

4. A communication system uses the following signal to transmit a data ‘0’

$$\begin{aligned} w_0[n] &= \cos(\pi n), 0 \leq n \leq 5 \\ &= \left\{ \underset{\uparrow}{1}, -1, 1, -1, 1, -1 \right\} \end{aligned}$$

and the following signal to transmit a data ‘1’

$$\begin{aligned} w_1[n] &= \cos\left(\frac{2}{3}\pi n\right), 0 \leq n \leq 5 \\ &= \left\{ \underset{\uparrow}{1}, -\frac{1}{2}, -\frac{1}{2}, 1, -\frac{1}{2}, -\frac{1}{2} \right\} \end{aligned}$$

Suppose that the following signal is received:

$$x[n] = \left\{ \underset{\uparrow}{1}, -\frac{3}{4}, \frac{1}{2}, \frac{3}{4}, -\frac{3}{4}, 0 \right\}$$

Given the received signal $x[n]$, which is more likely to have been transmitted, data ‘0’ or data ‘1’? To answer this question, please use the following procedure:

- (a) Compute the cross-correlation $r_{x,w_0}[n]$ between $x[n]$ and $w_0[n]$ by using a matched filter. In particular, compute $r_{x,w_0}[n] = x[n] * h_0[n]$ where

$$h_0[n] = w_0[-n] = \left\{ -1, 1, -1, 1, -1, \underset{\uparrow}{1} \right\}.$$

Do the calculation by hand, but you can use matlab to confirm your answer.

- (b) In a similar manner, find the cross-correlation $r_{x,w_1}[n]$ between $x[n]$ and $w_1[n]$.
- (c) Next, compare $r_{x,w_0}[n]$ with $r_{x,w_1}[n]$ to determine which signal is more likely. This can be done by comparing the two cross-correlation sequences and identifying which of the two has the largest maximum value. A more careful analysis tells us that it is sufficient to compare the two sequences at sample $n = 0$, i.e. the *middle* sample of the output. Thus, if $r_{x,w_0}[0] > r_{x,w_1}[0]$ you can conclude that data ‘0’ is more likely; otherwise data ‘1’ is more likely. State which is more likely: data ‘0’ or data ‘1’.