{\lambda k.(\mathsf{Mary}^{\rightsquigarrow}\ \lambda n.((\mathsf{thinks} \lhd (\mathsf{someone} \rhd \mathsf{left}))^{\rightsquigarrow}\ \lambda m.(k\ (m\ n))))}\ \underline{(\lambda x.x)} \\
\to_\beta^*\ & \mathsf{Mary}^{\rightsquigarrow}\ \lambda n.((\mathsf{thinks} \lhd (\mathsf{someone} \rhd \mathsf{left}))^{\rightsquigarrow}\ \lambda m.(m\ n)) \\
\equiv\ & \underline{\lambda k.(k\ \textsc{mary})}\ \underline{\lambda n.((\mathsf{thinks} \lhd (\mathsf{someone} \rhd \mathsf{left}))^{\rightsquigarrow}\ \lambda m.(k\ (m\ n))} \\
\to_\beta^*\ & (\mathsf{thinks} \lhd (\mathsf{someone} \rhd \mathsf{left}))^{\rightsquigarrow}\ \lambda m.(m\ \textsc{mary}) \\
\equiv\ & \underline{\lambda k.((\mathsf{thinks}^{\rightsquigarrow}\ \lambda m.((\mathsf{someone} \rhd \mathsf{left})^{\rightsquigarrow}\ \lambda n.(k\ (m\ n))))}\ \underline{\lambda m.(m\ \textsc{mary})} \\
\to_\beta^*\ & \mathsf{thinks}^{\rightsquigarrow}\ \lambda m.((\mathsf{someone} \rhd \mathsf{left})^{\rightsquigarrow}\ \lambda n.(m\ n\ \textsc{mary})) \\
\equiv\ & \underline{\lambda k.(k\ \textsc{thinks})}\ \underline{\lambda m.((\mathsf{someone} \rhd \mathsf{left})^{\rightsquigarrow}\ \lambda n.(m\ n\ \textsc{mary}))} \\
\to_\beta^*\ & (\mathsf{someone} \rhd \mathsf{left})^{\rightsquigarrow}\ \lambda n.(\textsc{thinks}\ n\ \textsc{mary}) \\
\equiv\ & \underline{\lambda k.(\mathsf{someone}^{\rightsquigarrow} \lambda n.(\mathsf{left}^{\rightsquigarrow} \lambda m.(k\ (m\ n))))}\ \underline{\lambda n.(\textsc{thinks}\ n\ \textsc{mary})} \\
\to_\beta^*\ & \mathsf{someone}^{\rightsquigarrow} \lambda n.(\mathsf{left}^{\rightsquigarrow} \lambda m.(\textsc{thinks}\ (m\ n)\ \textsc{mary}))) \\
\equiv\ & \exists\lambda n.(\underline{\lambda k.(k\ \textsc{left})}\ \underline{\lambda m.(\textsc{thinks}\ (m\ n)\ \textsc{mary}))}) \\
\to_\beta^*\ & \exists\lambda n.(\textsc{thinks}\ (\textsc{left}\ n)\ \textsc{mary}))
\end{aligned}
$$

2.2 Right-to-left incremental

$$
\begin{aligned}
& (\mathsf{Mary} \rhd (\mathsf{thinks} \lhd (\mathsf{someone} \rhd \mathsf{left})))^{\leftsquigarrow}\ (\lambda x.x) \\
\equiv\ & \underline{\lambda k.((\mathsf{thinks} \lhd (\mathsf{someone} \rhd \mathsf{left}))^{\leftsquigarrow}\ \lambda m.(\mathsf{Mary}^{\leftsquigarrow}\ \lambda n.(k\ (m\ n))))}\ \underline{(\lambda x.x)} \\
\to_\beta^*\ & (\mathsf{thinks} \lhd (\mathsf{someone} \rhd \mathsf{left}))^{\leftsquigarrow}\ \lambda m.(\mathsf{Mary}^{\leftsquigarrow}\ \lambda n.(m\ n)) \\
\equiv\ & (\mathsf{thinks} \lhd (\mathsf{someone} \rhd \mathsf{left}))^{\leftsquigarrow}\ \lambda m.(\underline{\lambda k.(k\ \textsc{mary})}\ \underline{\lambda n.(m\ n)}) \\
\to_\beta^*\ & (\mathsf{thinks} \lhd (\mathsf{someone} \rhd \mathsf{left}))^{\leftsquigarrow}\ \lambda m.(m\ \textsc{mary}) \\
\equiv\ & \lambda k.((\mathsf{someone} \rhd \mathsf{left})^{\leftsquigarrow}\ \lambda n.((\mathsf{thinks})^{\leftsquigarrow}\ \lambda m.(k\ (m\ n))))\ \lambda m.(m\ \textsc{mary}) \\
\equiv\ & \lambda k.((\mathsf{someone} \rhd \mathsf{left})^{\leftsquigarrow}\ \lambda n.(\underline{\lambda k.(k\ \textsc{thinks})}\ \underline{\lambda m.(k\ (m\ n))}))\ \lambda m.(m\ \textsc{mary}) \\
\to_\beta^*\ & \underline{\lambda k.((\mathsf{someone} \rhd \mathsf{left})^{\leftsquigarrow}\ \lambda n.(k\ (\textsc{thinks}\ n)))}\ \underline{\lambda m.(m\ \textsc{mary})} \\
\to_\beta^*\ & (\mathsf{someone} \rhd \mathsf{left})^{\leftsquigarrow}\ \lambda n.(\textsc{thinks}\ n\ \textsc{mary}) \\
\equiv\ & \lambda k.(\mathsf{left}^{\leftsquigarrow}\ \lambda m.(\mathsf{someone}^{\leftsquigarrow}\ \lambda n.(k\ (m\ n))))\ \lambda n.(\textsc{thinks}\ n\ \textsc{mary}) \\
\equiv\ & \underline{\lambda k.(\lambda k.(k\ \textsc{left})\ \lambda m.(\exists\lambda n.(k\ (m\ n))))}\ \underline{\lambda n.(\textsc{thinks}\ n\ \textsc{mary})} \\
\to_\beta^*\ & \underline{\lambda k.(k\ \textsc{left})}\ \underline{\lambda m.(\exists\lambda n.(\textsc{thinks}\ (m\ n)\ \textsc{mary}))} \\
\to_\beta^*\ & \exists\lambda n.((\textsc{thinks}\ (\textsc{left}\ n)\ \textsc{mary}))
\end{aligned}
$$

