

REASONABLE AGDA IS CORRECT HASKELL:

WRITING VERIFIED HASKELL USING AGDA2HS

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MOTIVATION: ISSUES WITH CURRENT PROGRAM EXTRACTORS

MAlonzo covers the entirety of Agda, but produces unreadable code:



```
insert : Nat → Tree → Tree
```

```
insert x Leaf = Node x Leaf Leaf
```

```
insert x (Node y l r) =
```

```
  case compare x y of λ where
```

```
    (LT _) → Node y (insert x l) r
```

```
    (EQ _) → Node y l r
```

```
    (GT _) → Node y l (insert x r)
```

```
{-# COMPILE AGDA2HS insert #-}
```

```
d_insert_1494 :: Integer -> Integer -> Integer
              -> T_Tree_1340 -> T_''8804'__1324
              -> T_''8804'__1324 -> T_Tree_1340
d_insert_1494 ~v0 ~v1 v2 v3 ~v4 ~v5 = du_insert_1494 v2 v3
du_insert_1494 :: Integer -> T_Tree_1340 -> T_Tree_1340
du_insert_1494 v0 v1 = case coe v1 of
  C_Leaf_1348 ->
    coe C_Node_1352 (coe v0) (coe C_Leaf_1348)
      (coe C_Leaf_1348)
  C_Node_1352 v2 v3 v4 ->
    coe MALonzo.Code.Haskell.Prim.du_case_of__54
      (coe d_compare_1474 (coe v0) (coe v2))
      (coe du_''46'extendedlambda0_1514 (coe v0) (coe v2)
        (coe v3) (coe v4))
  _ -> MALonzo.RTE.mazUnreachableError
```

MOTIVATION: ISSUES WITH CURRENT PROGRAM EXTRACTORS

Coq extracts more readable code, but still does not readily support typeclasses:



```
Class Monoid (a : Set) :=
  { mempty  : a
  ; mappend : a -> a -> a }.

Instance MonoidNat : Monoid nat :=
  { mempty := 0
  ; mappend i j := i + j }.

Fixpoint sumMon {a} `{Monoid a}
  (xs : list a) : a :=
  match xs with
  | [] => mempty
  | x :: xs => mappend x (sumMon xs)
  end.
```



```
data Monoid a = Build_Monoid a (a -> a -> a)

mempty :: (Monoid a1) -> a1
mempty = ...
mappend :: (Monoid a1) -> a1 -> a1 -> a1
mappend = ...
monoidNat :: Monoid Nat
monoidNat = Build_Monoid 0 add

sumMon :: (Monoid a1) -> (List a1) -> a1
sumMon h xs = case xs of {
  ([]) -> mempty h;
  (:) x xs0 -> mappend h x (sumMon h xs0)}
```

GOALS

1. Writing Haskell-like Agda (no need to cover the whole source language)
2. Verify your program using Agda's dependent types

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New point in the design space, enabled by:

- Agda very *similar* to Haskell
 - Agda's *dependent type system*
 - Agda's support for *erasure*
- + allows for **intrinsic verification!**



TREE EXAMPLE (EXTRINSIC VERSION)



```
data Tree : Set where
  Leaf  : Tree
  Node  : Nat → Tree → Tree → Tree
{-# COMPILE AGDA2HS Tree #-}

insert : Nat → Tree → Tree
insert x Leaf = Node x Leaf Leaf
insert x (Node y l r) =
  case compare x y of λ where
    (LT _) → Node y (insert x l) r
    (EQ _) → Node y l r
    (GT _) → Node y l (insert x r)
{-# COMPILE AGDA2HS insert #-}
```



```
data Tree = Leaf
          | Node Natural Tree Tree

insert :: Natural -> Tree -> Tree
insert x Leaf = Node x Leaf Leaf
insert x (Node y l r)
  = case compare x y of
      LT -> Node y (insert x l) r
      EQ -> Node y l r
      GT -> Node y l (insert x r)
```

TREE EXAMPLE (EXTRINSIC PROOFS)

⋮

@0 $_ \leq _$: $\text{Nat} \rightarrow \text{Tree} \rightarrow \text{Nat} \rightarrow \text{Set}$

$l \leq \text{Leaf} \quad \leq u = l \leq u$

$l \leq \text{Node } x \, t^l \, t^r \leq u = (l \leq t^l \leq x) \times (x \leq t^r \leq u)$

