Econ 110A: Lecture 7

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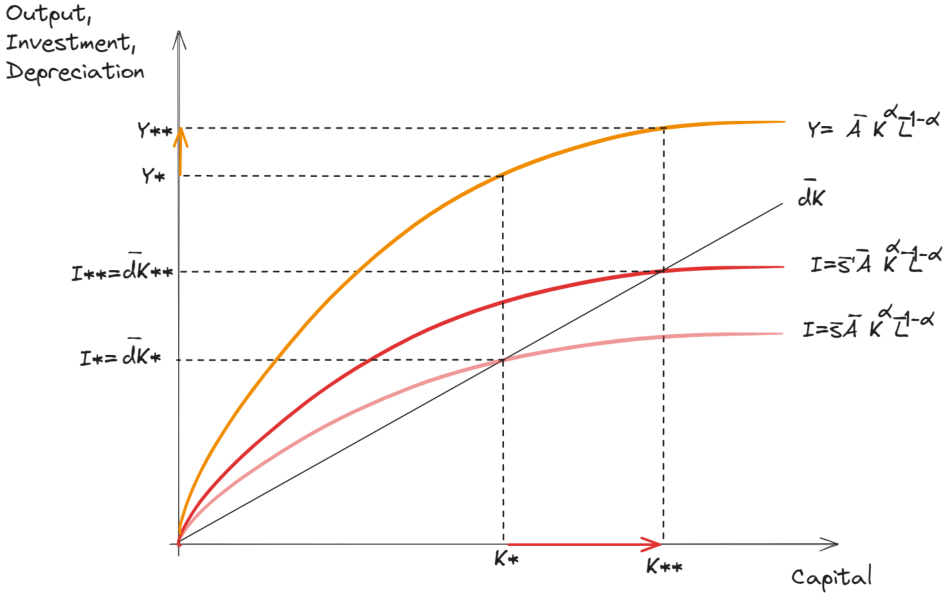
Experiments with the Solow Growth Model

We will use the Solow Model framework to perform a set of different "experiments" —i.e., see what happens if we change a given parameter in the model.

Experiment 1: Increase in Savings Rate

- Suppose the economy starts at the steady-state K^*, Y^*, I^*
- But then, the savings rate increases to $\bar{s}' > \bar{s}$
- What happens?
- The investment curve shifts up $I'(K) = \bar{s}'Y(K) > \bar{s}Y(K) = I(K)$
- Everything else stays the same because while the other variables will change, they will be changes along the curves, not changes of the curves.

Experiment 1: Increase in Savings Rate

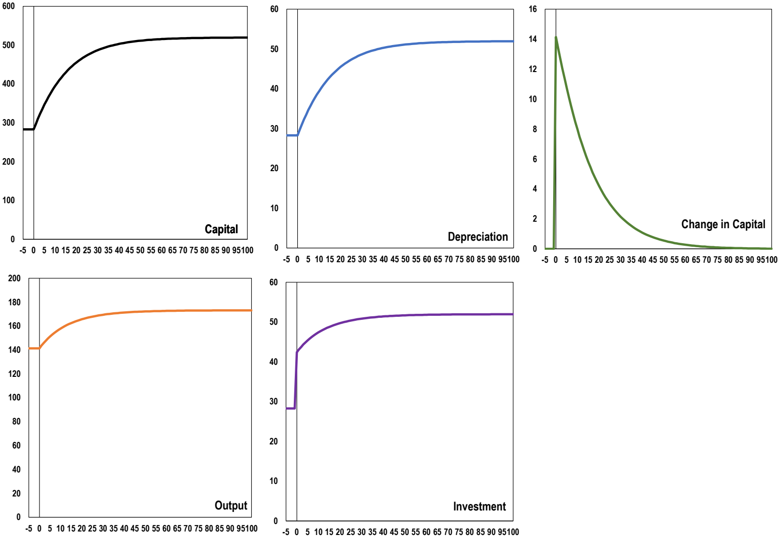


Experiment 1: Increase in Savings Rate

Predictions:

- Savings rate $\uparrow$
- Investment immediately $\uparrow$ (this is an immediate effect of increasing s)
- Capital stock $\uparrow$ over time (it is now worth it to have a higher capital stock)
- Investment $\uparrow$ over time (you do so through higher investment)
- Output $\uparrow$ over time, but not immediately

Experiment 1: Increase in Savings Rate, over time



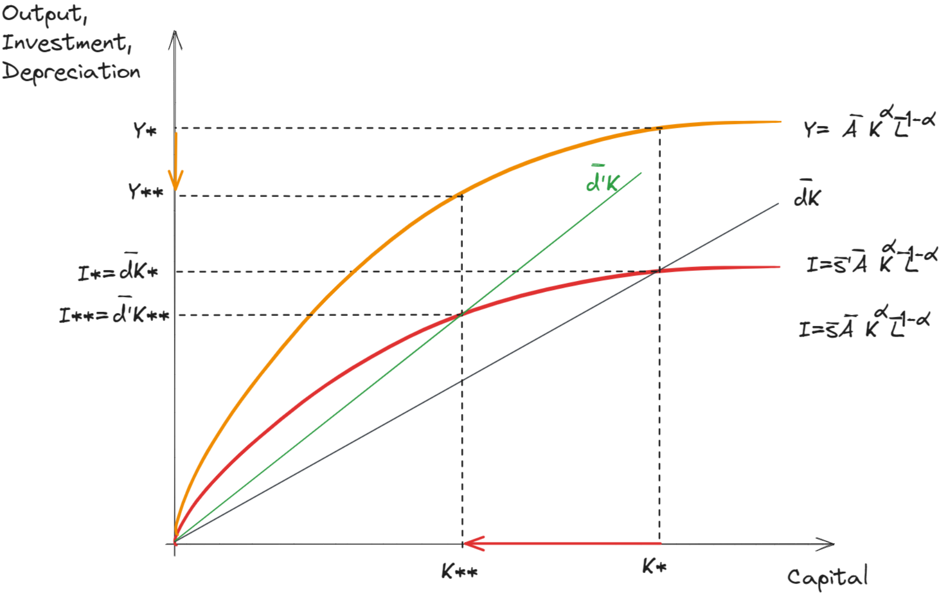
Experiment 1: Increase in Savings Rate, over time

Link to Excel File

Experiment 2: Increase in Depreciation Rate

- Suppose the economy starts at the steady-state K^*, Y^*, I^*
- But then, the depreciation rate increases to $\bar{d}' > \bar{s}$
- What happens?
- The depreciation curve shifts up $\bar{d}'K > \bar{d}K$
- Everything else stays the same because while the other variables will change, they will be changes along the curves, not changes of the curves.

Experiment 2: Increase in Depreciation Rate

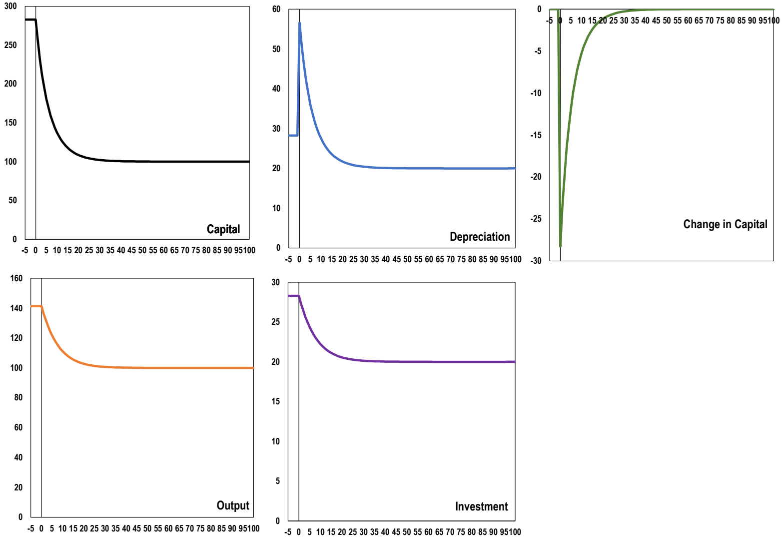


Experiment 2: Increase in Depreciation Rate

Predictions:

- Depreciation rate $\uparrow$
- Total depreciation increases $\uparrow$ immediately
- Capital stock $\downarrow$ over time (it is now optimal to have a smaller capital stock)
- Investment drops $\downarrow$ over time (you do so with smaller investment)
- Output $\downarrow$ over time (as a consequence you have smaller output)

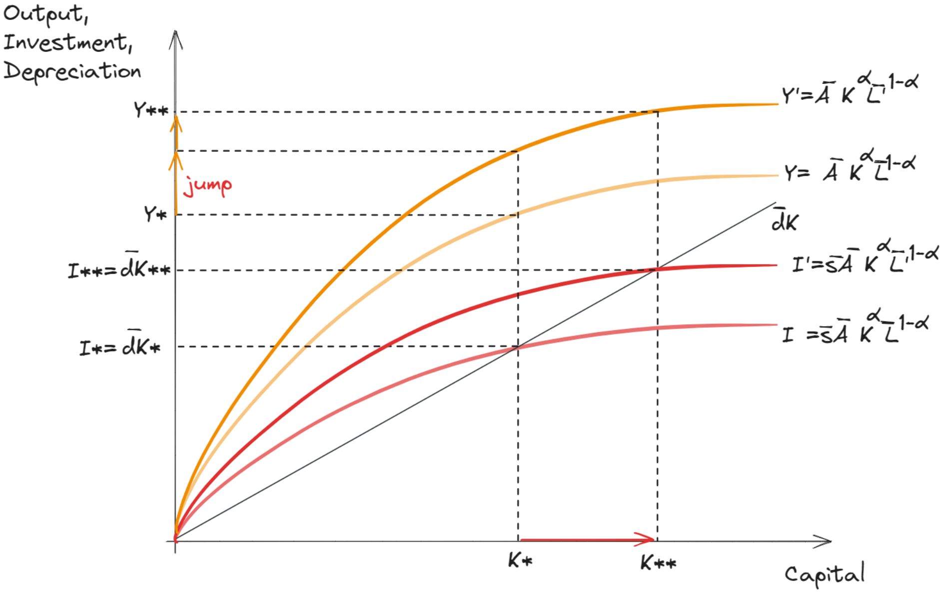
Experiment 2: Increase in Depreciation Rate, over time



Experiment 3: Increase in Population

- Suppose the economy starts at the steady-state K^*, Y^*, I^*
- But then, there is a change in the immigration law that increases immigration and the labor force increases to $\bar{L}' > \bar{L}$
- What happens?
- The output and the investment curves go up, since $\bar{L}$ is a parameter in both of these curves.
- Everything else stays the same because while the other variables will change, they will be changes along the curves, not changes of the curves.

