D^3 as a 2-MCFL

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1.3 Challenge

[This is not a hand-in exercise. If you can solve it by Dec 5, there will be a present for you!]

Let D^n be the language over an n-symbol alphabet, lexicographically ordered $a_1 < \cdots < a_n$, where words satisfy the following conditions:

1. each word contains an equal number of the n alphabet symbols
2. for every prefix p of a word, the number of a_i in $p \geq$ the number of a_{i+1} $(1 \leq i \leq n-1)$

D^n generalizes the familiar language of balanced brackets, in which case you have an alphabet of size 2, say $\{\mathsf{a}, \mathsf{b}\}$, with 'opening bracket' a preceding 'closing bracket' b in the lexicographic ordering.

The conjecture (Makoto Kanazawa, p.c.) is that for $n \geq 2$, D^n is the language of a non-wellnested $(n-1)$-MCFG.

Give a 2-MCFG for D^3, i.e. words over a 3-letter alphabet $\{\mathsf{a}, \mathsf{b}, \mathsf{c}\}$ (with the usual lexicographic order) satisfying conditions (1) and (2) above. Give the ACG encoding of your MCFG for D^3.

Reference M. Moortgat (2014), A note on multidimensional Dyck languages.

Some examples

Dyck words

- abc
- aabbcc
- abcabcabacbc

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$$a\,b\,a\,b\,a\,c\,b\,c\,a\,b\,c\,c$$

First-match policy

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$$ababacbcabcc$$

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First-match policy

$\mathbf{G}_0$: Grammar of triple insertions

$$\mathrm{S}(xy) \leftarrow \mathrm{W}(x, y). \tag{1}$$

$$\mathrm{W}(\epsilon, xy\mathbf{abc}) \leftarrow \mathrm{W}(x, y). \tag{2}$$

$$\mathrm{W}(\epsilon, x\mathbf{a}y\mathbf{bc}) \leftarrow \mathrm{W}(x, y). \tag{3}$$

$$\ldots\ldots$$

$$\mathrm{W}(\mathbf{ab}x\mathbf{c}y, \epsilon) \leftarrow \mathrm{W}(x, y). \tag{60}$$

$$\mathrm{W}(\mathbf{abc}xy, \epsilon) \leftarrow \mathrm{W}(x, y). \tag{61}$$

$$\mathrm{W}(\epsilon, \mathbf{abc}). \tag{62}$$

$$\mathrm{W}(\mathbf{a}, \mathbf{bc}). \tag{63}$$

$$\mathrm{W}(\mathbf{ab}, \mathbf{c}). \tag{64}$$

$$\mathrm{W}(\mathbf{abc}, \epsilon). \tag{65}$$

G_0: Grammar of triple insertions

$a \quad a \quad b \quad b \quad a \quad c \quad b \quad a \quad c \quad c \quad b \quad c$

Straddling counter-example

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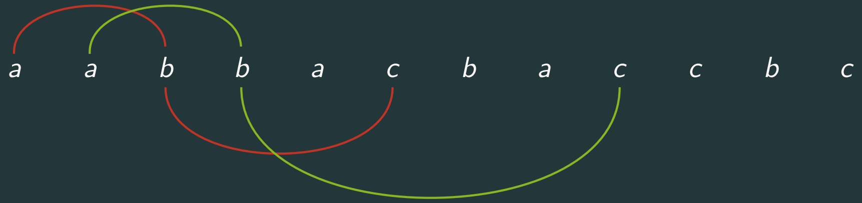
Straddling counter-example

G_0: Grammar of triple insertions



Straddling counter-example

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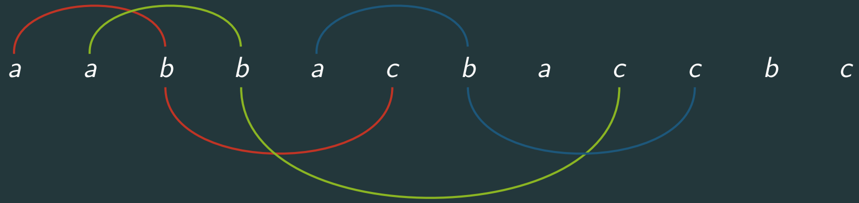