@0 $\text{insert-correct} : \forall \{t \, x \, l \, u\} \rightarrow l \leq t \leq u$

$\rightarrow l \leq x \rightarrow x \leq u \rightarrow l \leq \text{insert } x \, t \leq u$

$\text{insert-correct } \{\text{Leaf}\} _ \, l \leq x \, x \leq u = l \leq x, x \leq u$

$\text{insert-correct } \{\text{Node } y \, t^l \, t^r\} \{x\} \, (IH^l, IH^r) \, l \leq x \, x \leq u$

with $\text{compare } x \, y$

... | $\text{LT } x \leq y = \text{insert-correct } IH^l \, l \leq x \, x \leq y, IH^r$

... | $\text{EQ refl} = IH^l, IH^r$

... | $\text{GT } y \leq x = IH^l, \text{insert-correct } IH^r \, y \leq x \, x \leq u$

TREE EXAMPLE (INTRINSIC VERSION)



```
data Tree (@0 l u : Nat) : Set where
  Leaf  : (@0 pf: l ≤ u) → Tree l u
  Node  : (x : Nat) → Tree l x → Tree x u → Tree l u
{-# COMPILE AGDA2HS Tree #-}

insert : { @0 l u : Nat } (x : Nat) → Tree l u
        → @0 (l ≤ x) → @0 (x ≤ u) → Tree l u
insert x (Leaf _) l ≤ x x ≤ u =
  Node x (Leaf l ≤ x) (Leaf x ≤ u)
insert x (Node y l r) l ≤ x x ≤ u =
  case compare x y of λ where
    (LT x ≤ y) → Node y (insert x l l ≤ x x ≤ y) r
    (EQ x = y) → Node y l r
    (GT y ≤ x) → Node y l (insert x r y ≤ x x ≤ u)
{-# COMPILE AGDA2HS insert #-}
```

```
data Tree = Leaf
          | Node Natural Tree Tree

insert :: Natural -> Tree -> Tree
insert x Leaf = Node x Leaf Leaf
insert x (Node y l r)
  = case compare x y of
      LT -> Node y (insert x l) r
      EQ -> Node x l r
      GT -> Node y l (insert x r)
```


PRIMITIVES

- Export lowercase type variables to feel like home (i.e. **variable** $a\ b\ c\ \dots$: **Set**):

id : $a \rightarrow a$

id $x = x$

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e.g. `Agda.Builtin.Nat` \leftrightarrow `Numeric.Natural`

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- If not available in Agda, define them:

`infix -2 if_then_else_`

`if_then_else_ : Bool → a → a → a`

`if False then x else y = y`

`if True then x else y = x`

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REMEMBER

We want to cover as many Haskell features as possible, not Agda features.

Port Haskell's Prelude, staying faithful to the original functionality.

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```
error : (@0 i : ⊥) → String → a
error ()
```

```
head : (xs : List a) { @0 _ : NonEmpty xs } → a
```

```
head (x :: _) = x
```

```
head [] {p} = error i "empty list"
```

```
where @0 i : ⊥
```

```
      i = case p of λ ()
```

```
{-# COMPILER AGDA2HS head #-}
```



```
head :: [a] -> a
```

```
head (x : _) = x
```

```
head [] = error "empty list"
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  where @0 i : ⊥
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```
        i = case p of λ ()
```

```
{-# COMPILE AGDA2HS head #-}
```



```
head :: [a] -> a
```

```
head (x : _) = x
```

```
head [] = error "empty list"
```

Don't forget

On the Haskell side, we can feed `head` arbitrary input!

Correspondence with Agda's **instance arguments**.



<i>class definitions</i>	~	<i>record types</i>
<i>instance declarations</i>	~	<i>record values</i>
<i>constraints</i>	~	<i>instance arguments</i>

TYPECLASSES: CLASS DEFINITIONS ~ RECORD TYPES



```
record Monoid (a : Set) : Set where  
  field
```

```
    mempty : a
```

```
    mappend : a → a → a
```

```
    @0 left-identity : mappend mempty x ≡ x
```

```
    @0 right-identity : mappend x mempty ≡ x
```

```
    @0 associativity : mappend (mappend x y) z  
                      ≡ mappend x (mappend y z)
```

