1.3 Challenge

[This is not a hand-in exercise. If you can solve it by Dec 5, there will be a present for you!]

Let D^n be the language over an n-symbol alphabet, lexicographically ordered $a_1 < \cdots < a_n$, where words satisfy the following conditions:

1. each word contains an equal number of the n alphabet symbols
2. for every prefix p of a word, the number of a_i in $p \geq$ the number of a_{i+1} $(1 \leq i \leq n-1)$

D^n generalizes the familiar language of balanced brackets, in which case you have an alphabet of size 2, say $\{\mathsf{a}, \mathsf{b}\}$, with 'opening bracket' a preceding 'closing bracket' b in the lexicographic ordering.

The conjecture (Makoto Kanazawa, p.c.) is that for $n \geq 2$, D^n is the language of a non-wellnested $(n-1)$-MCFG.

Give a 2-MCFG for D^3, i.e. words over a 3-letter alphabet $\{\mathsf{a}, \mathsf{b}, \mathsf{c}\}$ (with the usual lexicographic order) satisfying conditions (1) and (2) above. Give the ACG encoding of your MCFG for D^3.

Reference M. Moortgat (2014), A note on multidimensional Dyck languages.

□