D^3 as a 2-MCFL

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1.3 Challenge

[This is not a hand-in exercise. If you can solve it by Dec 5, there will be a present for you!]

Let D^n be the language over an n-symbol alphabet, lexicographically ordered $a_1 < \cdots < a_n$, where words satisfy the following conditions:

1. each word contains an equal number of the n alphabet symbols
2. for every prefix p of a word, the number of a_i in $p \geq$ the number of a_{i+1} $(1 \leq i \leq n-1)$

D^n generalizes the familiar language of balanced brackets, in which case you have an alphabet of size 2, say $\{\mathsf{a}, \mathsf{b}\}$, with 'opening bracket' a preceding 'closing bracket' b in the lexicographic ordering.

The conjecture (Makoto Kanazawa, p.c.) is that for $n \geq 2$, D^n is the language of a non-wellnested $(n-1)$-MCFG.

Give a 2-MCFG for D^3, i.e. words over a 3-letter alphabet $\{\mathsf{a}, \mathsf{b}, \mathsf{c}\}$ (with the usual lexicographic order) satisfying conditions (1) and (2) above. Give the ACG encoding of your MCFG for D^3.

Reference M. Moortgat (2014), A note on multidimensional Dyck languages.

Some examples

DYCK WORDS

- abc
- aabbcc
- abcabcabacbc

Some examples

Dyck words

- abc
- aabbcc
- abcabcabacbc

Non-dyck words

- aabb

Some examples

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- aabbcc
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- aabbbcc

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$$a\,b\,a\,b\,a\,c\,b\,c\,a\,b\,c\,c$$

First-match policy

Some examples

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Non-dyck words

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- aabbbcc
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$$a\,b\,a\,b\,a\,c\,b\,c\,a\,b\,c\,c$$

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First-match policy

$\mathbf{G}_0$: Grammar of triple insertions

$$\mathrm{S}(xy) \leftarrow \mathrm{W}(x, y). \tag{1}$$

$$\mathrm{W}(\epsilon, xy\mathbf{abc}) \leftarrow \mathrm{W}(x, y). \tag{2}$$

$$\mathrm{W}(\epsilon, x\mathbf{a}y\mathbf{bc}) \leftarrow \mathrm{W}(x, y). \tag{3}$$

$$\ldots\ldots$$

$$\mathrm{W}(\mathbf{ab}x\mathbf{c}y, \epsilon) \leftarrow \mathrm{W}(x, y). \tag{60}$$

$$\mathrm{W}(\mathbf{abc}xy, \epsilon) \leftarrow \mathrm{W}(x, y). \tag{61}$$

$$\mathrm{W}(\epsilon, \mathbf{abc}). \tag{62}$$

$$\mathrm{W}(\mathbf{a}, \mathbf{bc}). \tag{63}$$

$$\mathrm{W}(\mathbf{ab}, \mathbf{c}). \tag{64}$$

$$\mathrm{W}(\mathbf{abc}, \epsilon). \tag{65}$$

$\mathbf{G}_0$: Grammar of triple insertions

$a \quad a \quad b \quad b \quad a \quad c \quad b \quad a \quad c \quad c \quad b \quad c$

Straddling counter-example

G_0: Grammar of triple insertions

$$a \quad a \quad b \quad b \quad a \quad c \quad b \quad a \quad c \quad c \quad b \quad c$$

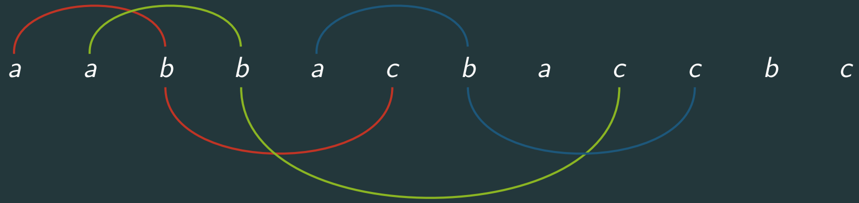
Straddling counter-example

$\mathbf{G}_0$: Grammar of triple insertions

$$a \quad a \quad b \quad b \quad a \quad c \quad b \quad a \quad c \quad c \quad b \quad c$$