```
open Monoid {...} public
```

```
{-# COMPILE AGDA2HS Monoid class #-}
```



```
class Monoid a where  
  mempty :: a  
  mappend :: a -> a -> a
```



instance

MonoidNat : Monoid Nat

MonoidNat = λ where

.mempty $\rightarrow 0$

.mappend $i\ j \rightarrow i + j$

.left-identity $\rightarrow \dots$

.right-identity $\rightarrow \dots$

.associativity $\rightarrow \dots$

{-# COMPILE AGDA2HS MonoidNat #-}



instance Monoid Nat where

mempty = 0

mappend i j = i + j

TYPECLASSES: CONSTRAINTS \sim INSTANCE ARGUMENTS



```
sumMon : {{ Monoid a }}  $\rightarrow$  List a  $\rightarrow$  a
sumMon [] = mempty
sumMon (x :: xs) = mappend x (sumMon xs)
{-# COMPILER AGDA2HS sumMon #-}
```



```
sumMon :: Monoid a => [a] -> a
sumMon [] = mempty
sumMon (x : xs) = mappend x (sumMon xs)
```

DEFAULT METHODS & MINIMAL COMPLETE DEFINITIONS



```
record Show (a : Set) : Set where
  field show      : a → String
        showsPrec : Nat → a → ShowS
        showList  : List a → ShowS
record Show1 (a : Set) : Set where
  field showsPrec : Nat → a → ShowS
  show x = showsPrec 0 x ""
  showList = defaultShowList (showsPrec 0)
record Show2 (a : Set) : Set where
  field show : a → String
  showsPrec _ x s = show x ++ s
  showList = defaultShowList (showsPrec 0)
open Show {!!}
{-# COMPILE AGDA2HS Show class Show1 Show2 #-}
```

```
class Show a where
  show :: a -> String
  showsPrec :: Nat -> a -> ShowS
  showList :: [a] -> ShowS
  {-# MINIMAL showsPrec | show #-}
  show x = showsPrec 0 x ""
  showList = defaultShowList
              (showsPrec 0)
  showsPrec _ x s = show x ++ s
```



instance

```
ShowMaybe : {{Show a}} → Show (Maybe a)
ShowMaybe {a = a} = record {Show1 s1}
  where
    s1 : Show1 (Maybe a)
    s1.Show1.showsPrec n = λ where
      Nothing → showString "nothing"
      (Just x) → showParen True
        ( showString "just " ∘ showsPrec 10 x )
{-# COMPILE AGDA2HS ShowMaybe #-}
```

```
instance (Show a)
  => Show (Maybe a) where
  showsPrec n = \case
    Nothing -> showString "nothing"
    (Just x) -> showParen True
      (showString "just " . showsPrec 10 x)
```

IOG's Cardano blockchain

- currently the 8th largest by market cap
- smart contracts written in PLUTUS, based on System F_{ω}^{μ}
- implemented in Haskell
- tested against Agda formalization

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IOG USE CASE: TYPE RENAMING & SUBSTITUTION



