Parallel Applications

Motivating Problems

Simulating Ocean Currents

- Regular structure, scientific computing

Simulating the Evolution of Galaxies

- Irregular structure, scientific computing

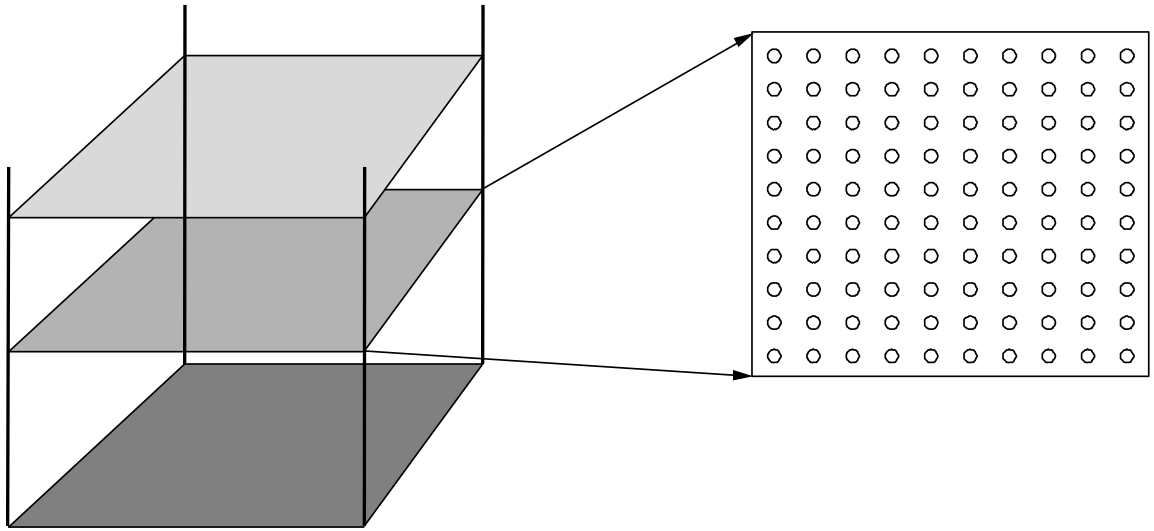
Rendering Scenes by Ray Tracing

- Irregular structure, computer graphics

Data Mining

- Irregular structure, information processing

Simulating Ocean Currents

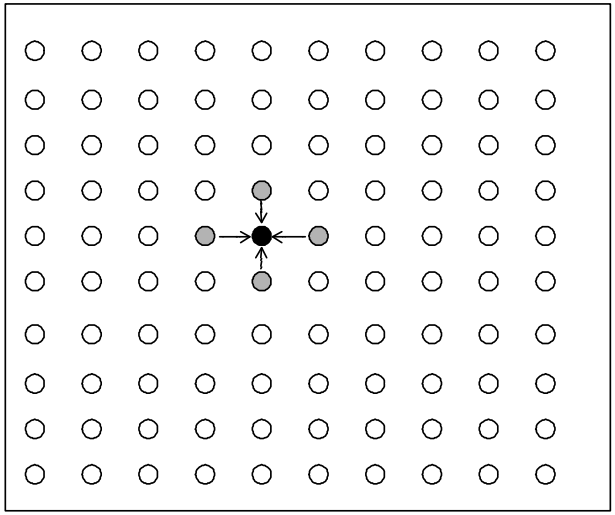


(a) Cross sections

(b) Spatial discretization of a cross section

- Model as two-dimensional grids
- Discretize in space and time
  - finer spatial and temporal resolution => greater accuracy
- Many different computations per time step
  - set up and solve equations
- Concurrency across and within grid computations

Grid Solver Example



Expression for updating each interior point:

$$A[i,j] = 0.2 \times (A[i,j] + A[i,j-1] + A[i-1,j] + A[i,j+1] + A[i+1,j])$$

- Simplified version of solver in Ocean simulation
- Gauss-Seidel (near-neighbor) sweeps to convergence
  - interior n-by-n points of (n+2)-by-(n+2) updated in each sweep
  - updates done in-place in grid, and diff. from prev. value computed
  - accumulate partial diffs into global diff at end of every sweep
  - check if error has converged (to within a tolerance parameter)
  - if so, exit solver; if not, do another sweep

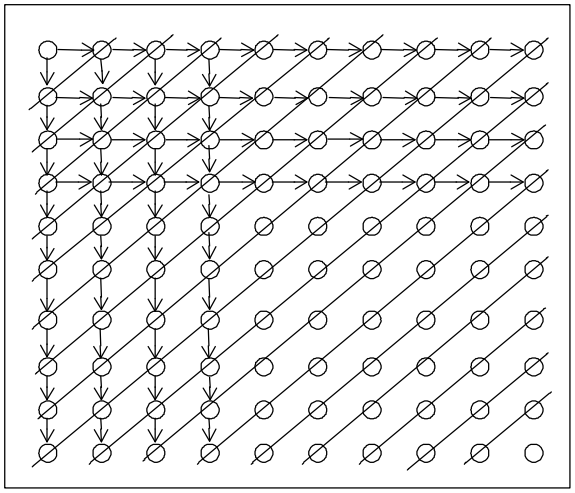
```
1.  int n;                                  /*size of matrix: (n + 2-by-n + 2) elements*/
2.  float **A, diff = 0;

3.  main()
4.  begin
5.    read(n) ;                             /*read input parameter: matrix size*/
6.    A ← malloc (a 2-d array of size n + 2 by n + 2 doubles);
7.    initialize(A);                        /*initialize the matrix A somehow*/
8.    Solve (A);                            /*call the routine to solve equation*/
9.  end main

10. procedure Solve (A)                     /*solve the equation system*/
11.   float **A;                            /*A is an (n + 2)-by-(n + 2) array*/
12. begin
13.   int i, j, done = 0;
14.   float diff = 0, temp;
15.   while (!done) do                      /*outermost loop over sweeps*/
16.     diff = 0;                           /*initialize maximum difference to 0*/
17.     for i ← 1 to n do                   /*sweep over nonborder points of grid*/
18.       for j ← 1 to n do
19.         temp = A[i,j];                  /*save old value of element*/
20.         A[i,j] ← 0.2 * (A[i,j] + A[i,j-1] + A[i-1,j] +
21.           A[i,j+1] + A[i+1,j]);         /*compute average*/
22.         diff += abs(A[i,j] - temp);
23.       end for
24.     end for
25.     if (diff/(n*n) < TOL) then done = 1;
26.   end while
27. end procedure
```

