Lecture 7: Government Consumption and Distortionary Taxation

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1 Characterization of the Equilibrium

Time is discrete. Households maximize the present discounted value of lifetime utility:

$$\begin{aligned} \max_{\{c_t,k_t,n_t\}_{t\geq 0}} \quad & \sum_{t=0}^{\infty} \beta^t \mathbb{E}_0 \left[\frac{c_t^{1-\sigma}}{1-\sigma} - \psi \frac{n_t^{1+\phi}}{1+\phi} + \Gamma(c_t^g) \right] \\ s.t. \quad & c_t + k_t = (1-\tau_t^n) w_t n_t + (1-\tau_t^k) r_t k_{t-1} + \tau_t^k \delta k_{t-1} + (1-\delta) k_{t-1} + \varphi_t + \Pi_t \end{aligned} \tag{1}$$

We can write the Lagrangian for this problem:

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\sigma}}{1-\sigma} - \psi \frac{n_t^{1+\phi}}{1+\phi} + \Gamma(g_t^c) + \lambda_t \Big((1-\tau_t^n) w_t n_t + (1-\tau_t^k) r_t k_{t-1} + \tau_t^k \delta k_{t-1} + \varphi_t + \Pi_t - c_t - k_t + (1-\delta) k_{t-1} \Big) \right]$$

whose first order conditions satisfy, for every $t \geq 0$:

$$\begin{aligned} c_t \quad &: \quad c_t^{-\sigma} = \lambda_t \\ n_t \quad &: \quad \psi n_t^{\phi} = \lambda_t (1-\tau_t^n) w_t \\ k_t \quad &: \quad \lambda_t = \beta \lambda_{t+1} \Big[r_{t+1} - \tau_{t+1}^k (r_{t+1} - \delta) + (1-\delta) \Big] \end{aligned}$$

From the first order conditions, we can derive the labor-leisure condition:

$$\psi n_t^{\phi} = c_t^{-\sigma} (1-\tau_t^n) w_t \tag{2}$$

which shows that labor taxes distort labor supply downwards. We can also derive Euler Equation:

$$\left(\frac{c_{t+1}}{c_t} \right)^{\sigma} = \beta \Big[r_{t+1} - \tau_{t+1}^k (r_{t+1} - \delta) + (1-\delta) \Big] \tag{3}$$

which shows that capital taxes distort savings (i.e., future consumption) downwards, under the restriction that $r_{t+1} > \delta$.

There is a representative firm endowed with a constant-returns to scale technology that maximizes the following objective function:

$$\max_{\{\tilde{k}_{t-1}, \tilde{n}_t\}_{t\geq 0}} \pi_t = y_t - r_t \tilde{k}_{t-1} - w_t n_t, \qquad s.t. \quad y_t = z_t \tilde{k}_{t-1}^{\alpha} \tilde{n}_t^{1-\alpha} \tag{4}$$

if an equilibrium exists, factor prices will equal their marginal products:

$$\alpha \frac{y_t}{\tilde{k}_{t-1}} = r_t \tag{5}$$

$$(1-\alpha)\frac{y_t}{\tilde{n}_t} = w_t \tag{6}$$

Finally, there is a government that collects labor, capital, and lump-sum taxes ($\varphi_t < 0$) and spends them on government consumption, government investment or lump-sum transfers ($\varphi_t > 0$). We assume that the government's budget balances for every period $t \geq 0$, i.e.:

$$c_t^g + \varphi_t = \tau_t^n w_t n_t + \tau_t^k (r_t - \delta) k_{t-1} \tag{7}$$

A **competitive equilibrium** with government consumption is characterized by a sequence of household allocations $\{c_t, k_t, n_t\}_{t\geq 0}$, with k_{-1} given; a sequence of firms allocations for capital and labor demand $\{\tilde{k}_t, \tilde{n}_t\}_{t\geq 0}$; a sequence of government consumption, taxes, and transfers $\{c_t^g, \tau_t^n, \tau_t^k, \varphi_t\}_{t\geq 0}$; and a sequence of factor prices $\{w_t, r_t\}_{t\geq 0}$, such that:

(a) given prices, household allocations solve the household's problem for every $t \geq 0$

(b) given prices, firm allocations solve the firm's problem for every $t \geq 0$;

(c) the government budget balances for every $t \geq 0$;

(d) factor markets clear ($k_{t-1} = \tilde{k}_{t-1}$ and $n_t = \tilde{n}_t$) for every $t \geq 0$;

(e) due to free entry, profits are zero ($y_t = r_t k_{t-1} + w_t n_t$) for every $t \geq 0$; and

(f) goods markets clear ($c_t + i_t + c_t^g = y_t$) for every $t \geq 0$.

1.1 Characterizing the Steady-State

Suppose that steady state that $z_t = z \forall t$ and steady-state government consumption to income ratio $\frac{c^g}{y} = \kappa^c$ is a constant. Let us solve for the steady state of this economy. We drop the time subscripts to indicate steady state values. Using the labor-leisure condition and the firm's labor demand:

$$\begin{aligned}
r &= \frac{1-\beta[1-(1-\tau^k)\delta]}{(1-\tau^k)\beta} \qquad \text{(from euler eqn)} \\
\frac{k}{y} &= \frac{\alpha}{r} \qquad \text{(from capital demand)} \\
\frac{i}{y} &= \delta \frac{k}{y} \qquad \text{(from capital accumulation)} \\
\frac{c^g}{y} &= \kappa^c \qquad \text{(exogenous)} \\
\frac{\varphi}{y} &= (1-\alpha)\tau^n + \tau^k (r-\delta)\frac{k}{y} - \frac{c^g}{y} \qquad \text{(from labor demand + govt budget cnst)} \\
\frac{c}{y} &= 1 - \frac{c^g}{y} - \frac{i}{y} \qquad \text{(from good markets clearance)} \\
n &= \left[\frac{1}{\psi}(c/y)^{-\sigma}(1-\tau^n)(1-\alpha)\right]^{\frac{1}{\phi+\sigma}} \left[z(k/y)^{\alpha}\right]^{\frac{1-\sigma}{(1-\alpha)(\phi+\sigma)}} \qquad \text{(see Appendix)} \\
y &= \left[z(k/y)^{\alpha} n^{1-\alpha}\right]^{\frac{1}{1-\alpha}} \qquad \text{(from production function)} \\
w &= (1-\alpha)\frac{y}{n} \qquad \text{(from labor demand)}
\end{aligned}$$

2 Different Kinds of Government Financing

2.1 What happens if government expenditure increases?

Note that the model above nests the simple neoclassical growth model we have seen if we set $\tau^k = \tau^n = \phi_t = g_t^c = 0 (\forall t)$. Suppose that government consumption increases permanently to some positive number. The government has three instruments to finance the new expenditures: taxes on capital; taxes on labor; and lump-sum taxes.

Now let us do some comparative statics. First, it is easy to see that r is increasing in τ^k:

$$\frac{\partial r}{\partial \tau^k} = \frac{\partial}{\partial \tau^k}\left[\frac{1-\beta}{\beta(1-\tau^k)} - \delta\right] = \frac{1-\beta}{\beta(1-\tau^k)^2} > 0$$

From this, we can see that steady-state capital stock is decreasing in τ^k:

$$\frac{\partial}{\partial \tau^k}\frac{k}{y} = \frac{\partial}{\partial r}\frac{\alpha}{r}\frac{\partial r}{\partial \tau^k} = -\frac{\alpha}{r^2}\frac{\partial r}{\partial \tau^k} < 0$$

Therefore, total wealth as a share of income in this economy (k/y) decreases if capital taxes increase. Realizing that steady state hours n and output y are increasing in k/y leads to the conclusion that n, y are decreasing in τ^k.

