APPROACHES IN DOA ESTIMATION

OMER BOKOBZA, EMAIL: omerbok@post.bgu.ac.il

ABSTRACT

Direction of Arrival (DOA) estimation is a critical component in various fields, such as radar, sonar, wireless communications, and acoustic source localization. This comprehensive review paper provides an analysis of diverse DOA estimation techniques, ranging from classical methods like MUSIC, LS, MVDR, and ESPRIT to cutting-edge approaches involving deep learning. We delve into the underlying principles, strengths, and limitations of each method and discuss their applications in real-world scenarios.

Index Terms— DOA.

1. INTRODUCTION

Direction of Arrival (DOA) refers to the angle at which a signal or wavefront arrives at a receiving antenna or sensor array. In the context of array signal processing, DOA estimation is the process of determining the angles from which signals are arriving at an antenna array. DOA estimation plays a crucial role in various applications, including radar, sonar, wireless communication, and array processing, where it is used to locate and track the sources of incoming signals. The DOA is typically measured with respect to a reference direction, often the broadside direction of the array. Various techniques, such as beamforming and spatial spectrum analysis, are employed to estimate the DOA and extract valuable spatial information from received signals.

2. METHODS OF DOA

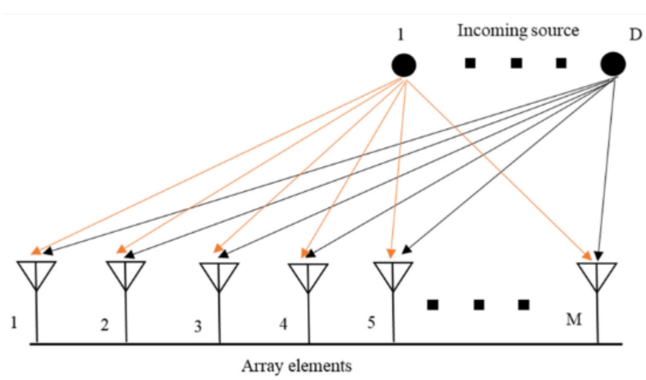

Fig. 1. Illustration of DOA estimation [1].

2.1. MUSIC And Root-MUSIC Algorithm

MUSIC (Multiple Signal Classification) algorithm and its variant, **ROOT-MUSIC (Root Multiple Signal Classification)** [2–5], are widely used techniques in array signal processing for estimating the direction of arrival (DOA) of multiple signals received by an antenna array.

For both algorithms, the input comprises the received signals $\mathbf{x}(t)$ from an antenna array, where $\mathbf{x}(t)$ is a vector of signal samples over time. The first step involves computing the sample covariance matrix $\mathbf{R}$ as:

$$\mathbf{R} = \frac{1}{N}\sum_{i=1}^{N}\mathbf{x}(i)\mathbf{x}^H(i)$$

Where:

- $\mathbf{R}$ is the sample covariance matrix.
- N is the number of samples.

Both MUSIC and ROOT-MUSIC then proceed to find the eigenvalues λ_i and eigenvectors $\mathbf{v}_i$ of $\mathbf{R}$. They calculate the spatial spectrum $P(\theta)$ as follows:

$$P(\theta) = \frac{1}{\sum_{i=1}^{N-M}\lambda_i|\mathbf{a}^H(\theta)\mathbf{v}_i|^2}$$

Where:

- $P(\theta)$ is the spectrum as a function of the DOA angle θ.
- $\mathbf{a}(\theta)$ represents the steering vector associated with the DOA angle θ.
- N is the number of array elements.
- M is the number of sources.

MUSIC is known for its ability to separate closely spaced sources, while ROOT-MUSIC provides precise DOA estimation. Both algorithms are employed in applications such as radar, sonar, and wireless communications for source localization and direction finding.