Decomposition

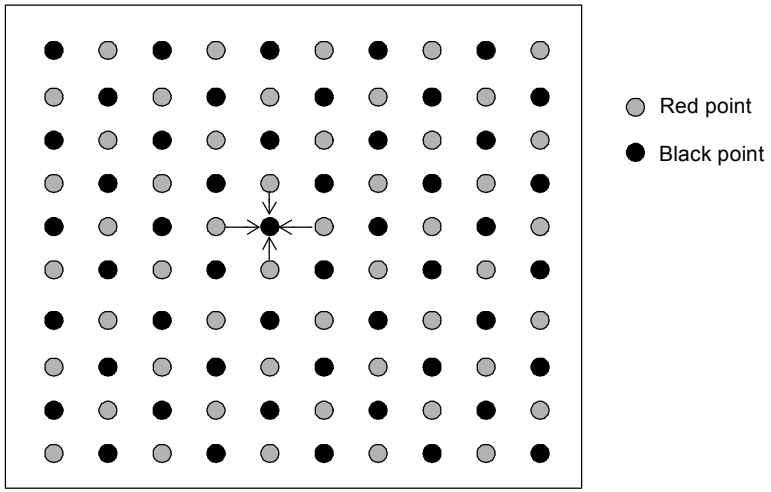
- Simple way to identify concurrency is to look at loop iterations
  - *dependence analysis*; if not enough concurrency, then look further
- Not much concurrency here at this level (all loops *sequential*)
- Examine fundamental dependences, ignoring loop structure



- Concurrency *O(n)* along anti-diagonals, serialization *O(n)* along diag.
- Retain loop structure, use pt-to-pt synch; Problem: too many synch ops.
- Restructure loops, use global synch; imbalance and too much synch

Exploit Application Knowledge

- Reorder grid traversal: red-black ordering



- Different ordering of updates: may converge quicker or slower
- Red sweep and black sweep are each fully parallel:
- Global synch between them (conservative but convenient)
- Ocean uses red-black; we use simpler, asynchronous one to illustrate
  - no red-black, simply ignore dependences within sweep
  - sequential order same as original, parallel program *nondeterministic*

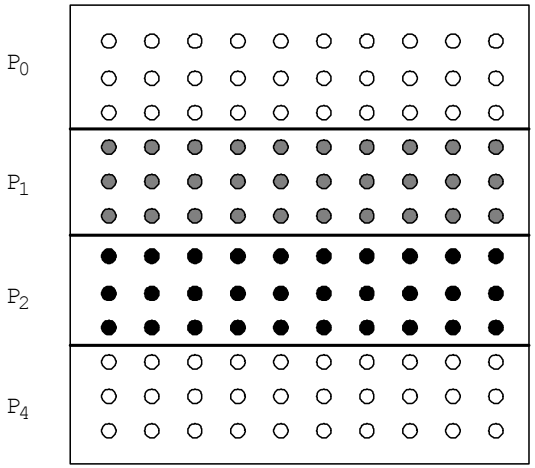
Decomposition Only

```
15. while (!done) do                    /*a sequential loop*/
16.   diff = 0;
17.   for_all i ← 1 to n do            /*a parallel loop nest*/
18.     for_all j ← 1 to n do
19.       temp = A[i,j];
20.       A[i,j] ← 0.2 * (A[i,j] + A[i,j-1] + A[i-1,j] +
21.         A[i,j+1] + A[i+1,j]);
22.       diff += abs(A[i,j] - temp);
23.     end for_all
24.   end for_all
25.   if (diff/(n*n) < TOL) then done = 1;
26. end while
```

- Decomposition into elements: degree of concurrency n^2
- To decompose into rows, make line 18 loop sequential; degree n
- `for_all` leaves assignment left to system
  - but implicit global synch. at end of `for_all` loop

Assignment

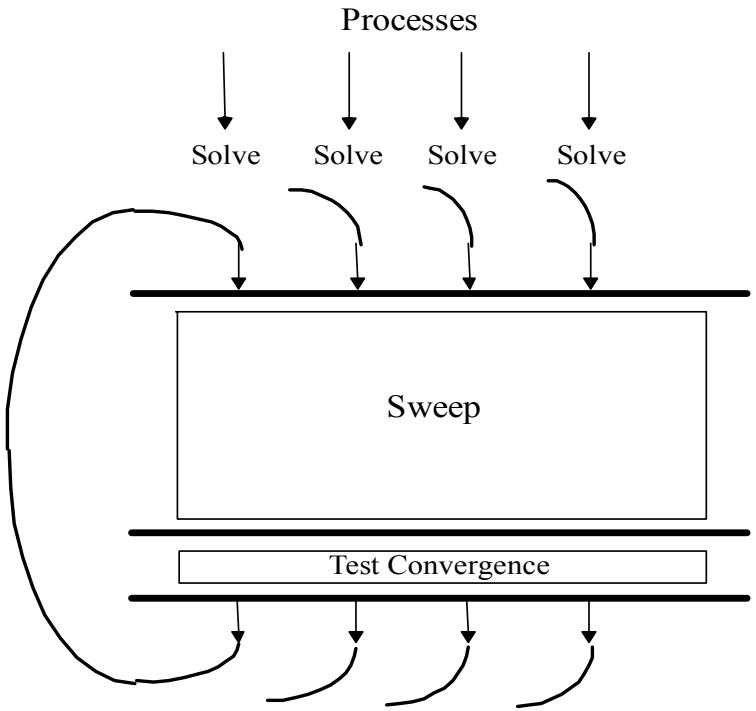
- Static assignments (given decomposition into rows)
  - block assignment of rows: Row i is assigned to process $\left\lfloor \frac{i}{p} \right\rfloor$
  - cyclic assignment of rows: process i is assigned rows i, $i+p$, and so on



- Dynamic assignment
  - get a row index, work on the row, get a new row, and so on
- Static assignment into rows reduces concurrency (from n to p)
  - block assign. reduces communication by keeping adjacent rows together
- Let's dig into orchestration under three programming models

Shared Address Space Solver

Single Program Multiple Data (SPMD)



