International Trade: Lecture 11

The Heckscher-Ohlin Model and the 4 Big Theorems of Modern Trade (ii)

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Last Class: Stolper-Samuelson Theorem

- Statement:
  If the relative price of one good increases, the ***real income*** *of the factor that is used intensively in production of the good will increase, while the other factor's real income falls.*

- Magnification effect:

$$\hat{w}_i > \hat{P}_C > \hat{P}_T > \hat{r}_i$$

(where $\hat{x} = (x' - x)/x$ is the percent change in variable x)

Putting everything together...

- We first mapped $w_i / r_i \mapsto K_{i,g} / L_{i,g}$:

$$\frac{K_{i,C}}{L_{i,C}} = \frac{\beta_C}{1-\beta_C} \times \frac{w_i}{r_i}, \qquad \frac{K_{i,T}}{L_{i,T}} = \frac{\beta_T}{1-\beta_T} \times \frac{w_i}{r_i}$$

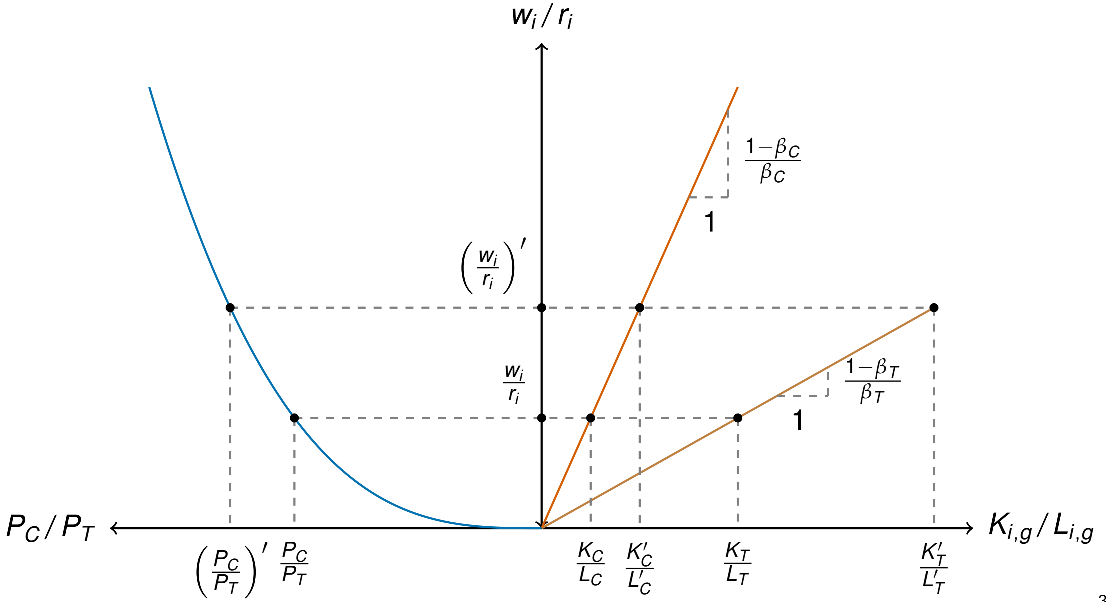
- We then mapped $P_C / P_T \mapsto w_i / r_i$:

$$\frac{w_i}{r_i} = \left(\frac{1}{\text{constant}} \times \frac{P_C}{P_T} \right)^{\frac{1}{\beta_T - \beta_C}}$$

- Putting these together, we can pin down $K_{i,g} / L_{i,g}$ from relative prices:

$$\frac{K_{i,g}}{L_{i,g}} = \text{constant}_g \times \left(\frac{P_C}{P_T} \right)^{\frac{1}{\beta_T - \beta_C}}$$

Signature chart



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- Question: How can both sectors raise the relative use of capital?
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- If you are unconvinced, that is fair
- I will try to convince otherwise first with algebra, then with charts

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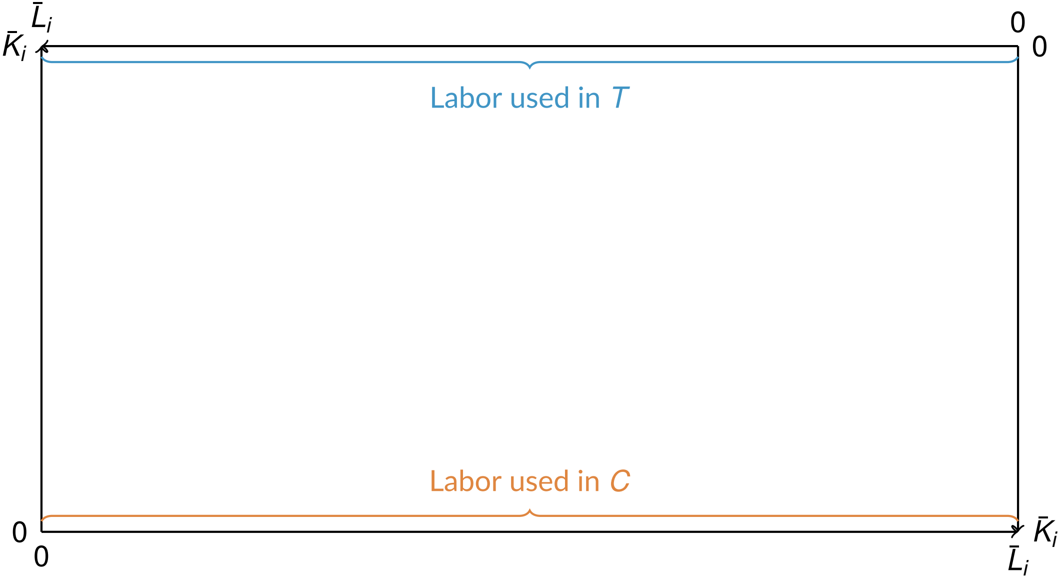
- Since $\frac{\bar{K}_i}{\bar{L}_i}$ is fixed and $\frac{K_{i,T}}{L_{i,T}} > \frac{K_{i,C}}{L_{i,C}}$, if both sectors increase their capital-to-labor ratio...