Experiment 3: Increase in Population

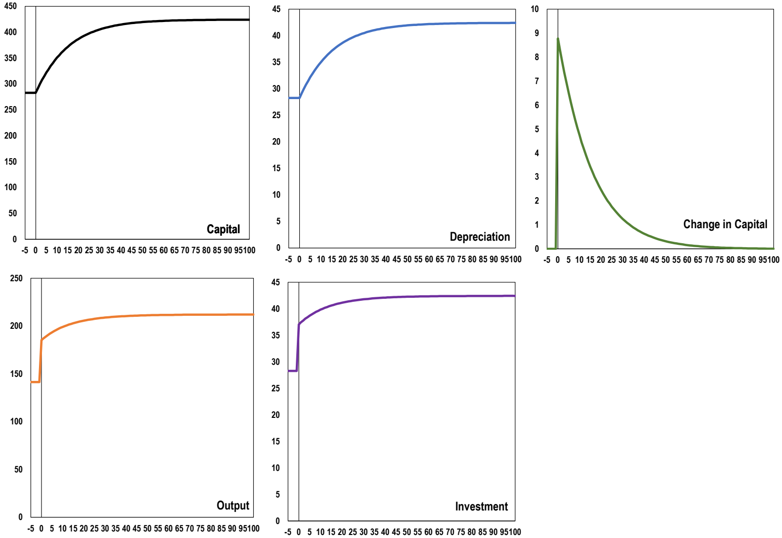


Experiment 3: Increase in Population

Predictions:

- Population $\uparrow$
- Output $\uparrow$ immediately (population is a factor of production!)
- Investment $\uparrow$ immediately (investment is a constant fraction of output!)
- Capital stock $\uparrow$ over time (more worth it to have more capital with larger population)
- Output $\uparrow$ over time

Experiment 3: Increase in Population, over time



Experiment 3: Increase in Population, over time

What is the effect on output per worker (GDP per capita)?

$$y^* = \frac{Y^*}{\bar{L}} = \bar{A}^{\frac{1}{1-\alpha}} \left(\frac{\bar{s}}{\bar{d}}\right)^{\frac{\alpha}{1-\alpha}} \cdot \bar{L} \cdot \frac{1}{\bar{L}} = \bar{A}^{\frac{1}{1-\alpha}} \left(\frac{\bar{s}}{\bar{d}}\right)^{\frac{\alpha}{1-\alpha}}$$

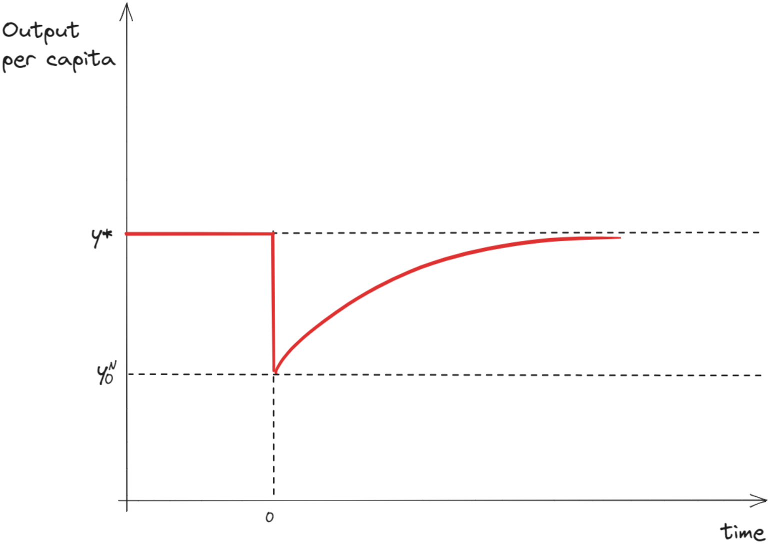
$\implies$ no impact on GDP per capita on the SS!

Over the short run:

$$y_0^* = \frac{\bar{A}(K^*)^\alpha \bar{L_N}^{1-\alpha}}{\bar{L_N}} = \bar{A}\left(\frac{K^*}{\bar{L_N}}\right)^\alpha < \bar{A}\left(\frac{K^*}{\bar{L}}\right)^\alpha = y^*$$

Intuition: due to **diminishing marginal returns on capital**, output per worker (wages in the background) go down as labor force goes up. As capital stock increases, this effect reverses.

