Econ 110A: Lecture 17

Carlos Góes[1]

[1]UC San Diego

UCSD

Consumption & Income

with and without prices

Two-Period Neoclassical Growth Model

The consumer maximizes lifetime utility subject to the intertemporal budget constraint

$$\max_{\{C_0, C_1\}} \quad U(C_0) + \beta U(C_1)$$

$$s.t. \qquad \underbrace{C_0 + \frac{C_1}{1+R}}_{\text{PDV of consumption}} = \underbrace{Y_0 + \frac{Y_1}{1+R}}_{\text{PDV of income}} \equiv \mathbb{W}$$

Euler Equation:

$$U'(C_0) = \beta(1+R)U'(C_1)$$

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Example: $\beta = 1$, $R = 0$, $U(c) = \ln(C)$:

$$\begin{aligned} \frac{1}{C_0} &= \frac{1}{C_1} \implies C_0 = C_1 = C^* \implies C^* + C^* = \mathbb{W} \implies C^* = \frac{1}{2}\mathbb{W} \\ S &= Y_0 - C^*, \quad S > 0 \implies \text{saving}, \quad S < 0 \implies \text{borrowing} \end{aligned}$$

Multi-Period Neoclassical Growth Model

The consumer maximizes lifetime utility subject to the intertemporal budget constraint

$$\max_{\{C_t\}_{t=0}^{69}} \quad \sum_{t=0}^{69} \beta^t U(C_t)$$

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- Hence:

$$\beta^t (1+R)^t U'(C_t) = \beta^{t+1}(1+R)^{t+1} U'(C_{t+1}) \iff U'(C_t) = \beta(1+R) U'(C_{t+1})$$

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$$\frac{1}{C_t} = \frac{1}{C_{t+1}} \implies C_t = C_{t+1} = C^* \implies \sum_{t=0}^{69} C^* = \mathbb{W} \implies C^* = \frac{1}{70}\mathbb{W}$$

Marginal Propensity to Consume (MPC)

The Marginal Propensity to Consume is the change in current consumption due to a change in current income.

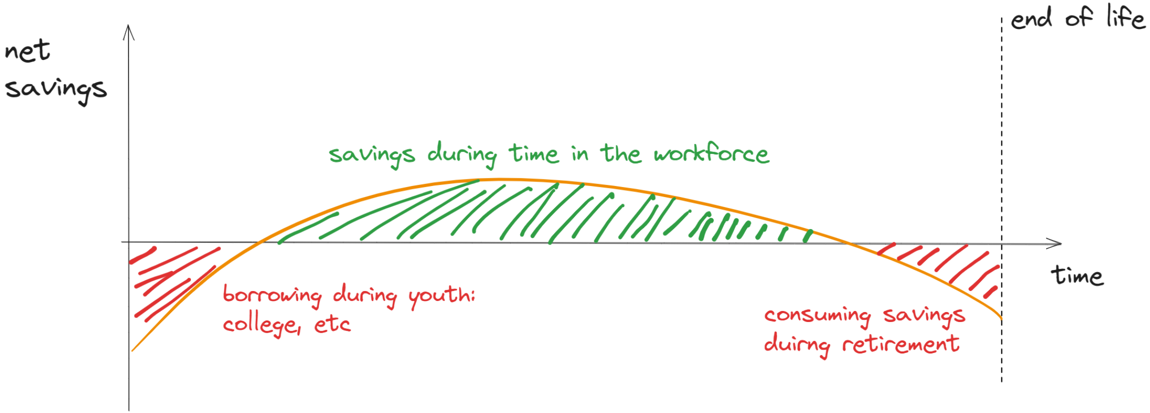
$$MPC \equiv \frac{\partial C_t}{\partial Y_t}$$

- Example of 2-period model: $C_0 = \frac{1}{2}\left(Y_0 + Y_1\right) \implies \frac{\partial C_1}{\partial Y_1} = \frac{1}{2}$
- Example of 70-period model: $C_0 = \frac{1}{70}\left(Y_0 + Y_1 + \cdots + Y_{69}\right) \implies \frac{\partial C_0}{\partial Y_0} = \frac{1}{70}$

Permanent Income Hypothesis

In the Neo-classical consumption model, per-period consumption is determined by the present discounted value of lifetime income, also known as permanent income. Hence, temporary changes in income have only small effects on per-period consumption, while changes that are perceived as permanent (or long-lasting) have stronger effects on consumption.