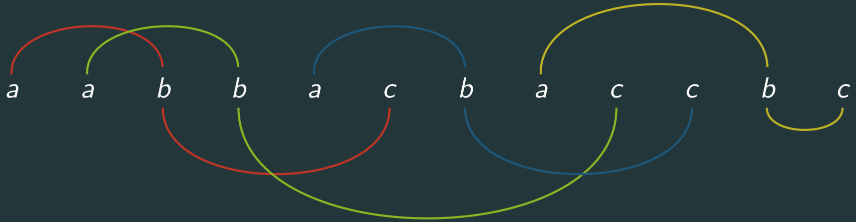
Straddling counter-example

G_0: Grammar of triple insertions



Straddling counter-example

G_0: Grammar of triple insertions



Straddling counter-example

Meta-grammars: Introduction

Notation

$$\mathcal{O}_m[\![\mathit{conclusion} \leftarrow \mathit{premises} \mid \{\mathit{partial\ orderings\ of\ inserted\ elements}\}]\!].$$

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Meta-grammar G_1

$$\left.\begin{array}{l} \mathcal{O}_2[\![W \leftarrow \epsilon \mid \{a < b < c\}]\!]. \\ \mathcal{O}_2[\![W \leftarrow W_{xy} \mid \{x < y,\ a < b < c\}]\!]. \end{array}\right\} \begin{array}{c} \text{TRIPLE} \\ \text{INSERTION} \end{array}$$

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Meta-grammar G_1

$$\left.\begin{array}{l} \mathcal{O}_2[\![W \leftarrow \epsilon \mid \{a < b < c\}]\!]. \\ \mathcal{O}_2[\![W \leftarrow W_{xy} \mid \{x < y,\ a < b < c\}]\!]. \end{array}\right\} \text{TRIPLE INSERTION}$$

$$+$$

$$\mathcal{O}_2[\![W \leftarrow W_{xy}, W_{zw} \mid \{x < y,\ z < w\}]\!]. \Big\} \text{INTERLEAVING WORDS}$$

G_2: Adding states

$$\left.\begin{array}{l}\mathcal{O}_2[\![A^+ \leftarrow \epsilon \mid \{a\}]\!].\\ \mathcal{O}_2[\![B^+ \leftarrow \epsilon \mid \{b\}]\!].\\ \mathcal{O}_2[\![C^+ \leftarrow \epsilon \mid \{c\}]\!].\end{array}\right\}\ \text{BASE CASES}$$

$$\left.\begin{array}{l}\mathcal{O}_2[\![C^- \leftarrow A^+, B^+ \mid \{x < y < z < w\}]\!].\\ \mathcal{O}_2[\![B^- \leftarrow A^+, C^+ \mid \{x < y < z < w\}]\!].\\ \mathcal{O}_2[\![A^- \leftarrow B^+, C^+ \mid \{x < y < z < w\}]\!].\\ \mathcal{O}_2[\![A^+ \leftarrow C^-, B^- \mid \{x < y < z < w\}]\!].\\ \mathcal{O}_2[\![B^+ \leftarrow C^-, A^- \mid \{x < y < z < w\}]\!].\\ \mathcal{O}_2[\![C^+ \leftarrow B^-, A^- \mid \{x < y < z < w\}]\!].\\ \forall\ \mathrm{K} \in \mathcal{S} \setminus \mathrm{W} :\ \mathcal{O}_2[\![\mathrm{K} \leftarrow \mathrm{K}_{xy}, \mathrm{W}_{zw} \mid \{x < y,\ z < w\}]\!].\end{array}\right\}\ \text{COMBINATIONS}$$

$$\left.\begin{array}{l}\mathcal{O}_2[\![W \leftarrow A^+, A^- \mid \{x < y < z < w\}]\!].\\ \mathcal{O}_2[\![W \leftarrow C^-, C^+ \mid \{x < y < z < w\}]\!].\end{array}\right\}\ \text{CLOSURES}$$

G_3: G_2 + Universal triple insertion

$$\begin{aligned}\mathsf{G}_3 = \mathsf{G}_2 + \forall\, \mathrm{K} \in \mathcal{S} \setminus \mathrm{W} :& \\ & \mathcal{O}_2[\![\mathrm{K} \leftarrow \mathrm{K}_{xy} \mid \{x < y,\ a < b < c\}]\!].\end{aligned}$$

Demo

Refining states

Example

$$A^+(x, y)$$

$$\swarrow \qquad \searrow$$

$$A^+_{left} \qquad\qquad A^+_{right}$$

Refining states

Example

$$
\begin{tikzcd}[column sep=small]
 & A^{+}(x, y) \arrow[dl] \arrow[dr] & \\
A^{+}_{left} & \vdots & A^{+}_{right}
\end{tikzcd}
$$

$$
\begin{tikzcd}
 & C^{-}(x, y) \arrow[dl] \arrow[d] \arrow[dr] & \\
C^{-}_{left} & C^{-}_{left,\ right} & C^{-}_{right}
\end{tikzcd}
$$

Refining states

Example

$$
\begin{array}{ccccc}
 & & A^{+}(x,y) & & \\
 & \swarrow & & \searrow & \\
 & A^{+}_{left} & & A^{+}_{right} & \\
 & & \vdots & & \\
 & & C^{-}(x,y) & & \\
 \swarrow & & \downarrow & & \searrow \\
C^{-}_{left} & & C^{-}_{left,\ right} & & C^{-}_{right}
\end{array}
$$

Why?

