International Trade: Lecture 3

Classical Ricardian Trade in General Equilibrium - Part I

Carlos Góes[1]

[1]George Washington University

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Ricardian Model: Preliminaries

- 2 countries $i \in \{US, COL\}$ and 2 products $p \in \{C, R\}$
- In country i, there are L_i units of labor (worker-hours) available
- In country i, to produce one unit of good p, firms use $a_{i,p}$ units of labor
- Producers: $\max_{Y_{i,p}} P_{i,p} Y_{i,p} - w_i a_{i,p} Y_{i,p}$
- PPF: $a_{i,C} Y_{i,C} + a_{i,R} Y_{i,R} \leq L_i$
- Preferences: $Q_{i,C}^{\alpha_i} Q_{i,R}^{1-\alpha_i}$

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- It also means that we consider all markets together: the market for computers and roses affect each other...
- ... as does the market for factors of production (e.g., labor)

Production Technology

- In country i, firms producing good p maximize profits under perfect competition:

$$\max_{Y_{i,p}} \pi_{i,p} = \max_{Y_{i,p}} P_{i,p} Y_{i,p} - w_i a_{i,p} Y_{i,p}$$

- Since labor only one type of labor (mobile across sectors), there is a single wage w_i

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- Total labor endowment satisfies the PPF across sectors:

$$\underbrace{a_{i,C} \times Y_{i,C}}_{\substack{\text{labor used in} \\ \text{production of } C}} + \underbrace{a_{i,R} \times Y_{i,R}}_{\substack{\text{labor used in} \\ \text{production of } R}} \leq \underbrace{L_i}_{\substack{\text{total labor} \\ \text{available in } i}}$$

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$$P_{i,p} = w_i a_{i,p} \iff \frac{P_{i,p}}{a_{i,p}} = w_i \qquad \text{for } p \in \{C, R\}$$

Preferences

- In country i, consumers preferences over products p, represented by a utility function.

$$U_i(Q_C, Q_R) \equiv Q_C^{\alpha_i} Q_R^{1-\alpha_i}, \qquad \text{for } 0 < \alpha_i < 1$$

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- Consumers take prices $P_{i,R}$, $P_{i,C}$ as given and maximize:

$$\max_{\{Q_{i,C}, Q_{i,R}\}} U_i(Q_{i,C}, Q_{i,R}) \equiv Q_{i,C}^{\alpha_i} Q_{i,R}^{1-\alpha_i} \qquad s.t. \qquad P_{i,C}Q_{i,C} + P_{i,R}Q_{i,R} = w_i L_i$$

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- How to solve this?

Preferences

Preferences

- Replace in objective function and solve unconstrained max problem:

$$\max_{\{Q_{i,C}\}} Q_C^{\alpha_i} \left(\frac{w_i L_i}{P_{i,R}} - \frac{P_{i,C}}{P_{i,R}} Q_{i,C} \right)^{1-\alpha_i}$$

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- First order condition:

$$\alpha_i Q_{i,C}^{\alpha_i-1}\left(\underbrace{\frac{w_iL_i}{P_{i,R}}-\frac{P_{i,C}}{P_{i,R}}Q_{i,C}}_{=Q_{i,R}}\right)^{1-\alpha_i}+Q_{i,C}^{\alpha_i}(1-\alpha_i)\left(\underbrace{\frac{w_iL_i}{P_{i,R}}-\frac{P_{i,C}}{P_{i,R}}Q_{i,C}}_{=Q_{i,R}}\right)^{1-\alpha_i-1}\left(-\frac{P_{i,C}}{P_{i,R}}\right)=0$$

$$\boxed{Q_{i,R}=\frac{1-\alpha_i}{\alpha_i}\left(\frac{P_{i,C}}{P_{i,R}}\right)Q_{i,C}}$$

Preferences

- Rewrite budget constraint: $Q_{i,R} = \frac{w_i L_i}{P_{i,R}} - \frac{P_{i,C}}{P_{i,R}} Q_{i,C}$

- Replace in objective function and solve unconstrained max problem:

$$\max_{\{Q_{i,C}\}} Q_C^{\alpha_i} \left(\frac{w_i L_i}{P_{i,R}} - \frac{P_{i,C}}{P_{i,R}} Q_{i,C} \right)^{1-\alpha_i}$$

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$$\boxed{Q_{i,R} = \frac{1-\alpha_i}{\alpha_i} \left(\frac{P_{i,C}}{P_{i,R}} \right) Q_{i,C}}$$

Preferences

- Replace result in budget constraint:

$$Q_{i,R} = \frac{1-\alpha_i}{\alpha_i}\left(\frac{P_{i,C}}{P_{i,R}}\right)Q_{i,C} = \frac{w_i L_i}{P_{i,R}} - \frac{P_{i,C}}{P_{i,R}}Q_{i,C}$$

$$\boxed{Q_{i,C} = \alpha_i \frac{w_i L_i}{P_{i,C}}}$$

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- Cobb-Douglas preferences: demand is a fixed share of their income $(\alpha_i, 1-\alpha_i)$; demand inversely proportional to the price of that good.

- Holds regardless of whether consumers are in autarky or trade.