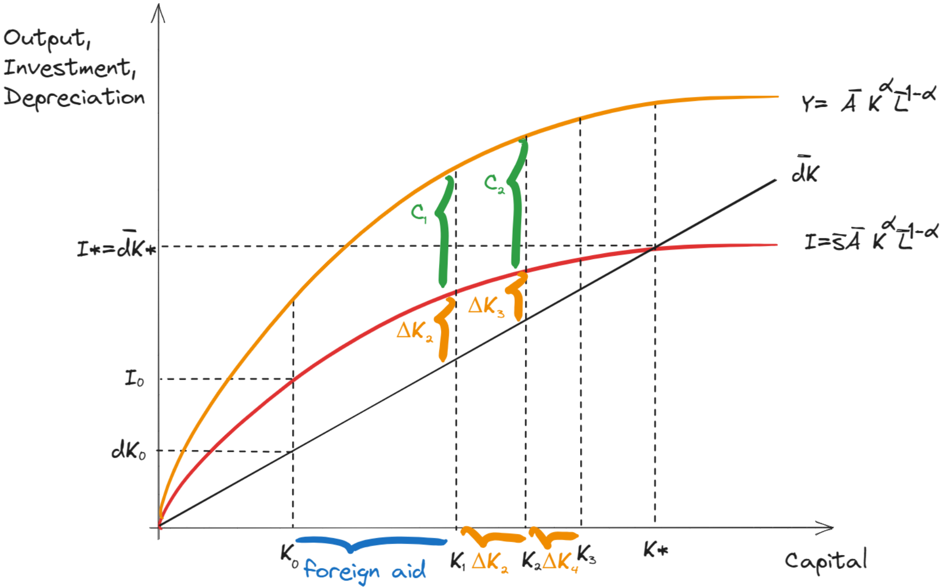
Experiment 3: Increase in Population, over time



Experiment 4: Foreign Aid

- Suppose the economy starts **below** of the steady-state.
- In the Solow Model, then, the economy is slowly accumulating capital and converging towards the SS.
- Your country becomes a part of a USAID program and receives a large foreign aid gift from the U.S. government of 100 billion U.S. dollars.
- What happens?
- No curves shift, but the capital stock moves closer to the steady state.

Experiment 4: Foreign Aid

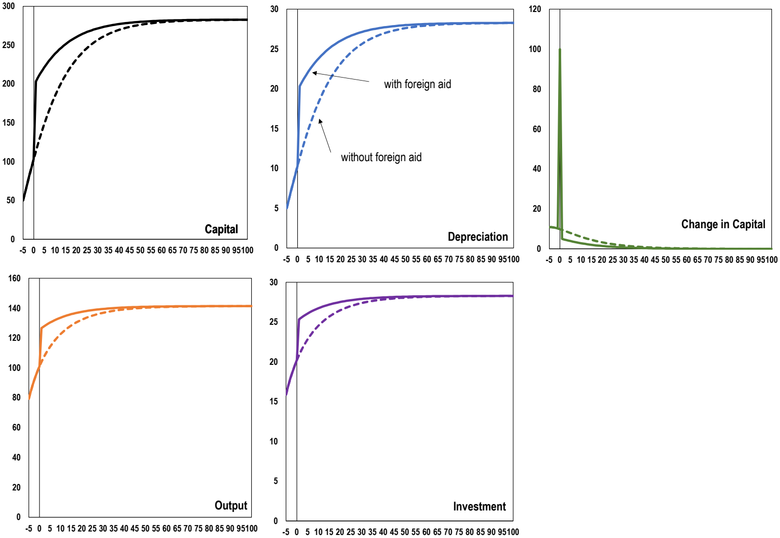


Experiment 4: Foreign Aid

Predictions:

- Foreign Aid $\uparrow$
- Capital Stock, Output, Investment, $\uparrow$ immediately (because of foreign aid)
- Capital Stock, Output, Investment keep converging towards steady state, but now do so at a slower pace than before —principle of transition dynamics

