International Trade: Lecture 11

The Heckscher-Ohlin Model and the 4 big Theorems of Modern Trade (i)

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Last class

- Diminishing marginal returns to scale
-

This and next class

- Do differences in endowment (rather than technology) drive international trade?
- Implications for the distribution of income across factors of production?
- What does trade integration do to difference in income across countrieS?
- Four main results of modern trade theory:
  - Stolper-Samuelson Theorem
  - Rybczynski Theorem
  - Hecksher-Ohlin Theorem
  - Factor Equalization Theorem

The Heckscher-Ohlin Trade Model

- Heckscher-Ohlin Model analyzes trade when
    - Resources can move costlessly across industries
    - Industries differ in the intensities of factor use
    - Countries differ in the relative endowments of factors

- Comparative advantage derives from:
    - relative factor abundance (in countries)
    - relative factor intensities (in production)

Heckscher and Ohlin

- Hecksher: Swedish economist, 1879-1952
- Published 1148 books and articles: $\sim$ 16 for every year he was alive!
- Ohlin: Swedish economist, 1899-1979
- His advisor was Eli Heckscher; won the Nobel Prize in 1977

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- Key force: Differences in factor intensities and endowments

Production

- Production of two sectors tech T and cloth C in country i use the same factors:

$$Y_{i,C} = K_{i,C}^{\beta_C} L_{i,C}^{1-\beta_C}, \qquad Y_{i,T} = K_{i,T}^{\beta_T} L_{i,T}^{1-\beta_T}$$

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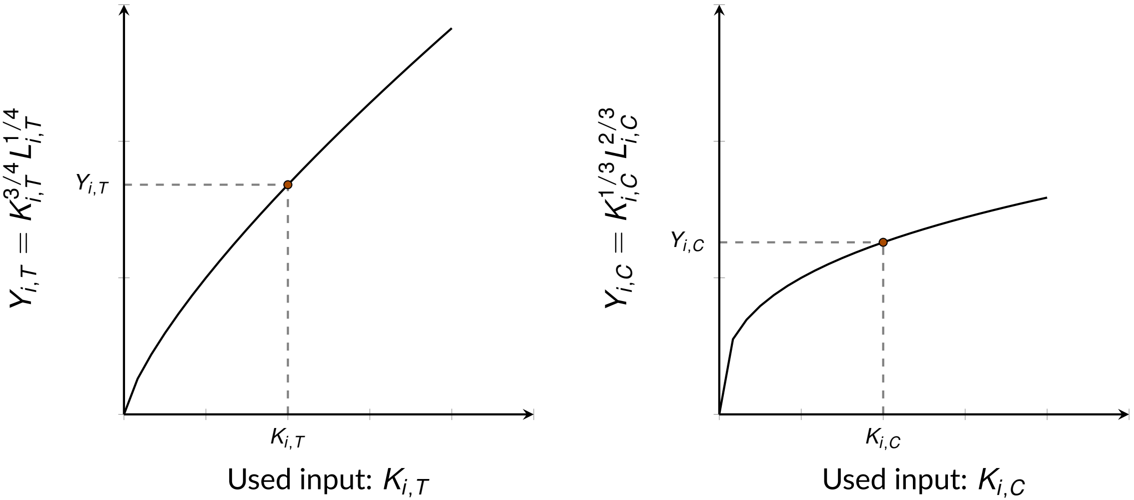
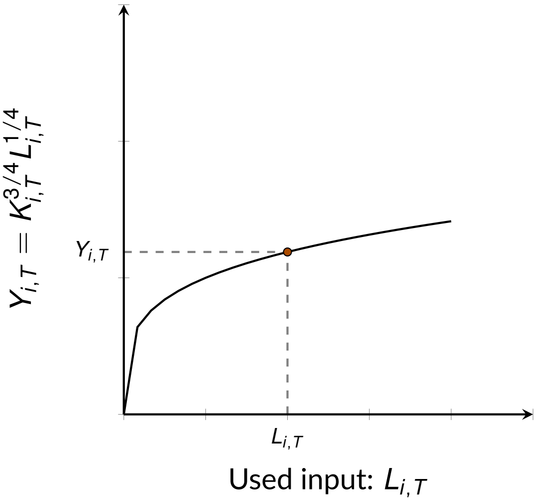
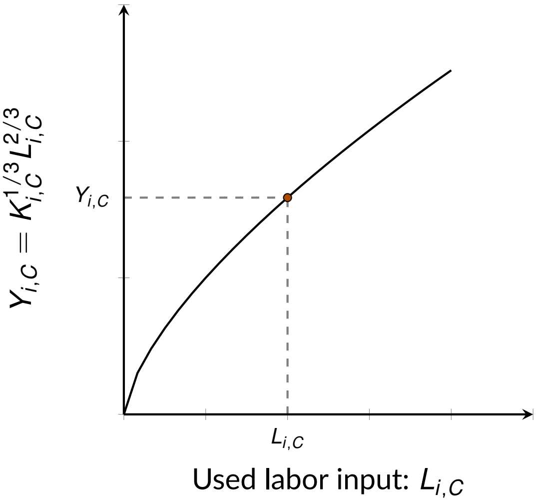
- Marginal return to capital in T is higher
- For same level of K, L, additional K more productive in tech

