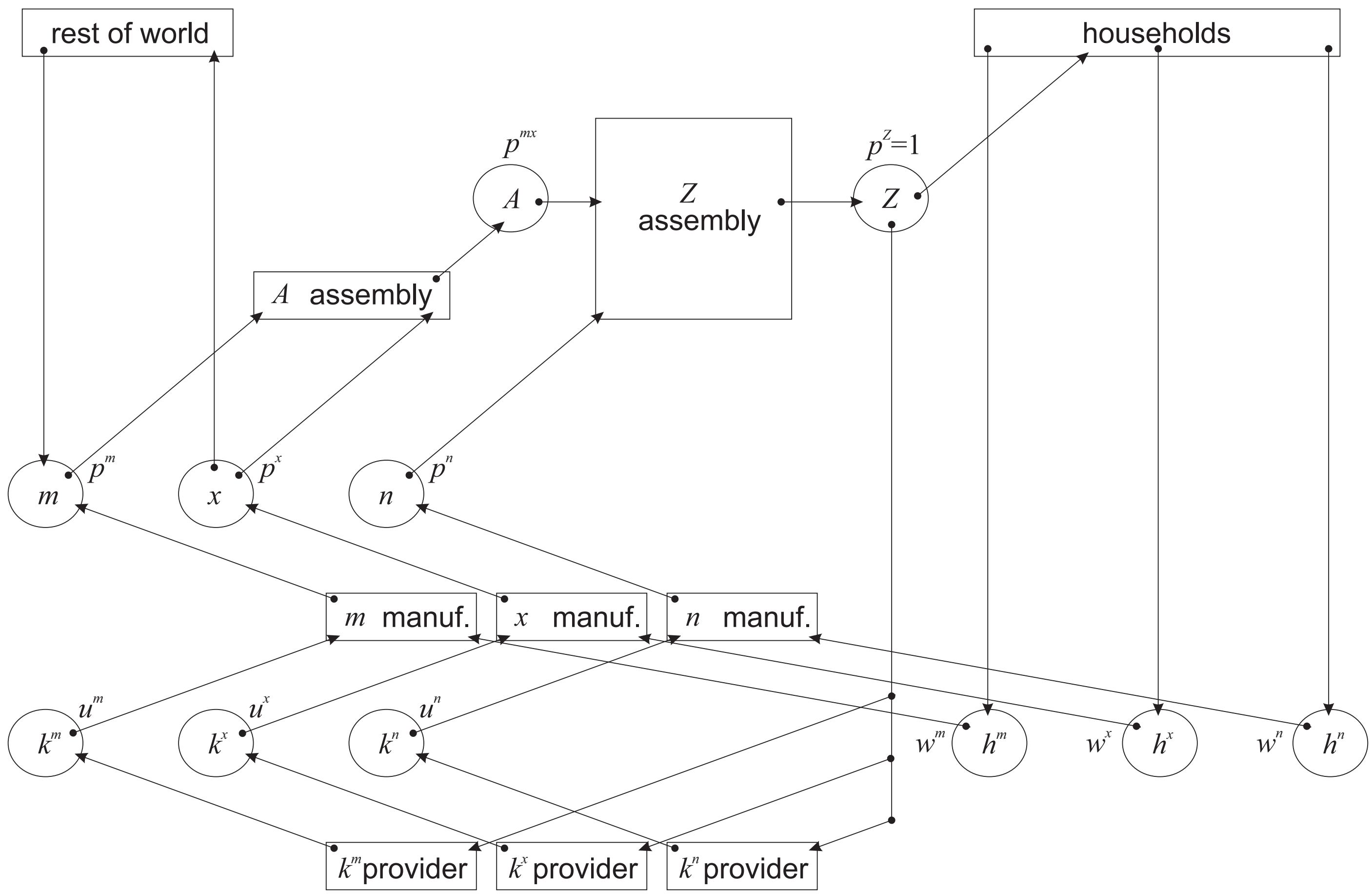
Globalization and Prosperity Lab

Computational Model: Theory

July 22, 2021

Small Open-economy MXN Model



Capital-Goods Providers: Sector Specific

- Capital-goods providers in sectors $j = m, x, n$ rent out capital at u_s^j
- Capital accumulation: $k_{s+1}^j = (1-\delta)k_s^j + i_s^j$ (so $i_s^j = k_{s+1}^j - (1-\delta)k_s^j$)
- Quadratic adjustment costs $\frac{\phi_j}{2}(k_{s+1}^j - k_s^j)^2$
- Dividend: $div_s^j = u_s^j k_s^j - i_s^j - \frac{\phi_j}{2}(k_{s+1}^j - k_s^j)^2$

$$= u_s^j k_s^j - k_{s+1}^j + (1-\delta)k_s^j - \frac{\phi_j}{2}(k_{s+1}^j - k_s^j)^2$$