- Assignment controlled by values of variables used as loop bounds

```
1.       int n, nprocs;          /*matrix dimension and number of processors to be used*/
2a.      float **A, diff;        /*A is global (shared) array representing the grid*/
                                 /*diff is global (shared) maximum difference in current
                                 sweep*/
2b.      LOCKDEC(diff_lock);     /*declaration of lock to enforce mutual exclusion*/
2c.      BARDEC (bar1);          /*barrier declaration for global synchronization between
                                 sweeps*/

3.    main()
4.    begin
5.          read(n); read(nprocs);      /*read input matrix size and number of processes*/
6.          A ← G_MALLOC (a two-dimensional array of size n+2 by n+2 doubles);
7.          initialize(A);              /*initialize A in an unspecified way*/
8a.         CREATE (nprocs–1, Solve, A);
8.       Solve(A);                      /*main process becomes a worker too*/
8b.         WAIT_FOR_END (nprocs–1);    /*wait for all child processes created to terminate*/
9.    end main

10.   procedure Solve(A)
11.      float **A;                                  /*A is entire n+2-by-n+2 shared array,
                                                     as in the sequential program*/
12.   begin
13.      int i,j, pid, done = 0;
14.      float temp, mydiff = 0;                     /*private variables*/
14a.     int mymin = 1 + (pid * n/nprocs);           /*assume that n is exactly divisible by*/
14b.     int mymax = mymin + n/nprocs - 1            /*nprocs for simplicity here*/

15.      while (!done) do                 /*outer loop over all diagonal elements*/
16.         mydiff = diff = 0;            /*set global diff to 0 (okay for all to do it)*/
16a.     BARRIER(bar1, nprocs);           /*ensure all reach here before anyone modifies diff*/
17.         for i ← mymin to mymax do     /*for each of my rows*/
18.               for j ← 1 to n do       /*for all nonborder elements in that row*/
19.               temp = A[i,j];
20.               A[i,j] = 0.2 * (A[i,j] + A[i,j-1] + A[i-1,j] +
21.                  A[i,j+1] + A[i+1,j]);
22.               mydiff += abs(A[i,j] - temp);
23.            endfor
24.         endfor
25a.        LOCK(diff_lock);              /*update global diff if necessary*/
25b.        diff += mydiff;
25c.        UNLOCK(diff_lock);
25d.        BARRIER(bar1, nprocs);        /*ensure all reach here before checking if done*/
25e.        if (diff/(n*n) < TOL) then done = 1;      /*check convergence; all get
                                                      same answer*/
25f.        BARRIER(bar1, nprocs);
26.      endwhile
27.   end procedure
```

Notes on SAS Program

- SPMD: not lockstep or even necessarily same instructions
- Assignment controlled by values of variables used as loop bounds
  - unique pid per process, used to control assignment
- Done condition evaluated redundantly by all
- Code that does the update identical to sequential program
  - each process has private mydiff variable
- Most interesting special operations are for synchronization
  - accumulations into shared diff have to be mutually exclusive
  - why the need for all the barriers?

Need for Mutual Exclusion

- Code each process executes:

```
load the value of diff into register r1
add the register r2 to register r1
store the value of register r1 into diff
```

- A possible interleaving:

P1	P2	
`r1 ← diff`		{P1 gets 0 in its r1}
	`r1 ← diff`	{P2 also gets 0}
`r1 ← r1+r2`		{P1 sets its r1 to 1}
	`r1 ← r1+r2`	{P2 sets its r1 to 1}
`diff ← r1`		{P1 sets cell_cost to 1}
	`diff ← r1`	{P2 also sets cell_cost to 1}

- Need the sets of operations to be atomic (mutually exclusive)

Mutual Exclusion

Provided by LOCK-UNLOCK around *critical section*

- Set of operations we want to execute atomically
- Implementation of LOCK/UNLOCK must guarantee mutual excl.

Can lead to significant serialization if contended

- Especially since expect non-local accesses in critical section
- Another reason to use private mydiff for partial accumulation

Global Event Synchronization

BARRIER(nprocs): wait here till nprocs processes get here

- Built using lower level primitives
- Global sum example: wait for all to accumulate before using sum
- Often used to separate phases of computation

Process P_1	*Process P_2*	*Process P_nprocs*
set up eqn system	set up eqn system	set up eqn system
Barrier (name, nprocs)	**Barrier** (name, nprocs)	**Barrier** (name, nprocs)
solve eqn system	solve eqn system	solve eqn system
Barrier (name, nprocs)	**Barrier** (name, nprocs)	**Barrier** (name, nprocs)
apply results	apply results	apply results
Barrier (name, nprocs)	**Barrier** (name, nprocs)	**Barrier** (name, nprocs)

- Conservative form of preserving dependences, but easy to use

WAIT_FOR_END (nprocs-1)

Pt-to-pt Event Synch (Not Used Here)

One process notifies another of an event so it can proceed

- Common example: producer-consumer (bounded buffer)
- Concurrent programming on uniprocessor: semaphores
- Shared address space parallel programs: semaphores, or use ordinary variables as flags

P_1	P_2
	`A = 1;`
	`b: flag = 1;`
`a: while (flag is 0) do nothing;`	
`print A;`	

- *Busy-waiting* or *spinning*

Group Event Synchronization

Subset of processes involved

- Can use flags or barriers (involving only the subset)
- Concept of producers and consumers

Major types:

- Single-producer, multiple-consumer
- Multiple-producer, single-consumer
- Multiple-producer, single-consumer