Decreasing marginal returns in capital (holding labor fixed)

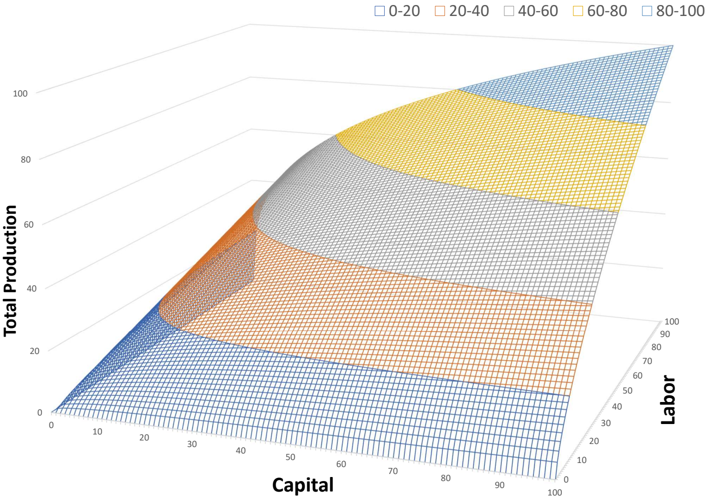


Decreasing marginal returns in labor (holding capital fixed)





Capital and Labor jointly



$$Y = K^{1/3}L^{2/3}$$

Cobb-Douglas is Constant Returns to Scale in Capital and Labor jointly... but diminishing marginal returns while holding the other factor fixed...

Optimality conditions

- At their optimal points, factor prices equal their marginal (revenue) product for labor...

$$
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P_T \times MPL_{i,T} = P_T \times \frac{\partial Y_{i,T}}{\partial L_{i,T}} &= w_i \\
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- ... and for capital, respectively:

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- Note there since factors are mobile, there are common factor prices $\{w_i, r_i\}$. Why?

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- Same reasoning is valid for r_i

Factor mobility

- Wage rage w_i will be such that

$$L_{i,C} + L_{i,T} = \bar{L}_i \qquad \text{and}$$
$$P_T \times MPL_{i,T} = w_i = P_C \times MPL_{i,C}$$
$$P_C \times (1-\beta_C) \times \left(\frac{K_{i,C}}{L_{i,C}}\right)^{\beta_C} = w_i = P_T \times (1-\beta_T) \times \left(\frac{K_{i,T}}{L_{i,T}}\right)^{\beta_T}$$