Prices in Autarky Equilibrium

- In equilibrium, **prices equal marginal cost** in each productive sector:

$$P_{i,p} = w_i a_{i,p} \iff \frac{P_{i,p}}{a_{i,p}} = w_i \qquad \text{for } p \in \{C, R\}$$

- We can pin down the relative price $P_{i,C}/P_{i,R}$:

$$\boxed{\frac{P_{i,C}}{a_{i,C}} = w_i = \frac{P_{i,R}}{a_{i,R}} \iff \frac{P_{i,C}}{P_{i,R}} = \frac{a_{i,C}}{a_{i,R}}}$$

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- In autarky, relative prices will reflect the **opportunity cost** within country i

Autarky Equilibrium

- Replacing $\frac{P_{i,p}}{a_{i,p}} = w_i \iff P_{i,C} = w_i a_{i,p}$ in demand functions solves in terms of parameters:

$$\boxed{Q_{i,C} = \alpha_i \frac{w_i L_i}{P_{i,C}} = \alpha_i \frac{L_i}{a_{i,C}}, \qquad Q_{i,R} = (1-\alpha_i)\frac{w_i L_i}{P_{i,R}} = (1-\alpha_i)\frac{L_i}{a_{i,R}}}$$

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- Can check choices satisfy the PPF:

$$\begin{aligned}
a_{i,C} \times Y_{i,C} + a_{i,R} \times Y_{i,R} &= L_i \\
a_{i,C} \times Q_{i,C} + a_{i,R} \times Q_{i,R} &= L_i && \text{(mkt clearing)} \\
a_{i,C} \times \alpha_i \frac{L_i}{a_{i,C}} + a_{i,R} \times (1-\alpha_i)\frac{L_i}{a_{i,R}} &= L_i && \text{(optimal demand)} \\
\alpha_i L_i + (1-\alpha_i) L_i &= L_i && \text{(checks out!)}
\end{aligned}$$

Numerical example: Autarky

Variable	United States (US)	Colombia (COL)
Labor endowment L_i	300 million	54 million
Preference parameter α_i	$1/2$	$3/4$
Unit labor requirement for computers $a_{i,C}$	3,000	5,400
Unit labor requirement for roses $a_{i,R}$	30	6
Max computers: $L_i/a_{i,C}$		
Max roses: $L_i/a_{i,R}$		
Opportunity cost $a_{i,C}/a_{i,R}$		
Demand for computers: $\alpha_i L_i/a_{i,C}$		
Demand for roses: $(1-\alpha_i)L_i/a_{i,R}$		

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Max roses: $L_i/a_{i,R}$	$300\text{m}/30 = 10\text{m}$	$54\text{m}/6 = 9\text{m}$
Opportunity cost $a_{i,C}/a_{i,R}$		
Demand for computers: $\alpha_i L_i/a_{i,C}$		
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Max roses: $L_i/a_{i,R}$	$300\text{m}/30 = 10\text{m}$	$54\text{m}/6 = 9\text{m}$
Opportunity cost $a_{i,C}/a_{i,R}$	$3{,}000/30 = 100$	$5{,}400/6 = 900$
Demand for computers: $\alpha_i L_i/a_{i,C}$	$0.5 \times 300\text{m}/3{,}000 = 50{,}000$	$0.75 \times 54\text{m}/5{,}400 = 7{,}500$
Demand for roses: $(1-\alpha_i)L_i/a_{i,R}$	$0.5 \times 300\text{m}/30 = 5\text{m}$	$0.25 \times 54\text{m}/6 = 2.25\text{m}$

Autarky Equilibrium

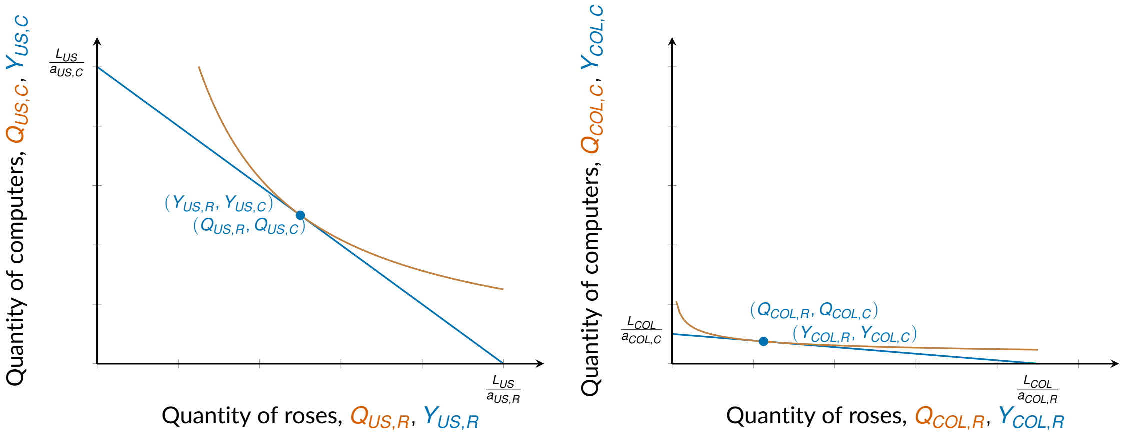

Figure: Autarky equilibrium