Trade, Growth, and Product Innovation

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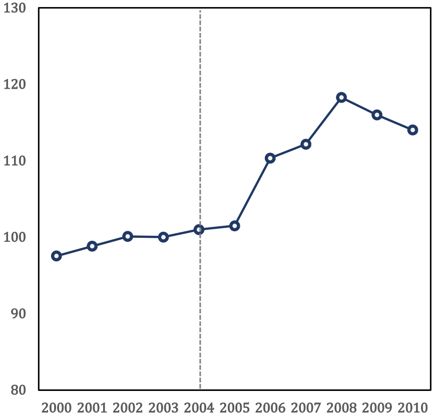
Can economic integration induce product innovation?



Total Number of Produced Varieties, Average Across New Member States (2004 Enlargement). Index, 2003 = 100. Source: Eurostat Prodcom.

- Opposing economic forces:
- **Specialization** or **selection** $\Longrightarrow$ ($\downarrow$) measure of varieties
- **Market size effect** $\Longrightarrow$ ($\uparrow$) measure of varieties

Life among product innovators

- Qualitative evidence on how managers think about the effects of market access on product innovation:
    - Sent qualitative questionnaire to 200+ firms; small number contacted me back

- **CEO of a Czech Biotech firm**:
  "Once we joined the EU, we were granted the same veterinary standards as producers in other member states. [...] This allowed us to increase our exports and fund our own genetic programmes."

- **Spokesperson of a Latvian liquor manufacturer**:
  "In 2004, we first started producing flavored variations of our signature vodka, which became a popular export product [...] and later started production of 18 new products."

Questionnaire Biotech Portfolio

This paper (so far)

- New quantitative framework integrating these opposing forces:
  - trade by comparative advantage;
  - endogenous growth & innovation in differentiated varieties;
  - i.e., specialization + market size effect.
- Causal evidence testing the mechanism of the model:
  - increased market access increases the prob. of starting to produce and export a given product
- (Summer plan) Quantification exercise:
  - (have it, but not calibrated) algorithm solves for unique BGP growth rate and endogenous variables given fundamentals
  - (planned, nontrivial) algorithm solves for transition dynamics

Contributions to the literature

- **Trade and growth** Grossman and Helpman 1989, 1990, 1991; Romer 1990; Rivera-Batiz and Romer 1991a, 1991b; Acemoglu and Ventura 2002; Atkeson and Burstein 2010; Perla, Tonetti, and Waugh 2015; Sampson 2016; Aghion et al. 2018; Grossman and Helpman 2018; Buera and Oberfield 2020; Hsieh and Klenow 2022; Trouvain 2022; Kleinman et al. 2023.
- **Theoretical and empirical understanding of the extensive margin of trade** Melitz 2003; Arkolakis, Ganapati, and Muendler 2020; Mayer, Melitz, and Ottaviano 2020; Klenow, Hsieh, and Shimizu 2022.
- **Documentation of changes in imported, exported, and produced varieties in trade and I.O.** Feenstra 1994; Hummels and Klenow 2005; Bernard et al. 2009; Goldberg et al. 2010; Kehoe and Ruhl 2013; Argente et al. 2020.
- **Welfare effects of economic integration** Eaton and Kortum 2002; Arkolakis, Costinot, and Rodríguez-Clare 2012; Ossa 2015; Melitz and Redding 2021; Caliendo et al. 2021.

Roadmap

- Theory
  - Description of the model
  - Main propositions regarding BGP
  - “Proof-of-concept” quantification algorithm
  - Comparative statics
- Empirics
  - Data and construction of the shock
  - Identification strategy
  - Results

Environment

- Nests the Romer and the Eaton Kortum models as special cases.
- Arbitrarily many source countries $s \in \mathbf{K}$.
- Three sectors:
  - competitive final goods $\omega \in [0, 1]$, traded internationally;
  - differentiated intermediate goods $\nu \in [0, M_s(t)]$, traded internationally;
  - R&D, use savings to fund new varieties.
- HH supply labor inelastically, consume, and invest in new varieties through equity markets.