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Factor intensities

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- With two sectors and two factors of production, the tech sector is relatively capital-intensive if, at any wage-rental ratio w_i / r_i,

$$\frac{K_{i,T}}{L_{i,T}} > \frac{K_{i,C}}{L_{i,C}}$$

From factor prices to optimal choices

- Recall:

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- Combining these:

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- Same valid for C. So both sectors' capital-to-labor ratios pinned down by relative factor prices:

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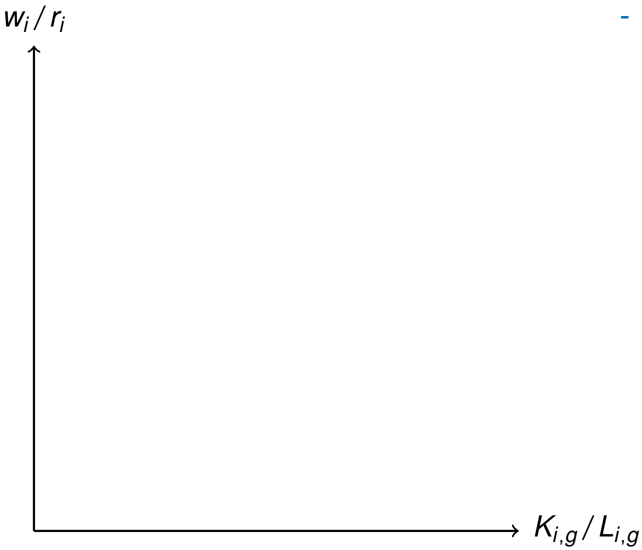
- Since $\beta_T > \beta_C$, for any prices $\frac{w_i}{r_i}$, it will be the case that $\frac{K_{i,T}}{L_{i,T}} > \frac{K_{i,C}}{L_{i,C}}$.

Graphical representation



- Let's place $K_{i,g}\,/\,L_{i,g}$ on the x-ais and $w_i\,/\,r_i$ on the y-axis

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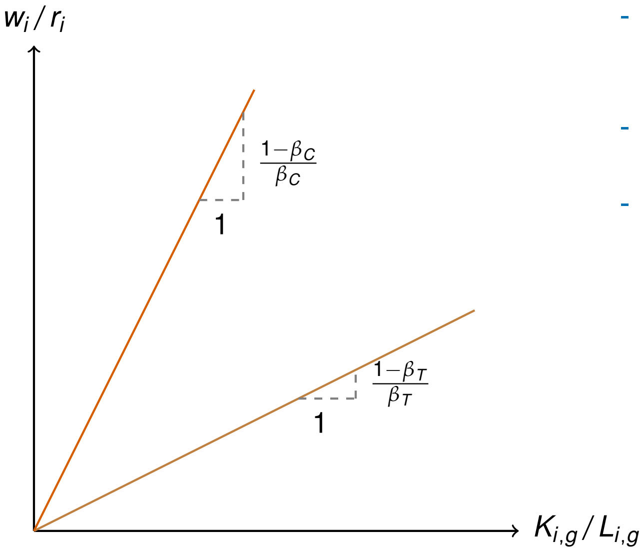

- Let's place $K_{i,g}\,/\,L_{i,g}$ on the x-ais and $w_i\,/\,r_i$ on the y-axis
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- Curve for C steeper than for T

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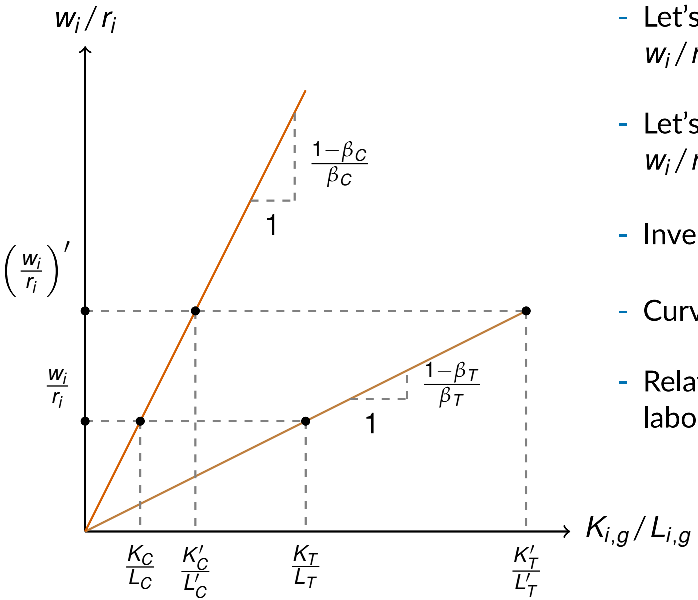