2.2. MVDR Algorithm

Minimum Variance Distortionless Response (MVDR) [5–7] algorithm is a powerful technique used in array signal processing for beamforming and direction of arrival (DOA) estimation. It aims to design a beamformer that minimizes the output power subject to the constraint of not distorting the signals arriving from specific directions.

Given an array of sensors receiving signals $\mathbf{x}(t)$, where $\mathbf{x}(t)$ is a vector of signal samples over time, the MVDR algorithm computes the spatial filter or weight vector $\mathbf{w}$ that minimizes the output power $\mathbf{w}^H\mathbf{R}\mathbf{w}$ subject to the constraint that $\mathbf{w}^H\mathbf{a}(\theta) = 1$ for a desired direction of arrival (DOA) angle θ. Here, $\mathbf{a}(\theta)$ represents the steering vector associated with the DOA angle θ, and $\mathbf{R}$ is the sample covariance matrix, which characterizes the statistics of the received signals.

The MVDR spatial filter is given by:

$$\mathbf{w}_{\text{MVDR}} = \frac{\mathbf{R}^{-1}\mathbf{a}(\theta)}{\mathbf{a}^H(\theta)\mathbf{R}^{-1}\mathbf{a}(\theta)}$$

This weight vector ensures that the output of the beamformer minimizes the power of noise and interference while preserving the signal from the desired DOA. MVDR is especially effective in scenarios with spatially correlated interference or when accurate DOA estimation is required for source localization.

In summary, MVDR is a key technique in array signal processing that optimally steers the antenna array to enhance the desired signal while suppressing interference and noise.

2.3. LS Algorithm

Least Squares (LS) [5, 8] algorithm is a widely used method in array signal processing for estimating the direction of arrival (DOA) of signals received by an antenna array. LS-based DOA estimation operates on the principle of minimizing the least squares error between the received signals and the estimated signals corresponding to different DOA angles.

Given an array of sensors receiving signals $\mathbf{x}(t)$, where $\mathbf{x}(t)$ is a vector of signal samples over time, and a steering matrix $\mathbf{A}$ that represents the array's response to different DOA angles, LS seeks to estimate the DOAs by solving the following optimization problem:

$$\hat{\boldsymbol{\theta}} = \arg\min_{\boldsymbol{\theta}} \|\mathbf{x}(t) - \mathbf{A}(\boldsymbol{\theta})\boldsymbol{\alpha}\|^2$$

Where:

- $\hat{\boldsymbol{\theta}}$ is the estimated DOA vector.
- $\boldsymbol{\theta}$ represents the vector of DOA angles to be estimated.
- $\mathbf{A}(\boldsymbol{\theta})$ is the steering matrix associated with the DOA angles $\boldsymbol{\theta}$.
- $\boldsymbol{\alpha}$ is the complex amplitude vector of the incoming signals.

LS seeks to find the DOA vector that minimizes the sum of squared errors between the received and estimated signals. While LS is straightforward and computationally efficient, it may be sensitive to noise and require a large number of snapshots for accurate DOA estimation. Despite its simplicity, LS remains a fundamental tool in DOA estimation applications such as radar, sonar, and wireless communications.

2.4. ESPRIT Algorithm

ESPRIT (Estimation of Signal Parameters via Rotational Invariance Techniques) [4, 5, 9] algorithm is a powerful method in array signal processing for estimating the directions of arrival (DOAs) of signals received by an antenna array. ESPRIT exploits the concept of rotational invariance to estimate DOAs accurately and efficiently.

Given an array of sensors receiving signals $\mathbf{x}(t)$, where $\mathbf{x}(t)$ is a vector of signal samples over time, ESPRIT first constructs a lower and upper sensor subarray by dividing the full array into two parts. The algorithm then computes the signal subspace by finding the eigenvalues and eigenvectors of the covariance matrix of the lower and upper subarrays.

