International Trade: Lecture 13

The Standard Trade Model, Gravity, and Welfare

Carlos Góes[1]

[1]George Washington University

Fall 2025

So far…

- Ricardian: differences in technology induce comparative advantage; constant returns

So far…

- Ricardian: differences in technology induce comparative advantage; constant returns
- Specific-Factors: specific factors $\rightarrow$ decreasing returns $\rightarrow$ distributional effects

So far…

- Ricardian: differences in technology induce comparative advantage; constant returns
- Specific-Factors: specific factors $\rightarrow$ decreasing returns $\rightarrow$ distributional effects
- Heckscher–Ohlin: same tech, different endowments $\rightarrow$ patterns of specialization

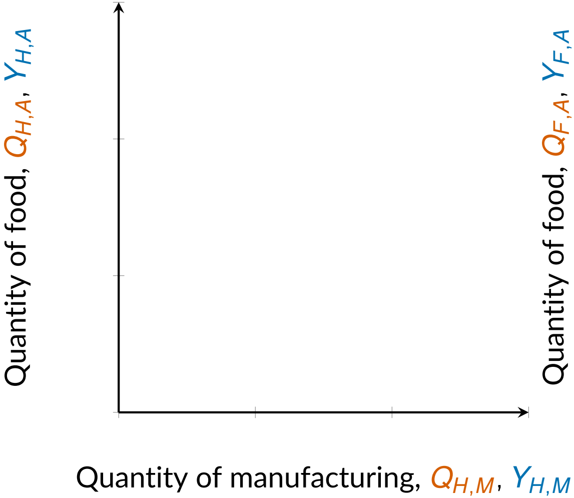
So far...

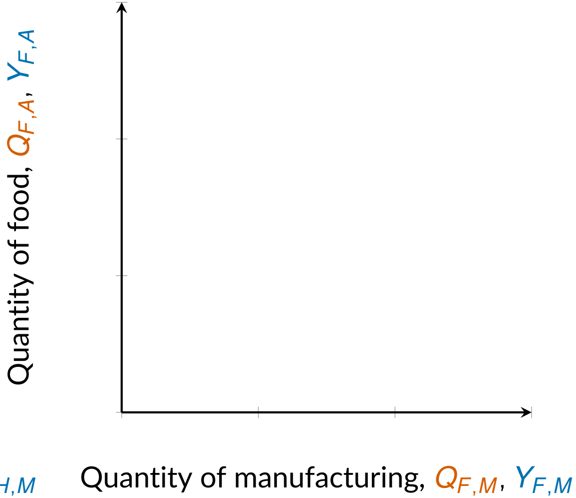
- Ricardian: differences in technology induce comparative advantage; constant returns
- Specific-Factors: specific factors $\rightarrow$ decreasing returns $\rightarrow$ distributional effects
- Heckscher–Ohlin: same tech, different endowments $\rightarrow$ patterns of specialization
- Standard Trade Model: trade happens due to differences $\rightarrow$ changes in terms of trade

This class

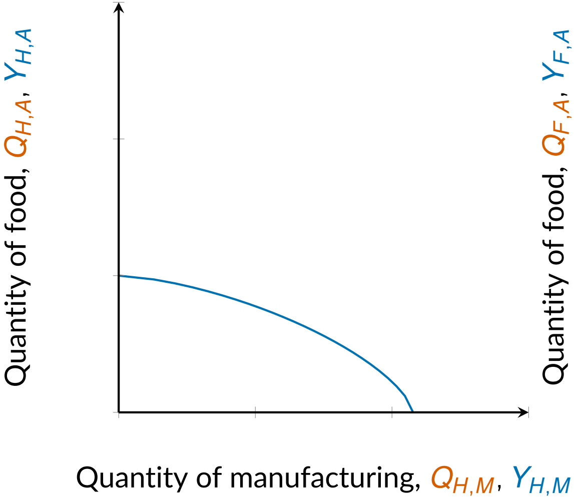
- Welfare gains from trade across models
- Changes in Terms of Trade (ToT) + growth bias
- The Gravity Equation
- Empirical estimates of welfare gains
- Data Lab will go hands-on later.

