

1. Consider an MPI program that uses a Reduce of  $n$  numbers followed by a Bcast of the result of the Reduce to all  $P$  processes. During the reduction phase, count any arithmetic as well as communication that occurs as taking 1 unit of time. Find the isoefficiency function.

2. **Prefix sums** are the  $n$  partial sums:

$$x_0, x_0 + x_1, x_0 + x_1 + x_2, \dots, x_0 + x_1 + \dots + x_{n-1}.$$

- Can you devise a parallel algorithm that requires only  $\log p$  communication phases using  $p$  processors?
- MPI provides a collective communication function, `MPI_Scan`, that can be used to compute prefix sums:

```
int MPI_Scan(
    void* sendbuf_p /* in */,
    void* recvbuf_p /* out */,
    int count /* in */,
    MPI_Datatype datatype /* in */,
    MPI_Op op /* in */,
    MPI_Comm comm /* in */);
```

It operates on arrays with count elements; both `sendbuf_p` and `recvbuf_p` should refer to blocks of count elements of type `datatype`. The `op` argument is the same as `op` for `MPI_Reduce`. Write an MPI program that generates a random array of count elements on each MPI process, finds the prefix sums, and prints the results.

3. The processes in an MPI program have data that looks Figure (a) below. For Figure (b), (c), and (d) give the name (you do not need to give the full function call or arguments) of the collective communication operation that leads to the data being communicated as shown.

P <sub>0</sub>	1	2	3	4	5
P <sub>1</sub>	6	7	8	9	10
P <sub>2</sub>	11	12	13	14	15
P <sub>3</sub>	16	17	18	19	20
P <sub>4</sub>	21	22	23	24	25

(a)

P <sub>0</sub>	55	60	65	70	75
P <sub>1</sub>	55	60	65	70	75
P <sub>2</sub>	55	60	65	70	75
P <sub>3</sub>	55	60	65	70	75
P <sub>4</sub>	55	60	65	70	75

(b)

P <sub>0</sub>	1	6	11	16	21
P <sub>1</sub>	2	7	12	17	22
P <sub>2</sub>	3	8	13	18	23
P <sub>3</sub>	4	9	14	19	24
P <sub>4</sub>	5	10	15	20	25

(c)

P <sub>0</sub>	1	2	3	4	5
P <sub>1</sub>	1	2	3	4	5
P <sub>2</sub>	1	2	3	4	5
P <sub>3</sub>	1	2	3	4	5
P <sub>4</sub>	1	2	3	4	5

(d)

4. One reason that leads the following program to have an isoefficiency of  $W = P \log P$  is that part of the code contains this section:

```
MPI_Comm_size(MPI_COMM_WORLD, &N);  
for (int i=0; i<N; i++) {  
    MPI_Reduce(b, c, n, MPI_INT, MPI_SUM, i, MPI_COMM_WORLD);  
}
```

Rewrite this to be a more efficient operation. (using a single MPI collective communication call)