Refining states

Example

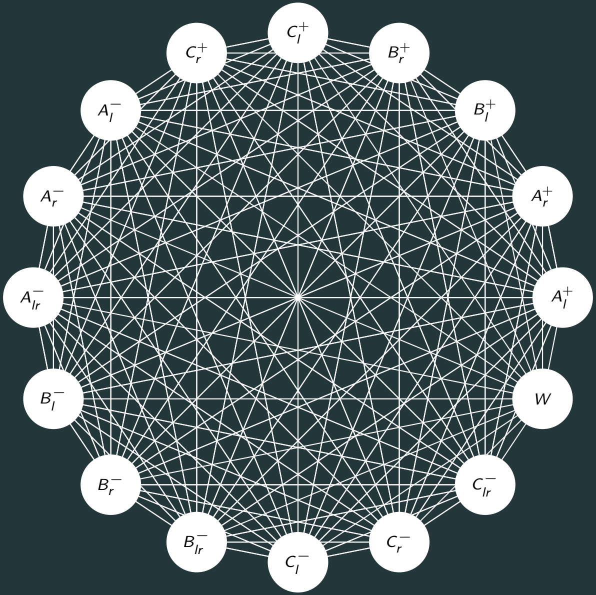
$$
\begin{array}{ccc}
 & A^{+}(x, y) & \\
\swarrow & & \searrow \\
A^{+}_{left} & & A^{+}_{right} \\
 & \vdots & \\
 & C^{-}(x, y) & \\
\swarrow & \downarrow & \searrow \\
C^{-}_{left} & C^{-}_{left,\ right} & C^{-}_{right}
\end{array}
$$

Why?

New orders in interactions

$$C^{-}(xz,\ wy) \leftarrow A^{+}_{left}(x,\ y),\ B^{+}(z,\ w).$$

Refining states: Interactions



G_4: Automatic Rule Inference

State descriptors $\mathcal{D}$

$$\mathrm{W} \mapsto (\epsilon, \epsilon)$$
$$\mathrm{A}_l^+ \mapsto (a, \epsilon)$$
$$\mathrm{A}_r^+ \mapsto (\epsilon, a)$$
$$\mathrm{B}_l^+ \mapsto (b, \epsilon)$$
$$\mathrm{B}_r^+ \mapsto (\epsilon, b)$$
$$\mathrm{C}_l^+ \mapsto (c, \epsilon)$$
$$\mathrm{C}_r^+ \mapsto (\epsilon, c)$$
$$\mathrm{A}_l^- \mapsto (bc, \epsilon)$$
$$\mathrm{A}_r^- \mapsto (\epsilon, bc)$$
$$\mathrm{A}_{lr}^- \mapsto (b, c)$$
$$\mathrm{B}_l^- \mapsto (ac, \epsilon)$$
$$\mathrm{B}_r^- \mapsto (\epsilon, ac)$$
$$\mathrm{B}_{lr}^- \mapsto (a, c)$$
$$\mathrm{C}_l^- \mapsto (ab, \epsilon)$$
$$\mathrm{C}_r^- \mapsto (\epsilon, ab)$$
$$\mathrm{C}_{lr}^- \mapsto (a, b)$$

Automatic Rule Inference: Example

$$\textsc{Case of } \mathrm{A}^{-}_{lr}(x_b,\ y_c) + \mathrm{B}^{-}_{lr}(z_a,\ w_c)$$

<u>PERMUTATION</u>

$\vdots$

$(zxw,\ y)$

$\vdots$

$(xzw,\ y)$

$\vdots$

Automatic Rule Inference: Example

$$\textsc{Case of}\ \mathrm{A}^{-}_{lr}(x_b,\ y_c) + \mathrm{B}^{-}_{lr}(z_a,\ w_c)$$

PERMUTATION		DESCRIPTOR
$\vdots$		$\vdots$
$(zxw,\ y)$	$\longrightarrow$	$(abc,\ c)$
$\vdots$		$\vdots$
$(xzw,\ y)$	$\longrightarrow$	$(bac,\ c)$
$\vdots$		$\vdots$