Households

$$
\begin{aligned}
\max_{[C_s(t), c_s(t,\omega)]_{\omega\in[0,1]}} \quad & \int_0^\infty \exp\{-\rho t\} \frac{C_s(t)^{1-\phi}}{1-\phi} dt \\
s.t. \quad P_s I_s + P_s(t) C_s(t) &= r_s(t) A_s(t) + w_s(t) L_s \\
C_s(t) &= \left[\int_0^1 c_s(t,\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right]^{\frac{\sigma}{\sigma-1}} \\
P_s(t) C_s(t) &= \int_0^1 p_s(t,\omega) c_s(t,\omega) d\omega
\end{aligned}
$$

Production

- Final goods producers have CRS technology resembling the Romer model:

$$y_s(t,\omega) = z_s(t,\omega)[\ell_s(t,\omega)]^{1-\alpha}\left(\frac{1}{\alpha}\sum_{k\in\mathbf{K}}\int_0^{M_k(t)}[x_{ks}(t,\omega,\nu)]^{\alpha}d\nu\right)$$

Production

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- Intermediate goods producers use final good as inputs. Optimal prices are destination-specific:

$$p_{ks}(t,\omega,\nu) = \frac{P_k(t)\tau_{ks}}{\alpha}$$

where trade costs τ_{ks} include tariffs and non-tariff barriers.

Production

- Can rewrite the final goods technology as

$$[z_s(t,\omega)]^{\frac{1}{1-\alpha}} \cdot \tilde{M}_s(t) \cdot \ell_s(t,\omega)$$

with $\tilde{M}_s(t) = \alpha^{-\frac{1}{1-\alpha}} \cdot \sum_{k\in\mathbf{K}} M_k(t) \cdot (\tau_{ks} P_k(t))^{-\frac{\alpha}{1-\alpha}}$.

Production

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- $\tilde{M}_s(t)$ is the effective measure of input varieties adjusted for marginal cost.

- $\tilde{M}_s(t)$ captures diffusion of non-rival intermediate goods.

Trade in Final Goods

Very similar to standard Eaton-Kortum assumptions:

- Perfect competition: $p_{ss}(\omega) = \underbrace{\frac{w_s^{1-\alpha}}{\bar{M}_s^{1-\alpha} z_s(\omega)}}_{\text{Marginal Cost}}$
- Destination prices: $p_d(\omega) = \underbrace{\min_s \{\tau_{sd} \cdot p_{ss}(\omega)\}}_{\text{Lowest Cost Supplier}}$
- $z_s(\omega)$ with Fréchet distribution: $F_s(z) = \exp\{-T_s z^{-\theta}\}$

Research and Development

- HHs use equity markets to invest $I_s(t)$ units of the final good in R&D;
- *Poisson* process success rate with flow arrival rate equal to $\psi I_s(t)dt$;
- Non-arbitrage condition:

$$\underbrace{r_s(t)}_{\text{return on savings}} = \underbrace{\frac{\psi \pi_s(t,\nu)}{P_s(t)}}_{\text{flow dividend rate on R\&D}} + \underbrace{\frac{\dot{P}_s(t)}{P_s(t)}}_{\text{capital gains}}$$

Derivation of non-arbitrage condition

Trade Equilibrium

- Trade in final goods governed by comparative advantage, but with an extensive margin shifter.

- Expenditure shares of goods from source country s at destination country d:

$$\lambda_{sd}^{EK} \equiv \frac{T_s (w_s \tau_{sd})^{-\theta}}{\sum_{k \in \mathbf{K}} T_k (w_k \tau_{kd})^{-\theta}}, \qquad \lambda_{sd}^{Melitz} \equiv \frac{(M_s)^{1-\sigma} (w_s \tau_{sd})^{-(1-\sigma)}}{\sum_{k \in \mathbf{K}} (M_k)^{1-\sigma} (w_k \tau_{kd})^{-(1-\sigma)}}$$

Trade Equilibrium

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$$\lambda_{sd}^{F}(t) = \frac{T_s\left(\tilde{M}_s(t)^{1-\alpha}\right)^{\theta}(w_s(t)^{1-\alpha}\tau_{sd})^{-\theta}}{\sum_{k\in\mathbf{K}} T_k\left(\tilde{M}_k(t)^{1-\alpha}\right)^{\theta}(w_k(t)^{1-\alpha}\tau_{kd})^{-\theta}}$$

- Can potentially use this model to “decompose” trade into Ricardo and Romer forces (cf. Klenow, Hsieh, and Shimizu 2022)

Equilibrium

- (Cross-sectional Eqm) Given $M_s(t)$, each t, system of N equations solves for wages:

$$w_s(t)L_s(t) = (1-\alpha)\sum_{d\in\mathbf{K}} \underbrace{\lambda^F_{sd}(t)P_d(t)Y_d(t)}_{\text{functions of } w_d(t)}$$

- (Dynamic Eqm) given wages, two differential equations for each country describe the dynamics:

$$\begin{aligned}\dot{C}_s(t) &= \frac{1}{\phi}\left[\frac{r_s(t)}{P_s(t)} - \rho\right] C_s(t) \\ \dot{M}_s(t) &= \frac{r_s(t)}{P_s(t)} M_s(t) + \psi \frac{w_s(t)}{P_s(t)} L_s - \psi C_s(t)\end{aligned}$$

where $r_s(t)$ and $P_s(t)$ are functions of $w_s(t)$.