Message Passing Grid Solver

- Cannot declare A to be shared array any more
- Need to compose it logically from per-process private arrays
  - usually allocated in accordance with the assignment of work
  - process assigned a set of rows allocates them locally
- Transfers of entire rows between traversals
- Structurally similar to SAS (e.g. SPMD), but orchestration different
  - data structures and data access/naming
  - communication
  - synchronization

```
1. int pid, n, b;                    /*process id, matrix dimension and number of
                                       processors to be used*/
2. float **myA;
3. main()
4. begin
5.      read(n);   read(nprocs);     /*read input matrix size and number of processes*/
8a.     CREATE (nprocs-1, Solve);
8b.     Solve();                     /*main process becomes a worker too*/
8c.     WAIT_FOR_END (nprocs-1);     /*wait for all child processes created to terminate*/
9. end main

10.  procedure Solve()
11.  begin
13.     int i,j, pid, n' = n/nprocs, done = 0;
14.     float temp, tempdiff, mydiff = 0;    /*private variables*/
6.    myA ← malloc(a 2-d array of size [n/nprocs + 2] by n+2);
                                     /*my assigned rows of A*/
7. initialize(myA);                  /*initialize my rows of A, in an unspecified way*/

15.  while (!done) do
16.     mydiff = 0;                  /*set local diff to 0*/
16a.    if (pid != 0) then SEND(&myA[1,0],n*sizeof(float),pid-1,ROW);
16b.    if (pid = nprocs-1) then
          SEND(&myA[n',0],n*sizeof(float),pid+1,ROW);
16c.    if (pid != 0) then RECEIVE(&myA[0,0],n*sizeof(float),pid-1,ROW);
16d.    if (pid != nprocs-1) then
          RECEIVE(&myA[n'+1,0],n*sizeof(float), pid+1,ROW);
                                     /*border rows of neighbors have now been copied
                                       into myA[0,*] and myA[n'+1,*]*/
17.     for i ← 1 to n' do           /*for each of my (nonghost) rows*/
18.       for j ← 1 to n do          /*for all nonborder elements in that row*/
19.         temp = myA[i,j];
20.         myA[i,j] = 0.2 * (myA[i,j] + myA[i,j-1] + myA[i-1,j] +
21.           myA[i,j+1] + myA[i+1,j]);
22.         mydiff += abs(myA[i,j] - temp);
23.       endfor
24.     endfor
                                     /*communicate local diff values and determine if
                                       done; can be replaced by reduction and broadcast*/
25a.      if (pid != 0) then                 /*process 0 holds global total diff*/
25b.        SEND(mydiff,sizeof(float),0,DIFF);
25c.        RECEIVE(done,sizeof(int),0,DONE);
25d.      else                               /*pid 0 does this*/
25e.        for i ← 1 to nprocs-1 do         /*for each other process*/
25f.          RECEIVE(tempdiff,sizeof(float),*,DIFF);
25g.        mydiff += tempdiff;              /*accumulate into total*/
25h.      endfor
25i       if (mydiff/(n*n) < TOL) then       done = 1;
25j.        for i ← 1 to nprocs-1 do         /*for each other process*/
25k.          SEND(done,sizeof(int),i,DONE);
25l.        endfor
25m.    endif
26.  endwhile
27.  end procedure
```

Should be !=

Notes on Message Passing Program

- Use of ghost rows
- Receive does not transfer data, send does
  - unlike SAS which is usually receiver-initiated (load fetches data)
- Communication done at beginning of iteration, so no asynchrony
- Communication in whole rows, not element at a time
- Core similar, but indices/bounds in local rather than global space
- Synchronization through sends and receives
  - Update of global diff and event synch for done condition
  - Could implement locks and barriers with messages
- Can use REDUCE and BROADCAST library calls to simplify code

```
      /*communicate local diff values and determine if done, using reduction and broadcast*/
25b.    REDUCE(0,mydiff,sizeof(float),ADD);
25c.    if (pid == 0) then
25i.      if (mydiff/(n*n) < TOL) then done = 1;
25k.    endif
25m.      BROADCAST(0,done,sizeof(int),DONE);
```

Orchestration: Summary

Shared address space

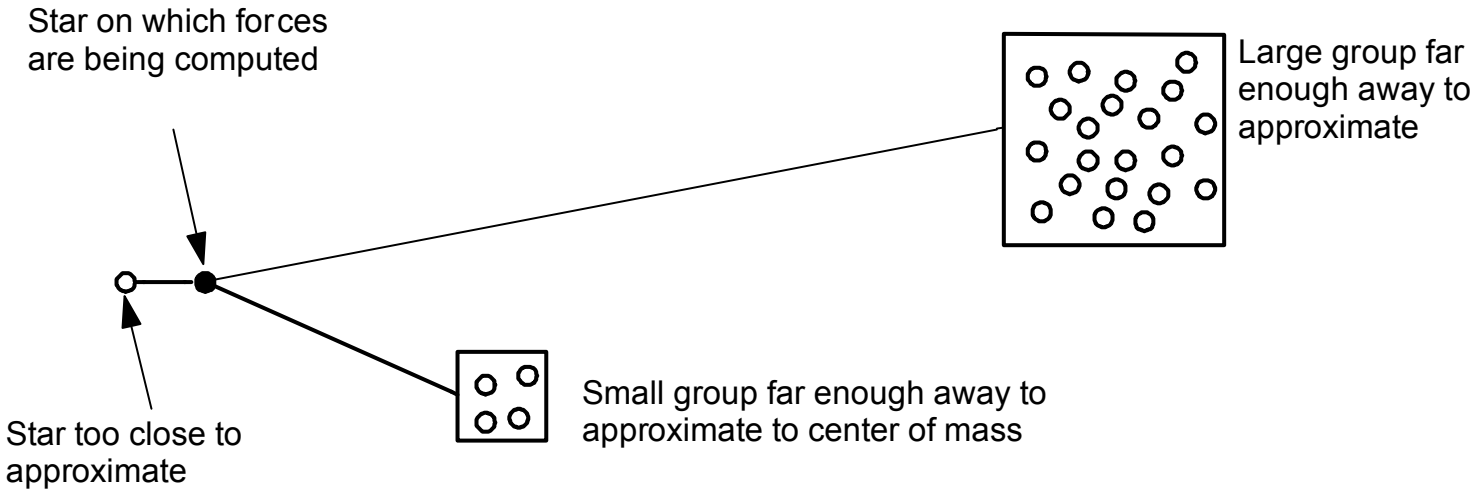
- Shared and private data explicitly separate
- Communication implicit in access patterns
- No *correctness* need for data distribution
- Synchronization via atomic operations on shared data
- Synchronization explicit and distinct from data communication

Message passing

- Data distribution among local address spaces needed
- No explicit shared structures (implicit in comm. patterns)
- Communication is explicit
- Synchronization implicit in communication

