Econ 110A: Lecture 3

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UCSD, Summer Session II

Growth Rate and Compounding

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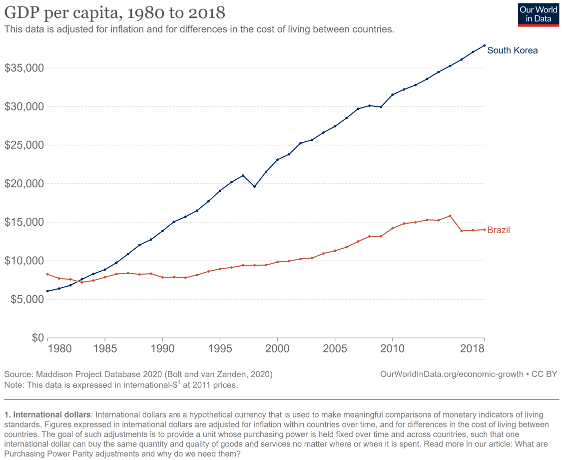
We can also iterate this backwards:

$$y_t = y_{t-1}(1 + g_t) = y_{t-2}(1 + g_{t-1})(1 + g_t) = y_0 \cdot \Pi_{k=0}^{t-1}(1 + g_{t-k})$$

We call this the property of compounding!

Small differences in growth rates make a HUGE difference over time

Brazil was richer than Korea in 1980... but Korea grew at at average of 4.5% per year since then, and Brazil at an average of 1.5% per year.



It is now 2.7x richer than Brazil!

Compounded Constant Growth Rate

If $g_t = g$ for all t, then:

$$y_t = y_0 \cdot \Pi_{k=0}^{t-1}(1 + g_{t-k})$$

becomes:

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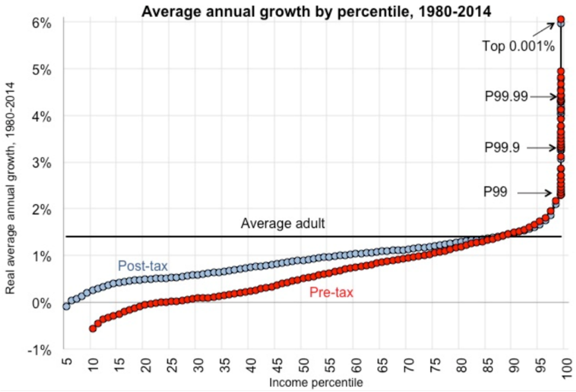
Just try it:

$$y_t = y_{t-1} \cdot (1 + g) = y_{t-2} \cdot (1 + g)^2 = y_{t-3} \cdot (1 + g)^3 = \cdots = y_0 \cdot (1 + g)^t$$

Example 1: Compounding and Inequality



- A: $\$1,000 \times (1 + 0.0\%)^{34} = \$1,000$
- B: $\$1,000 \times (1 + 1.5\%)^{34} = \$1,658$
- C: $\$1,000 \times (1 + 6.0\%)^{34} = \$7,251$

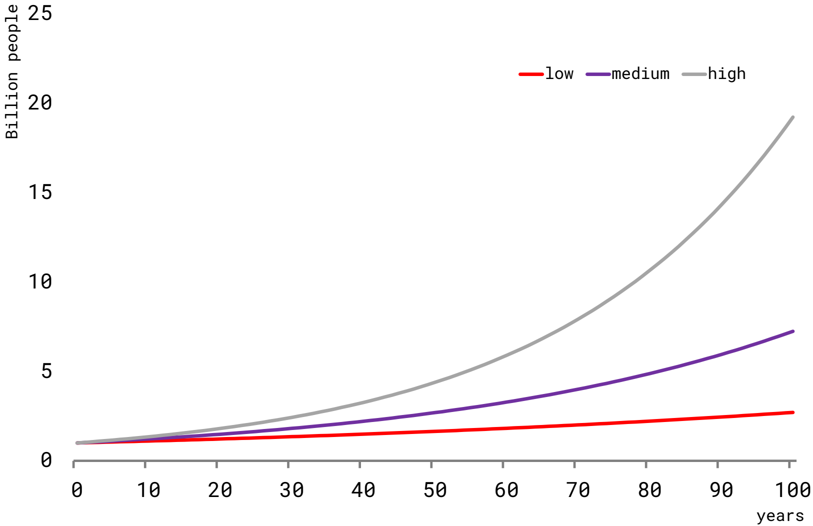
Example 2: Compounding and Population Growth

- Suppose the population of a country is $\bar{L} = 1$ billion.