Definition of Equilibrium

BGP, special cases

Proposition (BGP under autarky)

If $\tau_{sd} \to \infty$ for all $s \neq d$, then there is unique country specific growth rate $g_s^{autarky}$ such that, in every country $s \in \mathbf{K}$, $g_{M_s} = g_{Y_s} = g_{C_s} = g_{w_s} = g_{A_s} = g_s^{autarky}$.

Proposition (BGP under zero gravity)

If $\tau_{sd} = 1$ for all (s, d), then there is a unique world equilibrium growth rate $g^{zero\ gravity}$ such that, in every country $s \in \mathbf{K}$, $g_{M_s} = g_{Y_s} = g_{C_s} = g_{w_s} = g_{A_s} = g^{zero\ gravity}$.

$$g_k^{\text{autarky}} = \frac{1}{\phi}\left[(1-\alpha)\cdot\alpha\cdot\psi\cdot\left[\frac{w_k(t^*)L_k}{M_k(t^*)}\right]-\rho\right]$$

$$g_k^{\text{zero gravity}} = \frac{1}{\phi}\left[(1-\alpha)\cdot\alpha\cdot\psi\cdot\left[\frac{\sum_{s\in\mathbf{K}} w_s(t^*)L_s}{\sum_{n\in\mathbf{K}} M_n(t^*)}\right]-\rho\right]$$

- Intuition: **no trade costs** $\implies$ complete diffusion of non-rival inputs $\implies$ **growth happens as if world were a single country**.

BGP, general case

Proposition (BGP with finite trade costs)

If $\tau_{sd} \in (1, \infty)$ for all $s \neq d$, there exists a unique world equilibrium growth rate:

$$g_{M_s} = g_{Y_s} = g_{C_s} = g_{w_s} = g_{A_s} = g^* \qquad \forall s \in \boldsymbol{K}$$

and the world equilibrium growth rate g^* is pinned down by a vector of wages $\lambda_w \cdot [w_s(t^*)]_{s \in \boldsymbol{K}}$ and a vector of measures of varieties $\lambda_M \cdot [M_s(t^*)]_{s \in \boldsymbol{K}}$, up to the choice of scalars λ_w, λ_M. Furthermore, the real interest rates will equalize globally along the BGP.

Common Growth Rate, Sketch of Proof

BGP, general case

- Intuition:
- Trade works as a vehicle that links valuations of R&D stocks and returns;
- Countries with initially relatively many varieties $\implies$ lower relative price in final and intermediate goods ($\sim$ high productivity) $\implies \downarrow \pi$ and $\downarrow r \implies \downarrow g$ over transition path (and vice versa).
- Over BGP, real returns on assets and R&D equalize globally;
- This leads to a stable distribution of income $[w_s(t^*)]_{s\in\mathbf{K}}$ and varieties $[M_s(t^*)]_{s\in\mathbf{K}}$ along the BGP.

BGP, quantification: algorithm

- Assume $\phi = 1$. Euler equation + common g^* implies real interest rate equalization:

$$\frac{\dot{C}_s(t)}{C_s(t)} = g_s = g^* = \left[\frac{r_s(t)}{P_s(t)} - \rho\right] \qquad \forall s \in \mathbf{K}$$

- After some initial guess of $\mathbf{M}(t^*)$, solve for cross sectional eqm

- Calculate $\frac{r_s(t)}{P_s(t)}$. Countries with "too small" $M_s(t) \implies$ high returns for R&D (high $r\,/\,P$), and vice versa

- Then use:

$$\dot{M}_s(t) = \left(\frac{r_s(t)}{P_s(t)} M_s(t) + \psi \frac{w_s(t)}{P_s(t)} L_s - \rho M_s(t)\right)$$

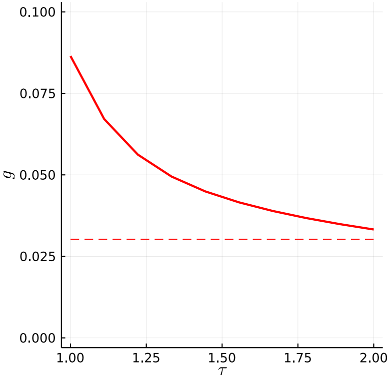
to update $M_s(t^*)$ until real interest rates equalize.