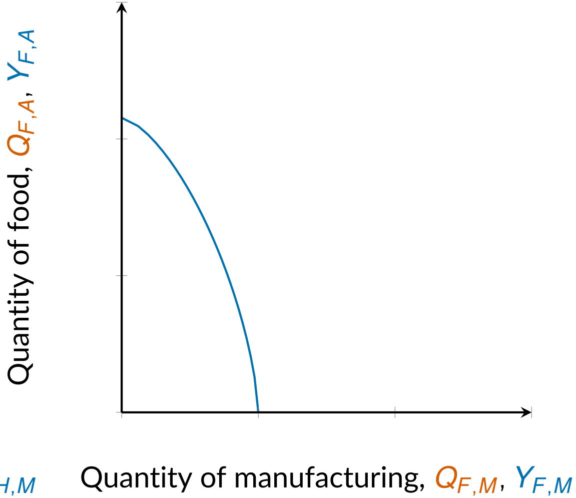
Graphical representation



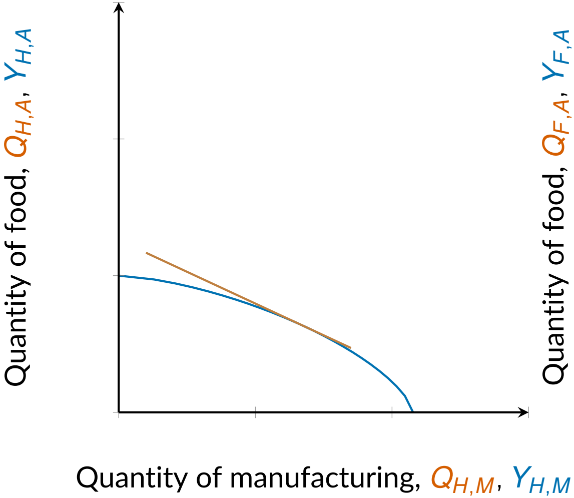


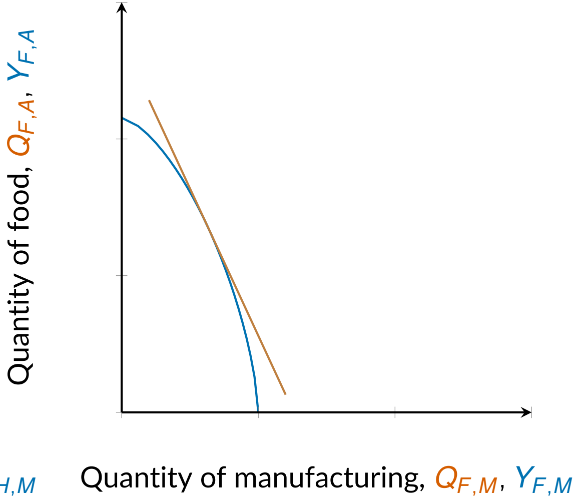
Graphical representation: PPF



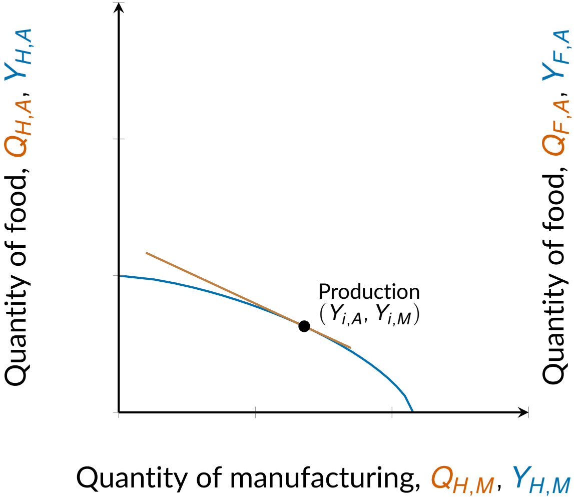


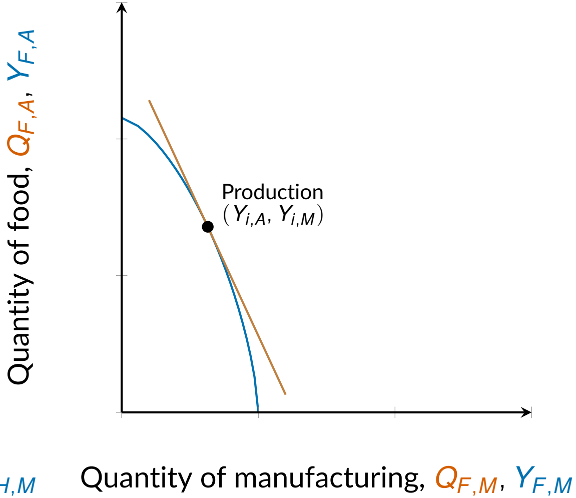
Graphical representation: autarky prices



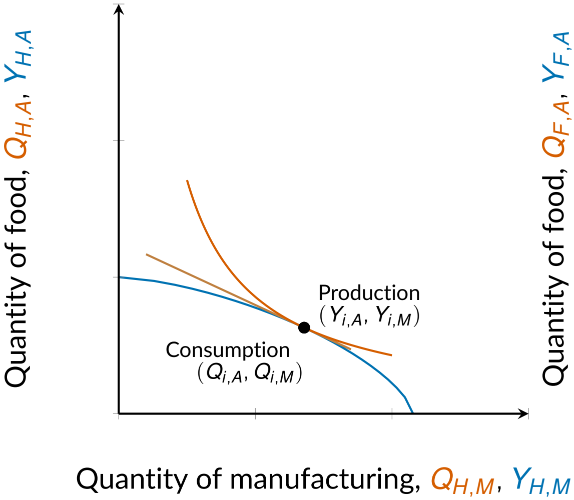


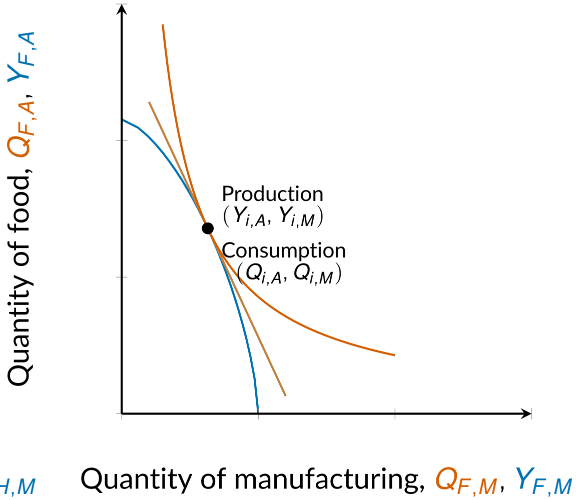
Graphical representation: autarky production



