Econ 110A: Lecture 8

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What are the takeaways from the Solow model?

- Determinants of long-run output per-capita: investment (saving) rate and TFP.
- TFP differences still main factor in per-capita income differences across countries
- Transition Dynamics helps understand differences in growth rates across countries
- It does NOT explain sustained long-run growth
- Differences in investment rates, TFP also not explained

What determines the Investment Rate?

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Should Ford build a new plant?

Ford Plant in Dearborn, MI

What determines the Investment Rate?

- Highly complex decision, many factors involved
- Our approach: use the principle of No-Arbitrage

"At market equilibrium, any two active investments must yield the same return."

No-Arbitrage Equation for Investment

Consider a firm thinking of investing in asset (think of it as a big machine) $\$P_{K,t}$ today.

The firm has two options:

- Deposit in a bank the dollar equivalent of $\$P_{K,t}$ in a bank today and earn the returns; or
- Buy the asset, rent it out, earn ($\bar{r}$), incur in depreciation ($\bar{d}$). Furthermore, the machine might change in price between today and tomorrow, so we need to account for the fact that in the change in returns $\$P_{K,t+1} - \$P_{K,t}$.

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By non-arbitrage, these two are equal (we will debate more why in a bit):

$$\underbrace{\$P_{K,t}(1+R) - \$P_{K,t}}_{\text{return on bank deposit}} \underbrace{=}_{\substack{\text{non-arbitrage}\\ \text{condition}}} \underbrace{\bar{r}\$P_{K,t} - \bar{d}\$P_{K,t} + \$P_{K,t+1} - \$P_{K,t}}_{\text{return on physical capital}}$$

Non-Arbitrage Equation for Investment

Let us manipulate this equation a bit to simplify our non-arbitrage condition.

$$\$P_{K,t}(1+R) - \$P_{K,t} = \bar{r}\$P_{K,t} - \bar{d}\$P_{K,t} + \$P_{K,t+1} - \$P_{K,t}$$

Let $\frac{\$P_{K,t+1} - \$P_{K,t}}{\$P_{K,t}} \equiv \frac{\Delta\$P_{K,t+1}}{\$P_{K,t}}$. Then, diving through by $\$P_{K,t}$:

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$$R = \bar{r} - \bar{d} + \Delta\$P_{K,t+1} \iff \bar{r} \equiv MPK = \underbrace{R + \bar{d} - \frac{\Delta\$P_{K,t+1}}{\$P_{K,t}}}_{\text{user cost of capital}}$$

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- R: opportunity cost of funds
- $\bar{d}$: depreciation cost
- $\frac{\Delta \$P_{K,t+1}}{\$P_{K,t}}$: capital gain (+) or loss (-)

What is the user cost of capital

Intuition:

Minimum return necessary to justify a given investment rather than putting it in the bank; or

Estimate of cost of increasing the firm's capital stock in one unit if the firm **owns** capital (marginal cost of capital) rather than rents it out in the market!

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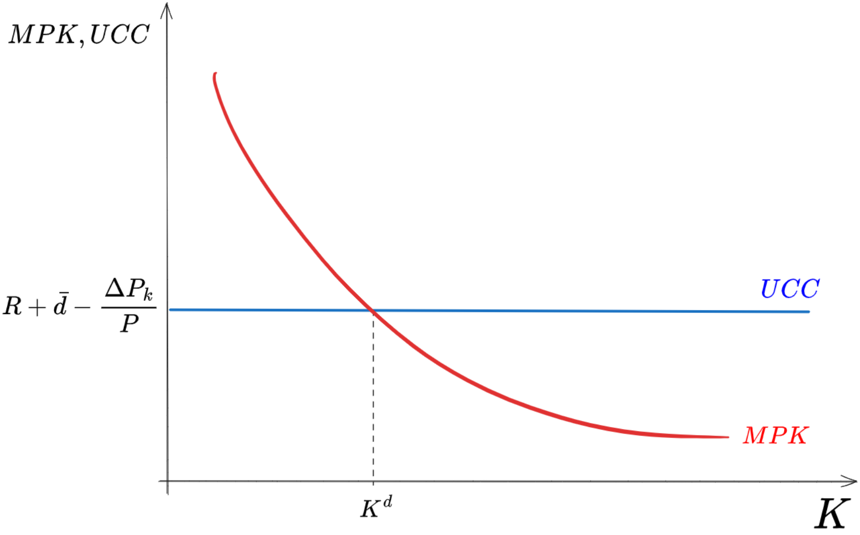
How does this determine investment?