Straddling counter-example

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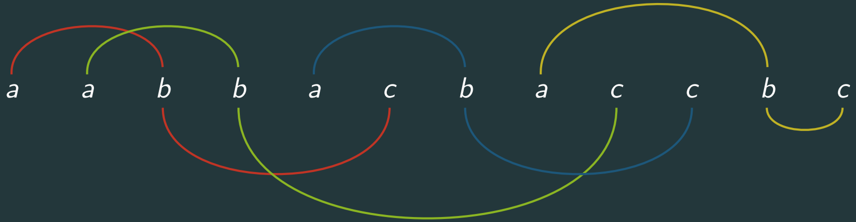

Straddling counter-example

Meta-grammars: Introduction

Notation

$$\mathcal{O}_m[\![\mathit{conclusion} \leftarrow \mathit{premises} \mid \{\mathit{partial\ orderings\ of\ inserted\ elements}\}]\!].$$

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Meta-grammar G_1

$$\left.\begin{array}{l} \mathcal{O}_2[\![W \leftarrow \epsilon \mid \{a < b < c\}]\!]. \\ \mathcal{O}_2[\![W \leftarrow W_{xy} \mid \{x < y,\ a < b < c\}]\!]. \end{array}\right\}\ \text{triple insertion}$$

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Meta-grammar $\mathrm{G_1}$

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$$+$$

$$\mathcal{O}_2[\![W \leftarrow W_{xy}, W_{zw} \mid \{x < y,\ z < w\}]\!].$$

$\mathbf{G}_2$: Adding states

$$\left.\begin{array}{l}\mathcal{O}_2[\![A^+ \leftarrow \epsilon \mid \{a\}]\!].\\ \mathcal{O}_2[\![B^+ \leftarrow \epsilon \mid \{b\}]\!].\\ \mathcal{O}_2[\![C^+ \leftarrow \epsilon \mid \{c\}]\!].\end{array}\right\}\text{ BASE CASES}$$

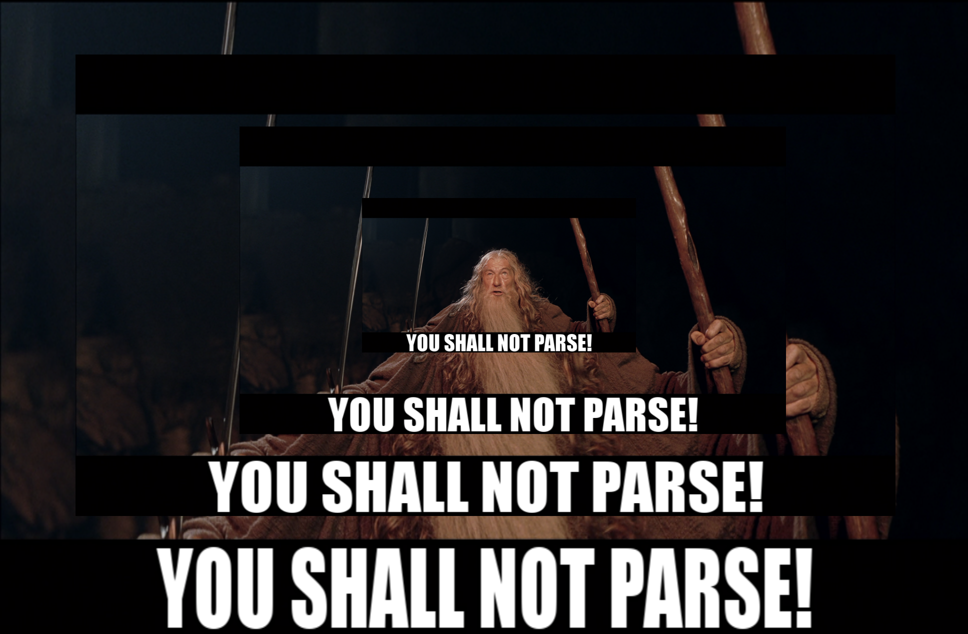
$$\left.\begin{array}{l}\mathcal{O}_2[\![C^- \leftarrow A^+, B^+ \mid \{x < y < z < w\}]\!].\\ \mathcal{O}_2[\![B^- \leftarrow A^+, C^+ \mid \{x < y < z < w\}]\!].\\ \mathcal{O}_2[\![A^- \leftarrow B^+, C^+ \mid \{x < y < z < w\}]\!].\\ \mathcal{O}_2[\![A^+ \leftarrow C^-, B^- \mid \{x < y < z < w\}]\!].\\ \mathcal{O}_2[\![B^+ \leftarrow C^-, A^- \mid \{x < y < z < w\}]\!].\\ \mathcal{O}_2[\![C^+ \leftarrow B^-, A^- \mid \{x < y < z < w\}]\!].\\ \forall\ \mathrm{K} \in \mathcal{S} \setminus \mathrm{W} :\ \mathcal{O}_2[\![\mathrm{K} \leftarrow \mathrm{K}_{xy}, \mathrm{W}_{zw} \mid \{x < y,\ z < w\}]\!].\end{array}\right\}\text{ COMBINATIONS}$$