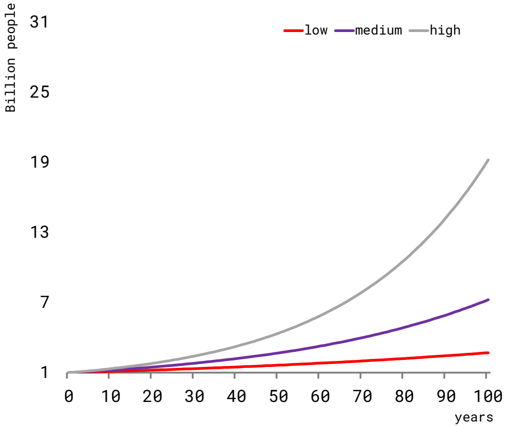
- You are a demographer who is trying to calculate the population of a this country $T = 100$ years from know.

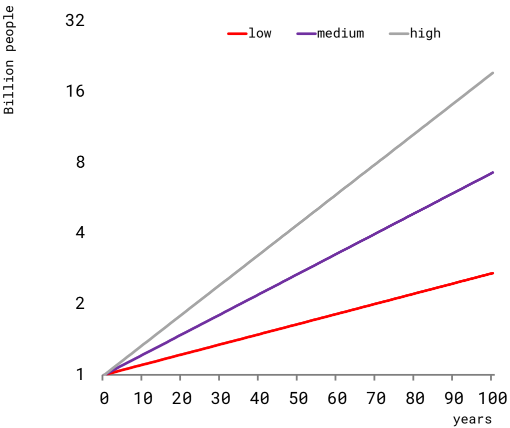
- You conjecture 3 scenarios. The population might growth $n_\ell = 1\%$ per year, $n_m = 2\%$, or $n_h = 3\%$ per year.
  - $\bar{L}(1.01)^{100} = 2.7$ billion
  - $\bar{L}(1.02)^{100} = 7.2$ billion
  - $\bar{L}(1.03)^{100} = 19.2$ billion

Example 2: Compounding and Population Growth



Ratio Scale or Log Scale





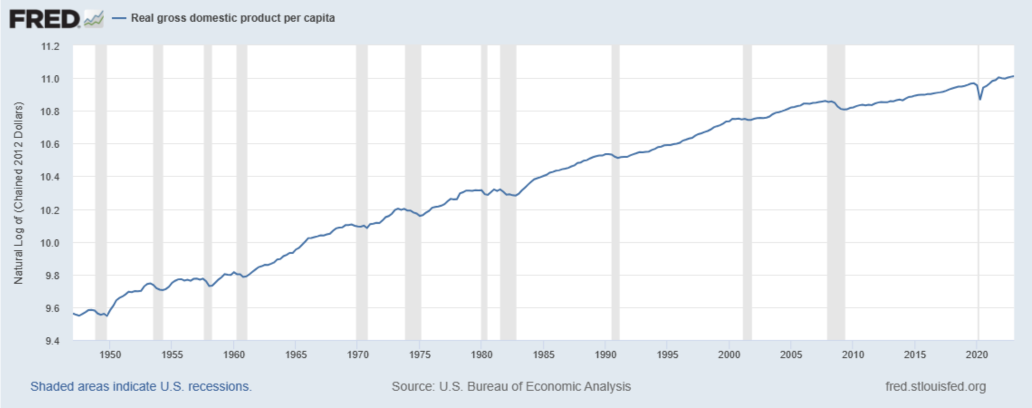
- key idea: re-scale vertical axis so each tick mark represents doubling of value
- important: the slope of the ratio scale graph of a variable that grows at a compounded constant growth rate g is constant and proportional to the growth rate g.

