International Trade: Lecture 7

Production in the Short and the Long Run

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Recap

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- Only one factor of production: labor
- Labor is mobile across sectors
- Economy can always adjust proportionately, workers gain (real income $\uparrow$)

Limits

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- Production often use more than one factor
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PeaceCorps HQ converted into apt building

1111 20th ST NW

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- Factors cannot always freely adjust
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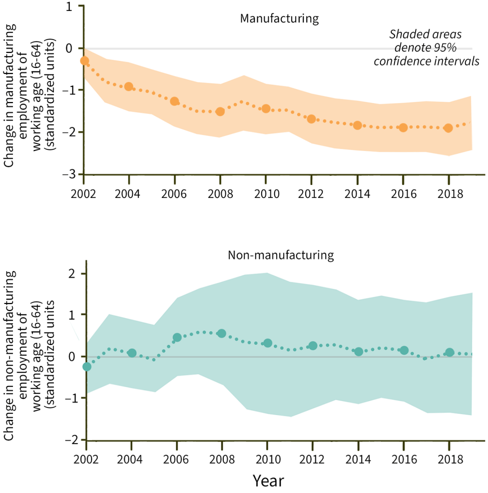
  - yes, but costly, lengthy
- What happens if we relax those assumptions?
  - some factor owners can be worse off from trade over short run
  - Ricardian logic can be though off as long run

Example: China trade shock

- China's share in world manufacturing exports rose from 3.1% in 1991 to 17.6% in 2015

Example: China trade shock



Source: Autor, Dorn & Hanson (2021)

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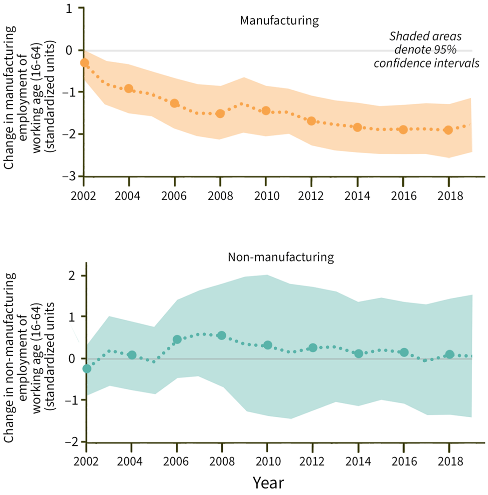

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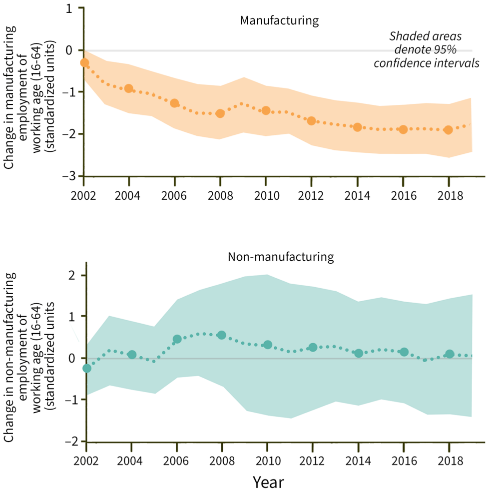

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- US regions more exposed to Chinese import competition saw (relative) losses in manufacturing employment growth
- Effects persist many years into the future, due to frictions for labor mobility
- Prices did fall about 1.5%: about 95% of US population saw real income growth due to shock; 5% losses

The Production Function

Gelato Factory

- Example: Gelato Factory

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- Intangibles:
  - License
  - Recipe
  - Business environment, Property rights

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- $F(\bar{Z}, K, L)$ is your production function. If $F(\cdot, \cdot, \cdot)$ is Cobb-Douglas, then:

$$Y = \bar{Z} \cdot K^{\beta} L^{1-\beta}, \qquad 0 \leq \beta \leq 1$$

Marginal Product: extra output produced by increasing one factor while keeping all the other factors fixed.

recall: economic agents make decision by “reasoning at the margin”