Experiment 4: Foreign Aid, over time

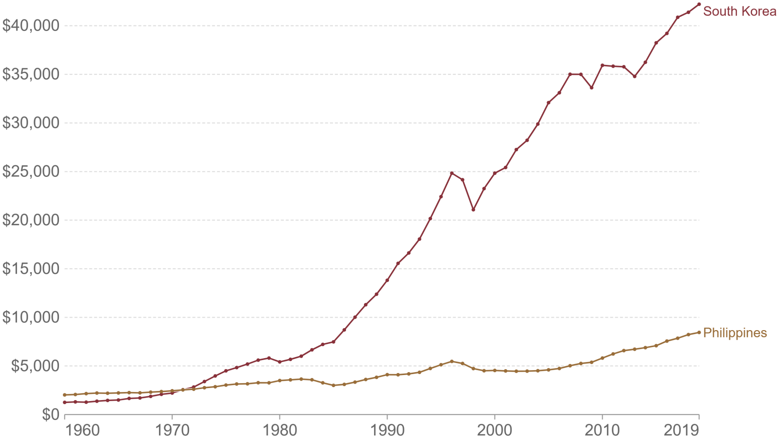


Two Pictures from 1960

GDP per capita, 1960 to 2019

This data is adjusted for inflation and for differences in the cost of living between countries.

Our World in Data

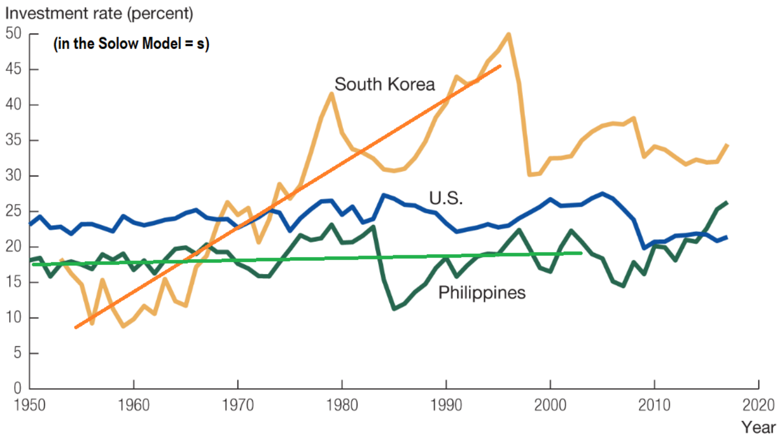


Source: Feenstra et al. (2015), Penn World Table (2021)

Note: This data is expressed in international-$[1] at 2017 prices, using multiple benchmark years to adjust for differences in the cost of living between countries over time.

1. **International dollars**: International dollars are a hypothetical currency that is used to make meaningful comparisons of monetary indicators of living

...what changed?



Source: Penn World Tables, Version 9.1.

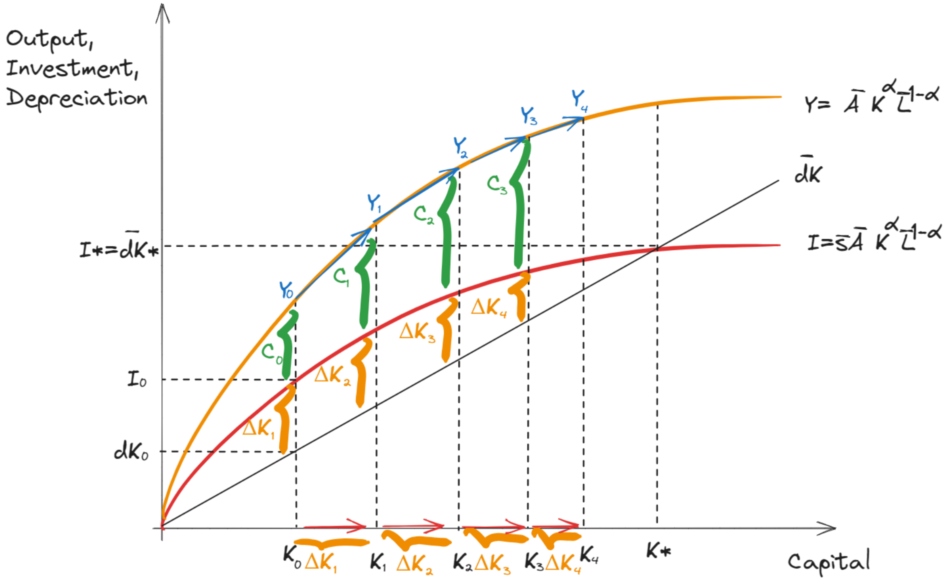
...what changed?