Furthermore, hours are decreasing in labor tax rates τ^n:

$$\frac{\partial n}{\partial \tau^n} = -\frac{n}{\left[\frac{1}{\psi}(c/y)^{-\sigma}(1-\tau^n)(1-\alpha)\right]} < 0$$

Since y is increasing in n, we can conclude that steady state output is also decreasing in labor tax rates τ^n.

Unlike taxes on capital or labor, lump-sum taxes do no create wedges on the optimality conditions. This does not mean, however, that nothing changes. Note that, at the steady-state:

$$\frac{c}{y} = 1 - \frac{c^g}{y} - \frac{i}{y}$$

which implies that we c/y decreases with a positive c^g. Since n, y are decreasing in c/y, government consumption will unambiguously increase steady-state output if financed through lump-sum taxes. If it is financed through labor or capital taxes, the net effect will depend on whether or not the wedge will dominate over the decrease in c/y.

2.2 Simulations

We compare the impulse responses of output, labor, and capital for three alternative financing mechanisms and discuss the economic intuition for the differences. We have calibrated the parameters with the following values. The three tax variables φ, τ^k, τ^n alternate between being endogenous variables or parameters, depending on the model. In each of the models below, we let one tax variable change to clear the government budget constraint while hold the others fixed. We further assume that government expenditure follows an $AR(1)$ process:

$$g_t^c = \delta + \rho g_{t-1}^c + \varepsilon_t$$

where $\frac{\delta}{1-\rho} = \kappa^c y_{ss}$. I calibrate the model parameters in the following way.

Variable	Description	Value
α	Capital Share	0.36
σ	Inverse of the IES	1
β	Discount Factor	0.99
δ	Private Depreciation	0.015
ψ	Labor Disutility Shifter	1
ϕ	Inverse of the Frisch Elasticity	0.25
ρ	AR(1) on government spending/apportionments	0.95

A The increase in government spending is financed by lump-sum taxes.