Production Function

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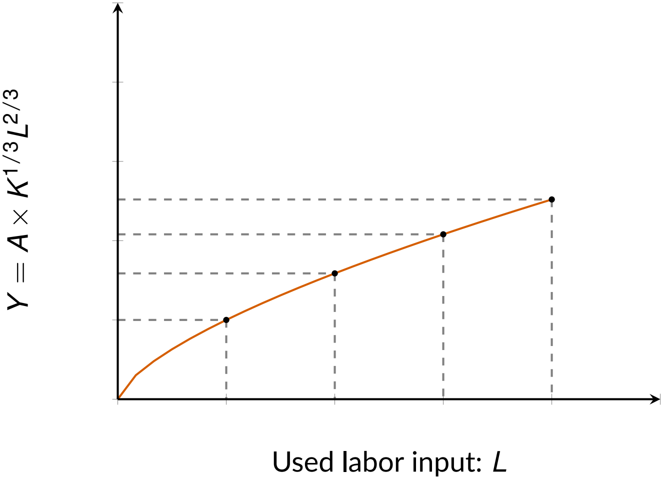
With Cobb-Douglas Technology, the Marginal Product of Labor and Capital are:

$$MPL \equiv \frac{\partial Y}{\partial L} = \underbrace{(1-\beta)\bar{Z}\left(\frac{K}{L}\right)^{\beta}}_{\text{decreasing in } L} \qquad MPK \equiv \frac{\partial Y}{\partial K} = \underbrace{\beta\bar{Z}\left(\frac{L}{K}\right)^{1-\beta}}_{\text{decreasing in } K}$$

Example: Labor

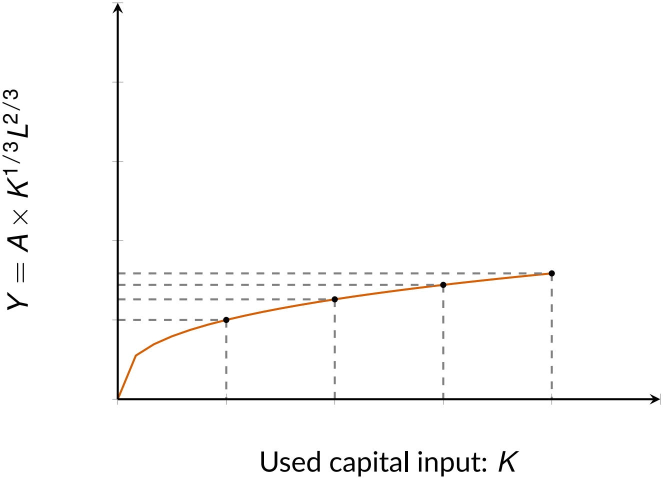


$$Y = \bar{Z} K^{1/3} L^{2/3}$$

L and Y when $K = 1$ and $\bar{Z} = 1$

L	Y
1	1
2	1.59
3	2.08
4	2.52

Example: Capital



$$Y = \bar{Z} K^{1/3} L^{2/3}$$

L and Y when $L = 1$ and $\bar{Z} = 1$

K	Y
1	1
2	1.26
3	1.44
4	1.59

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- If you start buying computers, the first ones will be very productive because they will be matched with idle employees.
- But at a certain point, new computers will start going idle for some time...
- And if you buy too many computers (say if your company has more computers than employees) any additional computers will be useless as they will be idle the whole time.

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- If you start buying computers, the first ones will be very productive because they will be matched with idle employees.
- But at a certain point, new computers will start going idle for some time...
- And if you buy too many computers (say if your company has more computers than employees) any additional computers will be useless as they will be idle the whole time.
- The production function of your firm exhibits diminishing returns in computers!

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for some $\lambda > 0$.

Returns to Scale

Claim: Cobb-Douglas is Constant Returns to Scale in (K,L)

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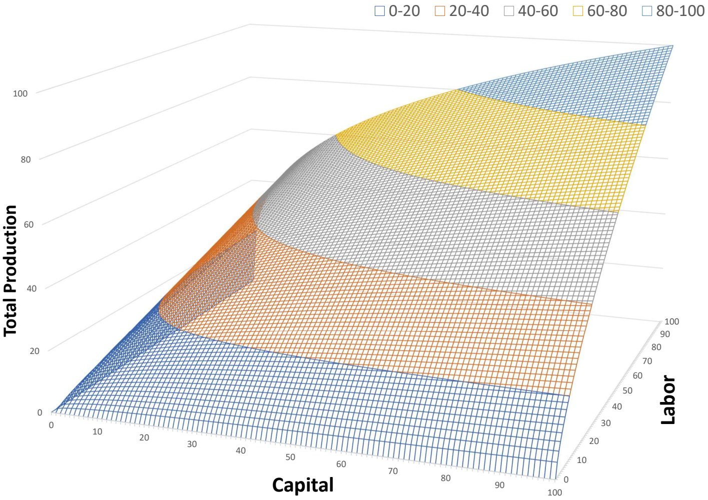
Proof.