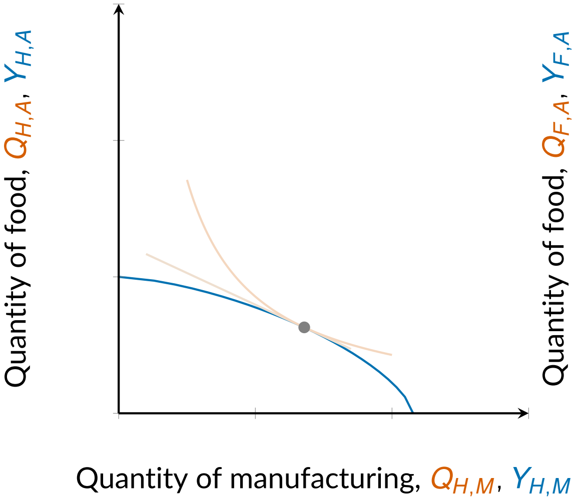


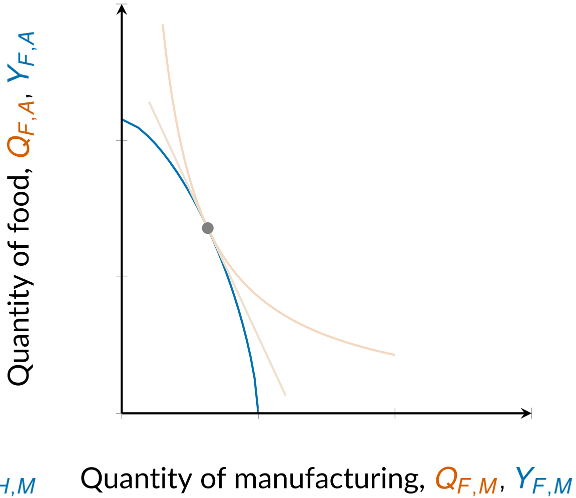
Graphical representation: autarky equilibrium (consumption + production)



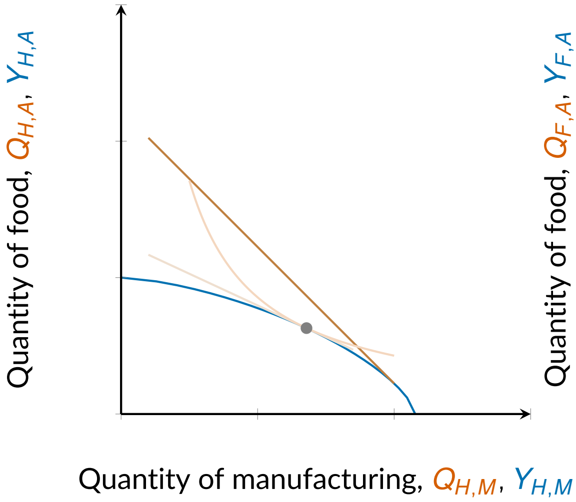


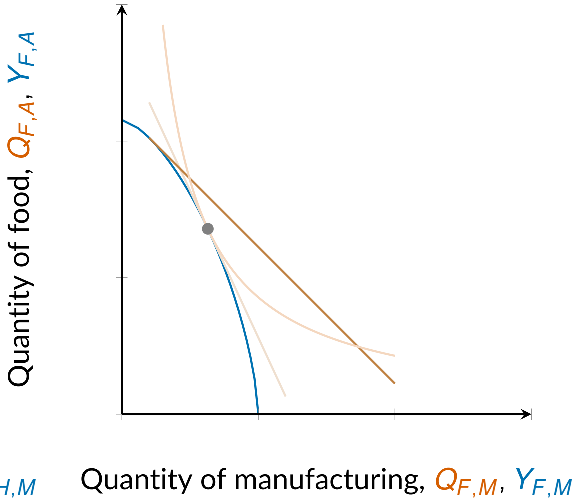
Graphical representation: autarky equilibrium (consumption + production)