Let $\mathbf{U}_L$ and $\mathbf{U}_U$ represent the eigenvector matrices corresponding to the lower and upper subarrays, respectively. The DOAs are then estimated by exploiting the relationship between $\mathbf{U}_L$ and $\mathbf{U}_U$:

$$\mathbf{U}_L = \mathbf{U}_U\mathbf{T}$$

Where: - $\mathbf{U}_L$ and $\mathbf{U}_U$ are the eigenvector matrices. - $\mathbf{T}$ is a diagonal matrix containing the complex exponential terms that correspond to the DOAs.

By solving for $\mathbf{T}$, ESPRIT obtains the DOAs directly. The algorithm is known for its ability to estimate DOAs with high resolution, even in the presence of closely spaced sources. ESPRIT is widely used in applications such as radar, sonar, and wireless communications for accurate source localization and direction finding.

3. MODEL - QUANTIZED DOA ESTIMATION

Quantization is a process of mapping a continuous range of input values to a finite set of discrete output values. My model is based on a quantization process for the signal entering the system - in this case one of the algorithms presented above. The process is characterized by the following steps:

1. **Determining Quantization Levels:** The number of quantization levels, denoted as N_q, is determined by the number of bits, n, used for quantization. It is calculated as $N_q = 2^n$, representing the total number of distinct quantization levels.

2. **Calculating Step Size:** The step size, denoted as Δ, is computed based on the signal's dynamic range, R, and the number of quantization levels: $\Delta = \frac{2R}{N_q}$.

3. **Quantization Process:** Given an input signal value, x, the quantized output, x_q, is obtained using the following equation:

$$x_q = \Delta \cdot \left\lfloor \frac{x}{\Delta} + \frac{1}{2} \right\rfloor$$

The signal x is divided by the step size, which scales it to fit within the quantization levels. The term $\left\lfloor \frac{x}{\Delta} + \frac{1}{2} \right\rfloor$ represents rounding to the nearest quantization level. The addition of $\frac{1}{2}$ is for proper rounding.

The choice of the number of bits, n, directly influences the granularity of quantization. A higher n results in finer quantization levels but requires more storage to represent the signal accurately. Algorithm 1 describes the process that includes performing quantization on the received signal and then using the quantized signal as input to each of the algorithms described in section 2.

Algorithm 1: Quantized Direction Of Arrival

Init: Common seed parameters - number of antennas, samples, sources and the antenna spacing
Input: Received Signal - based on randomized gaussian data

Create the received signal (steering vectors) with noise;

Apply quantization function to the received signal;

Apply each algorithm with the quantized signal;

Output: DOA spectrum of each algorithm

4. NUMERICAL STUDY

In this section I numerically evaluate my approach[1].I implemented the algorithms mentioned in section 2 in vanilla form, then I performed the quantization process on the received signal to these systems - and compared the performance (section 3). First, i will introduce the basis of these algorithms (subsection 4.1) and then analyze the results (subsection 4.2).

4.1. Common Seed

To implement Algorithm 1,presented a set of algorithms for Direction-of-Arrival (DOA) estimation. To evaluate and compare these algorithms, a common initialization and data generation process is employed. An array of m antennas (default - 8) is used as the sensor array configuration. A total of 1000 samples are generated to simulate the received signals with known DOAs and, optionally, known source amplitudes. The parameter *quantization_bits* - n (section 3) is set to 2 by default, indicating the number of bits used for signal quantization. The DOA estimation is performed over a range of 180 angular values spanning from $[-\frac{\pi}{2}$ to $\frac{\pi}{2})$ radians. The true DOAs and source amplitudes are defined beforehand for each simulation. The received signals are generated as a linear combination of source signals and added noise to simulate real-world scenarios. Additionally, the quantized received signals are computed for comparison. The results are analyzed in terms of Mean Squared Error (MSE) to evaluate the accuracy of the DOA estimation algorithms.

[1]The source code used in my experimental study, including all the hyper-parameters, is available online at https://github.com/omerbokobza/DOAsAlgorithmsImpl.