- ...then $\ell_{i,C}$ must increase to put a higher weight in the least capital intensive industry.

The Edgeworth Box

The Edgeworth Box



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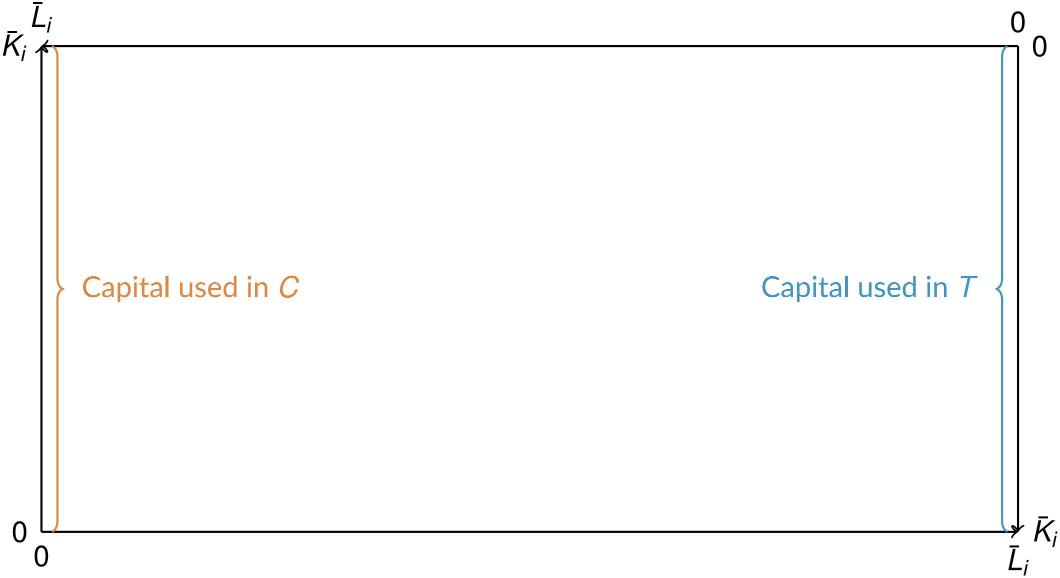

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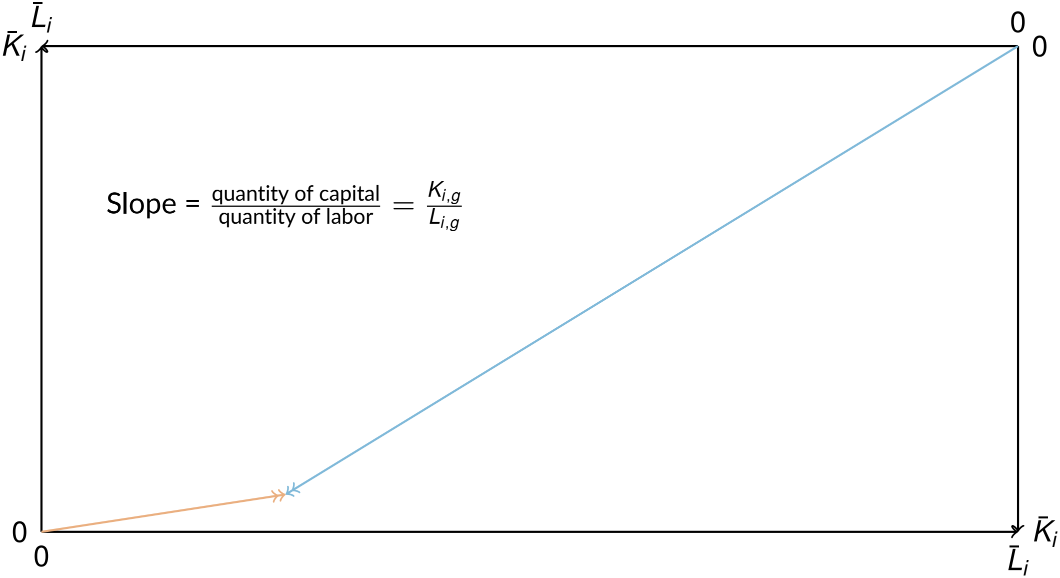

Slope = $\frac{\text{quantity of capital}}{\text{quantity of labor}} = \frac{K_{i,g}}{L_{i,g}}$