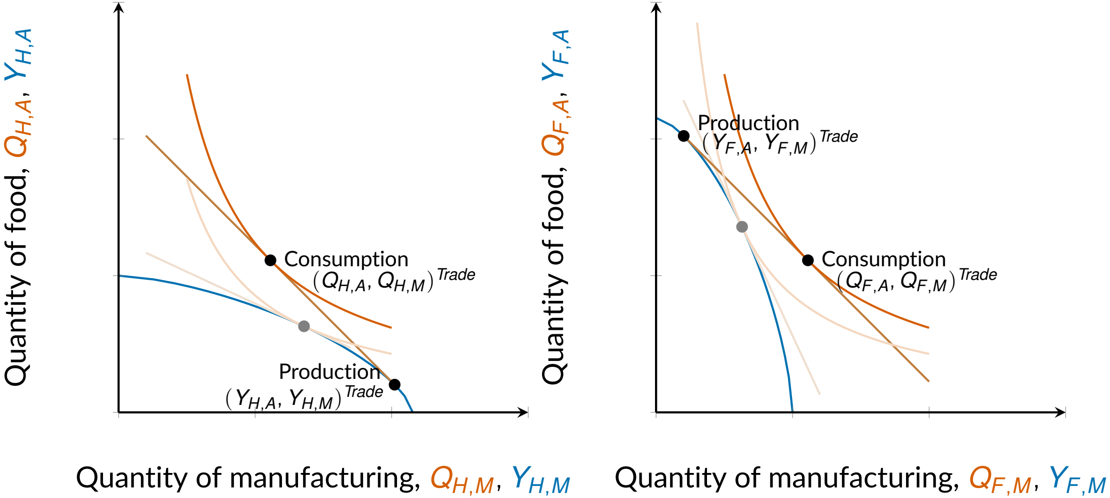


Graphical representation: free trade prices

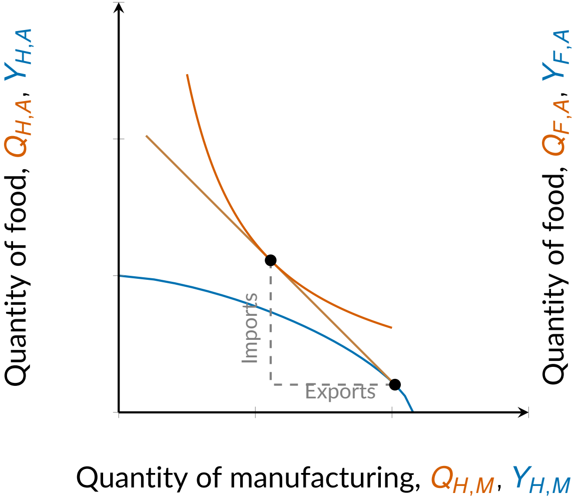


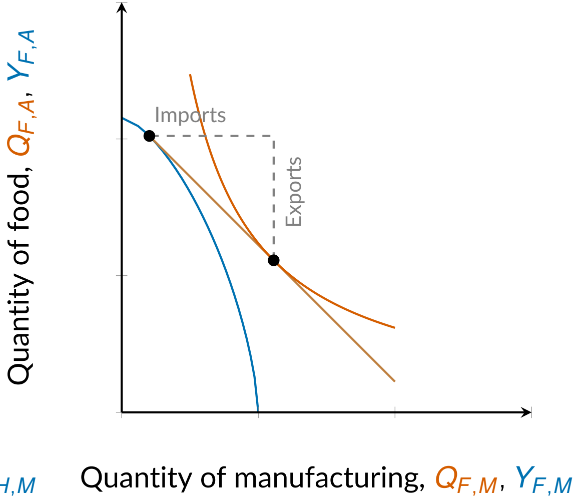


Graphical representation: trade equilibrium

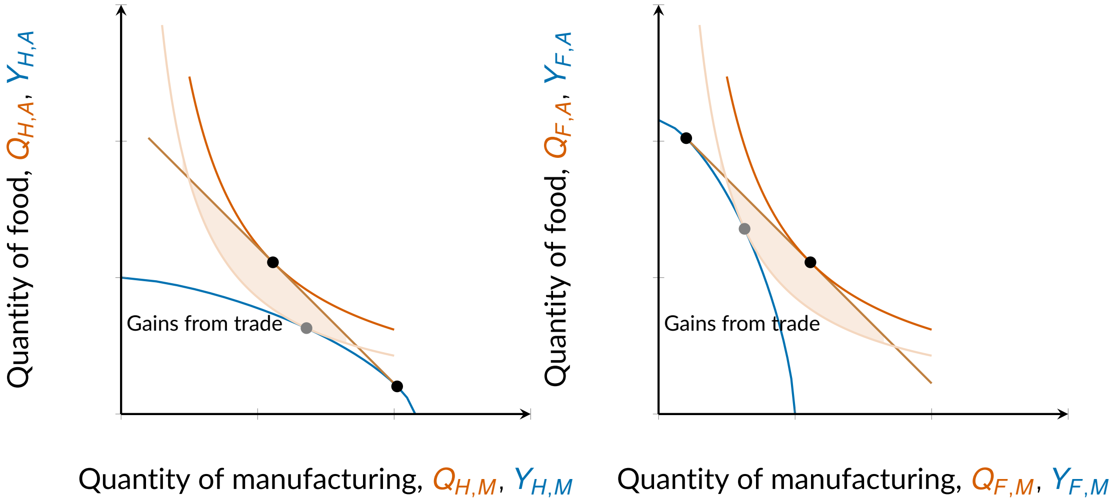


Graphical representation: specialization patterns





Graphical representation: gains from trade



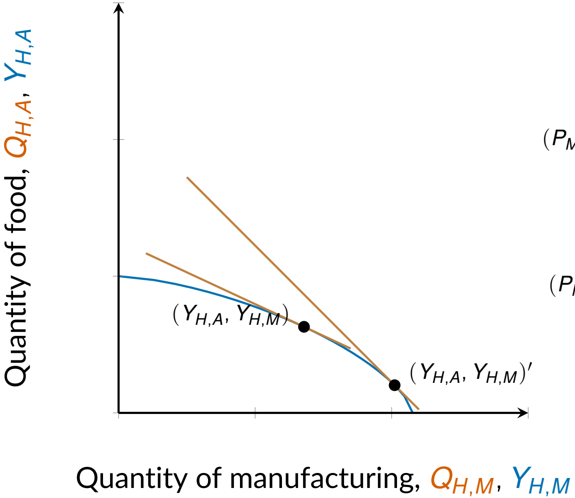
Key STM objects and notation

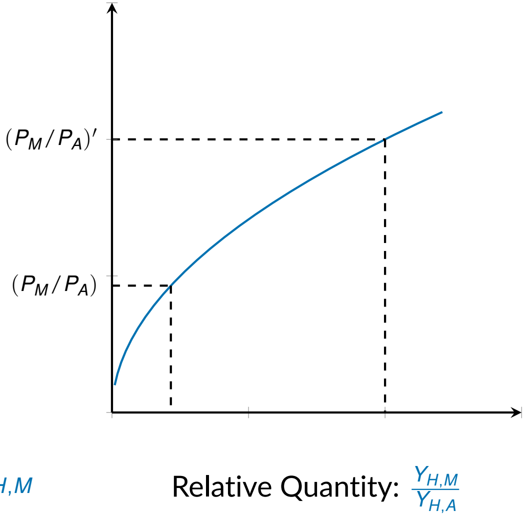
- Two goods: M (manufacturing) and A (agriculture/food). Relative price: $p \equiv P_M / P_A$
- Domestic relative supply: $RS(p) \equiv \frac{Y_M(p)}{Y_A(p)}$
- Domestic relative demand: $RD(p) \equiv \frac{Q_M(p,Y)}{Q_A(p,Y)}$
- World equilibrium: p^w s.t. $RS^{\text{world}}(p^w) = RD^{\text{world}}(p^w)$
- Terms of Trade: $\text{ToT} = \frac{P_{\text{exports}}}{P_{\text{imports}}}$
    - Do ToT of either country increase or decrease after they open up to trade in the STM?

From PPF to RS