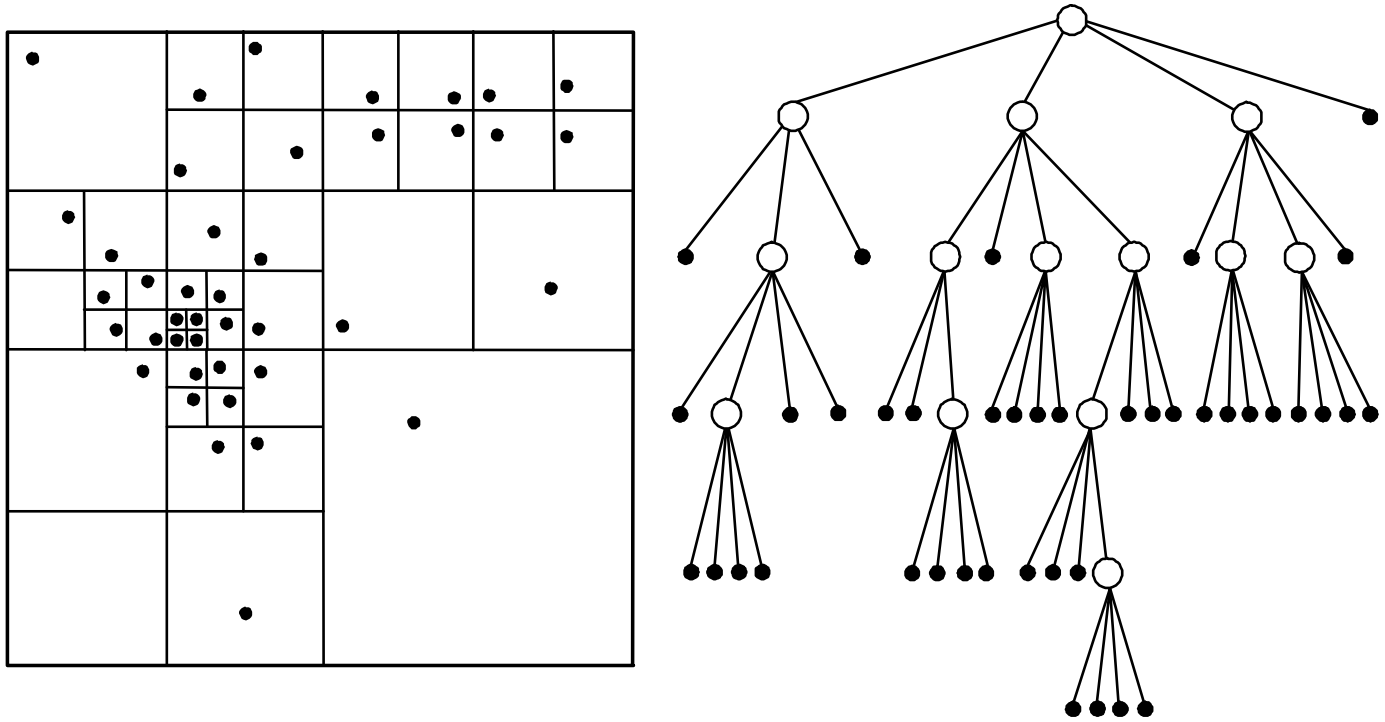
Simulating Galaxy Evolution (n-body Problem)

- Simulate the interactions of many stars evolving over time
- Computing forces is expensive
- $O(n^2)$ brute force approach
- Hierarchical Methods take advantage of force law: $G \frac{m_1 m_2}{r^2}$



- Many time-steps, plenty of concurrency across stars within one

Barnes-Hut



(a) The spatial domain

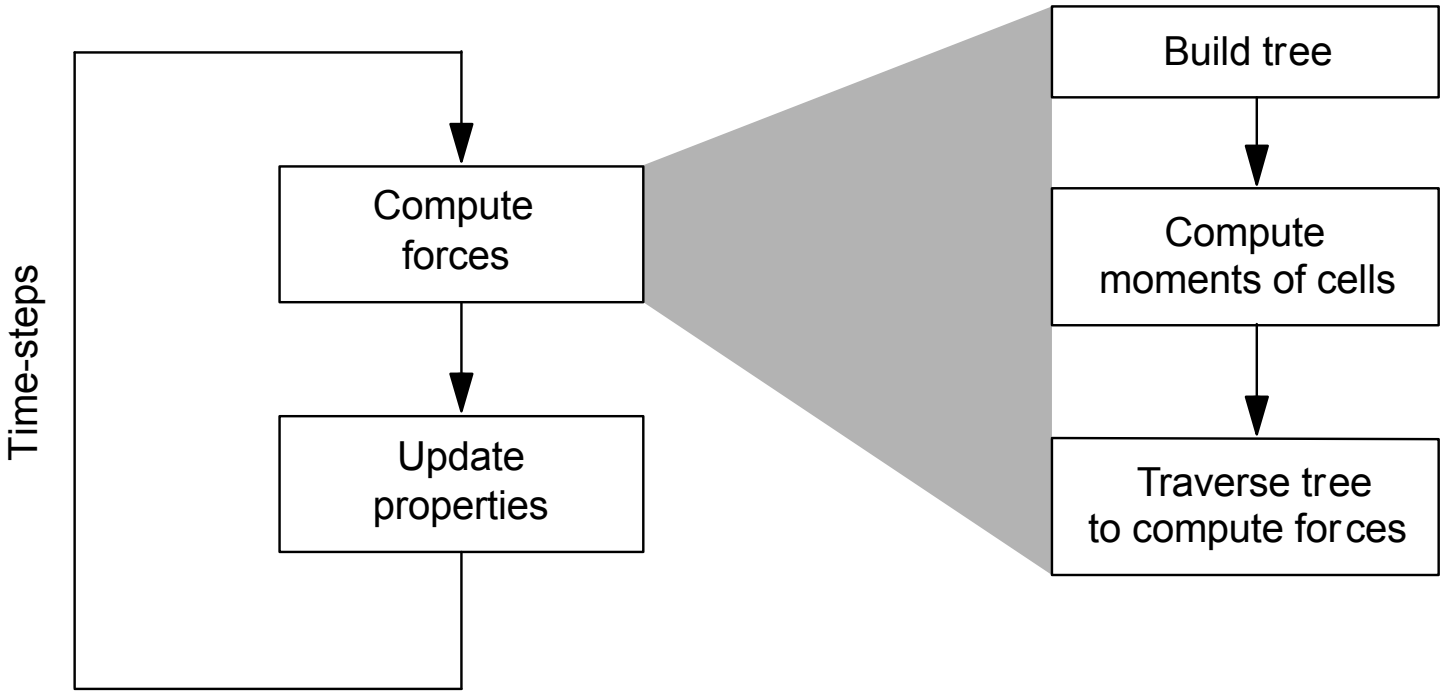
(b) Quadtree representation

Locality Goal:

- *Particles close together in space should be on same processor*

Difficulties: Nonuniform, dynamically changing

Application Structure



- Main data structures: array of bodies, of cells, and of pointers to them
  - Each body/cell has several fields: mass, position, pointers to others
  - pointers are assigned to processes

Partitioning

Decomposition: bodies in most phases, cells in computing moments

Challenges for assignment:

- Nonuniform body distribution => work and comm. Nonuniform
  - Cannot assign by inspection
- Distribution changes dynamically across time-steps
  - Cannot assign statically
- Information needs fall off with distance from body
  - Partitions should be spatially contiguous for locality
- Different phases have different work distributions across bodies
  - No single assignment ideal for all
  - Focus on force calculation phase
- Communication needs naturally fine-grained and irregular