Ratio Scale or Log Scale

Open Excel File

https://canvas.ucsd.edu/courses/48299/files/10222679?module_item_id=1893744

RGDP on the long run follows a constant growth rate, so the log of RGDP is linear!



Computing a Compounded Constant Growth Rate

Suppose we know y_0 (initial level) and y_t (current level). How do we compute the compounded constant growth rate g from 0 to t?

$$y_t = y_0 \cdot (1+g)^t \iff g = \left(\frac{y_t}{y_0}\right)^{\frac{1}{t}} - 1$$

In the U.S., take $y_0 = y_{1870}^{US} = \$5,000$, and $y_t = y_{2015}^{US} = \$50,800$. Then:

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$$g^{US} = \left(\frac{y_{2015}^{US}}{y_{1870}^{US}}\right)^{\frac{1}{165}} - 1 = \left(\frac{50,800}{5,000}\right)^{\frac{1}{165}} - 1 = 0.0193 \approx 2\%$$

Log Scale and Growth Rates

The logarithm of a variable that grows at a compounded constant growth rate g is linear in time and the slope is the growth rate g.

$$y_t = y_0 \cdot (1+g)^t$$

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$$\begin{aligned} y_t &= y_0 \cdot (1+g)^t \\ \ln[y_t] &= \ln[y_0 \cdot (1+g)^t] \\ \ln[y_t] &= \ln[(1+g)^t] + \ln[y_0] \end{aligned}$$

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\ln[y_t] &\approx t \cdot g + \ln[y_0]
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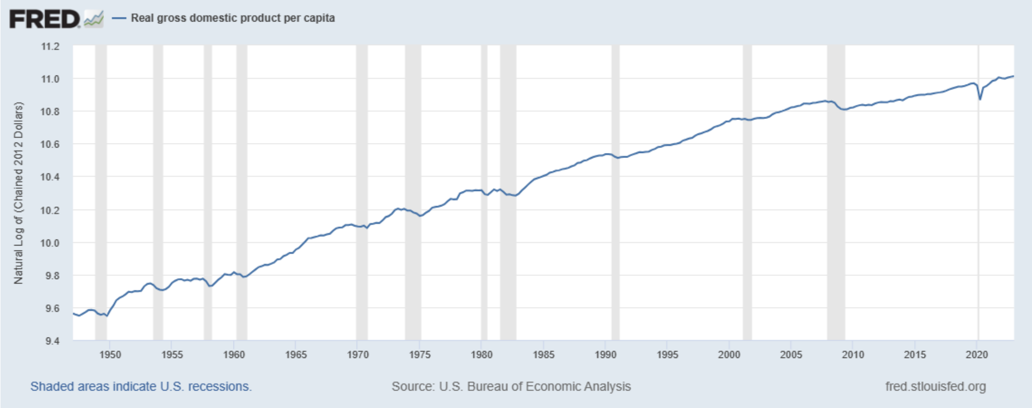
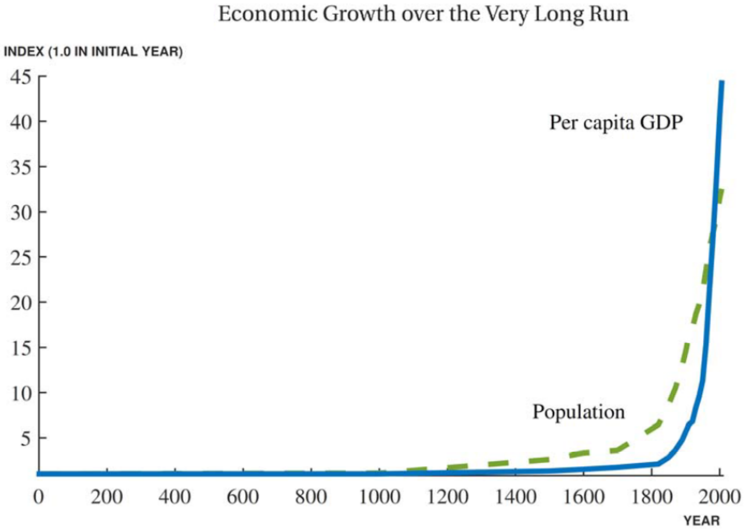
for g small because $log(1) = 0$, $dlog(1)/dx = x$, and $log(x)$ is continuous at 1.