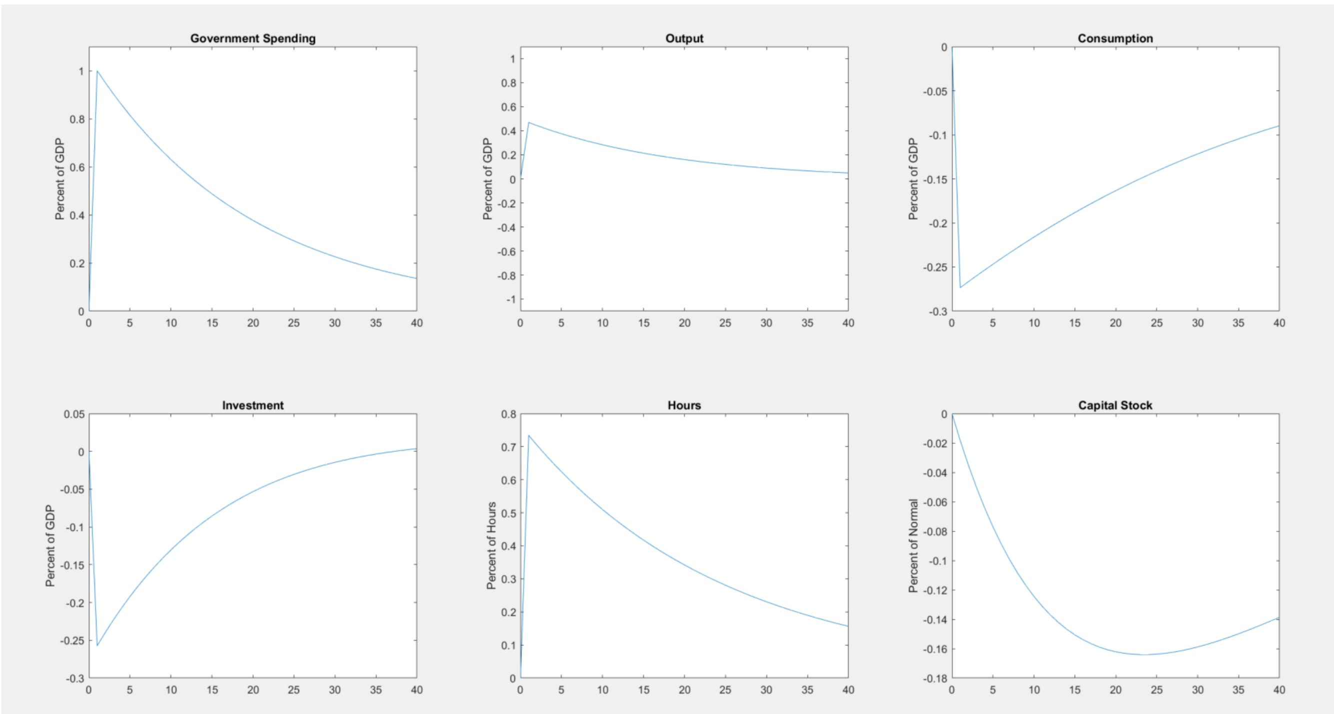

Figure 1: IRFs with lump-sum taxes adjusting

B The increase in government spending is financed by raising labor taxes.

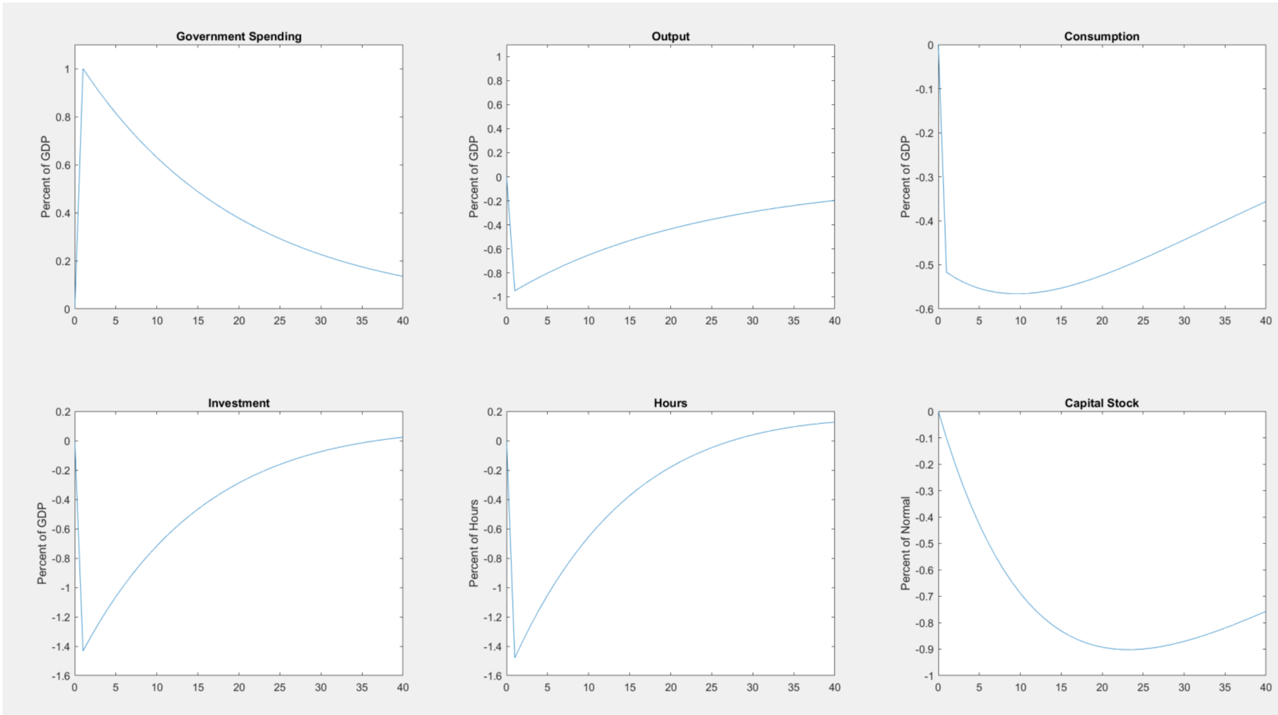

Figure 2: IRFs with labor income taxes adjusting

C The increase in government spending is financed by raising capital taxes.