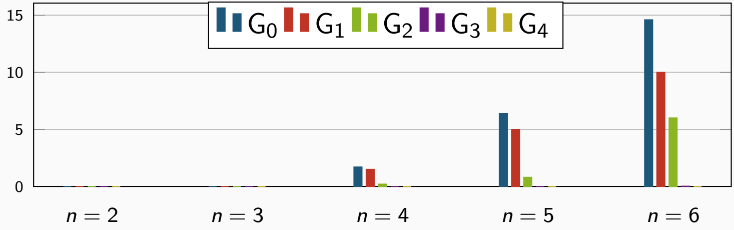
Automatic Rule Inference: Example

$$\textsc{Case of}\ \mathrm{A}^{-}_{lr}(x_b,\ y_c) + \mathrm{B}^{-}_{lr}(z_a,\ w_c)$$

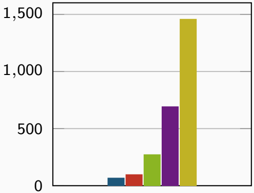
PERMUTATION		DESCRIPTOR		ELIMINATED
$\vdots$		$\vdots$		$\vdots$
			$\nearrow$	$(c,\ \epsilon) \hookleftarrow \mathrm{C}^{+}_{l}$
$(zxw,\ y)$	$\longrightarrow$	$(abc,\ c)$		
			$\searrow$	$(c,\ \epsilon) \hookleftarrow \mathrm{C}^{+}_{r}$
$\vdots$		$\vdots$		$\vdots$
$(xzw,\ y)$	$\longrightarrow$	$(bac,\ c)$	$\longrightarrow$	$(bac,\ c) \notin \mathcal{D}$
$\vdots$		$\vdots$		$\vdots$

Results

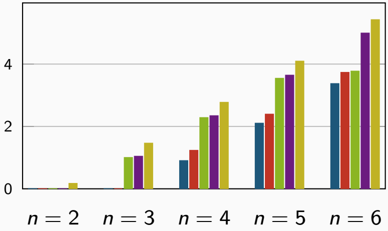
Counter-examples (%)



Rule size

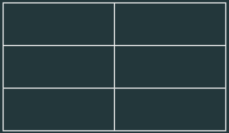


Computation Time (in log(sec))



Correspondences: Young Tableau

$a_1 \;\; a_2 \;\; b_3 \;\; b_4 \;\; c_5 \;\; c_6$



Correspondences: Young Tableau

$$a_1 \quad a_2 \quad b_3 \quad b_4 \quad c_5 \quad c_6$$

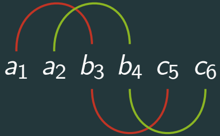
1	
3	
5	

Correspondences: Young Tableau

$$a_1 \; a_2 \; b_3 \; b_4 \; c_5 \; c_6$$

1	2
3	4
5	6

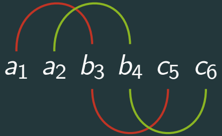
Correspondences: Promotion on Young Tableaux



1	2
3	4
5	6

	1
2	3
4	5

Correspondences: Promotion on Young Tableaux



1	2
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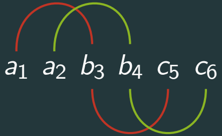
Correspondences: Promotion on Young Tableaux



1	2
3	4
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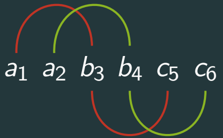

1	2
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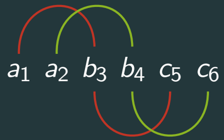

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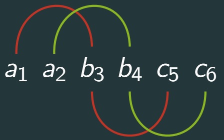

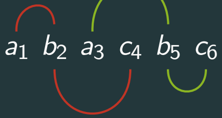

1	2
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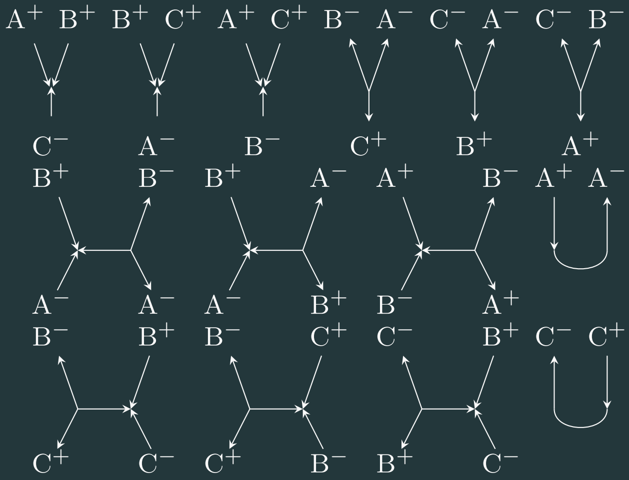
Growth rules



Road to completeness