Load Balancing

- Equal particles ≠ equal work.
  - Solution: Assign costs to particles based on the work they do

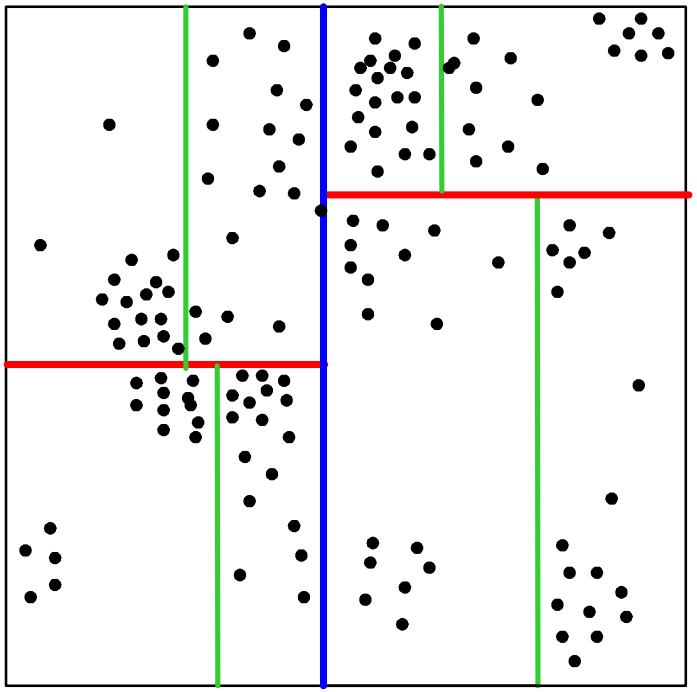
- Work unknown and changes with time-steps
  - Insight : System evolves slowly
  - Solution: *Count* work per particle, and use as cost for next time-step.

Powerful technique for evolving physical systems

A Partitioning Approach: ORB

Orthogonal Recursive Bisection:

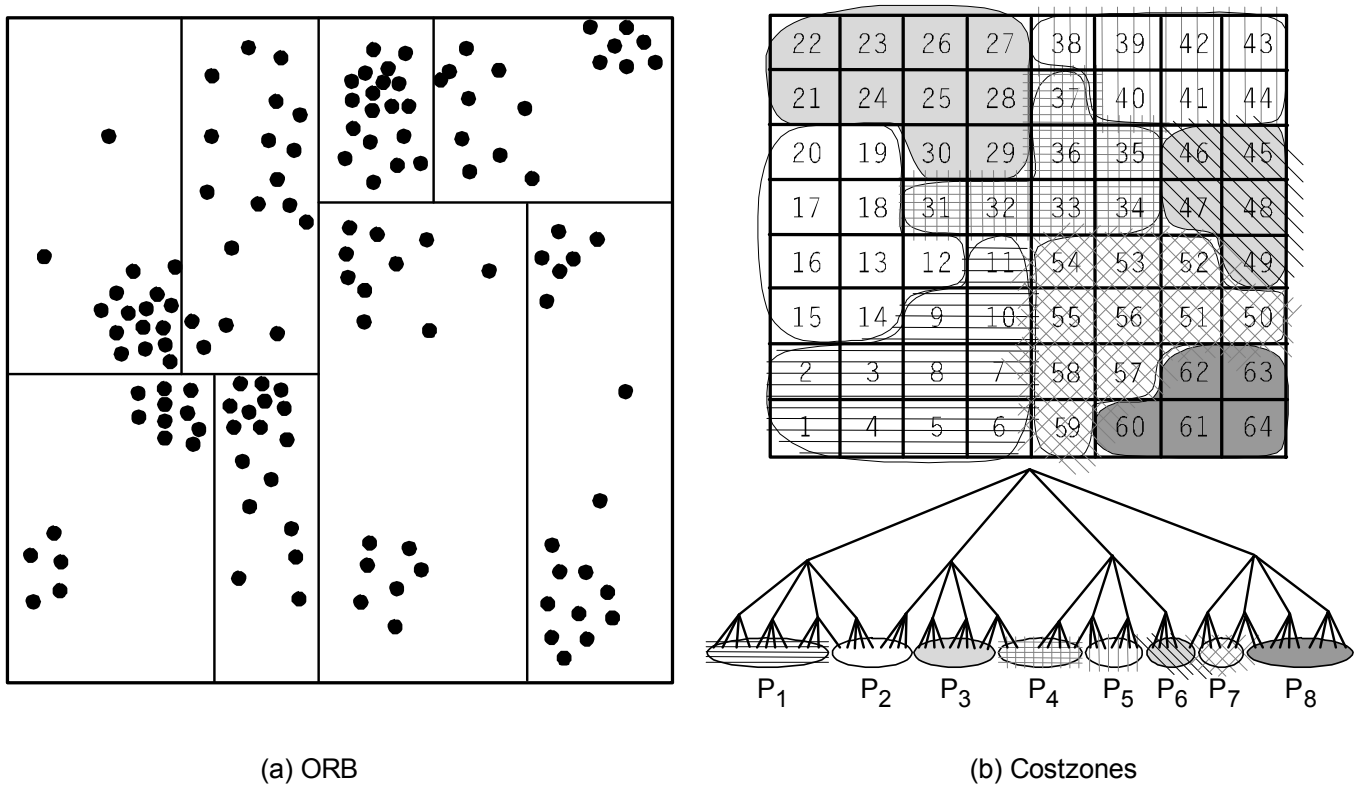
- Recursively bisect space into subspaces with equal work
  - Work is associated with bodies, as before
- Continue until one partition per processor



- **High overhead for large no. of processors**

Another Approach: Costzones

Insight: Tree already contains an encoding of spatial locality.