$$\left.\begin{array}{l}\mathcal{O}_2[\![W \leftarrow A^+, A^- \mid \{x < y < z < w\}]\!].\\ \mathcal{O}_2[\![W \leftarrow C^-, C^+ \mid \{x < y < z < w\}]\!].\end{array}\right\}\text{ CLOSURES}$$

G_3: G_2 + Universal triple insertion

$$\mathsf{G}_3 = \mathsf{G}_2 + \forall\, \mathrm{K} \in \mathcal{S} \setminus \mathrm{W} : \quad \mathcal{O}_2[\![\mathrm{K} \leftarrow \mathrm{K}_{xy} \mid \{x < y,\ a < b < c\}]\!].$$

Demo


YOU SHALL NOT PARSE!
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Refining states I

Example

$$A^{+}(x, y)$$

$$A^{+}_{left}(\ldots a \ldots, \ \ldots) \qquad A^{+}_{right}(\ldots, \ \ldots a \ldots)$$

Why?

Refining states I

Example

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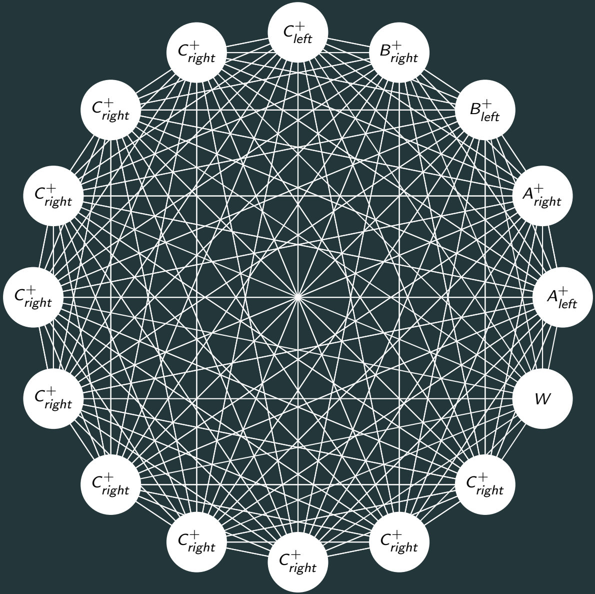
$$A^{+}_{left}(\dots a \dots,\ \dots) \qquad A^{+}_{right}(\dots,\ \dots a \dots)$$

Why?

New orders in interactions

$$C^{-}(xz,\ wy) \leftarrow A^{+}_{left}(x,\ y),\ B^{+}_{left}(z,\ w).$$

Refining states II



Automatic Rule Inference System

Overview

- Background
- $G0$: Triple insertion
- Meta-grammar notation
- G_0': Triple insertion (in O_2 notation)
- G_1: G0' + interleavings
- G_2: incomplete words
- G_3: G2 + 3-ins
- DEMO: **dyck**
- Refined states
- Constraints (notation)
- ARIS
- Results
- Road to completeness
- Correspondences
- DEMO: **dyckviz**