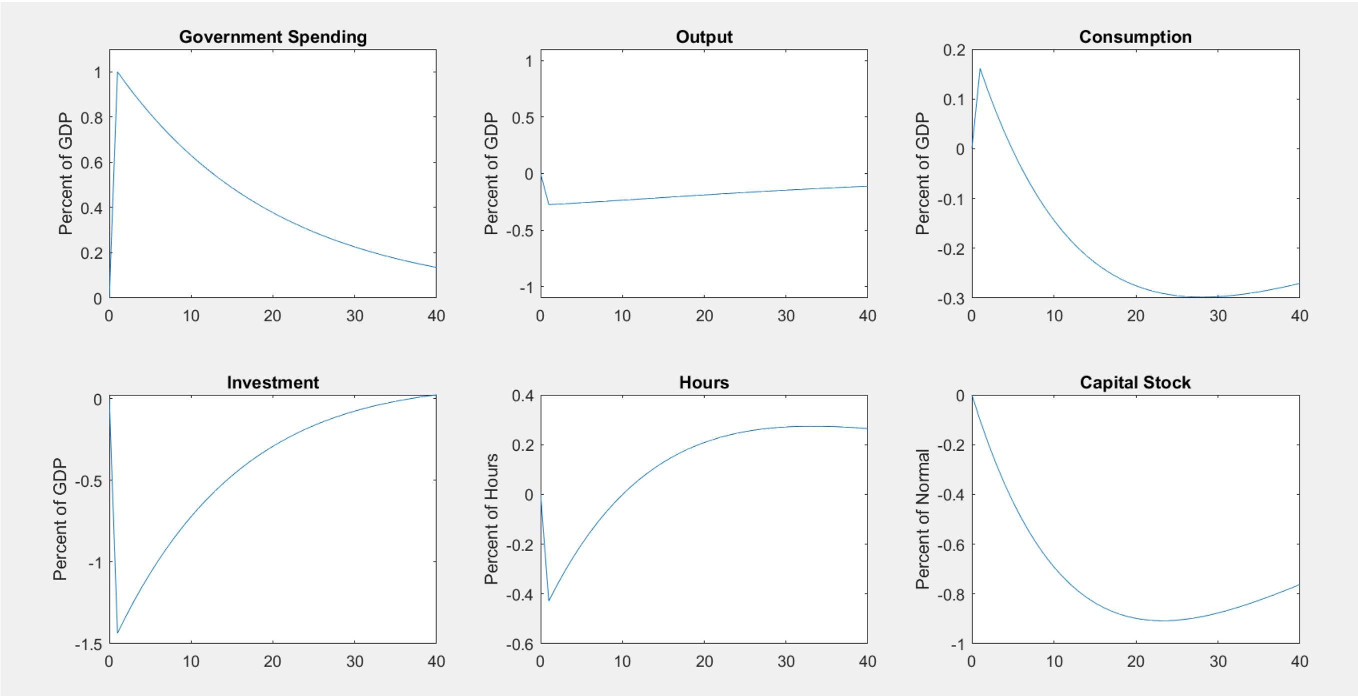

Figure 3: IRFs with capital income taxes adjusting

D **Discussion of intuition**:

When financed through lump-sum taxes, a shock to government spending still induces wedge-free optimal behavior of agents. In effect, the increase in government expenditures through lump-sum taxes is equivalent to shrinking the budget set available for private consumption. This negative income effects induces hours to increase. The increase in the interest rate induced by a higher MPK means that the path of consumption will be increasing after the initial shock, which induces disinvestment and a decrease of the capital stock. The short-run government spending multiplier is positive and smaller than one.