Capital-Goods Providers: Optimum

- Bellman: Value function $V(k_s^j)$ (with $V_s^j \equiv V(k_{s+1}^j)/(1+r_s)$)

$$V(k_s^j) = \max_{k_{s+1}^j} \left[u_s^j k_s^j - k_{s+1}^j + (1-\delta) k_s^j - \frac{\phi_j}{2}(k_{s+1}^j - k_s^j)^2 + \frac{1}{1+r_s} V(k_{s+1}^j) \right]$$

State variable: k_s^j, control variable: k_{s+1}^j

- First-order conditions: $-1 - \phi_j(k_{s+1}^j - k_s^j) + \frac{1}{1+r_s} V'(k_{s+1}^j) = 0$

  Envelope theorem: $V'(k_s^j) = u_s^j + (1-\delta) + \phi_j(k_{s+1}^j - k_s^j)$

- Optimal capital accumulation

  $1 + \phi_j(k_{s+1}^j - k_s^j) = \frac{1}{1+r_s}\left[u_{s+1}^j + (1-\delta) + \phi_j(k_{s+2}^j - k_{s+1}^j)\right]$

Importable, Exportable, and Nontradable Goods

- Technologies $j = m, x, n$: $F^j(k^j, h^j) = A^j \left(k^j\right)^{\alpha_j} \left(h^j\right)^{1-\alpha_j}$
  ($\alpha_m = \alpha_x = \alpha_{mx}$, $A^j = 1$)

- Profits: $p_t^j F^j(k_t^j, h_t^j) - w_t^j h_t^j - u_t^j k_t^j$ (zero in optimum)

- First-order conditions: $p_t^j \alpha_j A^j \left(\kappa_t^j\right)^{-(1-\alpha_j)} = u_t^j,$ ($\kappa_t^j \equiv k_t^j / h_t^j$)
  $p_t^j (1 - \alpha_j) A^j \left(\kappa_t^j\right)^{\alpha_j} = w_t^j$

- Market clearing: $m_t = -p_t^m (y_t^m - a_t^m)$, $x_t = p_t^x (y_t^x - a_t^x)$, $y_t^n = a_t^n$
  a_t^j: intermediate inputs into next-stage assembly

Final Goods Assembly

- CES Armington aggregator of importable and exportable goods

$$A(a_t^m, a_t^x) = \left[\chi_m \left(a_t^m\right)^{1-\frac{1}{\mu_{mx}}} + (1-\chi_m)\left(a_t^x\right)^{1-\frac{1}{\mu_{mx}}}\right]^{\frac{1}{1-\frac{1}{\mu_{mx}}}}$$

with $\mu_{mx} > 0, \chi_m \in (0,1)$

- Profits: $p_t^{mx} A(a_t^m, a_t^x) - p_t^m a_t^m - p_t^x a_t^x$ (zero in optimum)
- First-order conditions imply: $\frac{a_t^x}{a_t^m} = \left(\frac{1-\chi_m}{\chi_m}\right)^{\mu_{mx}} (ToT_t)^{-\mu_{mx}}$
- Market clearing: $a_t^{mx} = A(a_t^m, a_t^x)$

with $p_t^{mx} = \left[(\chi_m)^{\mu_{mx}} \left(p_t^m\right)^{1-\mu_{mx}} + (1-\chi_m)^{\mu_{mx}} \left(p_t^x\right)^{1-\mu_{mx}}\right]^{\frac{1}{1-\mu_{mx}}}$

Final Goods Assembly

- CES Armington aggregator of traded and nontradable goods

$$Z(a_t^{mx}, a_t^n) = \left[(1-\chi_n)\left(a_t^{mx}\right)^{1-\frac{1}{\mu_n}} + \chi_n\left(a_t^n\right)^{1-\frac{1}{\mu_n}}\right]^{\frac{1}{1-\frac{1}{\mu_n}}}$$

with $\mu_n > 0, \chi_n \in (0,1)$

- Profits: $Z(a_t^{mx}, a_t^n) - p_t^{mx} a_t^{mx} - p_t^n a_t^n$ (zero in optimum)