$$
\begin{aligned}
F(A, \lambda K, \lambda L) &= \bar{Z}(\lambda K)^{\beta}(\lambda L)^{1-\beta} \\
&= \lambda \bar{Z}(K)^{\beta}(L)^{1-\beta} \\
&= \lambda F(A, K, L)
\end{aligned}
$$

$\square$

Example: Capital and Labor jointly



$$Y = \bar{Z}K^{1/3}L^{2/3}$$
$$\bar{Z} = 1$$

Cobb-Douglas is Constant Returns to Scale in Capital and Labor jointly... but diminishing marginal returns while holding the other factor fixed...

Profit Maximization

$$\max_{\{K,L\}} \pi = \underbrace{P \cdot F(K,L)}_{\text{revenues}} - \left(\underbrace{w \cdot L + r \cdot K}_{\text{costs}} \right)$$

where

- P: price of the output good (if there is only one sector, we can normalize this $P = 1$, numéraire)
- $F(K,L)$: production function
- w: wages
- r: rental rate on capital

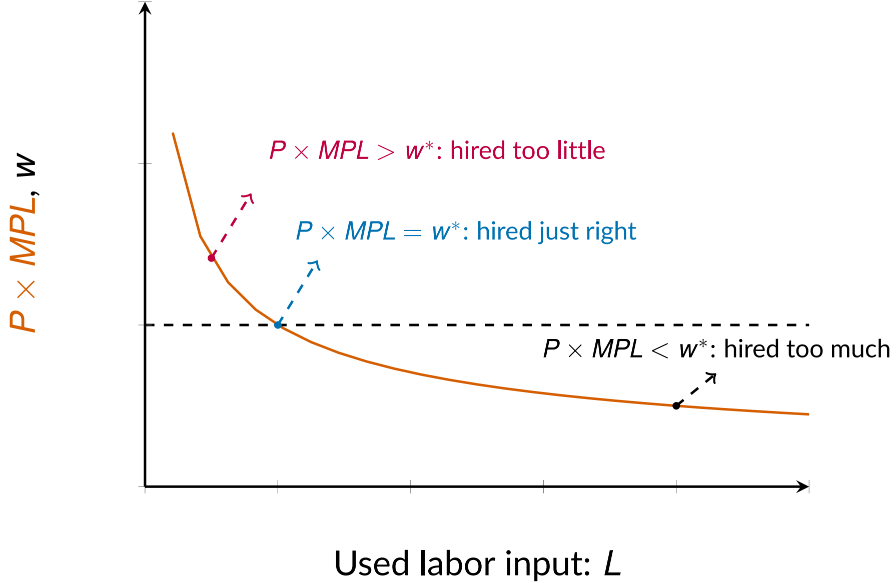
Profit Maximization

$$\max_{\{K,L\}} \pi = \underbrace{P \cdot F(K,L)}_{\text{revenues}} - \left(\underbrace{w \cdot L + r \cdot K}_{\text{costs}} \right)$$

First Order Conditions imply that, at the optimal:

$$\begin{aligned} \frac{\partial \pi}{\partial L} &= 0 \implies P \cdot \frac{\partial F(K,L)}{\partial L} = P \cdot MPL = w \\ \frac{\partial \pi}{\partial K} &= 0 \implies P \cdot \frac{\partial F(K,L)}{\partial K} = P \cdot MPK = r \end{aligned}$$