When financed through labor-income taxes, the increase in government spending induces a change in the relative price of leisure. It then becomes optimal to, for every unit of present consumption, to offer a smaller amount of hours. Therefore, hours drop substantially. Even though every unit of capital becomes more productive, investment drops in order to smooth out consumption intertemporally. The short-run government spending multiplier is negative and large.

When financed through capital-income taxes, the increase in government spending induces desinvestment since the return on capital has been effectively reduced. This leads to an initial increase in consumption and reduction in hours (i.e., consumption out of savings and decrease of the capital stock). The short-run government spending multiplier is negative and but not large.

A Additional Mathematical Derivations

To derive the steady-state of output and hours, first write the labor-leisure condition without the time subscripts and replacing the wage from the firm's FOC and solve for n:

$$\begin{aligned}
\psi n^{\phi} &= c^{-\sigma}(1-\tau^{n})(1-\alpha)\frac{y}{n} \\
n &= \left[\frac{1}{\psi}(c/y)^{-\sigma}(1-\tau^{n})(1-\alpha)\right]^{\frac{1}{1+\phi}} y^{\frac{1-\sigma}{1+\phi}}
\end{aligned}$$

Now replace this into the production function:

$$\begin{aligned}
y &= zk^{\alpha}n^{1-\alpha} \\
y &= z(k/y)^{\alpha}\left[(c/y)^{-\sigma}(1-\tau^{n})(1-\alpha)\right]^{\frac{1-\alpha}{1+\phi}} y^{\frac{\alpha(1+\phi)+(1-\alpha)(1-\sigma)}{1+\phi}} \\
y^{\frac{(1-\alpha)(1+\phi)-(1-\alpha)(1-\sigma)-\theta(1+\phi)}{1+\phi}} &= z(k/y)^{\alpha}\left[\frac{1}{\psi}(c/y)^{-\sigma}(1-\tau^{n})(1-\alpha)\right]^{\frac{1-\alpha}{1+\phi}} \\
y^{\frac{(1-\alpha)(\phi+\sigma)}{1+\phi}} &= z(k/y)^{\alpha}\left[\frac{1}{\psi}(c/y)^{-\sigma}(1-\tau^{n})(1-\alpha)\right]^{\frac{1-\alpha}{1+\phi}} \\
y &= \left[z(k/y)^{\alpha}\right]^{\frac{1+\phi}{(1-\alpha)(\phi+\sigma)}}\left[\frac{1}{\psi}(c/y)^{-\sigma}(1-\tau^{n})(1-\alpha)\right]^{\frac{1}{\phi+\sigma}}
\end{aligned}$$

which solves for y in terms of parameters. Replacing this back into the expression for n delivers:

$$\begin{aligned}
n &= \left[\frac{1}{\psi}(c/y)^{-\sigma}(1-\tau^{n})(1-\alpha)\right]^{\frac{1}{1+\phi}}\left\{\left[z(k/y)^{\alpha}\right]^{\frac{1+\phi}{(1-\alpha)(\phi+\sigma)}}\left[\frac{1}{\psi}(c/y)^{-\sigma}(1-\tau^{n})(1-\alpha)\right]^{\frac{1}{\phi+\sigma}}\right\}^{\frac{1-\sigma}{1+\phi}} \\
n &= \left[\frac{1}{\psi}(c/y)^{-\sigma}(1-\tau^{n})(1-\alpha)\right]^{\frac{1}{\sigma+\phi}}\left[z(k/y)^{\alpha}\right]^{\frac{1-\sigma}{(1-\alpha)(\sigma+\phi)}}
\end{aligned}$$