- First-order conditions imply: $\dfrac{a_t^n}{a_t^{mx}} = \left(\dfrac{\chi_n}{1-\chi_n}\right)^{\mu_n}\left(\dfrac{p_t^n}{p_t^{mx}}\right)^{-\mu_n}$

- Market clearing: $c_t + \sum_{j=m,x,n} i_t^j + \frac{\phi_j}{2}(k_{t+1}^j - k_t^j)^2 = Z(a_t^{mx}, a_t^n)$

with $\left[(1-\chi_n)^{\mu_n}\left(p_t^{mx}\right)^{1-\mu_n} + (\chi_n)^{\mu_n}\left(p_t^n\right)^{1-\mu_n}\right]^{\frac{1}{1-\mu_n}} = 1$

Household: Total Financial Wealth

- Financial wealth Q_t at the beginning of period t consists of foreign wealth and domestic firm ownership (only capital-goods producers matter)

$$Q_t \equiv p_t^{mx} b_t + \sum_{j=m,x,n} v_{t-1}^j \theta_t^j,$$

  - b_t: foreign wealth, denominated in traded goods prices p_t^{mx}
  - θ_t^j: shares in domestic firms (sectors $j = m, x, n$)
  - v_t^j: ex-dividend share price at the end of the period. $v_{t-1}^j \theta_t^j = k_t^j$ pays for each firm's capital

- Current account: $ca_t = p_t^{mx}(b_{t+1} - b_t) = (Q_{t+1} - Q_t) - \sum_{j=m,x,n} i_t^j$

$$= (Q_{t+1} - Q_t) - \sum_{j=m,x,n} \left(v_t^j \theta_{t+1}^j - v_{t-1}^j \theta_t^j \right)$$

Household: Dynamic Programming Problem

- Bellman: $U(b_s, \{\theta_s^j\}_{j=m,x,n}) = \max_{b_{s+1}, \{\theta_{s+1}^j\}} \Big[u(c_s, \{h_s^j\}) + \beta U(b_{s+1}, \{\theta_{s+1}^j\}) \Big]$

  ($U(\cdot)$ is the value function. Consider $\{h_s^j\}$ as given for now)

- State variables: $b_s, \{\theta_s^j\}$, control variables: $b_{s+1}, \{\theta_{s+1}^j\}_{j=m,x,n}$

- Period budget constraint:

$$c_s = p_s^{mx}\Big[(1+r_s)b_s - b_{s+1}\Big] + \sum_{j=m,x,n} \Big(v_s^j(\theta_s^j - \theta_{s+1}^j) + div_s^j \theta_s^j + w_s^j h_s^j \Big)$$

Household: Dynamic Programming Problem

- First-order conditions for $j = m, x, n$

  (i) $-u_c(c_s, \{h_s^j\})\, p_s^{mx} + \beta U_b(b_{s+1}, \{\theta_{s+1}^j\}) = 0$

  (ii) $-u_c(c_s, \{h_s^j\})\, v_s + \beta U_j(b_{s+1}, \{\theta_{s+1}^j\}) = 0$

  (iii) $\lim_{T\to\infty} \beta^T u_c(c_{t+T}, \{h_{t+T}^j\})\, p_{t+T}^{mx} \cdot b_{t+T+1} \geq 0$ (transversality condition)

  (iv) $\lim_{T\to\infty} \beta^T u_c(c_{t+T}, \{h_{t+T}^j\})\, v_{t+T}^j \cdot \theta_{t+T+1}^j \geq 0$ (transversality conditions)

- Envelope theorem $j = m, x, n$

  (i) $U_b(b_s, \{\theta_s^j\}) = p_s^{mx}(1 + r_s)\, u_c(c_s, \{h_s^j\})$

  (ii) $U_j(b_s, \{\theta_s^j\}) = (v_s^j + div_s^j)\, u_c(c_s, \{h_s^j\})$

Household: Intertemporal Optimum

- Euler conditions $j = m, x, n$ Ownership

(i) $u_c(c_s, \{h_s^j\}) = \beta(1 + r_{s+1})\frac{p_{s+1}^{mx}}{p_s^{mx}}u_c(c_{s+1}, \{h_{s+1}^j\})$ $\theta_s^j = 1$

(by $v_{s-1}^j\theta_s^j = k_s^j$, $v_{s-1}^j = k_s^j$)