How does convergence happen in the Solow Model?

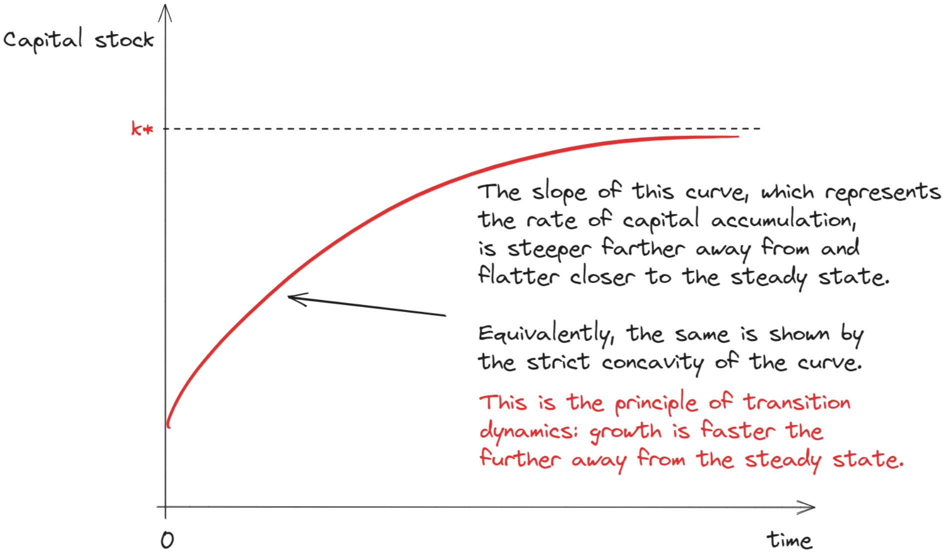
$$\frac{y^*_{Korea}}{y^*_{US}} = \left(\frac{\bar{A}_{Korea}}{\bar{A}_{US}}\right)^{\frac{3}{2}} \times \left(\frac{\bar{s}_{Korea}}{\bar{s}_{US}}\right)^{\frac{1}{2}}$$

- All else equal, an increase in $\bar{s}_{Korea}$ implies a **positive increase** in Korea's SS.
- In that case, as Korea would be **farther away from the SS**, by the **principle of transition dynamics**, it would start to grow faster through the lens of the Solow Model

Solow Model: The Complete Diagram, tracing out the dynamics



Solow Model: principle of transition dynamics



Solow Model: principle of transition dynamics

The mathematics behind the principle

$$\Delta K_{t+1} \quad = \quad \bar{s} Y_t - \bar{d} K_t$$

The mathematics behind the principle

$$
\begin{aligned}
\Delta K_{t+1} &= \bar{s}Y_t - \bar{d}K_t \\
\Delta \frac{K_{t+1}}{K_t} &= \bar{s}\frac{Y_t}{K_t} - \bar{d}
\end{aligned}
$$

Solow Model: principle of transition dynamics

The mathematics behind the principle

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\Delta K_{t+1} &= \bar{s}Y_t - \bar{d}K_t \\
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\Delta \frac{K_{t+1}}{K_t} &= \bar{s}\frac{Y_t}{K_t} - \bar{s}\frac{Y^*}{K^*} \qquad \left(\because \frac{K^*}{Y^*} = \frac{\bar{s}}{\bar{d}} \implies \bar{d} = \bar{s}\frac{Y^*}{K^*}\right)
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Solow Model: principle of transition dynamics
The mathematics behind the principle

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\Delta \frac{K_{t+1}}{K_t} &= \bar{s} \times \left(\frac{Y_t}{K_t} - \frac{Y^*}{K^*}\right)
\end{aligned}
$$