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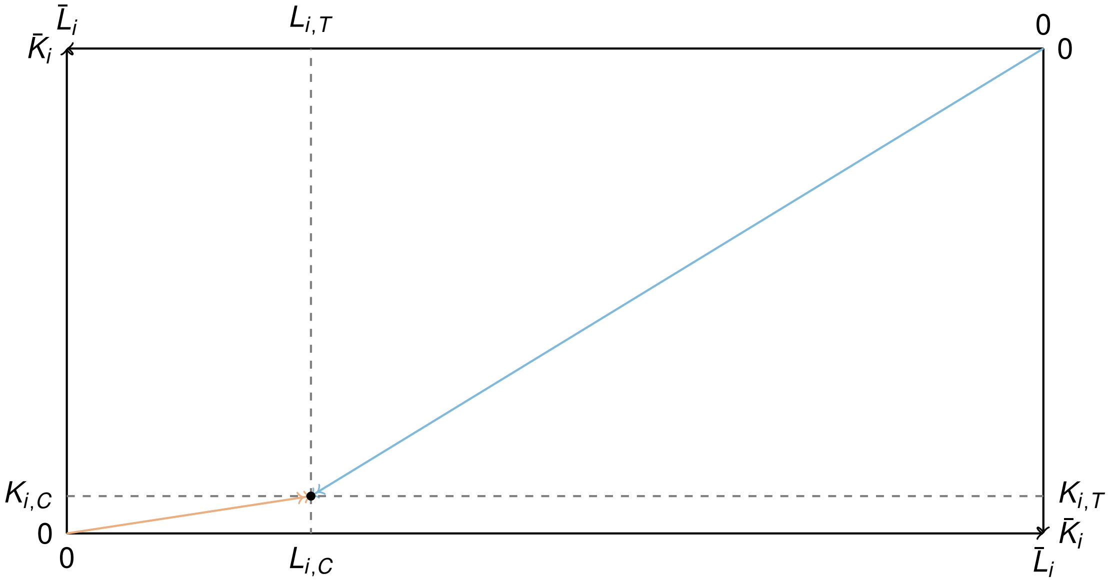


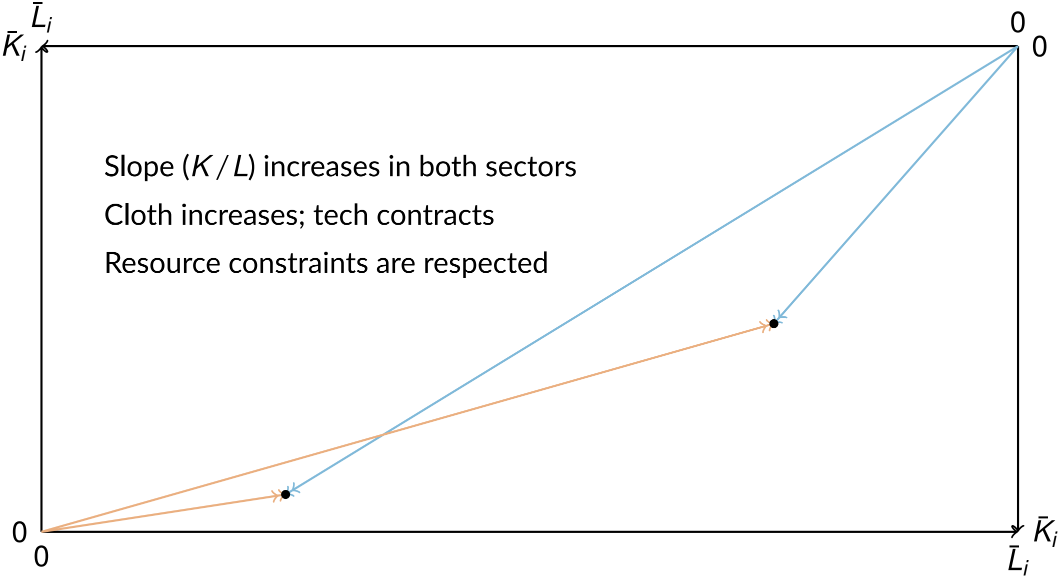
The Edgeworth Box

- Initial equilibrium shows a smaller cloth sector relative to tech.

- Slopes of the red and blue arrows denote the capital-to-labor ratios
  (remember: divide the rise – capital input – over run – labor input)

- Cloth sector uses very capital relative to labor (the slope is very flat) while the tech sector is, as expected, relatively more capital intensive (the slope is steeper)

The Edgeworth Box



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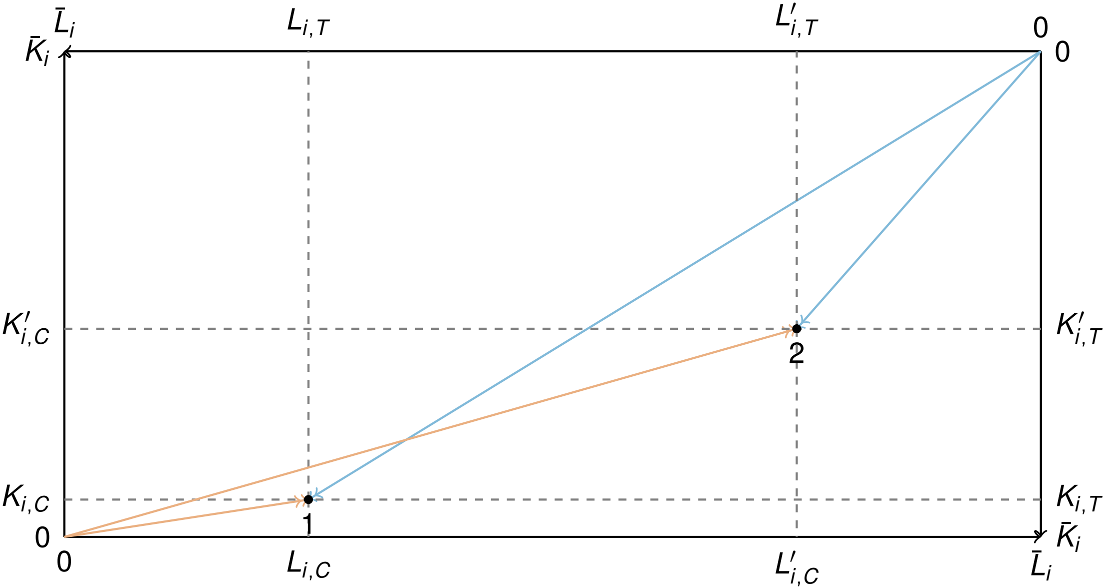