(ii) $v_s^j = \frac{div_{s+1}^j + v_{s+1}^j}{1+r_{s+1}}$

- Current account (income cash flow less consumption)

$$ca_s = p_s^{mx}[b_{s+1} - b_s] = r_s p_s^{mx} b_s + \sum_{j=m,x,n}\left(div_s^j + w_s^j h_s^j\right) - c_s$$

- Trade balance (output less absorption)

$$x_s - m_s = y_s - c_s - \sum_{j=m,x,n}\left(i_s^j + \frac{\phi_j}{2}(k_{s+1}^j - k_s^j)^2\right) = -r_s p_s^{mx} b_s + ca_s$$

Household: Endogenous Labor Supply

- Period utility: $u(c, \{h^j\}) = \frac{\left[c - \sum_{j=m,x,n} (h^j)^\omega / \omega\right]^{1-\sigma} - 1}{1-\sigma}$
- Bellman: Maximize over $\{h_s^j\}_{j=m,x,n}, b_{s+1}, \{\theta_{s+1}^j\}_{j=m,x,n}$
- First-order conditions with $u_c(c,h) = \left[c - \sum_{j=m,x,n} (h^j)^\omega / \omega\right]^{-\sigma}$

(i) $u_c(c_s, \{h_s^j\}) = \beta(1 + r_{s+1}) \frac{p_{s+1}^{mx}}{p_s^{mx}} u_c(c_{s+1}, \{h_{s+1}^j\})$

(ii) $v_s^j = \frac{div_{s+1}^j + v_{s+1}^j}{1 + r_{s+1}}$

(iii) $(h_s^j)^{\omega-1} = w_s^j$

(and transversality conditions)

Uncertainty

- Stochastic evolution of ToT AR(1):

$$\ln \frac{ToT_{t+1}}{ToT} = \rho \frac{ToT_t}{ToT} + \eta\, \epsilon_t.$$

Persistence ρ, standard deviation η, innovation ϵ_t (zero mean, unit var)

- Deterministic External Debt-Elastic Interest Rate (EDEIR):

$$r_t = r_0^* + \bar{p} + \psi \left(\exp\{-(b_t - \bar{b})\} - 1\right) \quad \text{and} \quad r^* = r_0^* + \bar{p}.$$

Interest rate risk premium: $\bar{p} + \psi \left(\exp\{-(b_t - \bar{b})\} - 1\right)$.

- Steady-state interest-rate premium $\bar{p}$ for poor/emerging countries

Equilibrium Conditions

- Capital: $1+\phi_j(k^j_{t+1}-k^j_t) = \frac{1}{1+r_t}\left[u^j_{t+1} + (1-\delta) + \phi_j(k^j_{t+2}-k^j_{t+1})\right]$
- Labor demand and supply: $p^j_t(1-\alpha_j)A^j_t\left(k^j_t/h^j_t\right)^{\alpha_j} = (h^j_t)^{\omega-1}$
- Euler condition: $u_c(c_t,\{h^j_t\}) = \beta(1+r_{t+1})\frac{p^{mx}_{t+1}}{p^{mx}_t}u_c(c_{t+1},\{h^j_{t+1}\})$
- Budget: $p^{mx}_t(b_{t+1}-b_t) + \sum_{j=m,x,n}(k^j_{t+1}-k^j_t)$
  $= r_t p^{mx}_t b_t + Z(\cdot) - c_t - \sum_{j=m,x,n}\left(\delta k^j_t + \frac{\phi_j}{2}(k^j_{t+1}-k^j_t)^2\right)$
- Define $k^{j,f}_t \equiv k^j_{t+1}$ to transform second-order difference equation
- **Eleven** first-order difference eqns. in **eleven unknowns** $b_t, k^j_t, k^{j,f}_t, h^j_t, c_t$

Steady-state Equilibrium Conditions

- Capital: $1 = \beta \left[p^j \alpha_j A^j \left(k^j / h^j \right)^{-(1-\alpha_j)} + (1-\delta) \right]$ and $u^j = u$
- Labor demand and supply: $(1-\alpha_j) A^j \left(k^j / h^j \right)^{\alpha_j} = (h^j)^{\omega - 1}$
- Euler condition: $1 = \beta(1 + r^*)$ and $b = \overline{b}$
- Budget: $Z(\cdot) = c + \sum_{j=m,x,n} \delta k^j - r^* p^{mx} \overline{b}$
- Interest rate risk premium is $\bar{p}$ in steady-state (for $b = \overline{b}$, given r_0^*)
- **Nine equations** in **twelve parameters** $\delta, r_0^*, \bar{p}, \beta, \alpha_j, A^j, \overline{b}, \omega$ and model with **twelve more** $\sigma, \mu_{mx}, \mu_n, \chi_m, \chi_n, ToT, \phi_j, \psi, \rho, \eta$