3 Plotkin

CONSTANT	SOURCE TYPE	TARGET VALUE	TARGET TERM
	A	$\lceil A \rceil$	type: $\overline{A} = (\lceil A \rceil \multimap \bot) \multimap \bot$
Mary	np	e	$\lambda k.(k\ \textsc{mary})$
someone	np	e	$\exists$
left	$np\backslash s$	$e \multimap (t \multimap \bot) \multimap \bot$	$\lambda k'.(k'\ \lambda x.\lambda k.(k\ (\textsc{left}\ x)))$
thinks	$(np\backslash s)/s$	$t \multimap ((e \multimap (t \multimap \bot) \multimap \bot) \multimap \bot) \multimap \bot$	$\lambda k.(k\ \lambda t.\lambda k'.(k'\ \lambda x.\lambda c.(c\ (\textsc{thinks}\ t\ x))))$

First, we compute the inner interpretations:

$$
\begin{aligned}
\overline{\textsf{someone} \triangleright \textsf{left}} &\equiv \lambda k.(\lambda k'.(k'\ \lambda x.\lambda k''.(k''\ (\textsc{left}\ x)))\ \lambda m.(\exists \lambda n.(m\ n\ k))) \\
&\to_\beta^* \lambda k.(\exists \lambda n.(k\ (\textsc{left}\ n))) \qquad (1)
\end{aligned}
$$

$$
\begin{aligned}
\overline{\textsf{thinks} \triangleleft (\textsf{someone} \triangleright \textsf{left})} &\equiv \lambda k.(\overline{\textsf{thinks}}\ \lambda m.(\overline{\textsf{someone} \triangleright \textsf{left}}\ \lambda n.(m\ n\ k))) \\
&\equiv \lambda k''.(\lambda k.(k\ \lambda t.\lambda k'.(k'\ \lambda x.\lambda c.(c\ (\textsc{thinks}\ t\ x))))\ \lambda m.(\overline{\textsf{someone} \triangleright \textsf{left}}\ \lambda n.(m\ n\ k''))) \\
&\to_\beta^* \lambda k''.(\overline{\textsf{someone} \triangleright \textsf{left}}\ \lambda n.(k''\ \lambda x.\lambda c.(c\ (\textsc{thinks}\ n\ x)))) \\
&\overset{(1)}{\equiv} \lambda k''.(\lambda k.(\exists \lambda n.(k\ (\textsc{left}\ n)))\ \lambda n.(k''\ \lambda x.\lambda c.(c\ (\textsc{thinks}\ n\ x)))) \\
&\to_\beta^* \lambda k''.(\exists \lambda n.(k''\ \lambda x.\lambda c.(c\ (\textsc{thinks}\ (\textsc{left}\ n)\ x))) \\
&\to_\alpha \lambda k.(\exists \lambda n.(k\ \lambda x.\lambda c.(c\ (\textsc{thinks}\ (\textsc{left}\ n)\ x))) \qquad (2)
\end{aligned}
$$

We can now compute the interpretation by giving the empty context (ϵ) as the initial continuation:

$$
\begin{aligned}
&\overline{\textsf{Mary} \triangleright (\textsf{thinks} \triangleleft (\textsf{someone} \triangleright \textsf{left}))}\ \epsilon \\
\equiv\ & \lambda k.(\overline{\textsf{thinks} \triangleleft (\textsf{someone} \triangleright \textsf{left})}\ \lambda m.(\lambda k.(k\ \textsc{mary})\ \lambda n.(m\ n\ k)))\ (\lambda x.x) \\
\to_\beta^*\ & \overline{\textsf{thinks} \triangleleft (\textsf{someone} \triangleright \textsf{left})}\ \lambda m.(\lambda k.(k\ \textsc{mary})\ \lambda n.(m\ n\ (\lambda x.x))) \\
\overset{(2)}{\equiv}\ & \lambda k.(\exists \lambda n.(k\ \lambda x.\lambda c.(c\ (\textsc{thinks}\ (\textsc{left}\ n)\ x)))\ \lambda m.(\lambda k.(k\ \textsc{mary})\ \lambda n.(m\ n\ (\lambda x.x))) \\
\to_\beta^*\ & \lambda k.(\exists \lambda n.(k\ \lambda x.\lambda c.(c\ (\textsc{thinks}\ (\textsc{left}\ n)\ x)))\ \lambda m.(m\ \textsc{mary}\ (\lambda x.x)) \\
\to_\beta^*\ & \exists \lambda n.(\textsc{thinks}\ (\textsc{left}\ n)\ \textsc{mary})
\end{aligned}
$$