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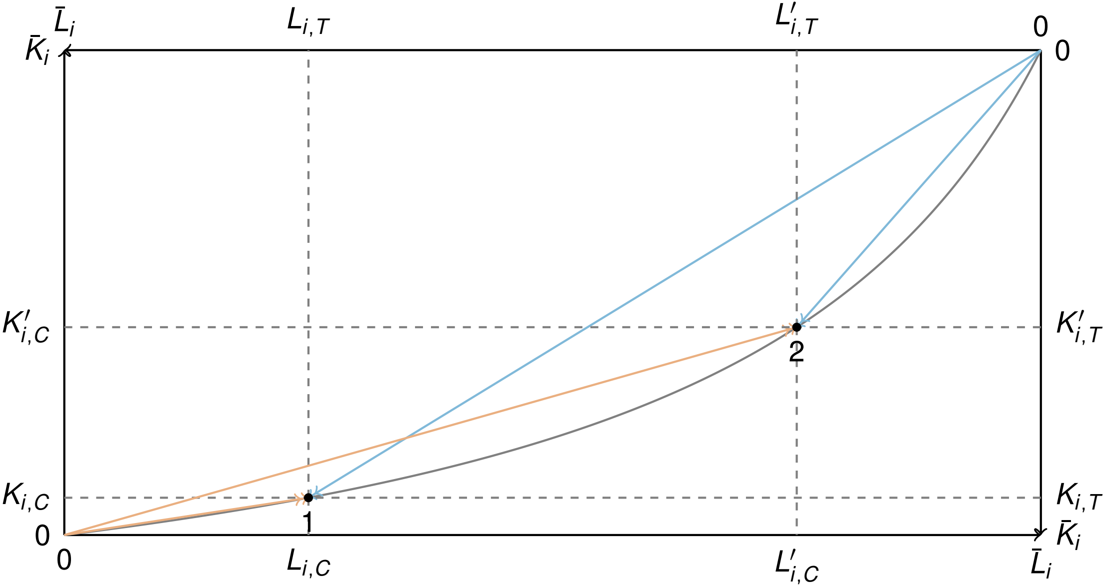

The Edgeworth Box

- Once prices P_C / P_T increase, the equilibrium moves to point 2.
- Cloth sector expands and the tech sector contracts, so the red arrow becomes longer and the blue arrow becomes shorter.
- Both arrows become steeper $\rightarrow$ both industries are more capital intensive
- Gray curve is called the *contract curve*.
- Denotes every possible equilibrium with choices of $\{L_{i,C}, K_{i,C}, L_{i,T}, K_{i,T}\}$

Rybczynski Theorem

- Statement:
  With both goods produced and fixed relative goods prices, an increase in the endowment of the factor used intensively in a given sector raises the output of that sector and decreases the output of the other sector

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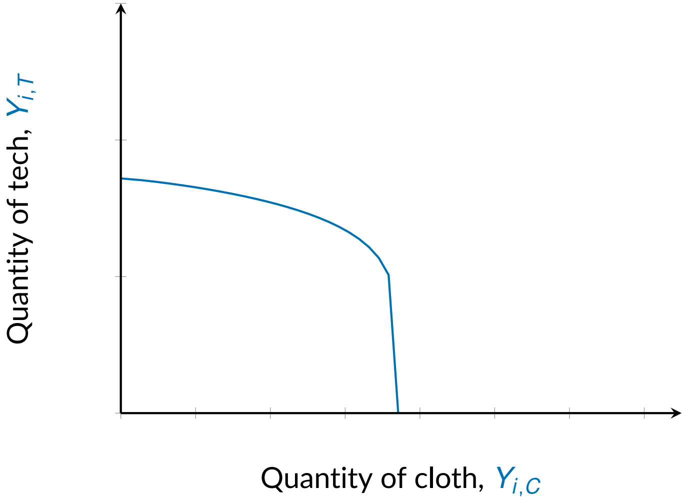
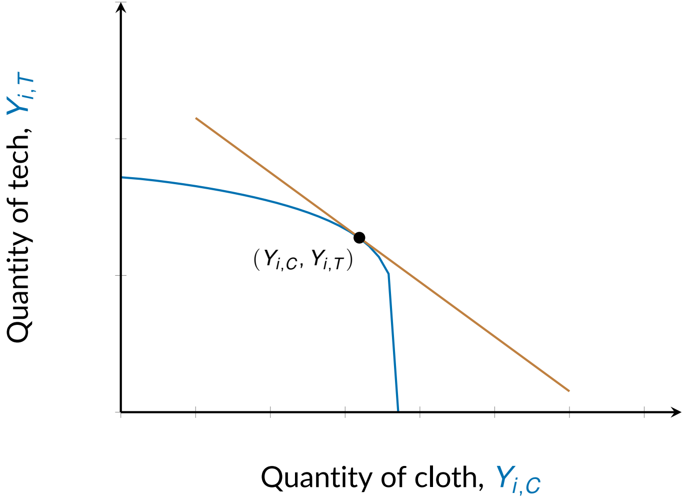
- Note that since relative goods prices P_C / P_T are assumed to be fixed, w_i / r_i also fixed
  (the latter are pinned down by the former)

Rybczynski Theorem: Graphical Representation



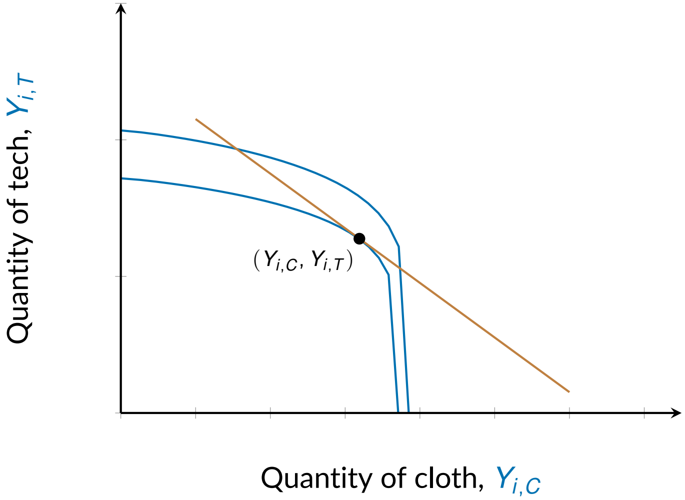
Rybczynski Theorem: Graphical Representation