- Relative prices and other underlying forces (factor mobility, diminishing product) characterize production
- At optimum: $MRT_{C,T} = \frac{dQ_A}{dQ_M} = -\frac{P_M}{P_A} = -p$
- As p rises, production tilts to M:
    - RS is upward sloping

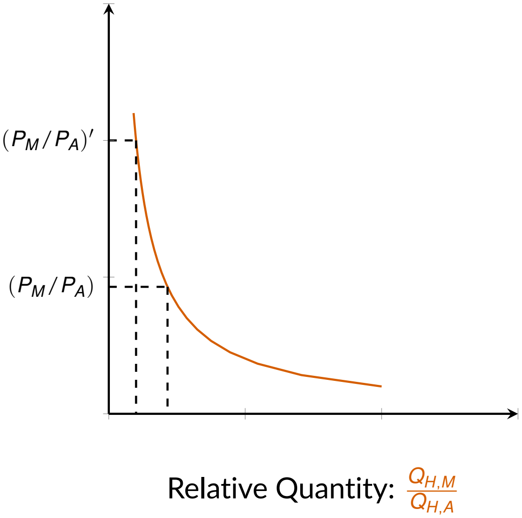
Relative Supply





From preferences to RD

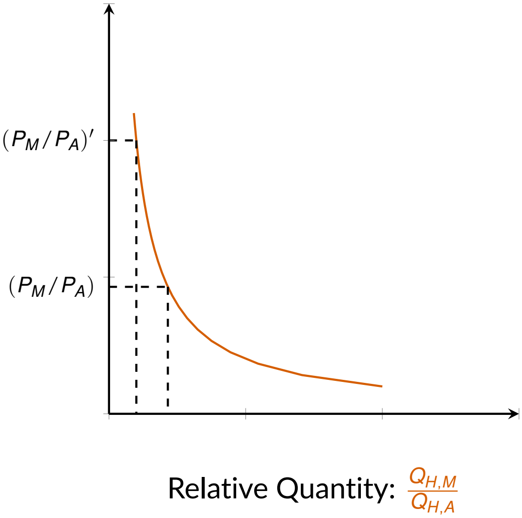
- Consumer maxmiize $Q_M^{\alpha_i} Q_A^{1-\alpha_i}$ s.t. $P_M Q_M + P_A Q_M = I_i$



From preferences to RD

- Consumer maxmiize $Q_M^{\alpha_i} Q_A^{1-\alpha_i}$ s.t. $P_M Q_M + P_A Q_M = I_i$

- As $p = P_M / P_A$ rises, A becomes relatively expensive $\Rightarrow$ RD falls in p



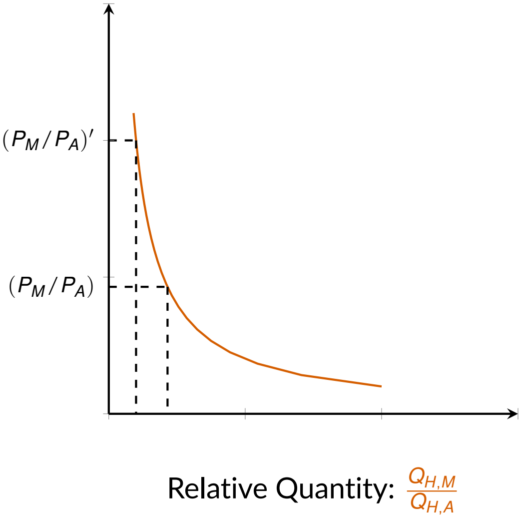
From preferences to RD

- Consumer maxmiize $Q_M^{\alpha_i} Q_A^{1-\alpha_i}$ s.t. $P_M Q_M + P_A Q_M = I_i$

- As $p = P_M / P_A$ rises, A becomes relatively expensive $\Rightarrow$ RD falls in p

- Recall optimal demand functions are:

$$Q_{H,M} = \alpha_H \frac{I_H}{P_M}, \qquad Q_{H,A} = (1 - \alpha_H) \frac{I_H}{P_A}$$