BGP, quantification

Parameters	
$\psi = 0.004$	
$\theta = 7$	
$\sigma = 2$	
$\phi = 1$	
$\rho = 0.03$	
$\alpha = 1/3$	
$\mathbf{L} = [1, 1, 1]$	
$\tau_{sd} = 1.1 \qquad (\forall s \neq d)$	
$\mathbf{T} = [1, 1, 1]$	$\mathbf{T} = [1, 2, 3]$
Stationary Distributions	
$\frac{w(t^*)}{P(t^*)} = [1, 1, 1]$ $M(t^*) = [1/3, 1/3, 1/3]$	$\frac{w(t^*)}{P(t^*)} = [1, 1.17, 1.29]$ $M(t^*) = [.25, .33, .39]$
Growth rates	
$g^* = 1.2\%$	$g^* = 1.58\%$

Comparative Statics

As $\tau \to 1$, the growth rate increases.



Proposition (Comparative statics of the BGP growth rate)

If $\tau_{sd} \in (1, \infty)$ for all $s \neq d$, along the BGP, $\frac{\partial g}{\partial \tau_{sd}} < 0$ for an arbitrary τ_{sd}.

Welfare

- Let $\phi = 1$, then $u(c) = \log(C)$ and, along BGP:

$$\int_{t^*}^{\infty} \exp\{-\rho t\} \log\left(\exp\{g^* t\} C_s(t^*)\right) dt = \underbrace{\frac{\log\left(C_s(t^*)\right)}{\rho}}_{\text{static}} + \underbrace{\frac{g^*}{\rho^2}}_{\text{dynamic}}$$

- Margin of this mechanism first over component is ambiguous...
- ...but it unambiguously increases dynamic gains from trade.

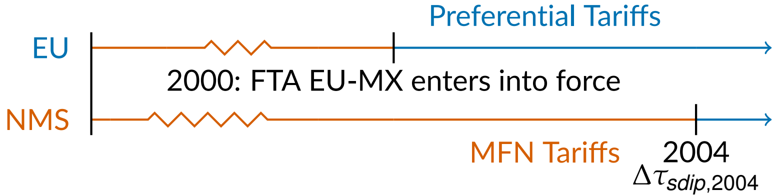
Testing the mechanism

- Mechanism: Trade liberalization $\rightarrow \uparrow$ Expected Exports $\rightarrow \uparrow$ Expected Profits $\rightarrow \uparrow$ Investment in R&D $\rightarrow \uparrow$ Product innovation

- I will test this causal link using reduced-form evidence

Testing the mechanism

- Mechanism: Trade liberalization $\rightarrow \uparrow$ Expected Exports $\rightarrow \uparrow$ Expected Profits $\rightarrow \uparrow$ Investment in R&D $\rightarrow \uparrow$ Product innovation
- I will test this causal link using reduced-form evidence
- Strategy: exploit 2004 enlargement of the EU and access of New Member States (NMS) of previously negotiated EU trade-agreements.
- Example: EU-Mexico Trade Agreement (FTA EU-MX):