$$K_{t+1} = K^d = K_t - \bar{d}K_t + I_t \implies I_t = K^d - (1 - \bar{d})K_t$$

Numerical example

Part C (15 pts): The User Cost of Capital and Investment

Tesla Inc. is considering how much to invest into physical capital for the next period. The company current physical capital is $K_t = 10$, which will be used in production at time *t*, and will depreciate at the estimated rate of $\bar{d} = 15\%$. The price of *unused* physical capital is expected to change at the rate of $\frac{\Delta p_K}{p_K}$ =5% between *t* and *t+1*, and the current financial interest rate at which the company could alternatively invest between the current and the next period is $R = 3\%$. The tax rate τ on returns to capital are assumed to be zero. The capital accumulation equation is $K_{t+1} = (1 - \bar{d})K_t + I_t$.

C.1 (8 pts) Please provide an expression for the user cost of capital as defined in class. For each term please explain what it captures. Next, use the expression to compute the user cost of capital for Tesla Inc. using the formula you provided.

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UCC &= 3\% + 15\% - 5\% = 13\%
\end{aligned}
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Numerical example

C.2 (7 pts) Suppose that Tesla Inc. produces output according to the production function $Y_t = K_t^{0.5}$ for any t. Assume that Tesla would like to invest in physical capital at time t until the marginal product of capital *in the next period, t+1,* is equal to the user cost of capital you computed above. What should the investment in physical capital I_t be for Tesla? Please show all your work and explain clearly all the steps in your argument.

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$$\begin{aligned} K_{t+1} &= K_t + I_t - \bar{d}K_t = I_t + (1-\bar{d})K_t \\ \iff I_t &= K_{t+1} - (1-\bar{d})K_t \\ &= 14.8 - (1-0.15)10 \\ &= 14.8 - 8.5 = 6.3 \end{aligned}$$

Numerical example

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Numerical example

What if R increases to $R' = 5\% > R = 3\%$?

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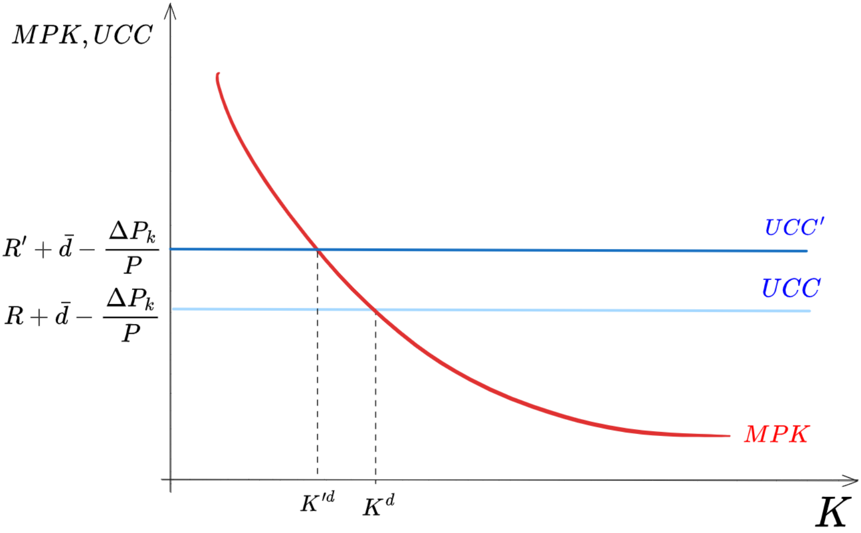
An increase in R decreases investment



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- So when the Fed increases interest rates, that depresses investment (and will eventually influence inflation, as you see in ECON110B)
- Two parts for the intuition:
  - Financial cost: it is more expensive to borrow money to finance investments
  - Opportunity cost: returns on financial investments are higher, so funds move away from real investments into financial investments up to the point that they are equal (nonarbitrage)