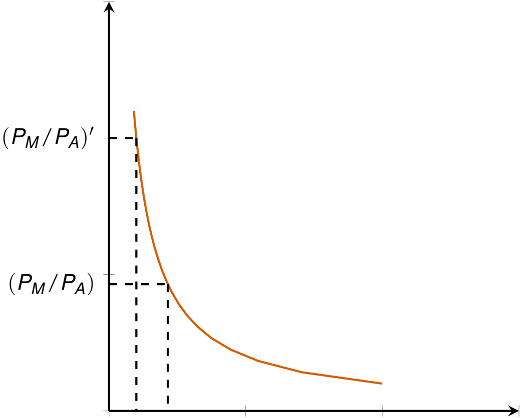
From preferences to RD

- Consumer maxmiize $Q_M^{\alpha_i} Q_A^{1-\alpha_i}$ s.t. $P_M Q_M + P_A Q_M = I_i$
- As $p = P_M / P_A$ rises, A becomes relatively expensive $\Rightarrow$ RD falls in p
- Recall optimal demand functions are:

$$Q_{H,M} = \alpha_H \frac{I_H}{P_M}, \qquad Q_{H,A} = (1 - \alpha_H) \frac{I_H}{P_A}$$

- Therefore:

$$\begin{aligned} \frac{Q_{H,M}}{Q_{A,M}} &= \frac{\alpha_H}{1-\alpha_H} \frac{I_H / P_M}{I_H / P_A} = \frac{\alpha_H}{1-\alpha_H} \frac{1}{P_M / P_A} \\ &= \frac{\alpha_H}{1-\alpha_H} \frac{1}{p} \end{aligned}$$



Relative Quantity: $\frac{Q_{H,M}}{Q_{H,A}}$

Domestic equilibrium and gains from trade

- Autarky: p^A at $RS(p) = RD(p)$
- With trade: price p^w pins production at RS and consumption on budget line with slope $-p^w$
- Gains: trade moves consumption to higher indifference curve if $p^w \neq p^A$

World RS and RD

- World RS: (roughly) horizontal aggregation of country supplies $\Rightarrow RS^W(p)$ is upward sloping with plateaus at specialization ranges

- World RD: aggregation of demands; with homothetic preferences, same shape as domestic RD

- World price p^w: intersection $RS^W(p)$ and $RD^W(p)$

STM: World Equilibrium

figure

From the Standard Trade Model to Trade Flows

- Production side: two countries (US, COL), two goods (C, R). Each good p in country i requires $a_{i,p}$ units of labor per unit output.

$$P_{i,p} = a_{i,p} w_i \quad \text{(zero profits)}$$

- Demand side: identical Cobb–Douglas preferences:

$$U_i = Q_{i,C}^{\alpha_i} Q_{i,R}^{1-\alpha_i}, \quad Q_{i,C} = \alpha_i \frac{w_i L_i}{P_{i,C}}.$$

Each country spends a fixed share α_i of income on computers and $1 - \alpha_i$ on roses.

From the Standard Trade Model to Trade Flows

- Production side: two countries (US, COL), two goods (C, R). Each good p in country i requires $a_{i,p}$ units of labor per unit output.

$$P_{i,p} = a_{i,p} w_i \quad \text{(zero profits)}$$

- Demand side: identical Cobb–Douglas preferences:

$$U_i = Q_{i,C}^{\alpha_i} Q_{i,R}^{1-\alpha_i}, \quad Q_{i,C} = \alpha_i \frac{w_i L_i}{P_{i,C}}.$$

  Each country spends a fixed share α_i of income on computers and $1 - \alpha_i$ on roses.

- Iceberg trade cost: to deliver one unit abroad, exporters must ship $\tau_{s \to d} \geq 1$ units.

From the Standard Trade Model to Trade Flows

- Production side: two countries (US, COL), two goods (C, R). Each good p in country i requires $a_{i,p}$ units of labor per unit output.

$$P_{i,p} = a_{i,p} w_i \quad \text{(zero profits)}$$

- Demand side: identical Cobb–Douglas preferences:

$$U_i = Q_{i,C}^{\alpha_i} Q_{i,R}^{1-\alpha_i}, \quad Q_{i,C} = \alpha_i \frac{w_i L_i}{P_{i,C}}.$$

  Each country spends a fixed share α_i of income on computers and $1 - \alpha_i$ on roses.

- Iceberg trade cost: to deliver one unit abroad, exporters must ship $\tau_{s \to d} \geq 1$ units.
- Delivered price in the destination:

$$P_{C,d} = \tau_{s \to d} \, a_{C,s} \, w_s$$

From Quantities to the Gravity Equation

- Resulting import demand (quantities)

$$Q_{s\to d} = \alpha_d \frac{w_d L_d}{\tau_{s\to d}\, a_{C,s}\, w_s} = \underbrace{\frac{1}{a_{C,s}\, w_s}}_{\text{source factors}} \times \underbrace{\alpha_d w_d L_d}_{\text{destination factors}} \times \underbrace{\frac{1}{\tau_{s\to d}}}_{\text{bilateral trade costs}}$$

(Imports rise with importer income, and fall with trade and production costs.)

From Quantities to the Gravity Equation

- Resulting import demand (quantities)

$$Q_{s\to d} = \alpha_d \frac{w_d L_d}{\tau_{s\to d}\, a_{C,s}\, w_s} = \underbrace{\frac{1}{a_{C,s}\, w_s}}_{\text{source factors}} \times \underbrace{\alpha_d w_d L_d}_{\text{destination factors}} \times \underbrace{\frac{1}{\tau_{s\to d}}}_{\text{bilateral trade costs}}$$

(Imports rise with importer income, and fall with trade and production costs.)

  - increasing in the importing country's income ($w_d L_d$),
  - decreasing in bilateral trade costs ($\tau_{s\to d}$),
  - decreasing in the exporter's unit labor requirement ($a_{p,s}$), and
  - decreasing in the exporter's wage (w_s), which represent production costs.