RGDP on the long run follows a constant growth rate, so the log of RGDP is linear!



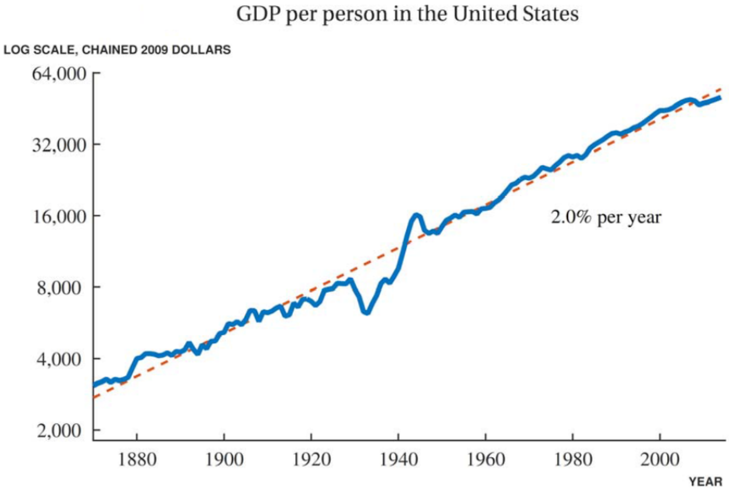
Some facts about long run growth

Fact 1: growth is a relatively recent phenomenon



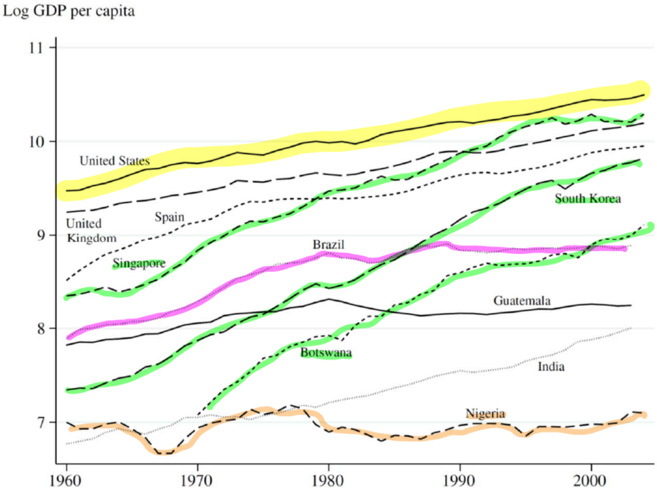
Note: Data are from Maddison (2008) for the “West,” i.e. Western Europe plus the United States. A similar pattern holds using the “world” numbers from Maddison.

Fact 2: continued persistent growth at the “frontier”



Note: Data for 1929–2014 are from the U.S. Bureau of Economic Analysis, NIPA Table 7.1. Data before 1929 are spliced from Maddison (2008).

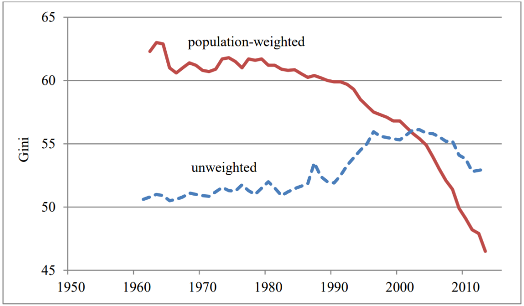
Fact 3: we observe heterogeneous growth experiences



Reproduced from Ch. 1 of Daron Acemoglu, “Introduction to Modern Economic Growth,” 2014

Fact 4: Average GDP per person diverged until 2000 and has been converging since then