Optimal production: $|P_C / P_T| = |MRT|$



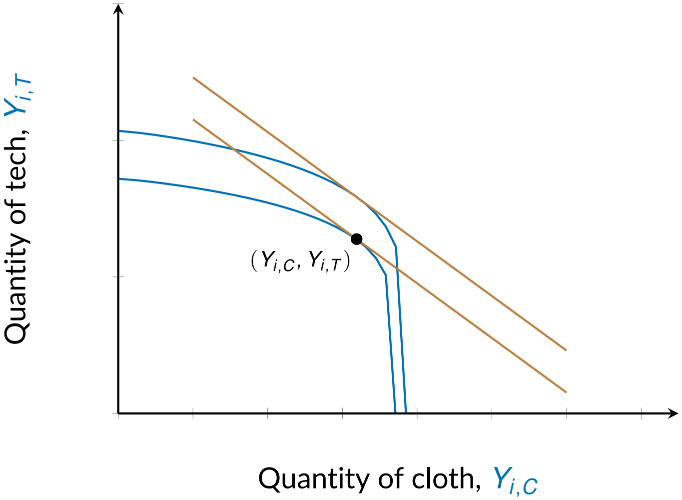
Rybczynski Theorem: Graphical Representation

Expansion in $\bar{K}_i' > \bar{K}_i$: potential T increases more than C (why?)



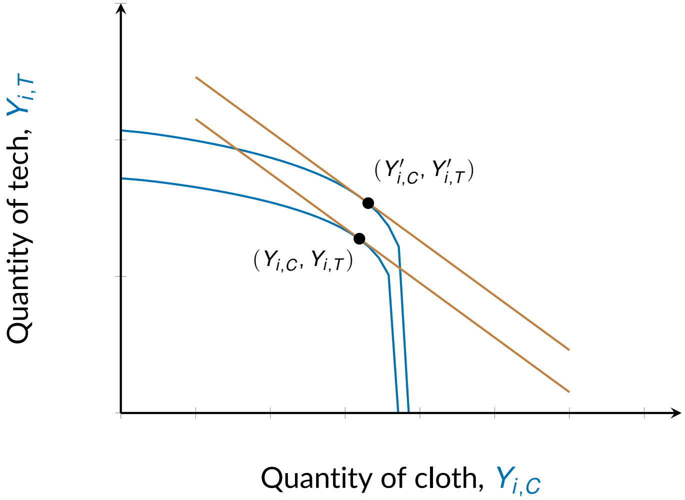
Rybczynski Theorem: Graphical Representation

Fixed P_C / P_T: parallel price lines



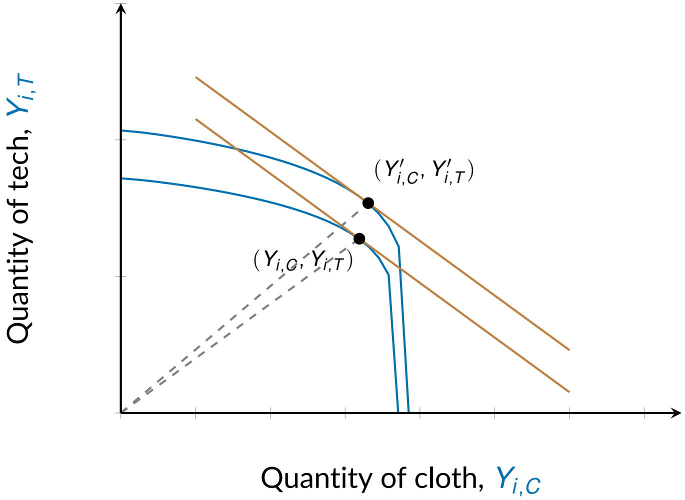
Rybczynski Theorem: Graphical Representation

New production: tangent between prices and new PPF



Rybczynski Theorem: Graphical Representation

New production point: tech grows faster than cloth (compare rays)



Rybczynski Theorem: Algebra

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\end{aligned}
$$

- Let $b_g \equiv \frac{\beta_g}{1-\beta_g}$. Solving for $\ell_{i,C}$:

$$
\ell_{i,C} = \frac{r_i / w_i \times \bar{L}_i \times b_T - \bar{K}_i}{r_i / w_i \times \bar{L}_i \times (b_T - b_C)}
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- Let $b_g \equiv \frac{\beta_g}{1-\beta_g}$. Solving for $\ell_{i,C}$:

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$$

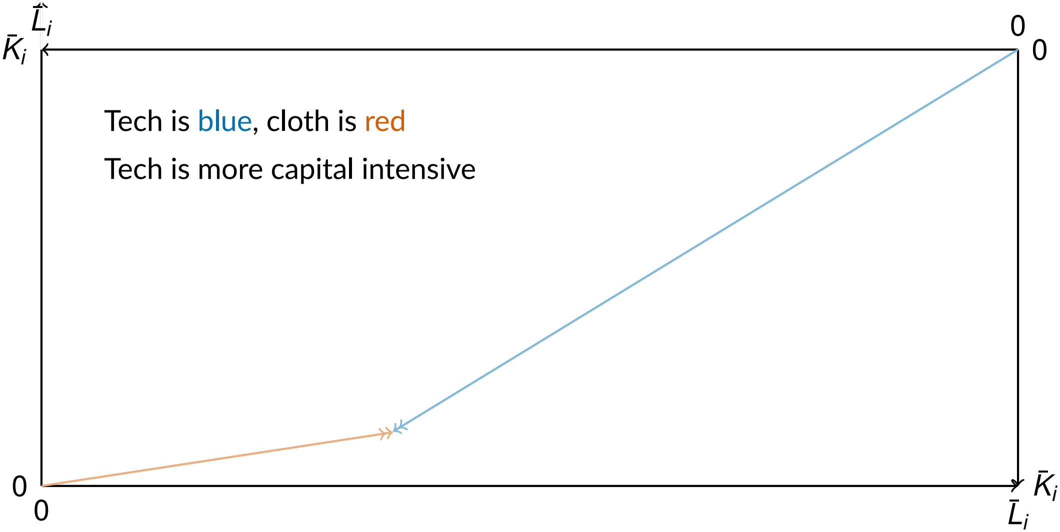
- What happens to $\ell_{i,C}$ if $\bar{K}_i$ increases?

$$
\left.\frac{\partial \ell_{i,C}}{\partial \bar{K}_i}\right|_{w_i/r_i} = -\frac{1}{r_i/w_i \times \bar{L}_i \times (b_T - b_C)} < 0
$$

Rybczynski Theorem: Predictions

- Suppose $\{P_C/P_T, w_i/r_i, K_{i,C}/L_{i,C}, K_{i,T}/L_{i,T}\}$ are all fixed
- If $\bar{K}_i$ ($\bar{L}_i$) increases, then:
  - The tech (cloth) sector expands, since it is capital (labor) intensive
  - The cloth (tech) sector contracts, since it is capital (labor) intensive
- Is this even feasible?
- Let us use an Edgewoth Box...

The Edgeworth Box



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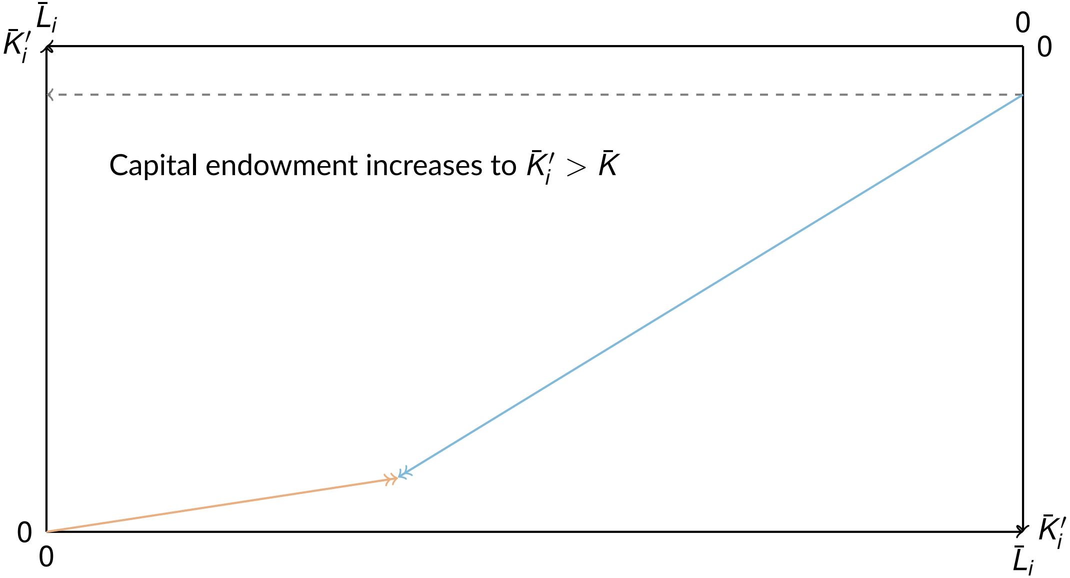

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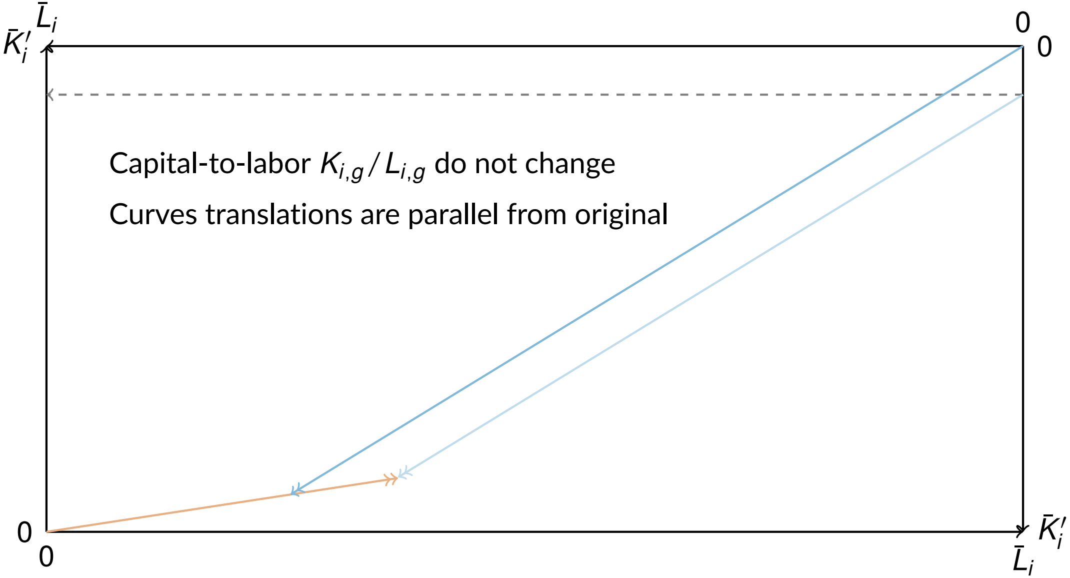