From Quantities to the Gravity Equation

- Resulting import demand (quantities)

$$Q_{s\to d} = \alpha_d \frac{w_d L_d}{\tau_{s\to d}\, a_{C,s}\, w_s} = \underbrace{\frac{1}{a_{C,s}\, w_s}}_{\text{source factors}} \times \underbrace{\alpha_d w_d L_d}_{\text{destination factors}} \times \frac{1}{\underbrace{\tau_{s\to d}}_{\text{bilateral trade costs}}}$$

(Imports rise with importer income, and fall with trade and production costs.)

  - increasing in the importing country's income ($w_d L_d$),
  - decreasing in bilateral trade costs ($\tau_{s\to d}$),
  - decreasing in the exporter's unit labor requirement ($a_{p,s}$), and
  - decreasing in the exporter's wage (w_s), which represent production costs.

- In general, exports from the s to d will satisfy:

$$X_{sd} \propto \frac{\underbrace{\omega_s}_{\text{source factors}} \times \underbrace{\gamma_d}_{\text{destination factors}}}{\underbrace{\tau_{sd}^{\theta}}_{\text{bilateral trade costs}}}$$

Gravity: Physics vs. Economics

Newton's Law of Gravitation

$$F_{ij} = G \frac{m_i \, m_j}{r_{ij}^2}$$

- m_i, m_j: physical masses
- r_{ij}: distance between objects
- G: universal gravitational constant

Gravity: Physics vs. Economics

Newton's Law of Gravitation

$$F_{ij} = G \frac{m_i \, m_j}{r_{ij}^2}$$

- m_i, m_j: physical masses
- r_{ij}: distance between objects
- G: universal gravitational constant

Trade Gravity (reduced form)

$$X_{ij} = K \frac{Y_i \, Y_j}{d_{ij}^{\theta}}$$

- Y_i, Y_j: *economic masses* (GDP/total expenditure)
- d_{ij}: bilateral distance (proxy for trade costs)
- $\theta > 0$: trade-cost elasticity (empirical)
- K: scaling constant / fixed effects

Why gravity?

- Connects trade flows to economic size and trade costs

- Workhorse for counterfactuals (tariffs, borders, infrastructure) and for mapping STM shocks to data

Why gravity?

- Connects trade flows to economic size and trade costs
- Workhorse for counterfactuals (tariffs, borders, infrastructure) and for mapping STM shocks to data
- It works!

Why gravity?

- Connects trade flows to economic size and trade costs

- Workhorse for counterfactuals (tariffs, borders, infrastructure) and for mapping STM shocks to data

- It works!

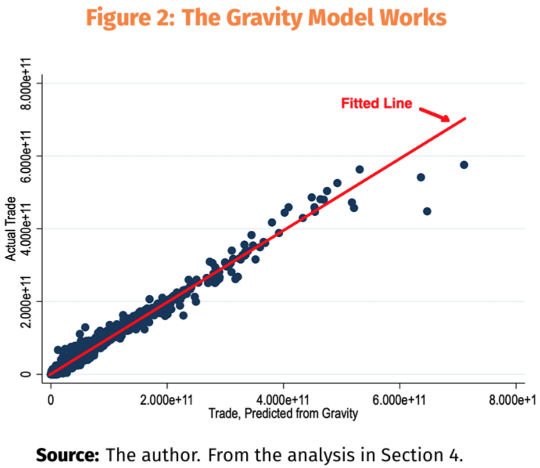


Figure: (Yotov, 2025)

Trade Gravity: Generalizing

- Note that:

$$X_{ij} = K \frac{Y_i \, Y_j}{d_{ij}^{\theta}}$$

  is a special case of:

$$X_{sd} \propto \frac{\underbrace{\omega_s}_{\text{source factors}} \times \underbrace{\gamma_d}_{\text{destination factors}}}{\underbrace{\tau_{sd}^{\theta}}_{\text{bilateral trade costs}}}$$

- So the Standard Trade Models (and most trade models we use) satisfy the gravity equation

What are bilateral trade costs?

- We have been using τ_{sd} to denote "bilateral trade costs"

What are bilateral trade costs?

- We have been using τ_{sd} to denote "bilateral trade costs"
- Conceptually, if producers want one unit of a good to arrive at the destination, they prduce $\tau_{sd} \geq 1$.

What are bilateral trade costs?

- We have been using τ_{sd} to denote "bilateral trade costs"
- Conceptually, if producers want one unit of a good to arrive at the destination, they prduce $\tau_{sd} \geq 1$.
- But what could influence?
  - Distance
  - Common/different language
  - Shared colonial history
  - Shared border
  - Geographical features
  - Transportation methods
  - Pirates/bandits, etc
  - Tariffs

What are bilateral trade costs?

- We have been using τ_{sd} to denote "bilateral trade costs"
- Conceptually, if producers want one unit of a good to arrive at the destination, they prduce $\tau_{sd} \geq 1$.
- But what could influence?
  - Distance
  - Common/different language
  - Shared colonial history
  - Shared border
  - Geographical features
  - Transportation methods
  - Pirates/bandits, etc
  - Tariffs
- We will explore these more during one of the data labs. For now, will give you some flavor on why trade costs matter.

Measuring Trade Costs: *Railroads of the Raj*

- Goal: Quantify how India's 19th–20th century railroad expansion reduced trade costs (and raised welfare).