Permanent Income Hypothesis



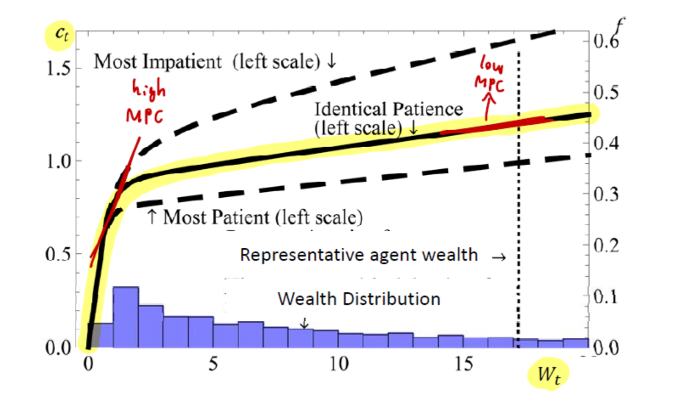
Permanent Income Hypothesis

Milton Friedman
1912-2006

Nobel Prize in Economics, 1976

Permanent Income Hypothesis? What are the empirics?

MPC and Wealth

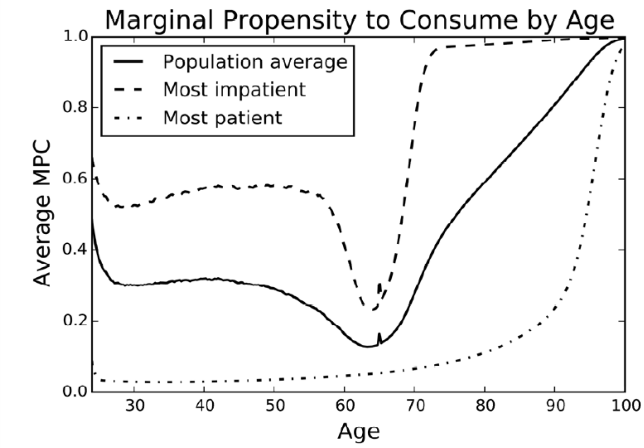


MPC is the slope of the black lines which represent per-period consumption as a function of wealth, $c_t = C(W_t)$

Source: Carroll (2017): The Distribution of Wealth and the Marginal Propensity to Consume

Permanent Income Hypothesis? What are the empirics?

MPC by Age



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Why MPC is not small as predicted by PIH?

- Consumers face **Borrowing Constraints**
- Consumers face **Uncertainty**

Borrowing Constraints (BC)

Consumers cannot borrow against their future labor income. Therefore, the intertemporal budget constraint in the Neoclassical Consumption Model is not the relevant constraint for consumption decisions.
$\Longrightarrow$ **Note:** consumers are **not** at their optimal allocation decisions! They would rather borrow and consume more!

Borrowing Constraints (BC)

- Add the constraint: $C_0 \leq Y_0$ (BC), which given $S = Y_0 - C_0, \implies S \geq 0$

- At the optimal, in our example: $C_0 = C_1 = C^* = \frac{1}{2}(Y_0 + Y_1)$.

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- Suppose $Y_0 < Y_1$. This implies that, at the optimal:

$$S^* = Y_0 - C^* = Y_0 - \frac{1}{2}(Y_0 + Y_1) = \frac{1}{2}Y_0 - \frac{1}{2}Y_1 < 0 \qquad (\because Y_0 < Y_1)$$

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- What is the MPC here? $MPC \equiv \frac{\partial C_0}{\partial Y_0} = 1$

Borrowing Constraints (BC)

How do we know not this is not the optimal choice?

- At the optimal $C_0 = C_1$, here $C_0 < C_1$
- Or, in terms of marginal utilities:

$$\frac{1}{C_0} > \frac{1}{C_1}, \qquad U'(C_0) > \beta(1+R)U'(C_1)$$

- In English, this means that you would be better off by moving some consumption from the future to the initial but you can't because you hit your borrowing constraint!