(a) ORB

(b) Costzones

- **Costzones is low-overhead and very easy to program**

Orchestration and Mapping

Spatial locality: Very different than in Ocean, like other aspects

- Data distribution is much more difficult than
  - Redistribution across time-steps
  - Logical granularity (body/cell) much smaller than page
  - Partitions contiguous in physical space does not imply contiguous in array
  - But, good temporal locality, and most misses logically non-local anyway
- Long cache blocks help within body/cell record, not entire partition

Temporal locality and working sets:

- Slow growth rate, and fits in second-level caches, unlike Ocean

Synchronization:

- Barriers between phases
- No synch within force calculation: data written different from data read
- Locks in tree-building, pt. to pt. event synch in center of mass phase

Mapping: ORB maps well to hypercube, costzones to linear array

Rendering Scenes by Ray Tracing

- Shoot rays into scene through pixels in image plane
- Follow their paths
  - they bounce around as they strike objects
  - they generate new rays: ray tree per input ray
- Result is color and opacity for that pixel
- Parallelism across rays

Partitioning

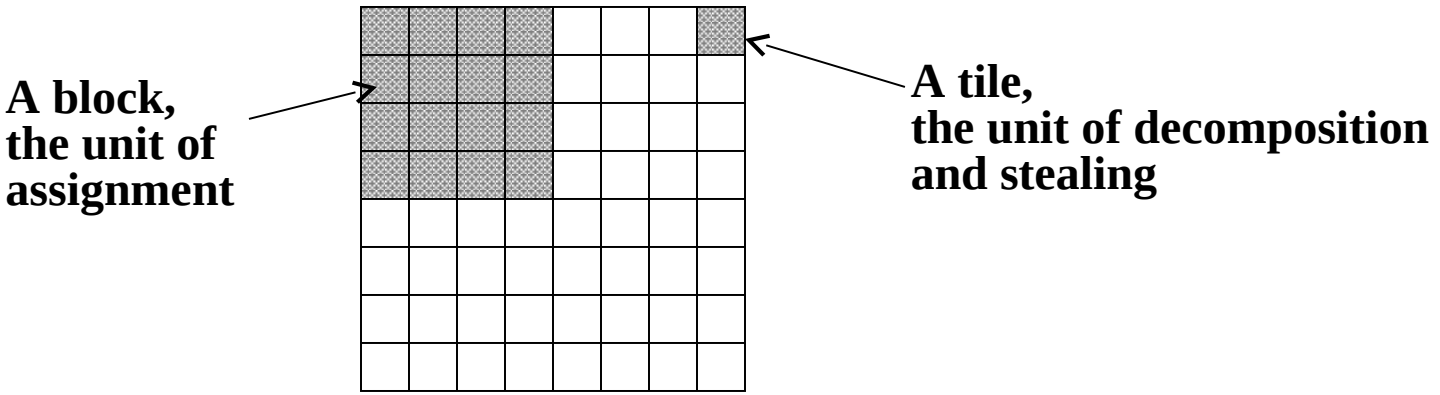
Scene-oriented approach

- Partition scene cells, process rays while they are in an assigned cell

Ray-oriented approach

- Partition primary rays (pixels), access scene data as needed
- Simpler; used here

Need dynamic assignment; use contiguous blocks to exploit spatial coherence among neighboring rays, plus tiles for task stealing

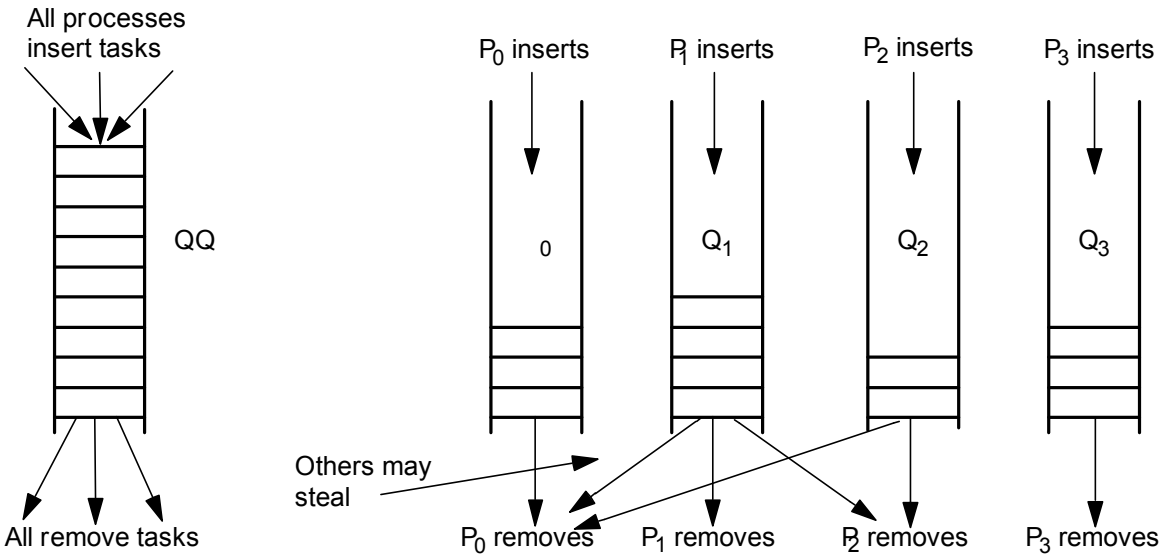


Dynamic Scheduling with Task Queues

Centralized versus distributed queues

Task stealing with distributed queues

- Can compromise comm and locality, and increase synchronization
- Whom to steal from, how many tasks to steal, ...
- Termination detection
- Maximum imbalance related to size of task



(a) Centralized task queue

(b) Distributed task queues (one per pr ocess)

Orchestration and Mapping

Spatial locality

- Proper data distribution for ray-oriented approach very difficult
- Dynamically changing, unpredictable access, fine-grained access
- Better spatial locality on image data than on scene data

Temporal locality

- Working sets much larger and more diffuse than Barnes-Hut
- But still a lot of reuse in modern second-level caches

Synchronization:

- One barrier at end, locks on task queues

Mapping: natural to 2-d mesh for image, but likely not important

Association Rule Problem

Given a database of transactions:

Transaction	Items
t_1	Bread,Jelly,PeanutButter
t_2	Bread,PeanutButter
t_3	Bread,Milk,PeanutButter
t_4	Beer,Bread
t_5	Beer,Milk

- Find all the association rules:

$X \Rightarrow Y$	s	α
Bread $\Rightarrow$ PeanutButter	60%	75%
PeanutButter $\Rightarrow$ Bread	60%	100%
Beer $\Rightarrow$ Bread	20%	50%
PeanutButter $\Rightarrow$ Jelly	20%	33.3%
Jelly $\Rightarrow$ PeanutButter	20%	100%
Jelly $\Rightarrow$ Milk	0%	0%

Association Rule Definitions

- $I=\{i_1, i_2, ..., i_n\}$: a set of all the items
- Transaction T: a set of items such that $T \subseteq I$
- Transaction Database D: a set of transactions
- A transaction $T \subseteq I$ contains a set $X \subseteq I$ of some items, if $X \subseteq T$
- *An Association Rule*: is an implication of the form $X \Rightarrow Y$, where $X, Y \subseteq I$

Association Rule Definitions

- A set of items is referred as an itemset. A itemset that contains k items is a k-itemset.
- The support s of an itemset X is the percentage of transactions in the transaction database D that contain X.
- The support of the rule $X \Rightarrow Y$ in the transaction database D is the support of the items set $X \cup Y$ in D.
- The confidence of the rule $X \Rightarrow Y$ in the transaction database D is the ratio of the number of transactions in D that contain $X \cup Y$ to the number of transactions that contain X in D.