Intuition for optimality result



Optimality Conditions for Demand with Cobb-Douglas

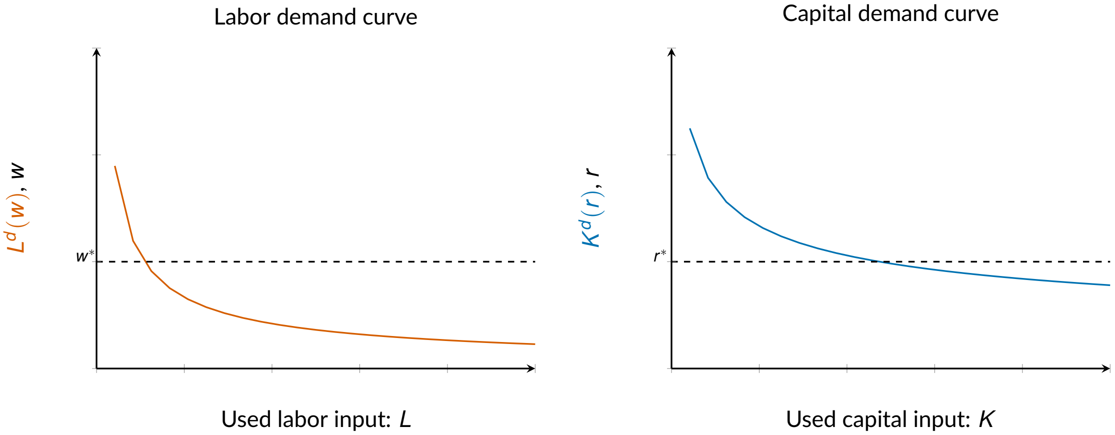
$$
\begin{aligned}
Y &= \bar{Z}(K^d)^{\beta}(L^d)^{1-\beta} \\
P(1-\beta)\bar{Z}\left(\frac{K^d}{L^d}\right)^{\beta} &= w \\
P\beta\bar{Z}\left(\frac{L^d}{K^d}\right)^{1-\beta} &= r
\end{aligned}
$$

Optimality Conditions for Demand with Cobb-Douglas

Note that we can derive a labor demand and capital demand schedule from each of those, which are decreasing in factor prices:

$$\begin{aligned} L^d &= \left(\frac{(1-\beta)\cdot A\cdot P}{w}\right)^{\frac{1}{\beta}}\cdot K^d \\ K^d &= \left(\frac{\beta\cdot A\cdot P}{r}\right)^{\frac{1}{1-\beta}}\cdot L^d \end{aligned}$$

Demand

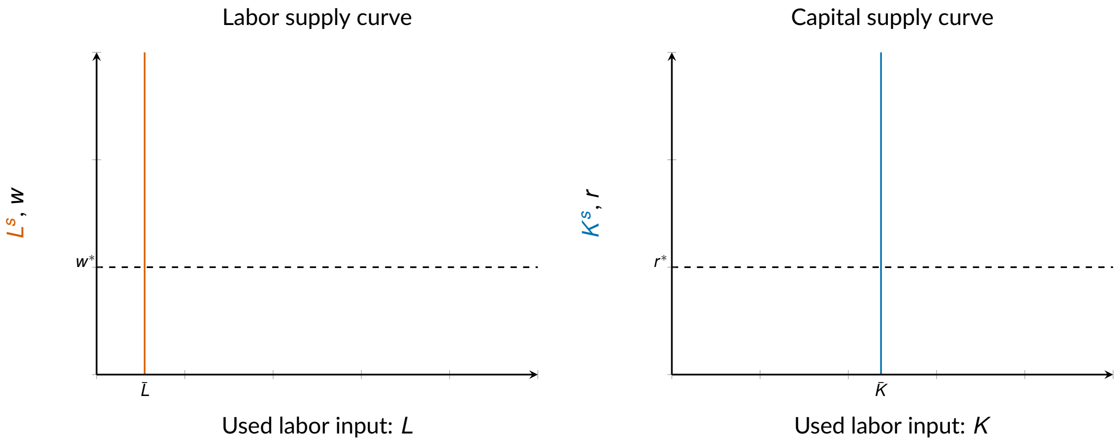

Labor demand curve
$L^d(w)$, w
w^*
Used labor input: L
Capital demand curve
$K^d(r)$, r
r^*
Used capital input: K

Supply side of the economy is simple

- Households supply labor and capital inelastically.
- Prices adjust to ensure that supply equals demand (market clearing condition)

$$
\begin{aligned}
L^d &= L^s = \bar{L} \qquad \text{(parameter)} \\
K^d &= K^s = \bar{K} \qquad \text{(parameter)}
\end{aligned}
$$

Supply



Labor supply curve
L^s, w
w^*
$\bar{L}$
Used labor input: L
Capital supply curve
K^s, r
r^*
$\bar{K}$
Used labor input: K