Figure 2: Global income inequality between countries, 1960-2013

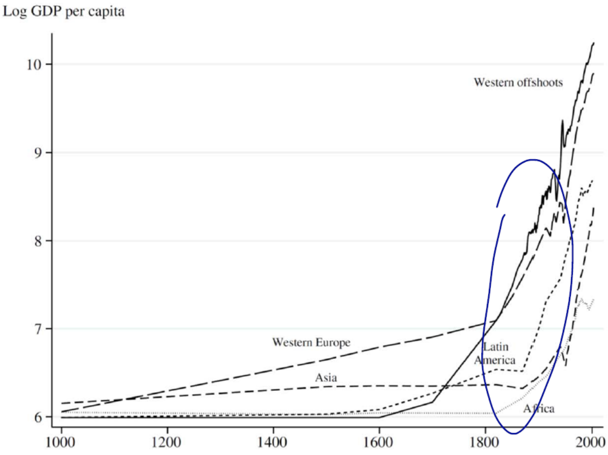


Source: Milanovic (2016).

Note: The graph shows inequality between countries (measured by Gini values estimated from countries' real per capita GDP)

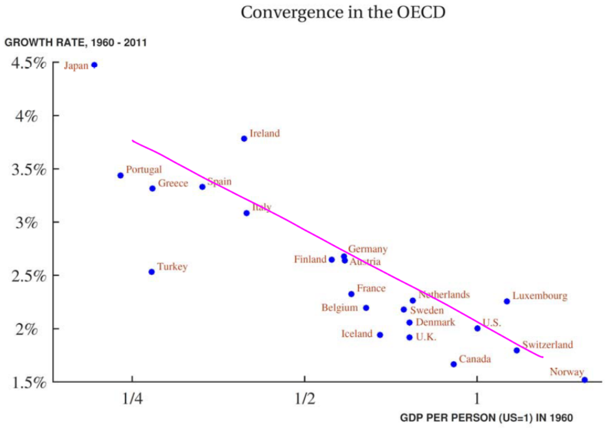
Milanovic, Branko (2016). Global Inequality: A New Approach for the Age of Globalization. Cambridge, Massachusetts: Harvard University Press.

Fact 5: Initial divergence seems to be as old as the Industrial Revolution



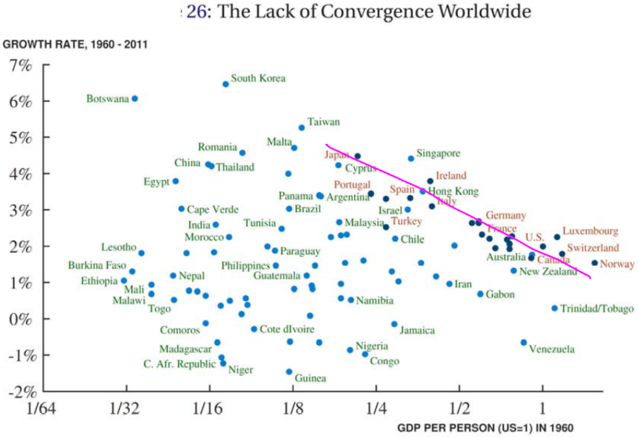
The evolution of average GDP per capita in Western offshoots, Western Europe, Latin America, Asia, and Africa, 1000–2000.

Fact 6: Conditional Convergence?



Source: The Penn World Tables 8.0. Countries in the OECD as of 1970 are shown.

Fact 6: Yes, but it doesn't generalize: Lack of Conditional Convergence!



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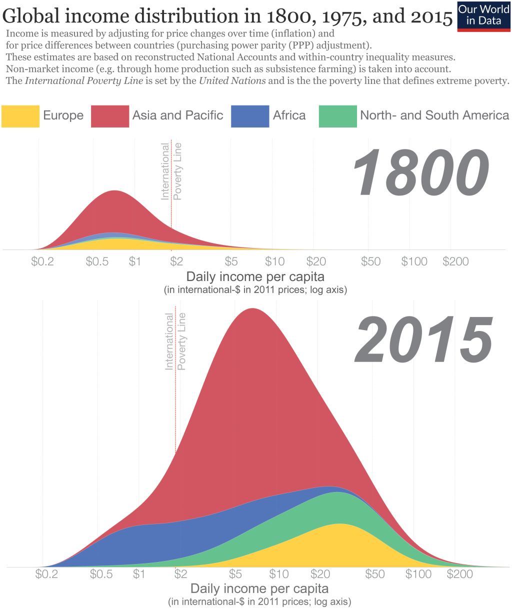
Fact 7: Lower Fraction of World Population Living in Poverty



