International Trade: Lecture 16
Firms and Trade: The Krugman Model

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Takeaways from last class

- Demand for differentiated goods

- Elasticity of substitution (σ)

- For all good, optimal choice implies MRS = relative prices

- Production under monopolistic competition, fixed costs, and IRS = profit opportunities

- Monopoly power implies $p^* > MC$

- In our framework, prices = mark up over MC.

Demand functions

After some algebra, we can solve for demand functions:

$$q_i(\varphi) = \underbrace{\left(\frac{p_i(\varphi)}{P_i}\right)^{-\sigma}}_{\text{relative price}} \times \underbrace{\frac{I_i}{P_i}}_{\text{real income}}$$

(check handout for step by step derivation)

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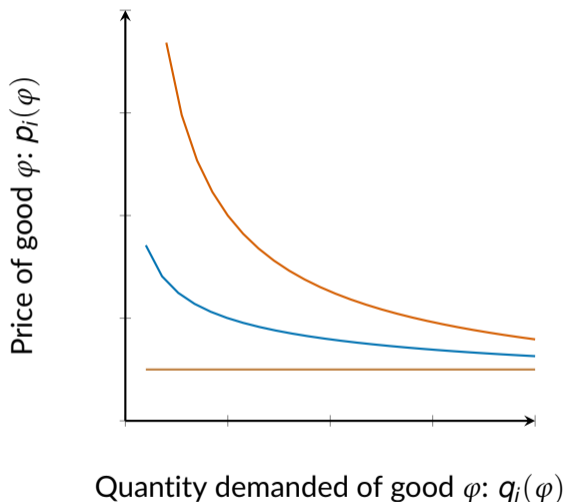Quantity demanded of good φ: $q_i(\varphi)$

Figure: Demand curve with different elasticities: $\sigma = 1.5, \sigma = 3, \sigma = \infty$

Production

- Producers of each good φ have a monopoly over the production of their good

- They also will have to pay a fixed cost $\bar{f}$ to set up shop (only if they enter the market)

- To produce a given quantity $q_i(\varphi)$, labor used is:
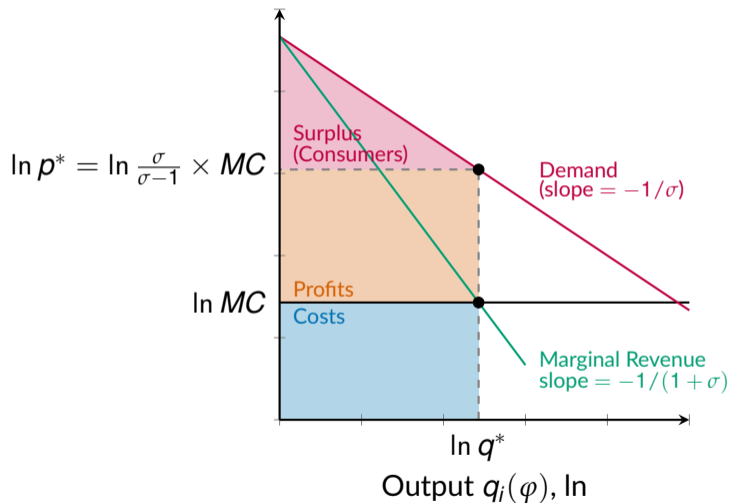
$$\ell = \bar{f} + a^* q_i(\varphi) \iff q_i(\varphi) = \frac{1}{a^*}(\ell - \bar{f})$$

- Optimal price = mark-up $\times$ marginal cost.

$$p^* = \frac{\sigma}{\sigma - 1} \times MC$$

- Note $\frac{\sigma}{\sigma-1} > 1$. We call this a mark-up.
  (what happens when $\sigma \to \infty$?)

Monopolistic competition

This class

- How to solve for the equilibrium of this model?

- What are the prices $\{p^*, P_i\}$, quantities demanded $\{q^*, Q_i\}$, goods in eqm $\{N\}$?

- What happens once we open up to trade?

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    - make a profit!

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- What is this point?

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\pi_i(\varphi) &= (p^* - MC)q^* - w_i \bar{f} = 0 \\
&= \left(\frac{\sigma}{\sigma - 1} a^* w_i - a^* w_i \right) q^* - w_i \bar{f} = 0
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- Solving for q^*:

$$
\boxed{q^* = (\sigma - 1) \times \frac{\bar{f}}{a^*}}
$$

(note: quantity sold by firm is identical)

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- a^* input labor requirement, maps from labor input to quantities

Free entry in equilibrium

- A surprising result is that, since all firms are identical, none of them will make positive profits in equilibrium!

- They maximize marginal profits, but free-entry + fixed costs wipes them away.

- Each additional good gives shoppers another option, so demand for every incumbent variety falls.

- Number of units sold per firm shrinks.

Love of variety

- What about the total basket Q_i? We can explicitly calculate it:

$$Q_i = \left[\sum_{\varphi \in \Phi_i} q_i(\varphi)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} = \left[\left(\underbrace{q^* + \cdots + q^*}_{N^* \text{ times}} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} = \left[N^*(q^*)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} = (N^*)^{\frac{\sigma}{\sigma-1}} q^*$$

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- People like having variety of options when purchase