```
data Kind : Set where
```

```
  Star : Kind
```

```
  _:=>_ : Kind → Kind → Kind
```

```
data Type (n : Set) : Set where
```

```
  TyVar : n → Type n
```

```
  TyFun : Type n → Type n → Type n
```

```
  TyForall : Kind → Type (Maybe n) → Type n
```

```
  TyLam : Type (Maybe n) → Type n
```

```
  TyApp : Type n → Type n → Kind → Type n
```

```
ren : (n → n') → Type n → Type n'
```

```
sub : (n → Type n') → Type n → Type n'
```

```
data Kind
```

```
  = Star
```

```
  | Kind :=> Kind
```

```
data Type n
```

```
  = TyVar n
```

```
  | TyFun (Type n) (Type n)
```

```
  | TyForall Kind (Type (Maybe n))
```

```
  | TyLam (Type (Maybe n))
```

```
  | TyApp (Type n) (Type n) Kind
```

```
ren :: (n -> n') -> Type n -> Type n'
```

```
sub :: (n -> Type n') -> Type n -> Type n'
```


`ren` is a *functorial map* on `Type`.

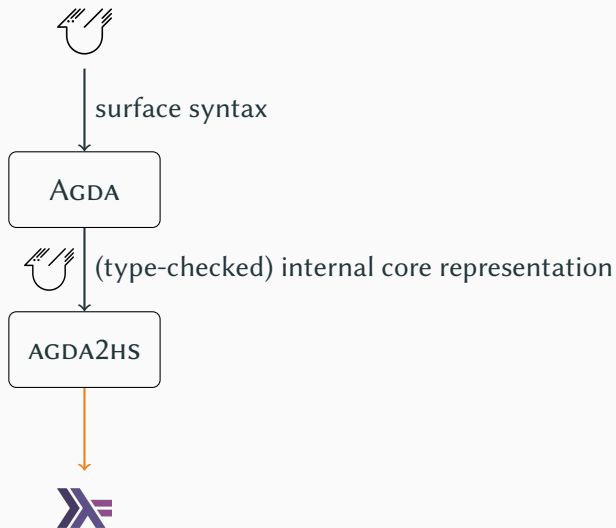
- `ren-id`: $(ty : \text{Type } n) \rightarrow \text{ren id } ty \equiv ty$
- `ren-comp`: $(ty : \text{Type } n) (\rho : n \rightarrow n') (\rho' : n' \rightarrow n'') \rightarrow \text{ren } (\rho' \circ \rho) ty \equiv \text{ren } \rho' (\text{ren } \rho ty)$

`ren` is a *functorial map* on `Type`.

- `ren-id`: $(ty : \text{Type } n) \rightarrow \text{ren id } ty \equiv ty$
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`sub` is a *monadic bind* on `Type`.

- `sub-id`: $(t : \text{Type } n) \rightarrow \text{sub TyVar } t \equiv t$
- `sub-var`: $(x : n) (\sigma : n \rightarrow \text{Type } n') \rightarrow \text{sub } \sigma (\text{TyVar } x) \equiv \sigma x$
- `sub-comp`: $(ty : \text{Type } n) (\sigma : n \rightarrow \text{Type } n') (\sigma' : n' \rightarrow \text{Type } n'') \rightarrow \text{sub } (\text{sub } \sigma' \circ \sigma) ty \equiv \text{sub } \sigma' (\text{sub } \sigma ty)$



IMPLEMENTATION: **WHERE** CLAUSES

Surface 

$f : \text{Nat} \rightarrow \text{Nat}$

$f\ x = \text{go}$

where

$\text{go} : \text{Nat}$

$\text{go} = \text{TODO}$

-- may use x

Intermediate 

$\text{go} : \text{Nat} \rightarrow \text{Nat}$

$\text{go}\ x = \text{TODO}$

$f : \text{Nat} \rightarrow \text{Nat}$

$f\ x = \text{go}\ x$

Output 

$f :: \text{Natural} \rightarrow \text{Natural}$

$f\ x = \text{go}$

where

$\text{go} :: \text{Natural}$


$\text{go} = \text{TODO}$

Is our translation **sound**?

1. Agda that typechecks produces valid Haskell
2. Translation preserves behaviour/semantics

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No formal proof  ...yet!

c.f. Cockx's NWO grant: *A Trustworthy and Extensible Core Language for Agda*

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2. Translation preserves behaviour/semantics

No formal proof 🤔 ...yet!

c.f. Cockx's NWO grant: *A Trustworthy and Extensible Core Language for Agda*

- Trust the ported Prelude and defined primitives
- Ensure all dependent types appear under *erased* positions
 - enforced by the AGDA2HS backend
- Translation of each language construct has equivalent behaviour
 - most cases blindingly obvious

NB: total functions + strong normalisation \Rightarrow evaluation order doesn't matter

Still many unsupported Haskell features:

- GADTs
- pattern guards, views
- 32-bit arithmetic
- Monadic code
- Infinite data
- Non-termination, general recursion

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Extra goodies:

- Generate **runtime checks** for decidable properties
- **QuickCheck** postulated properties
- HS2AGDA: inverse translation ⇒ streamline porting of **existing** libraries

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Extra goodies:

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More **applications** + **comparisons** with LiquidHaskell, hs-to-coq, etc..

AGDA2HS was developed during the last two **Agda Implementors' Meetings**

- biannual event where Agda users of all levels hack on Agda, its ecosystem, etc..

AIM XXXI in Edinburgh November 10-16, will include:

- talks
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- a hike along the Scottish seaside 🍷



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QUESTIONS?