4.2. Results

According to figure 2 it can be seen that the minimum error was obtained for the LS algorithm and for the MVDR, for these parameters. It can also be seen that the quantization did not overly affect the MSE, even when I tried to play with the number of bits I quantize.

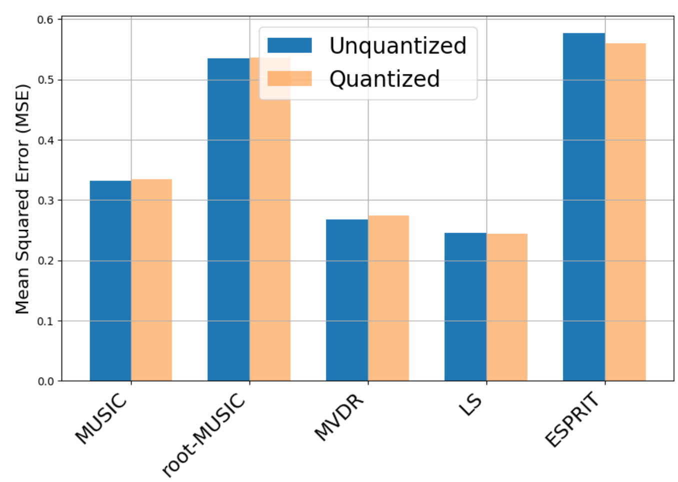

Fig. 2. Comparison of DOA Estimation Algorithms MSE (Unquantized vs. Quantized).

It is possible that the way in which I performed the quantization is not good, or that the quantization of the signal before the execution of the algorithm affects the performance negatively.

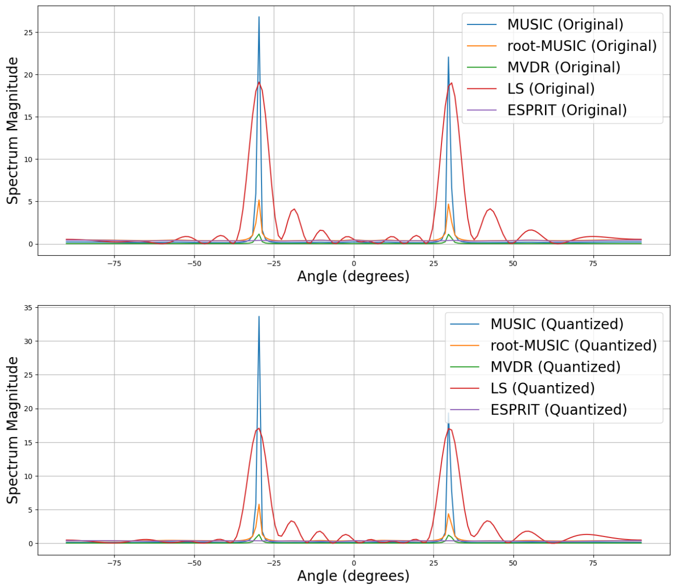

Fig. 3. Comparison of DOA Spectra (Original vs. Quantized).

In figure 3, The DOA spectrum can be seen comparing before and after quantization. Each peak in the DOA spectrum corresponds to a direction from which a signal source is emitting or a reflection is coming. The amplitude of the peak can indicate the strength of the signal coming from that particular direction.

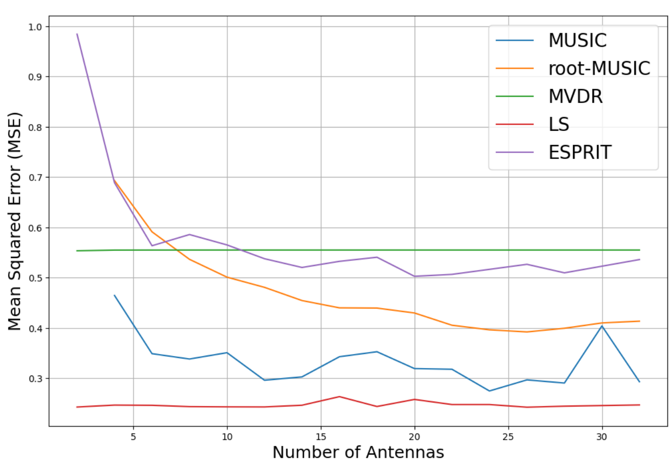

Fig. 4. Performance Comparison of DOA Estimation Algorithms with Varying Number of Antennas for 2 Sources (Unquantized)