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- Curve for C steeper than for T
- Relative factor prices pin down capital to labor ratio

From goods prices to optimal choices

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- Combining these:

$$\frac{P_C}{P_T} = \frac{MPL_{i,T}}{MPL_{i,C}} \iff \frac{P_C}{P_T} = \frac{(1-\beta_T) \times \left(\frac{K_{i,T}}{L_{i,T}}\right)^{\beta_T}}{(1-\beta_C) \times \left(\frac{K_{i,C}}{L_{i,C}}\right)^{\beta_C}}$$

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- But we have just shown K/L is a function of w/r. Hence:

$$\frac{P_C}{P_T} = \frac{(1-\beta_T) \times \left(\frac{\beta_T}{1-\beta_T} \times \frac{w_i}{r_i}\right)^{\beta_T}}{(1-\beta_C) \times \left(\frac{\beta_C}{1-\beta_C} \times \frac{w_i}{r_i}\right)^{\beta_C}} \iff \frac{P_C}{P_T} = \frac{(1-\beta_T)^{1-\beta_T}\beta_T^{\beta_T}}{(1-\beta_C)^{1-\beta_C}\beta_C^{\beta_C}} \times \left(\frac{w_i}{r_i}\right)^{\beta_T-\beta_C}$$

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(where $\hat{x} = (x' - x)/x$ is the percent change in variable x)

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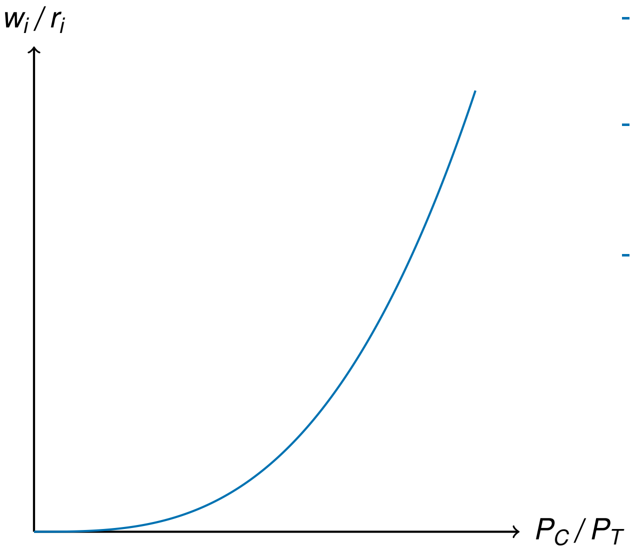
- Using our specific function form:

$$\frac{P_C}{P_T} = \text{constant} \times \left(\frac{w_i}{r_i}\right)^{\beta_T - \beta_C}, \qquad \text{with } 0 < \beta_C < \beta_T < 1$$

Graphical representation



- Let's place $P_C\,/\,P_T$ on the x-ais and $w_i\,/\,r_i$ on the y-axis
- Inverting equation,
  $$\frac{w_i}{r_i} = \left(\frac{1}{\text{constant}} \times \frac{P_C}{P_T}\right)^{\frac{1}{\beta_T - \beta_C}}$$
- Convex function: $w_i\,/\,r_i$ grows more than proportionately in $P_C\,/\,P_T$

Putting everything together...