Calibration

- Set initial values for desired parameters.
  Select data moments to match such as σ_h, σ_i, corr$(i_t, i_{t-1}), \sigma_{tb}$ corr$(tb_t, tb_{t-1}), \sigma_y$, corr$(y_t, y_{t-1})$.

  (a) Alter parameter guesses;

  (b) Simulate model and compute model-generated moments;

  (c) Check how close model-generated moments are to data moments;

  (d) If not close, repeat from (a), else stop.

  MATLAB: `obj_to_min.m` (not provided with textbook).

Partial Calibration by Category

Category A. Set like in one-sector model: σ, δ, r_0^*, $\bar{p}$, α_{mx}, α_n, ω.
And $\beta = 1/(1 + r_0^* + \bar{p})$, $ToT = 1$, $A^m = A^n = 1$.

Category B. Get from output and trade data directly: μ_{mx}, μ_n, χ_m, χ_n, $\bar{b}$, A^x.

Category C. Get from country-by-country estimation: ρ, η.
Get from country-by-country calibration: ϕ_j, ψ.

Parameter Values Common to One-Sector Models

$\bar{\sigma}$	δ	r^*	α_{mx}	α_n	ω	ToT	$A^m = A^n$	β
2	0.1	0.11	0.35	0.25	1.455	1	1	$1/(1+r^*)$

Source: Uribe/Schmitt-Grohé (2017, Ch. 8).

- Time unit one year
- Parameter values chosen partly for comparability
- Sector-specific capital shares α_{mx}, α_n from labor share data

Parameter Values Based on Sectoral Output and Trade Data

μ_{mx}	μ_n	χ_m	χ_n	$\bar{b}$	A^x
1	0.5	0.8980	0.5640	-0.0078	$A^m = A^n = 1$

Source: Uribe/Schmitt-Grohé (2017, Ch. 8).

- μ_{mx}: $\mu_{mx} < 1$ estimates at quarterly frequency, $\mu_{mx} > 1$ estimates at 5-10 year horizon. At annual frequency $\mu_{mx} = 1$ reasonable. Literature survey (by Ö. Akinci 2011): atemporal $\mu_n = 1/2$ plausible.

- χ_m, χ_n, $\bar{b}$: match averages across countries of share of the trade balance in GDP $s_{tb} = 0.01$, share of exports in GDP $s_x = 0.20$, and share of services in GDP $s_n = 0.50$; ratio of export to import sector size $(p^m y^m)/(p^x y^x) = 1$ (so x production share 0.25).

Non-steady-state Parameter Values Estimated or Calibrated

	ρ	η	ϕ_m	ϕ_x	ϕ_n	ψ
Median	0.53	0.09	21.2	0.8	13.4	0.9

Source: Uribe/Schmitt-Grohé (2017, Ch. 7 and 8). *Note*: ρ, η medians across 51 poor and emerging countries over the period 1980-2011 (Uribe/Schmitt-Grohé (2017, Ch. 7). ϕ_m,ϕ_x,ϕ_n,ψ medians across 38 poor and emerging countries over the period 1980-2011 (Uribe/Schmitt-Grohé (2017, Ch. 8).