BC Example

Let $u(C) = \ln(C)\,, R = 0,\ \beta = 1.$ Consider two consumers, Anna and Bill, with the following income profiles

$$\textbf{Anna}: Y_1^A = 100, Y_2^A = 50\ ; \qquad \textbf{Bill}: Y_1^B = 50,\ Y_2^B = 100$$

Compute Anna's and Bill's consumption and saving (or borrowing). What are Anna's and Bills' MPC's?

Anna

$$C_1^A = \frac{1}{2} W^A = \frac{1}{2}(100 + 50) = 75$$

$$S^A = 100 - 75 = 25$$

$$MPC^A = \frac{\partial C_1^A}{\partial Y_1^A} = \frac{1}{2}$$

Bill

$$C_1^B = \frac{1}{2} W^B = \frac{1}{2}(50 + 100) = 75$$

$$S^B = 50 - 75 = -25$$

$$MPC^B = \frac{\partial C_1^B}{\partial Y_1^A} = \frac{1}{2}$$

BC Example

Suppose now that Anna and Bill are both borrowing constrained so that $C_1 \leq Y_1$. Compute consumption and saving (or borrowing). What are Anna's and Bills' MPC's?

- Nothing changes for Anna! , $MPC^A = 1/2$
- Bill : $C_1^B = Y_1^B = 50$, $MPC^B = 1$

Uncertainty

If future income is uncertain, consumers save more (i.e. borrow less) today. This is called precautionary saving. Once consumers have saved enough, any increase in current income is disproportionately consumed.

Knowing the MPC is crucial to understand the effect of economic policies.

Example: COVID-19 relief payments to household in March 2020 and December 2020. What is the predicted effect on consumption (and thus GDP) according to the consumption model?

- Under PIH: small or no effect
- If PIH does not hold: large effects of cash payment for consumers with low wealth (high MPC), small effect of cash payment for consumers with high wealth (low MPC).

Consumption & Income

with prices

Two-Period model with Monetary Prices

The household supplies labor L in both periods, at the monetary wage w_0 in period 0 and w_1 in period 1. There is only one type of consumption good in the economy, bread. The monetary price of bread is P_0 per pound in period 0, and P_1 in period 1. The interest rate on saving/borrowing in monetary units is i.

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\max_{\{C_0, C_1\}} \quad & U(C_0) + \beta U(C_1) \\
s.t. \quad & w_0 L = P_0 C_0 + S \\
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IBC (solve for S in either equation, replace in the other):

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- Recall: $\pi_t = \frac{P_t - P_{t-1}}{P_{t-1}}$, then: $\frac{P_t}{P_{t-1}} = 1 + \pi_t$.

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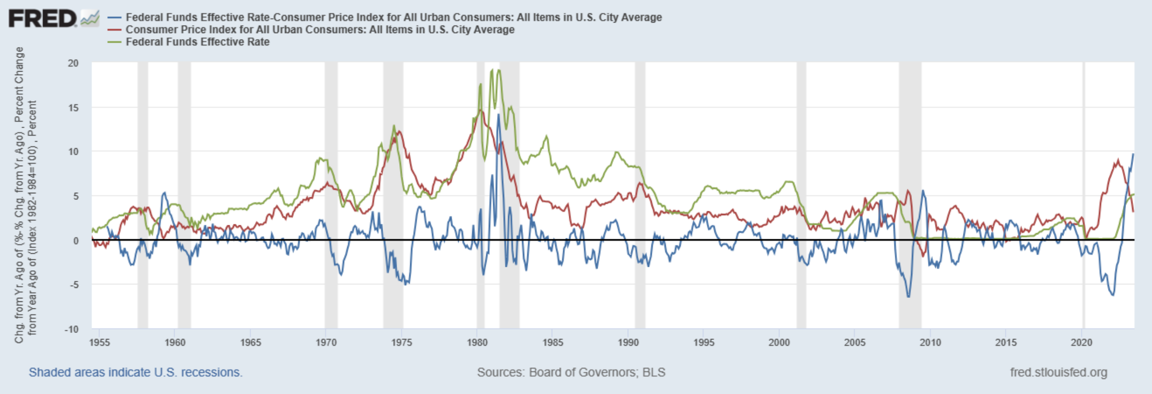
- Taking logs $R \approx i - \pi$. This is called the Fisher equation.

Fisher equation



red interest rates: R, nominal interest rates: i, inflation: π

`https://fred.stlouisfed.org/graph/?g=17AEy`