- We first mapped $w_i / r_i \mapsto K_{i,g} / L_{i,g}$:

$$\frac{K_{i,C}}{L_{i,C}} = \frac{\beta_C}{1-\beta_C} \times \frac{w_i}{r_i}, \qquad \frac{K_{i,T}}{L_{i,T}} = \frac{\beta_T}{1-\beta_T} \times \frac{w_i}{r_i}$$

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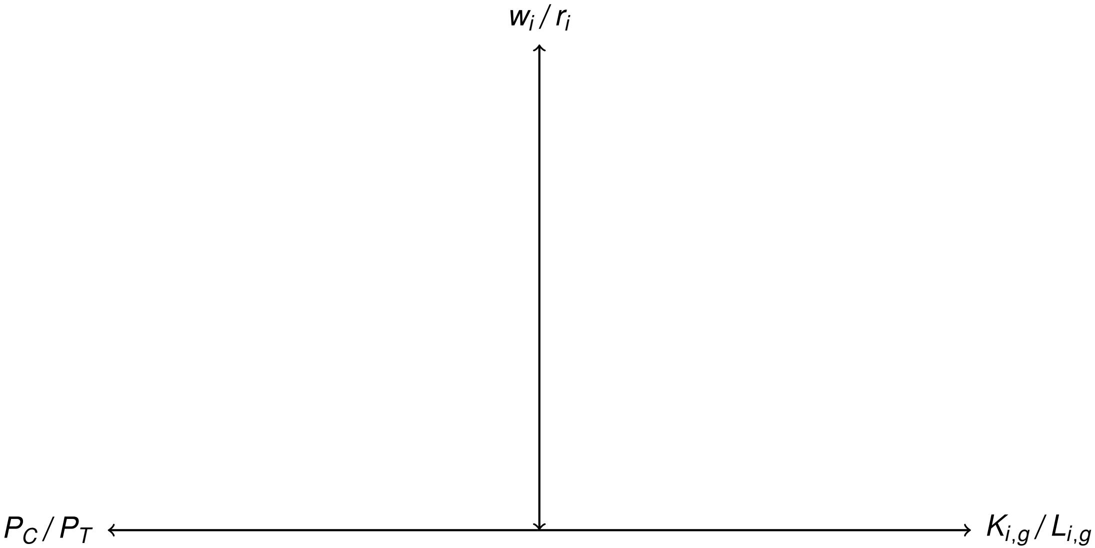
$$\frac{w_i}{r_i} = \left(\frac{1}{\text{constant}} \times \frac{P_C}{P_T} \right)^{\frac{1}{\beta_T - \beta_C}}$$

- Putting these together, we can pin down from relative prices $P_C / P_T \mapsto K_{i,g} / L_{i,g}$:

$$\frac{K_{i,g}}{L_{i,g}} = \text{constant}_g \times \left(\frac{P_C}{P_T} \right)^{\frac{1}{\beta_T - \beta_C}}$$