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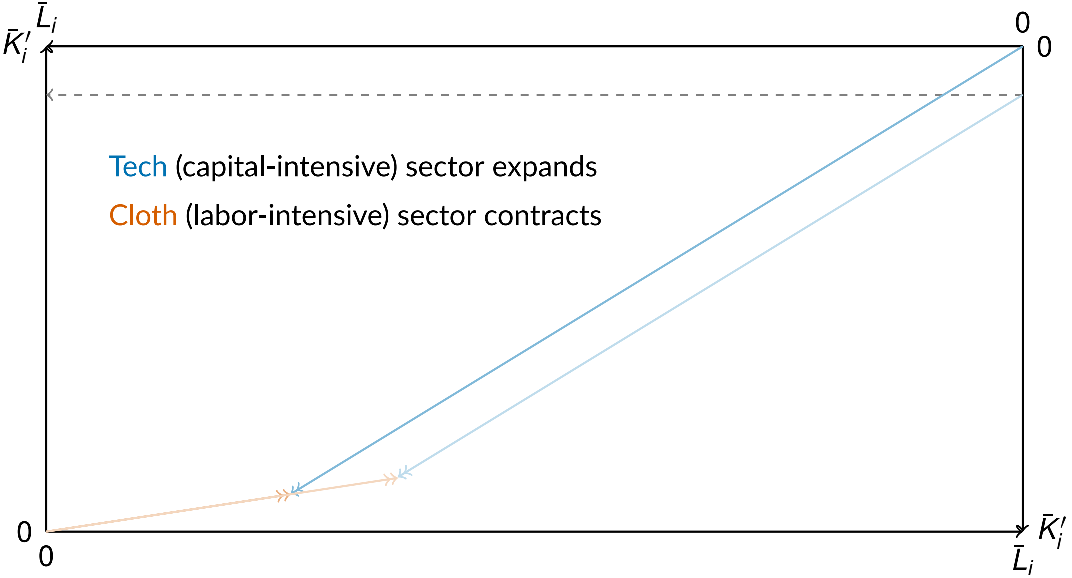

Demand

- For now, we only spoke about the production side of the economy
- Have taken prices as exogenous or only talked about an equilibrium in passing
- To have a full description of this economy, have to define preferences
- Luckily, they look familiar:

$$\max_{\{Q_{i,C}, Q_{i,T}\}} U_i(Q_{i,C}, Q_{i,T}) \equiv Q_{i,C}^{\alpha} Q_{i,T}^{1-\alpha} \qquad s.t. \qquad P_C Q_{i,C} + P_T Q_{i,T} = I_i = w_i \bar{L}_i + r_i \bar{K}_i$$

- With optimal demand functions:

$$Q_{i,C} = \alpha \frac{I_i}{P_C}, \qquad Q_{i,T} = (1-\alpha) \frac{I_i}{P_T}$$

Hecksher-Ohlin Theorem

- Statement:
  In a world economy of countries that are identical in their preferences and technologies but different in their endowments, countries will export goods that intensively utilize their relatively abundant factors of production and import goods that require intensive use of factors they possess in relative scarcity.

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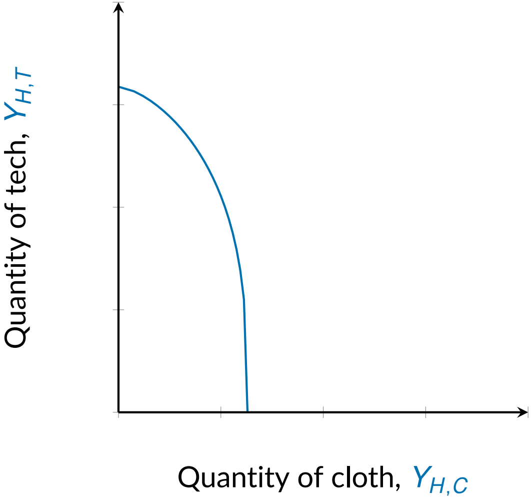
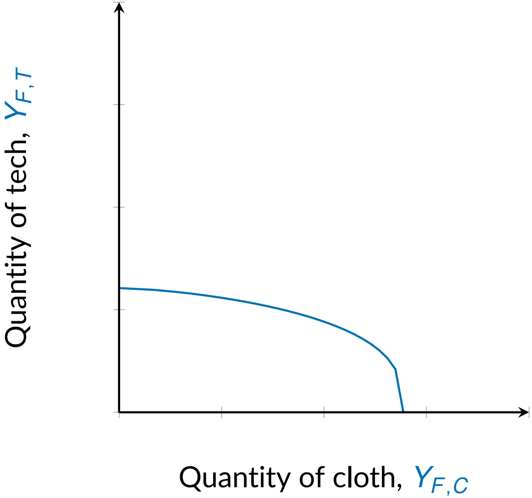
- Intuition:
  - From Rybzynski Theorem: since the countries are otherwise identical, for identical prices P_C / P_F, home produces more of tech relative to cloth (and vice versa)
  - Preferences (as controlled by α) are identical, consumers will want to consume identical shares of their income on each good
  - Corollary: capital intensive country H will export the capital intensive good T while the labor intensive country F will export the labor intensive good C.

