**INSTRUCTIONS:**

1. Submit your solution to Gradescope by the due time; no late submissions will be accepted.
2. Type your solution (except figures; figures can be hand-drawn and then scanned); no hand-written solutions will be accepted.

**Problem 1 (10 points).**

Construct the (implicit) suffix tree for string $s = aababab\$$ using the Ukkonen's algorithm. Show the intermediate steps (at least the implicit suffix tree after each phase and where each phase ends).

**Problem 2 (10 points).**

Prove that following a suffix link in an implicit suffix tree or a suffix tree the depth of a node (defined as the number of nodes from the root to this node) will decrease by at most 1.

**Problem 3 (10 points).**

You are given the suffix array $SA$ of an (unknown) string $S$ and an array $C$, of length $|\Sigma|$, where $C[c]$ says how many occurrences of letter $c \in \Sigma$ there are in $S$. Design an algorithm to reconstruct $S$ from $SA$ and $C$ in $O(|S|)$ time. Your answer should be fewer than 3 sentences.

**Problem 4 (10 points).**

Consider two variants of the skew algorithm (to construct the suffix array).

1. In step 7 of sorting $G_1$, use the procedure of step 3 (i.e., let $T = S$, sort all its tokens and then use a recursive call). Is this variant correct? If so, analyze its running time.
2. Partition all suffixes into 4 groups, sort the last 3 groups, followed by sorting the first group and merge the two sorted lists, all using the same procedure with the skew algorithm. Is this variant correct? If so, analyze its running time.

**Problem 5 (16 points).**

(We did not cover the construction of the LCP array in class; here is one.) You are the suffix array $SA$ of a string $S$ and the *inverse suffix array* $R$ where $SA[i] = j$ if and only if $R[j] = i$. (This inverse suffix array was named as "rank" array in the step 5 of the skew algorithm, as $R[j]$ gives the "ranking" of suffix-$j$.)

1. Prove that $LCP[R[i+1]-1] \geq LCP[R[i]-1]-1$, for any $1 \leq i \leq |S|-1$.
2. Use above to design an algorithm, running in $O(|S|)$ time, to construct the LCP array given $SA$ and $R$.

**Problem 6 (10 points).**

Give two distinct 9-letter words over the alphabet $\Sigma = \{a,b\}$ that are not cyclic rotations of each other but that have the same first 3 columns of the BWT matrix. *Hint:* consider paths in directed graph $G = (V,E)$ where $V = \Sigma^3$ and there is an edge from $xyz$ to $yzw$ for any $x,y,z,w \in \Sigma$.