Constructing the trade shock

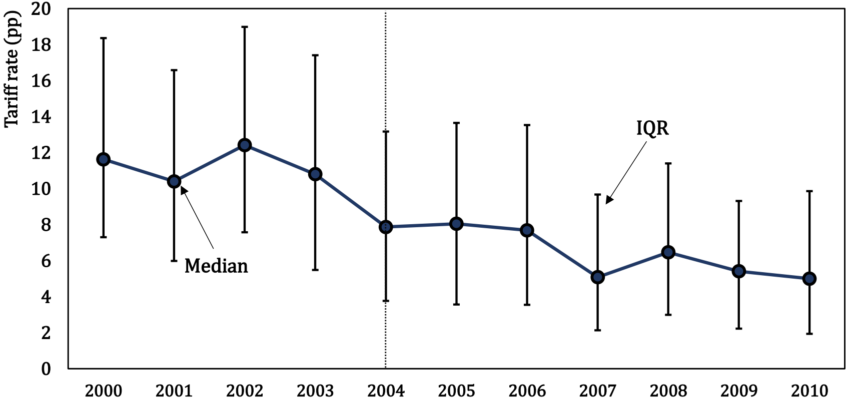
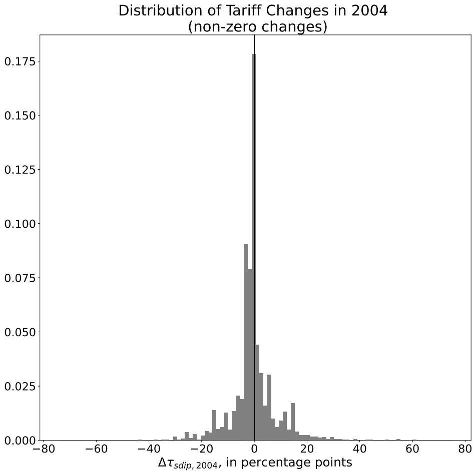


Figure: Interquartile Range Bilateral HS6-Product-Level Tariff Rates Between New Member States (2004 EU Enlargement) and Set of Countries that Concluded Trade Agreements with EU prior to 2004. Source: Constructed from WITS Preferential and MFN databases.

Constructing the trade shock



- Understanding the source of variation:

$$\underbrace{s}_{\text{9 NMS}} \times \underbrace{d}_{\text{13 trade partners}} \times \underbrace{p}_{\approx\text{2000 product codes}}$$

EU Enlargement Timeline Pre-2004 Trade Agreements List

Product List Example

- $> 200k$ total, $30k$ nonzero obs
- Granularity of the data $\implies$ focus on within source-destination-industry-horizon (across product) variation

Entry regressions

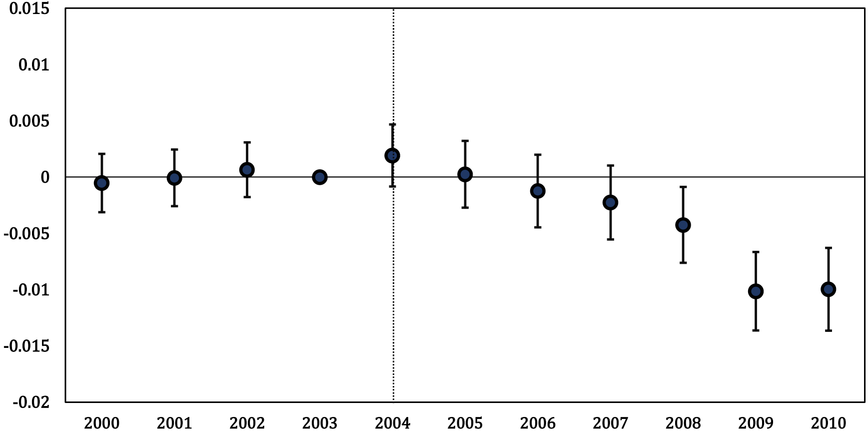
$$P(\underbrace{X_{sdip,h}}_{\text{Exports}} > 0 | \underbrace{Y_{s\cdot ip,2003}}_{\text{Production}} = 0) \quad = \quad \alpha_h + \beta_h \cdot \Delta\tau_{sdip,2004} + \gamma_{sdi,h} + \nu_{sdip,h}$$

$$\text{for each } h \in \{2000, \cdots, 2010\}$$

- Event-study design using local projections in a linear probability model
- Large-N in each cross-section permits many fixed-effects: $\gamma_{sdi,h}$ are source-destination-industry interactions for each h
- Intuition: identification is robust to Poland policymakers endogenously targeting EU accession to have preferential access to Mexico's car industry (relative to other industries and countries), but **not** if they want to have preferential access to compact cars relative to SUVs in Mexico.