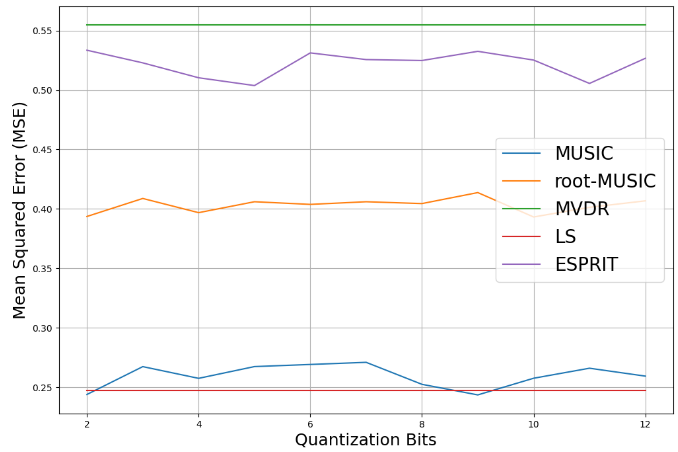

Fig. 5. Performance Comparison of DOA Estimation Algorithms at Different Quantization Bits for 8 Antennas and 2 Sources

In figure 4 and 5 I made a comparison of the number of hyper-parameters as a function of MSE vs the number of bits of quantization and vs the number of antennas in the array. It can be seen that the effect of the number of antennas in the array is large in most algorithms, because the MSE decreases. On the other hand, the quantization hardly affects the MSE (when increasing the number of bits in each algorithm).I have to admit it's strange because for more bits I get more information - so I would expect to get lower error and higher accuracy in the signal discretization.

After analyzing the results, I realized that the quantization does not affect the results too much, so I decided to try another new direction, without quantization.

5. WIND OF CHANGE - DEEP LEARNING APPROACH

The Deep Learning Approach for Direction of Arrival (DOA) Estimation leverages a feedforward neural network to estimate DOA angles from received signals in an antenna array. This approach is effective when traditional signal processing techniques encounter limitations. The key components of this approach include the neural network architecture, input data, and loss function. algorithm 2 describe the Net.

5.1. Neural Network Architecture

The neural network is a feedforward model comprising three layers: an input layer, two hidden layers, and an output layer. The input layer accepts a flattened representation of received signals from the antenna array. Each hidden layer consists of densely connected neurons, with the first hidden layer containing 64 neurons and the second hidden layer containing 32 neurons. These hidden layers employ the Rectified Linear Unit (ReLU) activation function to capture complex relationships within the data. The output layer provides estimates of the DOA angles.

Mathematically, the neural network's output can be represented as:

$$\hat{y} = f(W_{\text{out}} \cdot f(W_{\text{hidden2}} \cdot f(W_{\text{hidden1}} \cdot X + b_{\text{hidden1}}) + b_{\text{hidden2}}) + b_{\text{out}})$$

where: $\hat{y}$ represents the estimated DOA angles. X denotes the input data. W and b represent the weights and biases of the neural network layers, respectively. $f(\cdot)$ represents the ReLU activation function.

5.2. Input Data

The input to the system consists of received signals from the antenna array, collected from various sources, containing information about signal origins. Initially, the input data is flattened to create a consistent representation and is then standardized to ensure uniform scaling. For comparing performances with the other algorithms - the system was initialized according to subsection 4.1.