General Equilibrium

Endogeneous Variables: $Y, K, L, w, r,$ Numéraire: $P = 1$

Five equations for five unknowns

$$
\begin{aligned}
Y &= \bar{\bar{Z}}(K)^{\alpha}(L)^{1-\alpha} && (1)\\
P(1-\alpha)\bar{\bar{Z}}\left(\frac{K}{L}\right)^{\alpha} &= w && (2)\\
P\alpha\bar{\bar{Z}}\left(\frac{L}{K}\right)^{1-\alpha} &= r && (3)\\
L &= \bar{L} = L^{s} && (4)\\
K &= \bar{K} = K^{s} && (5)
\end{aligned}
$$

Solution to the Production Model

Replacing the market clearing condition in and normalizing $P = 1$ to be the numéraire of this economy:

$$
\begin{aligned}
Y^* &= \bar{Z}(\bar{K})^{\alpha}(\bar{L})^{1-\alpha} && (6)\\
w^* &= (1-\alpha)\bar{Z}\left(\frac{\bar{K}}{\bar{L}}\right)^{\alpha} && (7)\\
r^* &= \alpha\bar{Z}\left(\frac{\bar{L}}{\bar{K}}\right)^{1-\alpha} && (8)\\
L^* &= \bar{L} && (9)\\
K^* &= \bar{K} && (10)
\end{aligned}
$$

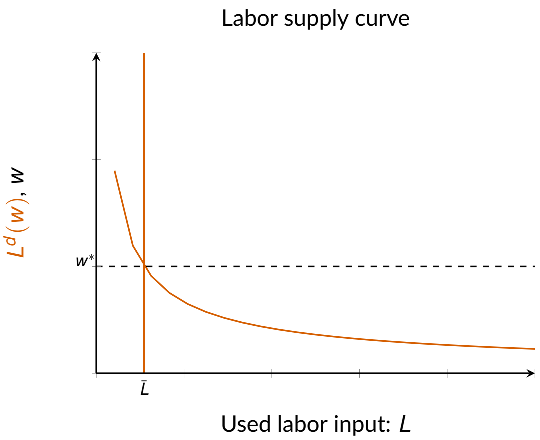
Note that everything on the right-hand side of the equations is a parameter, so this is indeed an explicit solution!

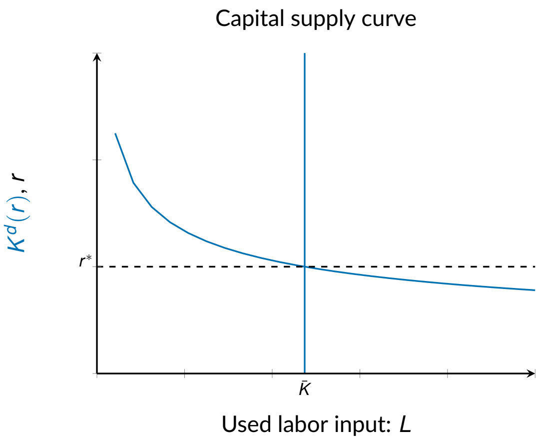
Numerical Example

Suppose $\bar{K} = 20$, $\bar{L} = 160$, $\bar{Z} = 1$, $\alpha = \frac{1}{3}$. What is the solution to the Production Model? Replacing in the set of equations before:

$$
\begin{aligned}
Y^* &= (20)^{\frac{1}{3}}(160)^{\frac{2}{3}} = 80 \\
w^* &= \frac{2}{3}\left(\frac{20}{160}\right)^{\frac{1}{3}} = \frac{1}{3} \\
r^* &= \frac{1}{3}\left(\frac{160}{20}\right)^{\frac{2}{3}} = \frac{4}{3} \\
L^* &= 160 \\
K^* &= 20
\end{aligned}
$$

Solution to the Model in General Equilibrium





Numerical Example

Now suppose the total available capital changes to $\bar{K}' = 10$. What happens?

$$
\begin{aligned}
Y^{**} &= (10)^{\frac{1}{3}}(160)^{\frac{2}{3}} = 63.5 \\
w^{**} &= \frac{2}{3}\left(\frac{10}{160}\right)^{\frac{1}{3}} = 0.26 \\
r^{**} &= \frac{1}{3}\left(\frac{160}{10}\right)^{\frac{2}{3}} = 2.11 \\
L^{**} &= 160 \\
K^{**} &= 10
\end{aligned}
$$

Numerical Example: Graphical Representation of Comparative Statics