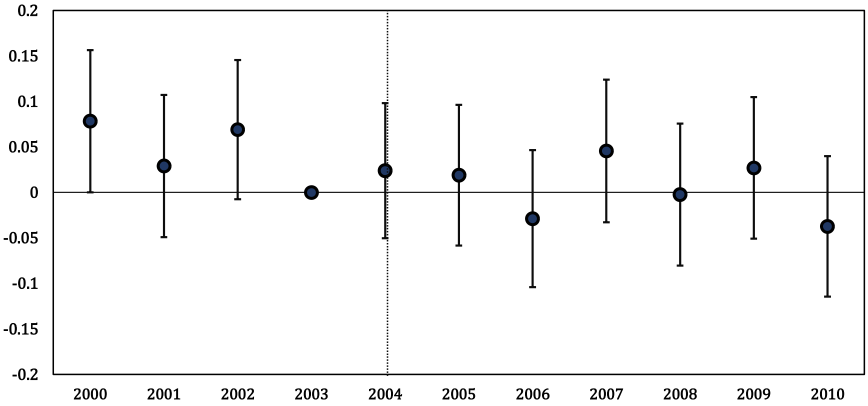
Entry regressions

$$P(\underbrace{X_{sdip,h}}_{\text{Exports}} > 0 | \underbrace{Y_{s\cdot ip,2003}}_{\text{Production}} = 0) = \alpha_h + \beta_h \cdot \Delta\tau_{sdip,2004} + \gamma_{sdi,h} + \nu_{sdip,h}$$



Continuation regressions

$$P(\underbrace{X_{sdip,h}}_{\text{Exports}} > 0 | \underbrace{Y_{s \cdot ip,2003}}_{\text{Production}} = 1) = \alpha_h + \beta_h \cdot \Delta\tau_{sdip,2004} + \gamma_{sdi,h} + \nu_{sdip,h}$$



Market access regressions

- An increase in market access by 1 percentage point increases the probability of starting to produce and export a given product by about 1 percent.
- $\approx$ one third conditional mean $\mathbb{E}\left[X_{sdip,h} > 0 \middle| Y_{s\cdot ip,2003} = 0, h > 2003\right] = 2.9\%$
- Conditional on initial production, additional market access has no impact on exports.

Conclusion and next steps

- Novel framework that integrates insights of trade and macroeconomics, reconciling mechanisms of specialization and market size.

- (Partial) quantitative framework that (so far) solves for endogenous BGP; hope to have transition dynamics.

- Documentation of novel facts and causal evidence consistent with mechanism of the model exploiting enlargement of the European Union.

- Next steps for summer (in terms of priority):
    - algo for transition dynamics;
    - calibration of quantitative exercise;
    - extending reduced-form work with firm-level patents OECD data.

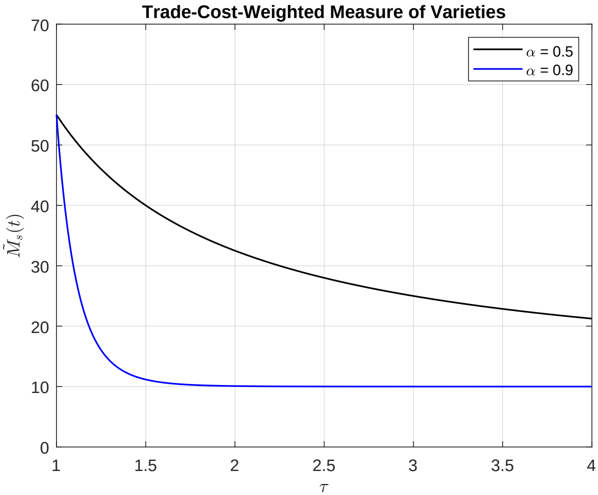
Transition dynamics

- Autarky: back to Romer model $\implies$ no transition dynamics (known result).
- Free trade: world economy becomes fully integrated Romer economy $\implies$ no transition dynamics.
- $\tau \in (1, \infty)$: transition dynamics! **why?**
- Have not yet fully solved transition dynamics in GE, some results in PE deliver important intuition.

Transition dynamics in PE (i)

- What is the technology state?
- Autarky: $\tilde{M}_s(t) \propto M_s(t)$, linear in own measure.
- Free trade: $\tilde{M}_s(t) \propto \sum_{k \in \mathbf{K}} M_k(t)$, linear in sum of measures.
- $\tau \in (1, \infty)$: $\tilde{M}_s(t) \propto \sum_{k \in \mathbf{K}} M_k(t) \cdot (\tau_{ks} P_k(t))^{-\frac{\alpha}{1-\alpha}}$, substitution across sources.

Transition dynamics in PE (ii)



Transition dynamics in GE

- Intuition:
- Countries with initially relatively many varieties $\implies$ lower relative price in final and intermediate goods ($\sim$ high productivity).
- $\implies \downarrow \pi$ and $\downarrow r \implies \downarrow g$ over transition path.
- At BGP, real returns equalize $\implies$ g equalize.