Signature chart



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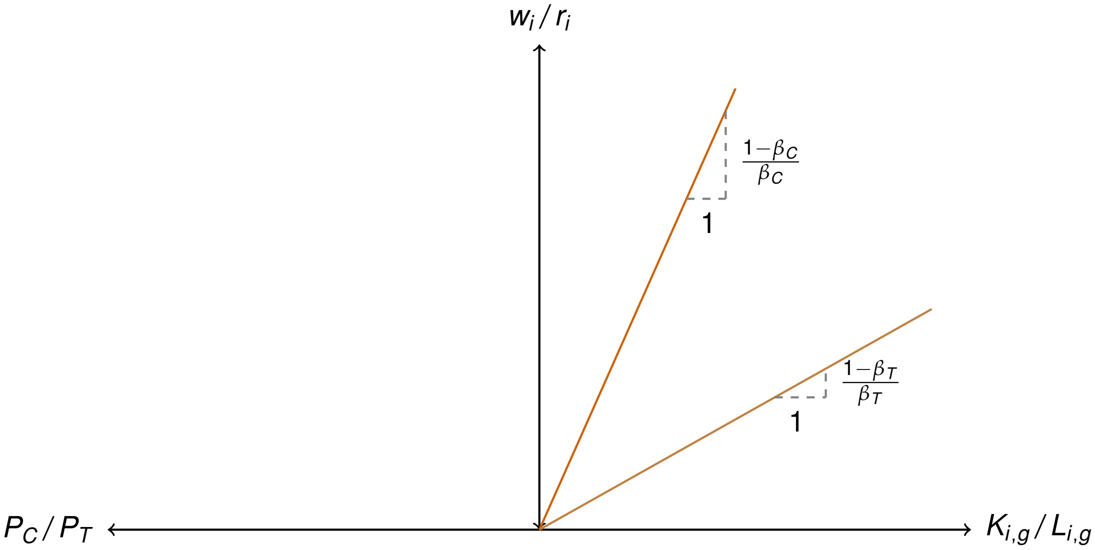

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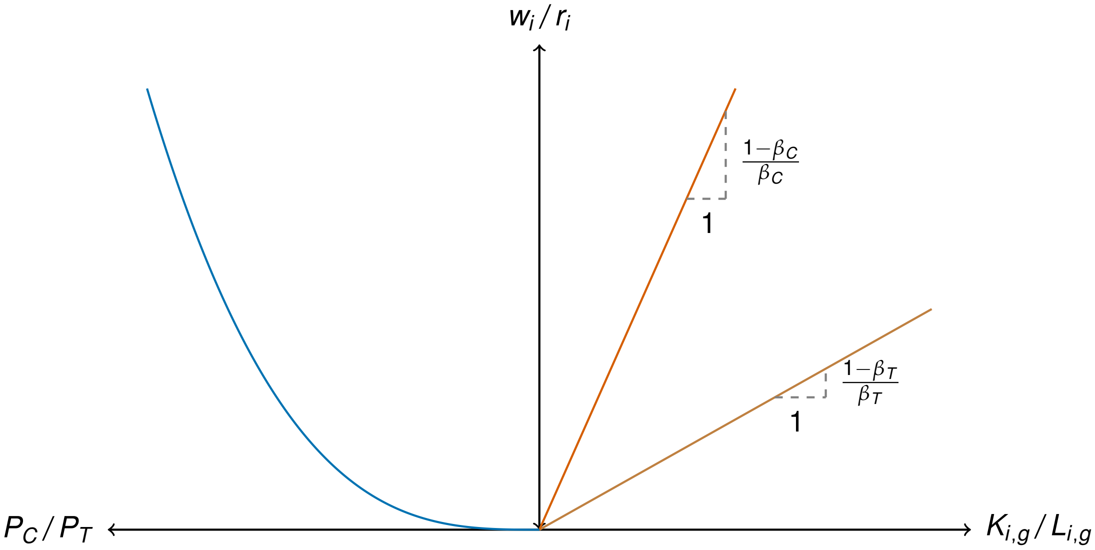