Graphical representation





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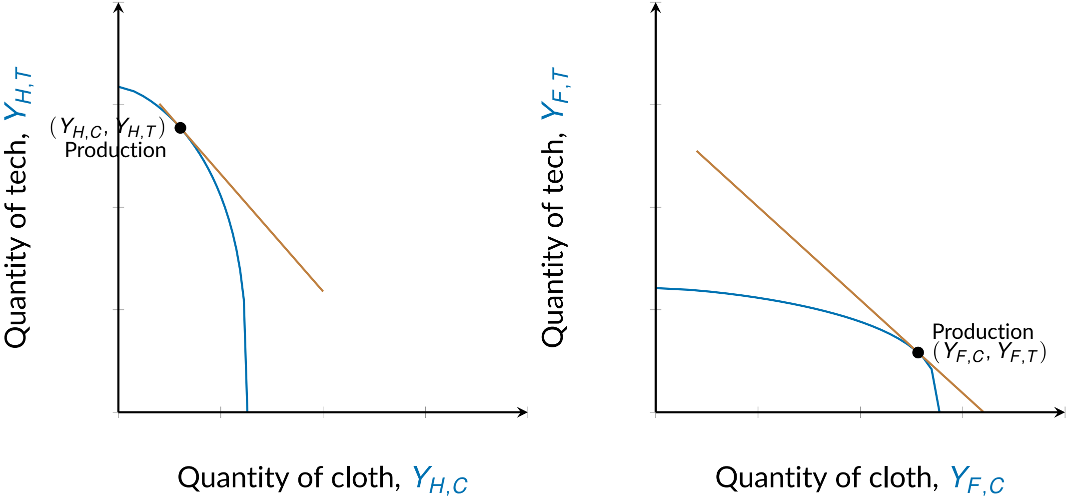

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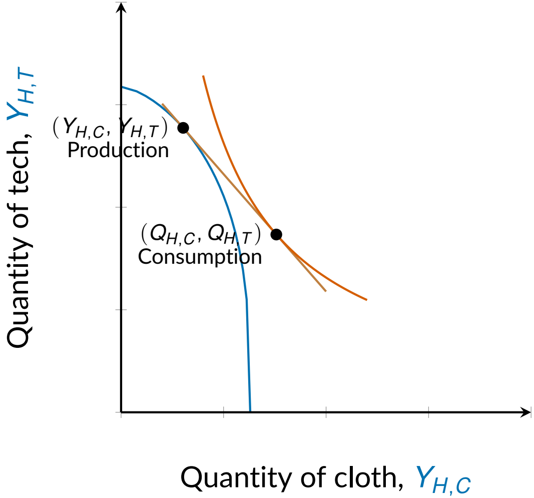

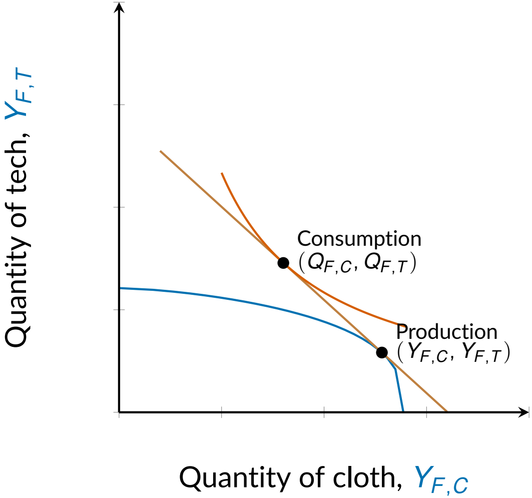

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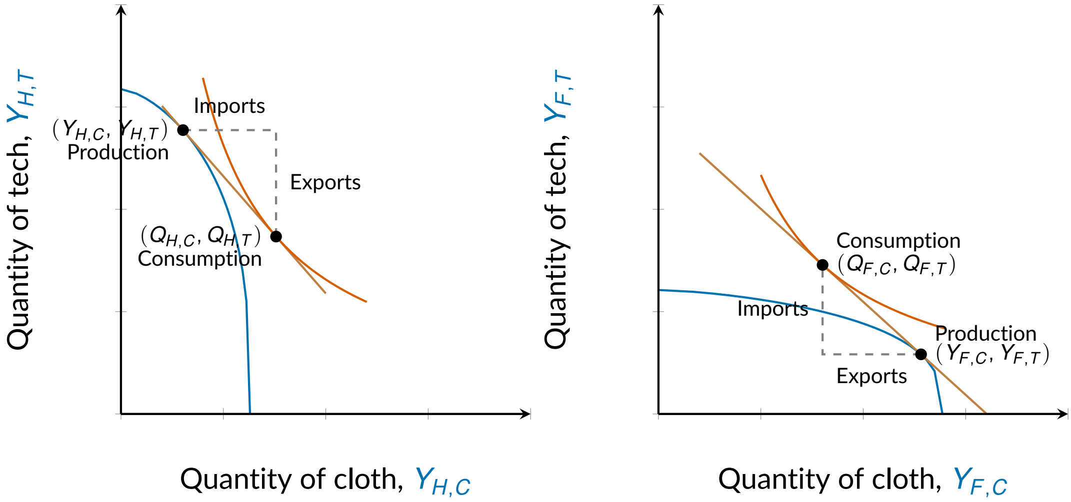

Factor equalization theorem

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- We saw w_i / r_i can be pinned from relative prices P_C / P_T

$$\frac{P_C}{P_T} = \frac{(1-\beta_T)^{1-\beta_T} \beta_T^{\beta_T}}{(1-\beta_C)^{1-\beta_C} \beta_C^{\beta_C}} \times \left(\frac{w_i}{r_i} \right)^{\beta_T - \beta_C}$$

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- Then, relative factor prices equalize under free trade:

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- Is this realistic? What are some limits of this model?

Conclusion and discussion