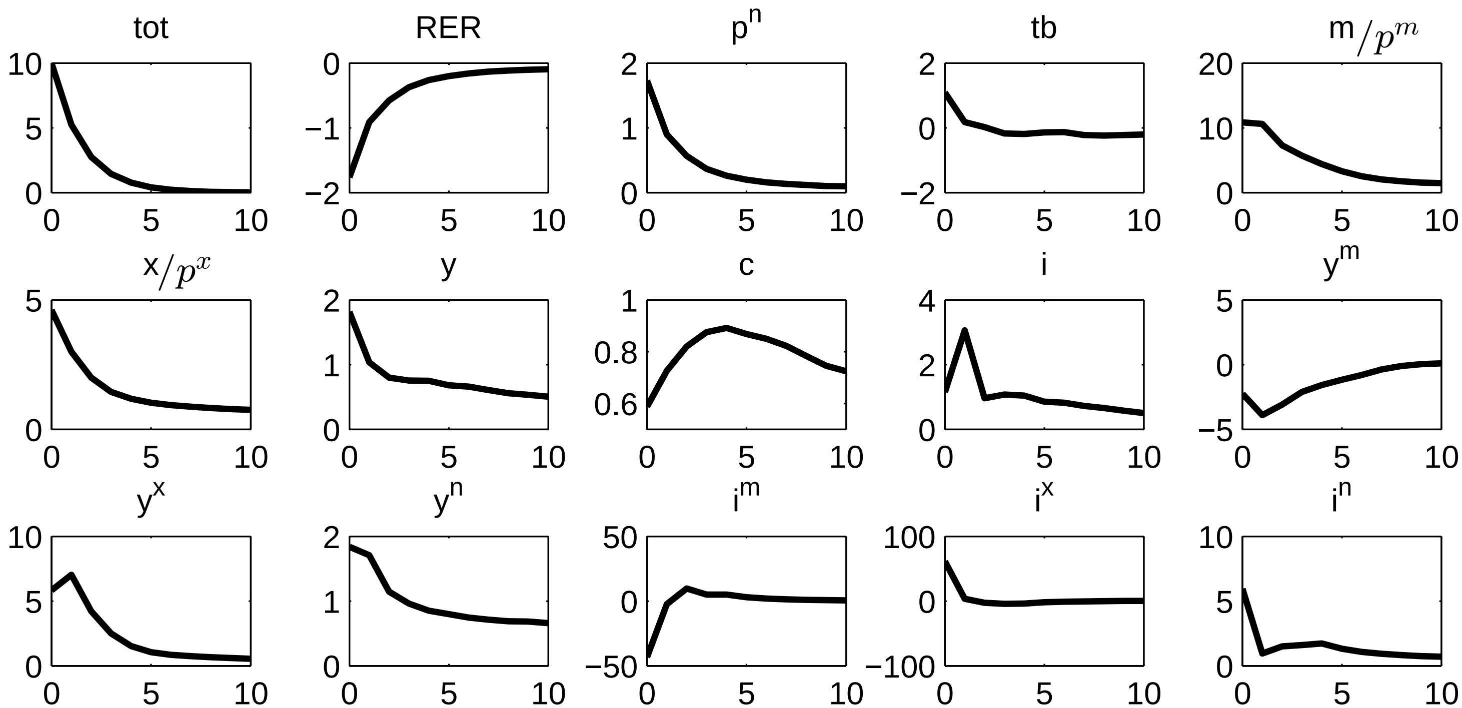
- ρ,η: estimated by OLS using country-level terms-of-trade data (WDI).

- ϕ_m,ϕ_x,ϕ_n,ψ: calibrated by requiring model to match investment-output volatility ratio and trade-balance-output volatility ratio for each of 38 poor/emerging countries 1980-2011; ratio of tradable sector to nontrabale sector investment volatility 1.5 in OECD.

Key Features for Importance of ToT Shocks

- ρ: more persistent ToT shock makes output more volatile.
- $s_n = 1/2$: larger nontraded sector reduces output effect of ToT shock.
- $s_x = 1/5$: larger export share augments output effect of ToT shock.

Impulse Responses to ToT Shock in SOE-MXN Model



Source: Uribe and Schmitt-Grohé (2017).

Note: Impulse responses to a 10-percent terms-of-trade shock computed as the median impulse response across countries period by period from the simulated SOE-MXN model. All variables except for trade balance and external debt expressed in percent deviations from steady state. The trade balance and external debt expressed in level deviations from steady state in percent of steady-state output.

Run-Down: Preceding Graph

- Positive terms-of-trade shock leads to appreciation of real exchange rate as relative price of non-tradeable goods increases
- Positive terms-of-trade shock reduces output in import-competing sector, raises output in export and non-traded sectors at impact, and raises overall output
- The simultaneous increase in output and prices of the non-traded goods indicates that additional demand leads to an expansion
- Marginal revenue product (and capital returns) in export and non-traded sectors rise at impact, so that total investment increases. The investment response is delayed, similar to the SVAR-RER model (though there is no dip at impact)
- Consumption increases less than proportionally compared to output over time
- At impact, the import response (in value) is weaker than the export expansion so that the trade balance *increases*