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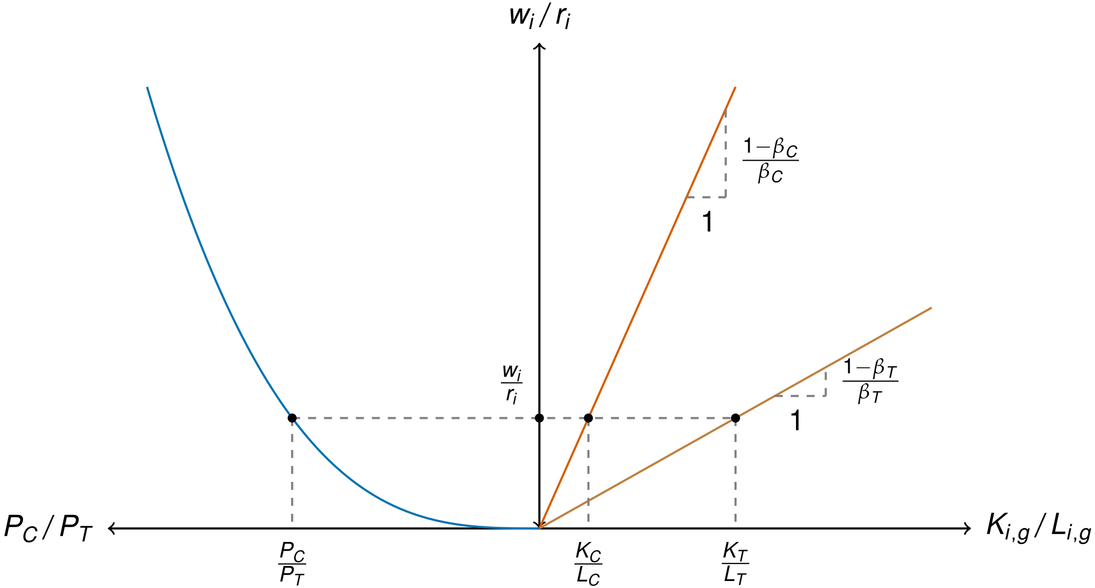

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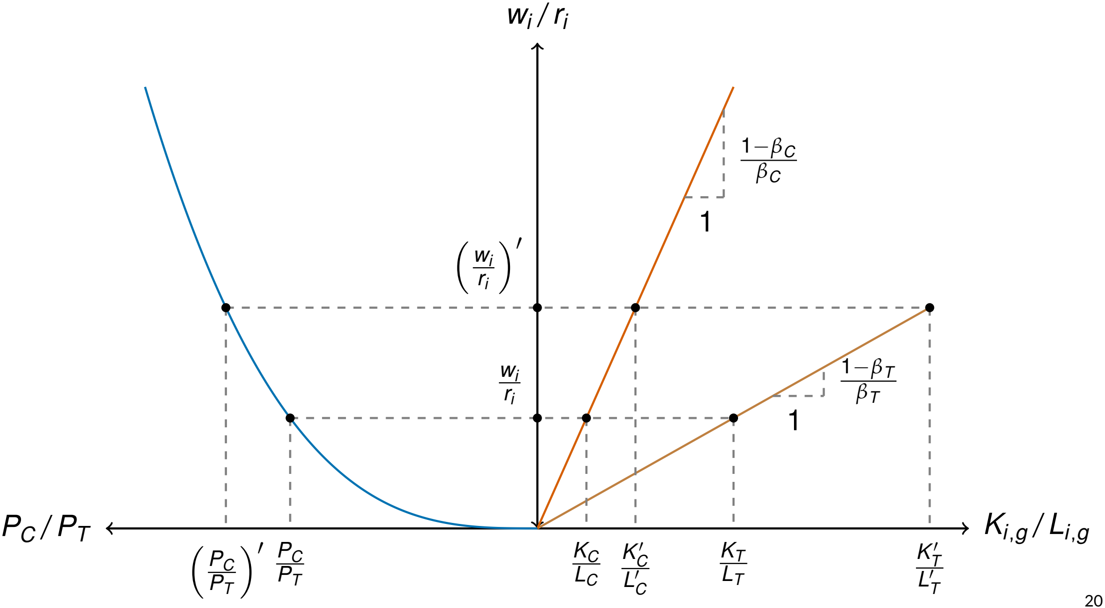

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- Question: How can both sectors raise the relative use of capital?

Putting everything together...

- If the relative price increases to $(P_C/P_T)' > P_C/P_T \rightarrow$ both sectors more capital intensive
- Intuition: labor becomes relatively expensive $\rightarrow$ firms substitute capital for labor
- Question: How can both sectors raise the relative use of capital?
- Cloth sector expands while tech sector contracts.
- We will understand why next class...