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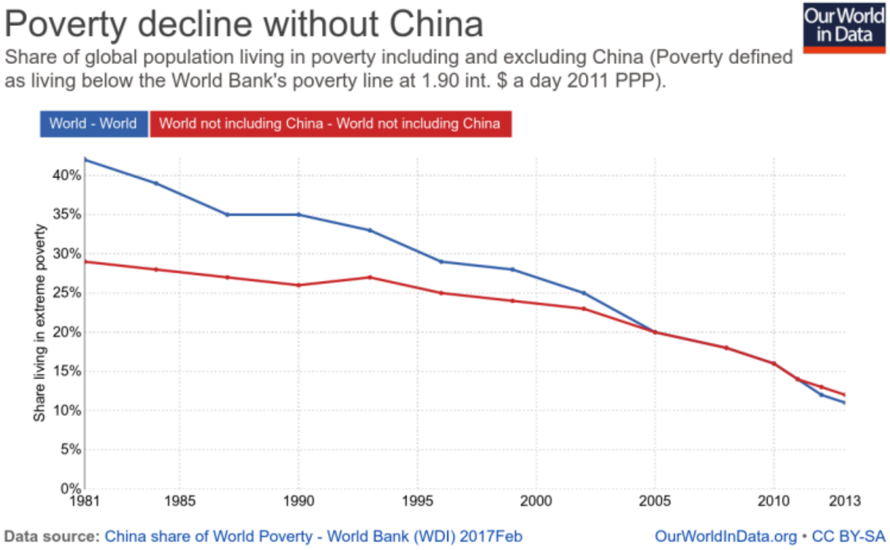

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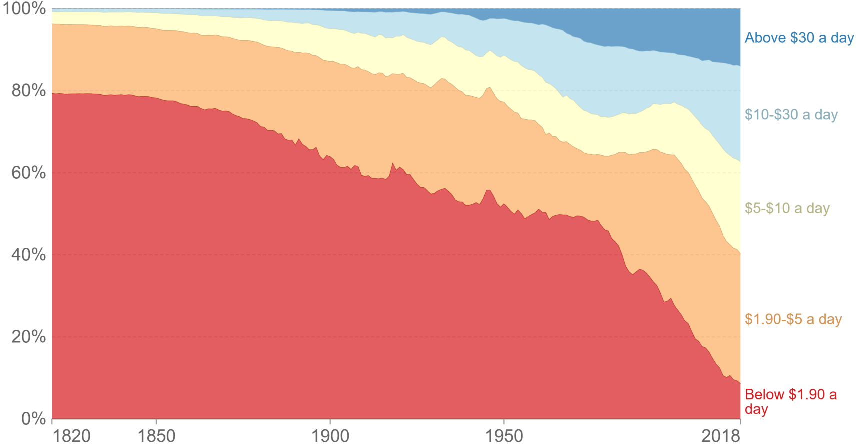
Distribution of population between different poverty thresholds, World, 1820 to 2018

Our World in Data

This data is adjusted for inflation and for differences in the cost of living between countries. Data after 1981 relates to household income or expenditure surveys collated by the World Bank; before 1981 it is based on historical reconstructions of GDP per capita and inequality data.



Source: Moatsos (2021)

ON THE MECHANICS OF ECONOMIC DEVELOPMENT*

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I do not see how one can look at figures like these without seeing them as representing *possibilities*. Is there some action a government of India could take that would lead the Indian economy to grow like Indonesia's or Egypt's? If so, *what*, exactly? If not, what is it about the 'nature of India' that makes it so? The consequences for human welfare involved in questions like these are simply staggering: Once one starts to think about them, it is hard to think about anything else.

What have we learned?

- Growth rates compound over time
- They can help us explain time series processes such as wealth accumulation and population growth
- Scale transformations such as the log scale can help us see growth rates better
- Economic growth is a recent phenomenon and has been persistent since the 19th century
- But growth experiences have been highly heterogeneous
- Income levels diverged, then started to converge
- Poverty levels collapsed and we might see a world without extreme poverty in the next decades