Qualitative Questionnaire

- After your country joined the European Union, did your company:
  - start producing more products/services or varieties;
  - start producing fewer products/services or varieties; or
  - keep producing about the same number of products/services or varieties?

- If your company changed the number of products/services or varieties after EU accession, how was the change implemented and what were the results? Please include any important information or relevant anecdotes.

- If your company changed the number of products/services or product/service varieties after EU accession, was the decision primarily motivated by access to new technologies/imports, access to new markets/exports, or both? Explain.

- After your country joined the European Union, did your company:
  - stay in the same industry;
  - expanded to another industry; or
  - move completely to a new industry?

- If your company expanded to another industry or moved to a new industry. Please explain whether the change was related to your country's EU accession.

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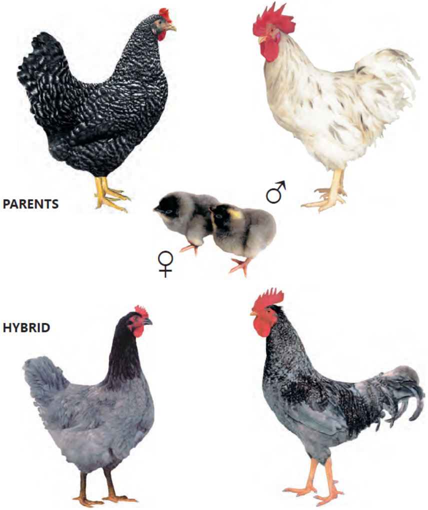
Czech Biotech Firm Products



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Enlargement of the EU

1957
- Belgium
- France
- Germany
- Italy
- Luxembourg
- Netherlands

1973
- Denmark
- Ireland
- UK

1981
- Greece

1986
- Portugal
- Spain

1995
- Austria
- Finland
- Sweden

2004
- Cyprus
- Czechia
- Estonia
- Hungary
- Latvia
- Lithuania
- Malta
- Poland
- Slovakia
- Slovenia

2007
- Bulgaria
- Romania

2013
- Croatia

Future?
- Albania
- Moldova
- Montenegro
- N. Macedonia
- Serbia
- Turkey
- Ukraine

EU Pre-2004 Trade Agreements

Partner	Signed	Provisional application	Full entry into force
Switzerland	1972		1973
Iceland	1992		1994
Norway	1992		1994
Turkey	1995		1995
Tunisia	1995		1998
Israel	1995	1996	2000
Mexico	1997		2000
Morocco	1996		2000
Jordan	1997		2002
Egypt	2001		2004
North Macedonia	2001	2001	2004
South Africa	1999	2000	2004
Chile	2002	2003	2005

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Prodcom list example

Prodcom Code	Description
…	…
10511133	Milk and cream of a fat content by weight of <= 1%, not concentrated nor containing added sugar or other sweetening matter, in immediate packings of a net content <= 2 l
10511137	Milk and cream of a fat content by weight of <= 1%, not concentrated nor containing added sugar or other sweetening matter, in immediate packings of a net content > 2 l
10511142	Milk and cream of a fat content by weight of > 1% but <= 6%, not concentrated nor containing added sugar or other sweetening matter, in immediate packings of a net content <= 2 l
10511148	Milk and cream of a fat content by weight of > 1% but <= 6%, not concentrated nor containing added sugar or other sweetening matter, in immediate packings of a net content > 2 l
10511210	Milk and cream of a fat content by weight of > 6% but <= 21%, not concentrated nor containing added sugar or other sweetening matter, in immediate packings of <= 2 l
10511220	Milk and cream of a fat content by weight of > 6% but <= 21%, not concentrated nor containing added sugar or other sweetening matter, in immediate packings of > 2 l
10511230	Milk and cream of a fat content by weight of > 21%, not concentrated nor containing added sugar or other sweetening matter, in immediate packings of <= 2 l
10511240	Milk and cream of a fat content by weight of > 21%, not concentrated nor containing added sugar or other sweetening matter, in immediate packings of > 2 l
10512130	Skimmed milk powder (milk and cream in solid forms, of a fat content by weight of <= 1,5%), in immediate packings of <= 2,5 kg
10512160	Skimmed milk powder (milk and cream in solid forms, of a fat content by weight of <= 1,5%), in immediate packings of > 2,5 kg
…	…

◂ back

R&D, details

- HHs use equity market to invest $I_s(t)$ units of the final good in R&D;
- *Poisson* process success rate with flow arrival rate equal to $\psi I_s(t)dt$;
- Value of an invention:

$$V_k(t,\nu) = \int_t^{\infty} \exp\left\{-\int_t^{\tau} r_k(s)ds\right\} \pi_k(\tau,\nu)d\tau$$