Love-of-variety: graphical representation
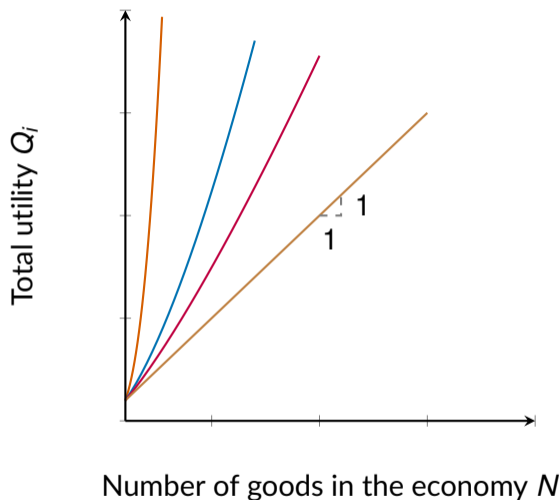

Number of goods in the economy N

Figure: Love of variety with different elasticities: $\sigma = 1.5$, $\sigma = 3$, $\sigma = \infty$

Love-of-variety: photographical representation

Figure: Boris Yeltsin, then president of the Russian Soviet Socialist Republic, visits America, 1989

Rationing and lack of variety

Figure: Rationing store in Cuba, 2012

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Intuition:
  - large $L_i \rightarrow$ large market $\rightarrow$ the more goods/firms in equilibrium;
  - large $\bar{f} \rightarrow$ larger IRS $\rightarrow$ firms larger $\rightarrow$ fewer firms in equilibrium.
  - large $\sigma \rightarrow$ more substitutable goods $\rightarrow$ markups smaller $\rightarrow$ surviving firms have to sell more units to be able to pay for fixed costs $\rightarrow$ fewer firms in equilibrium.
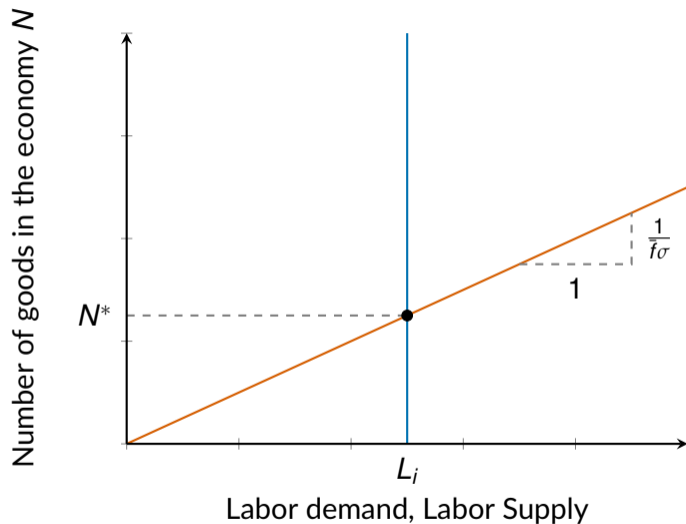
Labor market clearing

Figure: Labor market and number of firms equilibrium

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- Such that:
    - consumers maximize utility (behave optimally)
    - firms maximize profits (behave optimally)
    - factor markets clear: $L_i = N \times \ell^*$
    - goods markets clear: $I_i = P_i Q_i = N \times p^* q^*$ (check handout)

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    - $\Phi_i := \Phi_H \cup \Phi_F = \{H_1, H_2, \cdots, H_{N_H}, F_1, F_2, \cdots, F_{N_F}\}$: all goods available in country i.

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- Preferences:

$$\max_{\{q_i(\varphi)\}_{\varphi \in \Phi_i}} Q_i \equiv \left[\underbrace{\sum_{\varphi \in \Phi_H} q_i(\varphi)^{\frac{\sigma-1}{\sigma}}}_{\text{home goods}} + \underbrace{\sum_{\varphi \in \Phi_F} q_i(\varphi)^{\frac{\sigma-1}{\sigma}}}_{\text{foreign goods}} \right]^{\frac{\sigma}{\sigma-1}}$$

$$s.t. \quad P_i Q_i = \sum_{\varphi \in \Phi_i} p_i(\varphi) q_i(\varphi) \leq I_i = w_i L_i$$

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- How many firms exist in each country? Use the labor market clearing condition to find that:

$$N^* = \frac{L_i}{\sigma \bar{f}}$$

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- That becomes clear once we evaluate what happens with welfare/utility:

$$
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Q_i &\equiv \left[\sum_{\varphi \in \Phi_H} q_i(\varphi)^{\frac{\sigma-1}{\sigma}} + \sum_{\varphi \in \Phi_F} q_i(\varphi)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \\
&= \left[N^*(q^*)^{\frac{\sigma-1}{\sigma}} + N^*(q^*)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \\
&= (2N^*)^{\frac{\sigma}{\sigma-1}} q^*
\end{aligned}
$$

Welfare after trade

- Consumers are better off after trade openness:

$$Q_i^{\text{Trade}} = (2N^*)^{\frac{\sigma}{\sigma-1}} q^* > (N^*)^{\frac{\sigma}{\sigma-1}} q^* = Q_i^{\text{Autarky}}$$

- With identical countries, half of the new entrants locate in H and half in F

- Wach country *hosts* the same number of firms as before ($N^* = L/(\sigma \bar{f})$)...

- ...but consumers in both countries can now purchase *all* $2N^*$ varieties.

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- But why does trade even happen in this world?

Interindustry trade

- In the other models we saw (Ricardian, SFM or HO) identical identical countries would have no reason to trade

- Gains from trade hing on cross-country differences and differences in relative prices

- The Krugman model showcases horizontal intra-industry trade that arises even though the countries are identical

- Gains from trade come from the love-of-variety and the ability of increasing-returns firms to cover fixed costs in a larger market.

Takeaway

Trade in the Krugman model is fundamentally different from comparative advantage trade: it is horizontal and survives even when countries look exactly alike. Integration doubles the menu of goods, lowers the price index, and raises welfare without changing firm size, wages, or the markup.