Example

- **Given a database of transactions:**

Transaction	Items
t_1	Bread,Jelly,PeanutButter
t_2	Bread,PeanutButter
t_3	Bread,Milk,PeanutButter
t_4	Beer,Bread
t_5	Beer,Milk

- **Find all the association rules:**

$X \Rightarrow Y$	s	α
Bread $\Rightarrow$ PeanutButter	60%	75%
PeanutButter $\Rightarrow$ Bread	60%	100%
Beer $\Rightarrow$ Bread	20%	50%
PeanutButter $\Rightarrow$ Jelly	20%	33.3%
Jelly $\Rightarrow$ PeanutButter	20%	100%
Jelly $\Rightarrow$ Milk	0%	0%

Association Rule Problem

- Given:
  - a set I of all the items;
  - a database D of transactions;
  - minimum support s;
  - minimum confidence c;
- Find:
  - all association rules $X \Rightarrow Y$ with a minimum support s and confidence c, i.e. Find all Frequent Itsemsets.

Important Properties to Exploit

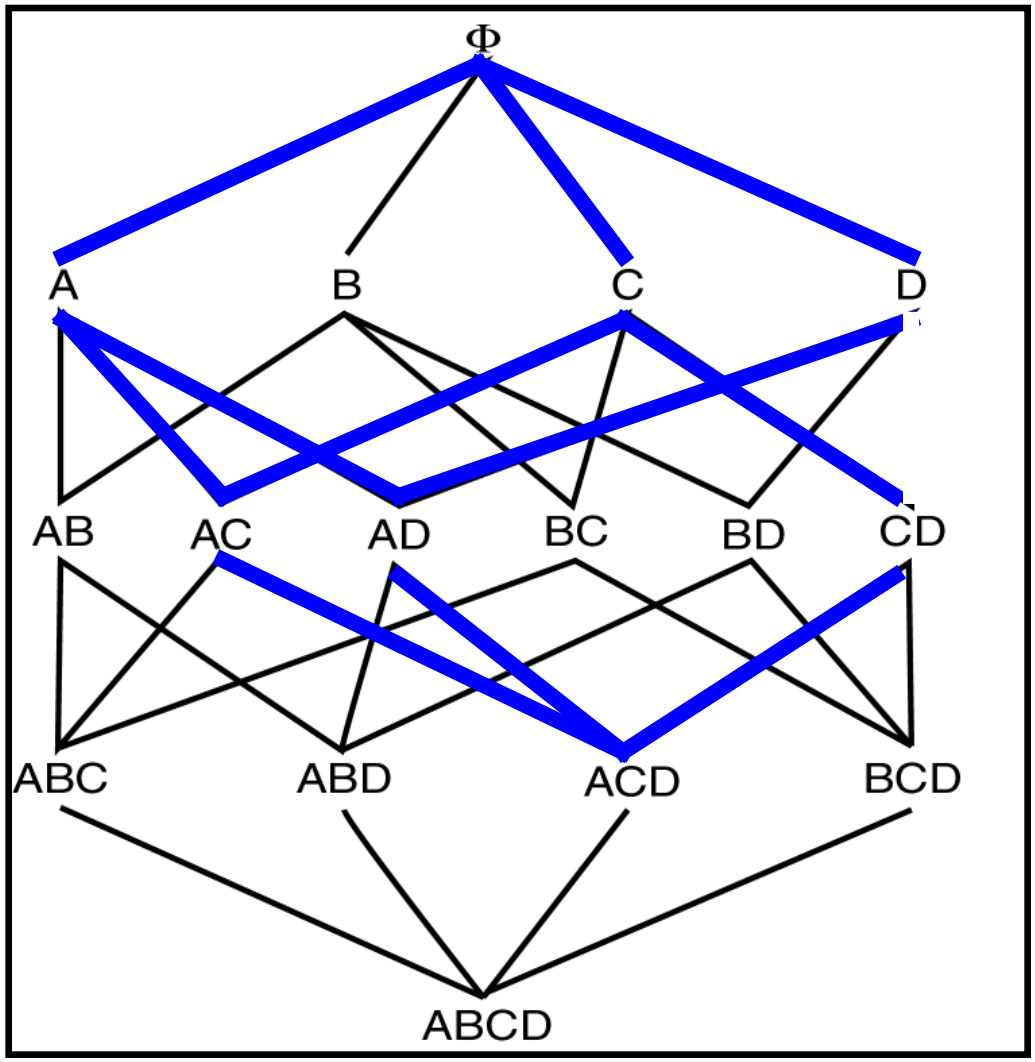
- *Frequent Itemset Property:*

Any subset of a frequent itemset is frequent.

- *Contrapositive:*

If an itemset is not frequent, none of its supersets are frequent.

Frequent Itemset Property



Sequential Algorithm

- L_k: Set of frequent itemsets of size k
- C_k: Set of candidate itemset of size k

L_1 = {frequent items};
for (k = 1; L_k !=∅; k++) do
C_{k+1} = candidates generated from L_k;
for each transaction t
increment the count of all candidates in C_{k+1} that are contained in t
L_{k+1} = frequent candidates in C_{k+1} with min_support

How do you generate C_{k+1}-

If we have large itemsets of length k, we can do joins on these (after lexicographically sorting them) to get possible itemsets of length k.

Equivalence Classes

L2 = { AB, AC, AD, BC, BD, CD, DE }

C3 = {ABC, ABD, ACD, BCD}

Parallel Implementation

Say we have Large Itemsets of Size 2

Reorganize the Database as follows:

{ISx, T1, T2, … }, {ISy, T1, T3, ….}, ….

Balanced assignment of Equivalence Classes Amongst the Processors

Each Processor can independently check for larger itemsets.

Can lead to some imbalance because of mismatch in equivalence classes for larger itemsets.