- Free-entry: $\psi V_k(t,\nu)I_k(t) - P_k(t)I_k(t) \geq 0 \implies V_k(t,\nu) = \frac{P_k(t)}{\psi}$
- Non-arbitrage condition:

$$\underbrace{r_k(t)}_{\text{return on savings}} = \underbrace{\frac{\psi\pi_k(t,\nu)}{P_k(t)} + \frac{\dot{P}_k(t)}{P_k(t)}}_{\text{return on R\&D}}$$

◂ back

Equilibrium

A world economy equilibrium is defined by allocations $[C_s(t), I_s(t), A_s(t), M_s(t), c_s(t,\omega), \ell_s(t,\omega), x_{sd}(t,\omega,\nu), \pi_s(t,\nu)]$ and prices $[w_s(t), r_s(t), P_s(t), p_{ss}(t,\omega), p_{sk}(t,\omega,\nu)]$ such that:

- given prices, $C_s(t), I_s(t), A_s(t), c_s(t,\omega)$ maximize utility;
- given prices, $\ell_s(t,\omega)$ and $x_{sd}(t,\omega,\nu)$ final goods firms maximize profits;
- given $[P_s(t), p_{ss}(t,\omega)]$ and demands $x_{sd}(t,\omega,\nu)$, intermediate firms choose $p_{sk}(t,\omega,\nu)$ to maximize profits;
- increase in the measure of varieties satisfies: $\dot{M}_s(t) = \psi I_s(t)$;
- labor, asset, and goods markets clear; and
- trade balances.

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Sketch of Proof, Common Growth Rate

$$
\begin{aligned}
\frac{w_s(t+1)L_s}{w_s(t)L_s} = g_{w_s} &= \frac{\sum_{s\in\mathbf{K}} \lambda_{sd}(t+1)P_d(t+1)Y_d(t+1)}{\sum_{s\in\mathbf{K}} \lambda_{sd}(t)P_d(t)Y_d(t)} \\
&= \sum_{s\in\mathbf{K}} \gamma_{sd}(t)\left(g_{\lambda_{sd}} \cdot g_{P_d} \cdot g_{Y_d}\right) \qquad (\forall t \geq t^*)
\end{aligned}
$$

where $\gamma_{sd}(t) \equiv \frac{\lambda_{sd}(t)P_d(t)Y_d(t)}{\sum_{s\in\mathbf{K}} \lambda_{sd}(t)P_d(t)Y_d(t)}$.

- By assumption, along the BGP, relative prices must be constant, so $g_{P_d} = 1$.
- Since $(\forall t)\lambda_{sd}(t) \in (0,1)$ for an equilibrium to exist, $g_{\lambda_{sd}} = 1$. Suppose $g_{\lambda_{sd}} > 1$. Then $\lim_{t\to\infty} \lambda_{sd}(t) = \infty \implies$ contradiction. Suppose $g_{\lambda_{sd}} < 1$ for some s. This implies that $g_{\lambda_{s'd}} > 1$ for some other s', $\implies$ contradiction. Therefore, $g_{\lambda_{sd}} = 1$ for all sd.

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Sketch of Proof, Common Growth Rate

Therefore:

$$g_{w_s} \quad = \quad \sum_{s \in \mathbf{K}} \gamma_{sd}(t) g_{Y_d} \qquad (\forall t \geq t^*)$$

where $\gamma_{sd}(t) \equiv \frac{\lambda_{sd}(t) P_d(t) Y_d(t)}{\sum_{s \in \mathbf{K}} \lambda_{sd}(t) P_d(t) Y_d(t)}$.

- The last step is showing that, since g_{Y_d}, g_{w_s} are all constant, the weights $\gamma_{sd}(t) = \gamma_{sd}$ must be fixed along the BGP and $(\forall s, d) g_{Y_d} = g_{Y_s} = g_{w_s} = g_{w_d}$.

- Suppose $g_{Y'_d} > g_{Y_d}$ for some d' compared to others d. In that case, $\gamma_{sd'}(t)$ will grow faster relative to other d. However, that implies that g_{w_s} is not a constant $\implies$ contradiction.

- It is easy to show that $g_{c_s} = g_{w_s} = g_{Y_s}$. Therefore, all countries grow at a common rate g^* along the BGP.

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