Measuring Trade Costs: *Railroads of the Raj*

- Goal: Quantify how India's 19th–20th century railroad expansion reduced trade costs (and raised welfare).
- Idea: When trade is costly, identical goods sell for different prices across regions:

$$p_i = p_j \times \tau_{ij} \quad \Longrightarrow \quad \ln(p_i / p_j) = \ln(\tau_{ij})$$

Measuring Trade Costs: *Railroads of the Raj*

- Goal: Quantify how India's 19th–20th century railroad expansion reduced trade costs (and raised welfare).
- Idea: When trade is costly, identical goods sell for different prices across regions:

$$p_i = p_j \times \tau_{ij} \quad \Longrightarrow \quad \ln(p_i/p_j) = \ln(\tau_{ij})$$

- Natural experiment: expansion of India's railway system during colonial times.

Measuring Trade Costs: *Railroads of the Raj*

- Goal: Quantify how India's 19th–20th century railroad expansion reduced trade costs (and raised welfare).
- Idea: When trade is costly, identical goods sell for different prices across regions:

$$p_i = p_j \times \tau_{ij} \quad \Longrightarrow \quad \ln(p_i/p_j) = \ln(\tau_{ij})$$

- Natural experiment: expansion of India's railway system during colonial times.
- Salt as a reference good: tradeable and homogeneous; limited local production.

Measuring Trade Costs: *Railroads of the Raj*

- Goal: Quantify how India's 19th–20th century railroad expansion reduced trade costs (and raised welfare).
- Idea: When trade is costly, identical goods sell for different prices across regions:

$$p_i = p_j \times \tau_{ij} \quad \Longrightarrow \quad \ln(p_i / p_j) = \ln(\tau_{ij})$$

- Natural experiment: expansion of India's railway system during colonial times.
- Salt as a reference good: tradeable and homogeneous; limited local production.
- Intuition: expansion of railroads reduce trade costs $\rightarrow$ prices of salt should converge.

Expansion of India’s railroad system

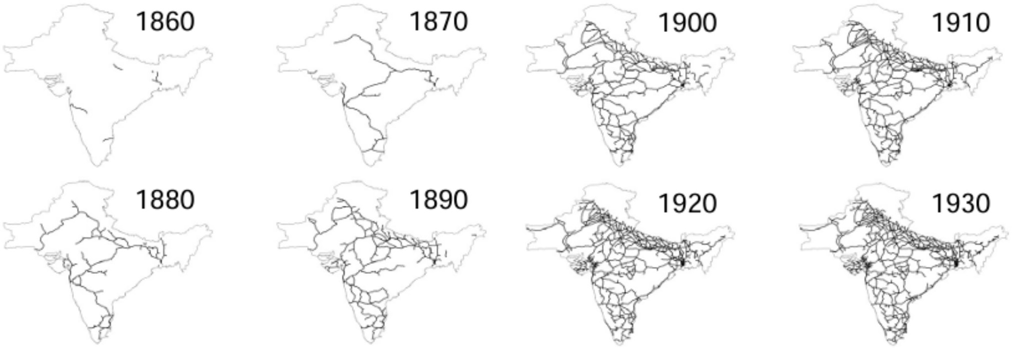


Figure 1: The evolution of India’s railroad network, 1860-1930: These figures display the decadal evolution of the railroad network (railroads depicted with thick lines) in colonial India (the outline of which is depicted with thin lines). The first railroad lines were laid in 1853. This figure is based on a GIS database in which each (approximately) 20 km long railroad segment is coded with a year of opening variable. Source: Author’s calculations based on official publications. See Appendix A for details.

Figure: Source: (Donaldson, 2018)

Measuring Trade Costs: *Railroads of the Raj*

- Donaldson (2018) uses price differences between district pairs as a *revealed measure* of trade frictions.
- Empirical specification:

$$\ln\left(\frac{p_{it}}{p_{jt}}\right) = \alpha + \beta \,\text{Effective Distance}_{ijt} + \gamma X_{ijt} + \varepsilon_{ijt}$$

- Effective distance:
    - shortest available travel distance, given the transport network available in a given year — taking into account which transport modes exist (rail, river, road) and their relative cost-efficiency.
- $\beta > 0 \Rightarrow$ rail connection $\Rightarrow$ smaller effective distance $\Rightarrow$ smaller price gaps $\Rightarrow$ lower trade costs.

Results

TABLE 2—RAILROADS AND TRADE COSTS: STEP 1

Dependent variable: log salt price at destination	(1)	(2)
log effective distance to source, along lowest-cost route	0.088	
(at historical freight rates)	(0.028)	
log effective distance to source, along lowest-cost route		0.169
(at estimated mode costs)		[0.062, 0.296]
Estimated mode costs per unit distance:		1
Railroad (normalized to 1)		N/A
Road		2.375
		[1.750, 10.000]
River		2.250
		[1.500, 6.250]
Coast		6.188
		[5.875, 10.000]
Observations	7,345	7,345
R^2	0.946	0.946

Notes: Regressions estimating equation (12) using data on 6 types of salt (listed in online Appendix A), from 133 districts in Northern India, annually from 1861 to 1930. Column 1 and column 2 estimated by OLS and NLS respectively; both include salt type × year and salt type × destination fixed effects. "Effective distance to source, along lowest-cost route" measures the railroad-equivalent kilometers (because railroad freight rate is normalized to 1) between the salt source and the destination district, along the lowest-cost route given relative mode costs per unit distance. "Historical freight rates" used are 4.5, 3.0, and 2.25 respectively for road, river, and coastal mode costs per unit distance, all relative to rail transport. Standard errors corrected for clustering at the destination district level are reported in parentheses of column 1, and bootstrapped 95 percent confidence intervals are reported in column 2.

Figure: Source: Donaldson, 2018