Logic and Language: Exercise (Week 5)

Orestis Melkonian [6176208], Konstantinos Kogkalidis [6230067]

1 LG: continuation semantics

1.1

TERM	TYPE	TYPE$^\ell$
$\lceil \textbf{some} \rceil$	$(np^\perp \otimes n)^\perp$	$(e^\perp \otimes e^\perp)^\perp$
$\lceil \textbf{popular} \rceil$	$(n^\perp \otimes n)^\perp$	$(e^{\perp^\perp} \otimes e^\perp)^\perp$
$\lceil \textbf{saint} \rceil$	n	$e^\perp$
$\lceil \textbf{arrived} \rceil$	$(np \otimes s^\perp)^\perp$	$(e \otimes t^\perp)^\perp$
α	$s^\perp$	$t^\perp$
z	np	e
y	n	$e^\perp$

1.2

SOURCE TYPE	CONSTANT	$\lceil . \rceil^\ell$
n/n	popular	$\lambda\langle c, y\rangle.(c\ (\lambda z.\ \wedge (y\ z)\ (\text{POPULAR}\ z)))$

1.3

1. We compute the interpretation below:

$$\lceil \ddagger \rceil = \lambda a_0.(\lceil \textbf{arrived} \rceil\ \langle \lambda\beta_0.(\lceil \textbf{some} \rceil\ \langle \beta_0, \lambda\gamma_0.(\lceil \textbf{popular} \rceil\ \langle \gamma_0, \lambda a_1.(\lceil \textbf{saint} \rceil\ a_1)\rangle)\rangle), a_0\rangle)$$

2. The adjucted $\cdot^\ell$ translations are the following:

$$\begin{aligned}
\lceil \textbf{some} \rceil^\ell &= \lambda\langle x, k\rangle.(\exists\lambda z.(\wedge\ (k\ \lambda\theta.(\theta\ z))\ (x\ z))) \\
\lceil \textbf{popular} \rceil^\ell &= \lambda\langle c, k\rangle.(c\ (\lambda z.(\wedge\ (\text{POPULAR}\ z)\ (k\ \lambda\theta.(\theta\ z))))) \\
\lceil \textbf{saint} \rceil^\ell &= \lambda c.(c\ \text{SAINT}) \\
\lceil \textbf{arrived} \rceil^\ell &= \lambda\langle k, c\rangle.(k\ \lambda z.(c\ (\text{ARRIVED}\ z)))
\end{aligned}$$

We can now corroborate the α-equivalence of the two $\cdot^{\ell}$ translations:

$$
\begin{aligned}
\lceil \ddagger \rceil^{\ell} = {} & \lambda a_0.(\underline{\lambda\langle k, c\rangle.(k\ \lambda z.(c\ (\text{ARRIVED}\ z)))} \\
& \quad \langle \lambda\beta_0.(\lambda\langle x, k\rangle.(\exists\lambda z.(\wedge\ (k\ \lambda\theta.(\theta\ z))\ (x\ z)))\ \langle \beta_0, \lambda\gamma_0.(\\
& \qquad \lambda\langle c, k\rangle.(c\ (\lambda z.(\wedge\ (\text{POP}\ z)\ (k\ \lambda\theta.(\theta\ z)))))\ \langle \gamma_0, \lambda a_1.(\lambda c.(c\ \text{SAINT})\ a_1)\rangle\rangle)), a_0\rangle) \\
\to^{*}_{\beta} {} & \lambda a_0.(\underline{\lambda\langle x, k\rangle.(\exists\lambda z.(\wedge\ (k\ \lambda\theta.(\theta\ z))\ (x\ z)))}\ \langle \lambda z.(a_0\ (\text{ARRIVED}\ z)), \lambda\gamma_0.(\\
& \qquad \lambda\langle c, k\rangle.(c\ (\lambda z.(\wedge\ (\text{POPULAR}\ z)\ (k\ \lambda\theta.(\theta\ z)))))\ \langle \gamma_0, \lambda a_1.(\lambda c.(c\ \text{SAINT})\ a_1)\rangle\rangle)) \\
\to^{*}_{\beta} {} & \lambda a_0.(\exists\lambda z.(\wedge\ (\underline{\lambda\langle c, k\rangle.(c\ (\lambda z.(\wedge\ (\text{POPULAR}\ z)\ (k\ \lambda\theta.(\theta\ z)))))} \\
& \qquad \langle \lambda\theta.(\theta\ z), \lambda a_1.(\lambda c.(c\ \text{SAINT})\ a_1)\rangle)\ (a_0\ (\text{ARRIVED}\ z))) \\
\to^{*}_{\beta} {} & \lambda a_0.(\exists\lambda z.(\wedge\ (\underline{\lambda\theta.(\theta\ z)}\ (\lambda z.(\wedge\ (\text{POPULAR}\ z)\ (\underline{\lambda c.(c\ \text{SAINT})}\ (\lambda\theta.(\theta\ z))))))\ (a_0\ (\text{ARRIVED}\ z))) \\
\to^{*}_{\beta} {} & \lambda a_0.(\exists\lambda z.(\wedge\ (\wedge\ (\text{POPULAR}\ z)\ (\text{SAINT}\ z))\ (a_0\ (\text{ARRIVED}\ z))) \\
\to_{\alpha} {} & \lambda c.(\exists\lambda z.(\wedge\ (\wedge\ (\text{POPULAR}\ z)\ (\text{SAINT}\ z))\ (c\ (\text{ARRIVED}\ z))) \\
\to_{\alpha} {} & \lambda c.(\exists\lambda x.(\wedge\ (\wedge\ (\text{POPULAR}\ x)\ (\text{SAINT}\ x))\ (c\ (\text{ARRIVED}\ x))) \\
= {} & \lceil \dagger \rceil^{\ell}
\end{aligned}
$$