Solow Model: principle of transition dynamics

The mathematics behind the principle

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\Delta \frac{K_{t+1}}{K_t} &= \bar{s} \frac{Y^*}{K^*} \times \left(\frac{Y_t / K_t}{Y^* / K^*} - 1 \right)
\end{aligned}
$$

Solow Model: principle of transition dynamics

The mathematics behind the principle

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\Delta \frac{K_{t+1}}{K_t} &= \bar{s} \frac{Y^*}{K^*} \times \left(\left[\frac{K^*}{K_t} \right]^{1-\alpha} - 1 \right) \qquad \left(\because Y^* = \bar{A}(K^*)^{\alpha}(\bar{L})^{1-\alpha}, \quad Y_t = \bar{A}(K_t)^{\alpha}(\bar{L})^{1-\alpha} \right)
\end{aligned}
$$

Solow Model: principle of transition dynamics

The mathematics behind the principle

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\Delta K_{t+1} &= \bar{s} Y_t - \bar{d} K_t \\
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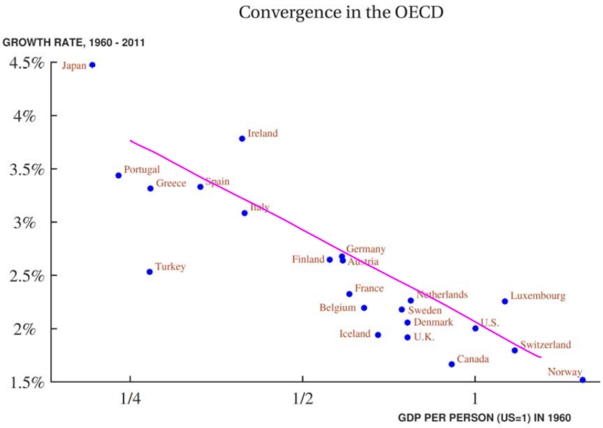
\end{aligned}
$$

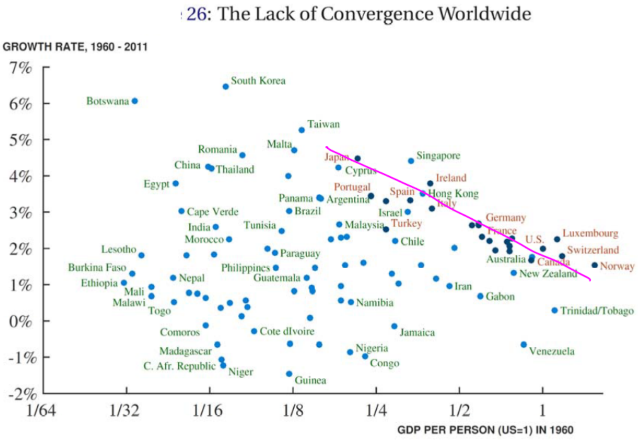
So $\Delta \frac{K_{t+1}}{K_t}$ will be large if the gap $\frac{K^*}{K_t}$ is large.

Fact 6: Conditional Convergence?



Source: The Penn World Tables 8.0. Countries in the OECD as of 1970 are shown.

Fact 6: Yes, but it doesn't generalize: Lack of Conditional Convergence!



Source: The Penn World Tables 8.0.

...what is missing?

- Korea was under de facto dictatorship from 1961 to 1979 (President Park assassination)
- The history of economic development of a country is always complicated. Korea is no exception.
- The spectacular growth that took off in the 1970's in Korea happened under a political repressive regime.
- At the same time, the government took a more central role in directing the development of heavy industry (steel, ships), which required large investments, made by forced "borrowing" from private citizens (which had access to better saving instruments compared to before).
- The Solow model offers that perspective, of an increased investment rate, associated with a higher accumulation of capital. But it does not explain how the investment rate increased and why.
- And it is completely oblivious to the respect of human and civil rights in the capital accumulation process.
- It also does not speak of industrial policy or human capital.