Interpretation of Model Responses to ToT Shock

- Supply side: Exportable production expands, importable and non-tradable production shrinks (given p^n)
- Demand side: Importable and nontradable demand rises, domestic exportable demand falls. ToT up raise overall demand (wealth effect)
- RER appreciates, p^n rises
- Exports and imports increase, with positive net effect (consistent with Harberger-Laursen-Metzler (HLM) effect)
- Aggregate investment increases less than proportionally on impact, with exportable investment rising, importable investment declining

Share of Variance Explained by ToT Shock in SOE, SVAR models

Variable	SOE-MX	SOE-MXN	SVAR-RER
Terms of Trade	100	100	100
Trade Balance	27	17	12
Output	18	14	10
Consumption	24	17	9
Investment	20	5	10
Real Exchange Rate		1	14

Source: Uribe/Schmitt-Grohé (2017, Ch. 8).
Note: Shares expressed as percentages. Cross-country median of fraction of variance explained by ToT shock. For SOE-MX and SOE-MXN models, numerator is variance conditional on ToT shock predicted by model for country-specific calibrations. Denominator is unconditional variance from SVAR model.

- SOE-MXN model similar to SVAR for output and investment variance. ToT shocks less important in data than in theory for TB, consumption.

Summary

- How do ToT affect trade balance among emerging countries?
  - In empirical SVAR-RER model, positively—reminiscent of HLM
  - In calibrated SOE-MXN model also positively
- How much do ToT shocks matter among emerging countries?
  - In SVAR-RER model, they explain 10 percent of variation
  - In calibrated SOE-MXN model similar for output, investment
- How important is measurement for variation?

Conclusion

- Important role of domestic technology shocks for SOEs.
  Modest role of ToT shocks (relative foreign-to-home technology shocks).

- Reasonable success of SOE-RBC and SOE-MXN models, capturing variation in data and impulse-responses of SVAR for median country.

- ToT crude summary measure of global shocks.
  Specific industries matter.

- Success of SOE models for individual countries up for investigation.

BACKUP

Derivation of Unit Cost of Composite Final Good

- Minimize cost for $\underline{Z}$ units of output:

$$\min_{a_t^{mx}, a_t^n} p_t^{mx} a_t^{mx} + p_t^n a_t^n \quad \text{s.t.} \quad Z(a_t^{mx}, a_t^n) \geq \underline{Z}$$

- First-order conditions: $a_t^k = (\chi_k)^{\mu_n} \left(\frac{p_t^k}{\xi_t}\right)^{-\mu_n} \underline{Z}$
  ($k = mx, n$; $\chi_{mx} \equiv 1 - \chi_n$)

- Lagrange multiplier ξ_t: Additional expenditure for additional output. Use optimal a_t^k in $Z(a_t^{mx}, a_t^n) = \underline{Z}$ to find

$$\xi_t = \left[\sum_{k=mx,n} (\chi_k)^{\mu_n} \left(p_t^k\right)^{1-\mu_n}\right]^{\frac{1}{1-\mu_n}} \equiv p_t^c \ (= 1).$$

Model Consistent Real GDP

- Ratio of nominal GDP to price level: real GDP$_t = \frac{\sum_{j=m,x,n} P_t^j y_t^j}{P_t}$

  Paasche price index(WDI): $P_t = \frac{\sum_{j=m,x,n} P_t^j y_t^j}{\sum_{j=m,x,n} P_0^j y_t^j}$ ("GDP at constant LCU")

- Therefore real GDP$_t = \sum_{j=m,x,n} P_0^j y_t^j$

- Replace Paasche base year with steady state prices:

  real GDP$_t = \sum_{j=m,x,n} p^j y_t^j$

- Model uses prices $p^j \equiv P^j / P^C$ relative to consumption good, so all variables require scaling P^C / P_t

Ratio of Variances Predicted by SOE-MXN Model

	y	c	i
$Var\left(x^{CPI}\right)/Var\left(x^{Paasche}\right)$	2.4	1.7	1.2

Source: Uribe/Schmitt-Grohé (2017, Ch. 8). *Note*: Proportions expressed as factors. Cross-country median of fraction of variance explained by ToT shock in SOE-MXN models, numerator is variance using units of current consumption (x^{CPI}) and denominator is variance using units of output (Paasche price deflator $x^{Paasche}$).

- ToT-shock predicted variances of output, consumption and investment larger when macro variables expressed in units of consumption as opposed to units of output.