5.3. Loss Function

The neural network is trained using the Mean Squared Error (MSE) loss function. The MSE measures the squared differences between the predicted DOA angles ($\hat{y}$) and the true DOA angles (y). The neural network minimizes this loss during training by adjusting its weights and biases using optimization techniques such as the Adam optimizer.

The MSE loss is defined as:

$$\text{MSE} = \frac{1}{N} \sum_{i=1}^{N} (\hat{y}_i - y_i)^2$$

where: N is the number of samples. $\hat{y}_i$ and y_i represent the predicted and true DOA angles for the i-th sample. This Deep Learning Approach for DOA Estimation employs a well-structured neural network architecture, suitable input data preprocessing, and the MSE loss function to provide accurate DOA angle estimation based on received signals from an antenna array. The neural network is trained to learn intricate patterns in the data and minimize the squared differences between predicted and true DOA angles, ultimately providing accurate DOA estimation.

Algorithm 2: DeepDOA

Init: Initialize neural network weights and biases, common seed parameters

Input: Received signals, true DOA angles

while *Training* **do**

- Forward propagation through a neural network with two hidden layers:
  1. Flatten input signals and compute activations.
  2. Apply ReLU activation functions.
  3. Estimate DOA angles.

  Compute Mean Squared Error (MSE) loss and perform back-propagation to update network parameters.

while *Testing* **do**

- Forward propagation for DOA estimation.

Output: Estimated DOA angles

Note: Training minimizes the MSE loss between estimated and true DOA angles.

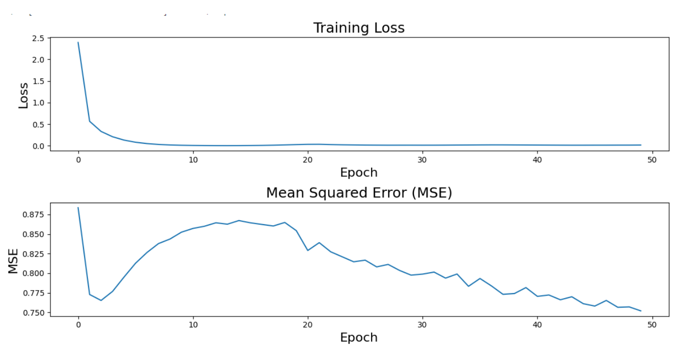

Fig. 6. Loss and MSE vs Epochs during the learning process

The model I built does converge, and the MSE is also decreasing, so I conclude that it works well. However, the error here is higher than some of the other models presented in the previous sections - I guess this can be solved by adjusting the hyper-parameters of the model itself or the parameters of the system such as the number of antennas, sources, etc.

6. APPLICATION - RADAR SYSTEMS!

Radar systems play a critical role in detecting and tracking objects in various applications, including air traffic control, weather monitoring, and defense. To enhance the capabilities of radar systems, integrating Direction of Arrival (DOA) algorithms becomes pivotal. DOA algorithms, such as MUSIC, LS, MVDR, ESPRIT, and even deep learning approaches, enable radar systems to accurately estimate the angles from which signals or objects arrive. This information can be invaluable for target localization, tracking, and discrimination.

By combining these DOA algorithms with radar systems, it becomes possible to extract richer spatial information about the environment, enabling better object detection, tracking, and situational awareness. This integration can significantly improve the performance of radar systems across various domains.

7. CONCLUSIONS

During this review, I implemented several algorithms in the world of DOA, and tried to introduce my own idea - a combination with quantization of the signal entering the system. I found that the idea did not succeed in the MSE aspects, as the results were not very impressive. It is possible that the signal entering the system is not suitable to be quantized (complex exponent) and there may be other reasons such as incorrect implementation of the quantization, incorrect adjustment of the system parameters. All in all, I really enjoyed studying and researching the above topic, as well as deepening and realizing another idea such as a neural network (even if not very complicated).

8. REFERENCES

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