2 Pregroups

2.1

(4)

$(\to)$

$$
\dfrac{\dfrac{}{1^l \to 1^l 1}\,(1) \qquad \dfrac{}{1^l 1 \to 1}\,(2)}{1^l \to 1}\,\to
$$

$(\leftarrow)$

$$
\dfrac{\dfrac{}{1 \to 1^l 1}\,(2) \qquad \dfrac{}{1^l 1 \to 1^l}\,(1)}{1 \to 1^l}\,\to
$$

(5) $(\to)$:

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A^{rl} \to A^{rl} 1}\,(1) \qquad \dfrac{}{1 \to A^r A}\,(2)}{A^{rl} \to A^{rl}(A^r A)}\,\to}{A^{rl} \to (A^{rl} A^r) A}\,(1) \qquad \dfrac{}{A^{rl} A^r \to 1}\,(2)}{A^{rl} \to 1A}\,\to \qquad \dfrac{}{1A \to A}\,(1)}{A^{rl} \to A}\,\to
$$

$(\leftarrow)$:

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A \to A1}\,(1) \qquad \dfrac{}{1 \to A^r A^{rl}}\,(2)}{A \to A(A^r A^{rl})}\,\to}{A \to (AA^r) A^{rl}}\,(1) \qquad \dfrac{}{AA^r \to 1}\,(3)}{A \to 1A^{rl}}\,\to \qquad \dfrac{}{1A^{rl} \to A^{rl}}\,(1)}{A \to A^{rl}}\,\to
$$

(6) $(\to)$:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{(AB)^l \to (1(AB))^l}\,\to \quad \dfrac{}{1 \to (B^lA^l)^r(B^lA^l)}\,(3)}{(AB)^l \to (((B^lA^l)^r(B^lA^l))(AB))^l}\,\to}{(AB)^l \to ((B^lA^l)^r((B^lA^l)(AB)))^l}\,(1)}{(AB)^l \to ((B^lA^l)^r(B^l(A^l(AB))))}\,(1)}{(AB)^l \to ((B^lA^l)^r(B^l((A^lA)B)))}\,(1) \quad \dfrac{}{A^lA \to 1}\,(2)}{(AB)^l \to ((B^lA^l)^r(B^l(1B)))^l}\,\to \quad \dfrac{}{1B \to B}\,(1)}{(AB)^l \to ((B^lA^l)^r(B^lB))^l}\,\to \quad \dfrac{}{B^lB \to 1}\,(2)}{(AB)^l \to ((B^lA^l)^r1)^l}\,\mathit{todo} \quad \dfrac{}{(B^lA^l)^r1 \to (}}{\dfrac{(AB)^l \to ((B^lA^l)^r)^l}{(AB)^l \to B^lA^l}\,(5)}$$

2.2

We first calculate the pregroup translation of the given sequent:

$$\begin{aligned}
&\overline{(p/((q/q)/r))/r} \\
=&\overline{(p/((q/q)/r))}\ r^\ell \\
=&p\ \overline{((q/q)/r)}^\ell\ r^\ell \\
=&p\ (\overline{(q/q)}\ r^\ell)^\ell\ r^\ell \\
=&p\ ((q\ q^\ell)\ r^\ell)^\ell\ r^\ell \\
=&p\ (q\ (q^\ell\ r^\ell))^\ell\ r^\ell && \{\textit{rule (1) from 2.1}\} \\
=&p\ (q^\ell\ r^\ell)^\ell\ q^\ell\ r^\ell && \{\textit{rule (6) from 2.1}\} \\
=&p\ r^{\ell^\ell}\ q^{\ell^\ell}\ q^\ell\ r^\ell && \{\textit{rule (6) from 2.1}\}
\end{aligned}$$

Finally, we prove the sequent by drawing a string diagram: