Trade, Growth, and Product Innovation

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Abstract

Can economic integration induce product innovation? I propose a new quantitative framework that integrates the opposing forces of specialization and market size to answer this question. This model encompasses an arbitrary number of asymmetric countries and nests the Eaton-Kortum model of trade and the Romer growth model as special cases. I provide an analytical expression for welfare gains from trade and show that its static and dynamic components operate through the two aforementioned opposing forces. In this framework, the product innovation growth rate increases with higher market access. I test this dynamic mechanism exploiting the 2004 Eastwards expansion of the European Union and show that a plausibly exogenous increase in market access increases the probability of starting production of and exporting a given product. Finally, I calibrate a quantitative version of the model and estimate that: (a) the EU expansion increased its long-run yearly growth rate by about 0.16%; and (b) dynamic gains account for more than 90% of welfare gains from trade.

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1 Introduction

The relationship between economic integration and economic growth is elusive. Over the last decades, the trade literature converged to a broad consensus regarding how to summarize the static gains from trade. But there is no similar consensus on how to measure dynamic gains from trade[1]. This paper delves into this topic by examining the mechanisms through which economic integration can induce product innovation.

Economic theory presents conflicting viewpoints regarding this question. *Canonical trade theory* typically suggests that increased economic integration should cause countries to produce a **smaller range** of produced goods[2]. Models that emphasize growth and innovation, such as those common in *macroeconomics*, often emphasize the role of market size for having an incentive to innovate and produce a **large range** of goods[3].

In this paper, I integrate these two traditions by proposing a new dynamic general equilibrium model of trade and growth that combines trade by comparative advantage, endogenous growth, and innovation in differentiated varieties. My model nests the Eaton and Kortum (2002) model of trade and the P. M. Romer (1990) growth model as special cases. By combining opposing forces, this model comprehensively analyzes the relationship between economic integration and product innovation.

I focus on *product innovation* for two key reasons, one theoretical and one empirical. From a theoretical standpoint, the new product margin can have large welfare implications. As shown by P. Romer (1994), in a simple trade model, adding extensive margin can make welfare costs of a 10% tariff increase from 1% to 20%. Empirically, around trade liberalization episodes, the bulk of trade creation comes from the extensive margin (Kehoe & Ruhl, 2013).

The main theoretical contribution of this paper lies in rationalizing market access as an avenue for growth and product innovation. In the model, I show that increased market access is related to a higher growth rate for a particular country and that, in cases where the model is solvable analytically, the steady-state equilibrium product innovation growth rate increases as countries open up to foreign trade. This finding highlights the positive impact of economic integration on fostering product innovation.

I also provide an analytical formula for dynamic gains from trade that subsumes the static results of Arkolakis et al. (2012) into a dynamic framework. In doing so, I will clarify that

[1]For a comprehensive review of the literature and the different mechanisms that link trade, growth, and innovation, see the paper by M. Melitz and Redding (2021)

[2]In the class of Ricardian models, this follows naturally: as a country opens up to trade, it specializes in a smaller set of goods. But this also happens in the class of Melitz models. As a country opens up to trade, due to the selection effect, the least productive firms of each country exit the market, which results in a smaller range of firms (or, equivalently, goods) in either market. This result holds with asymmetric populations and symmetric productivity distributions or even with productivity distributions, as long as the countries are not too dissimilar —see Demidova (2008).

[3]This is true of a very large class of endogenous growth models in macroeconomics, both with and without scale effects. See, for instance, Chapter 13 of Acemoglu (2008).

the dynamic and static components of welfare will often have opposing mechanisms. The reason is that the latter operates on households as consumers and the former on households as producers and investors.

My model combines the strengths of two sides of the trade literature. Like much of the trade and growth literature, it incorporates forward-looking dynamics. However, unlike many of those papers, it shies away from stylized simplifications, such as symmetric countries or two-country cases. It encompasses an arbitrary number of asymmetric countries and is fit for quantitative exercises. Therefore, it fits neatly into the tradition of quantitative trade models (Costinot & Rodríguez-Clare, 2014) in international trade or policy counterfactuals using dynamic stochastic general equilibrium models in macroeconomics (Christiano et al., 2018).

To validate the proposed mechanism of the model, I leverage the Eastwards enlargement of the European Union (EU). I start by documenting a set of facts and show that, compared to countries that selected into being candidates of the EU but were not yet members, candidates New Member States (NMS) of the EU, started: (a) producing more product varieties; (b) spending more on private research and development per capita; and (c) having larger trading values. All of these macro moments are consistent with the mechanisms of the model.

Later, in order to go beyond correlational analysis, I exploit the fact that, once NMS join the EU, they not only have preferential access to the European market, but they also have to adhere to the common trade policy of the European Union. NMS have immediate preferential access to third-party markets via pre-existing trade agreements between the EU and these third-party markets. Importantly, the NMS did not get to negotiate the tariff variation that they face —they are only a byproduct of the EU accession process.

Then, I utilize this plausibly exogenous product-level variation and trade-and-production matched data in an event-study approach. As a result, I demonstrate that a plausibly exogenous increase in market access leads to a higher probability of initiating production and exporting a given product, which is consistent with the main mechanism of the theoretical model.

Finally, I calibrate a quantitative version of the model that solves for the endogenous balanced growth path of the model with a simplified experiment of asymmetric country groups. I then use this framework and apply trade cost shocks to replicate the policy scenario of the 2004 Eastwards enlargement of the European Union. With this toolkit I estimate that, in this framework: (a) the EU expansion increased its long-run yearly growth rate by about 0.16%; and (b) dynamic gains from trade account for more than 90% of welfare gains from trade.

Contributions to the Literature This work lies on the intersection between macroeconomics and international trade and it makes contributions both to economic theory and empirical research. For that reason, its additions to the scientific literature span a range of research topics.

I add to the theoretical literature on trade and growth —and in particular to trade and product innovation. The literature can be traced back to the seminal paper by P. M. Romer (1990). While Romer does not develop a full model, he mentions in the paper that a natural extension of his model "pertain to its implications for growth, trade, and research."[4] Extensions of the Romer model of endogenous growth of product innovation to a two-country framework were later done by Rivera-Batiz and Romer (1991a) and Rivera-Batiz and Romer (1991b) as well as Grossman and Helpman (1990), in a very similar framework. I extend the Romer growth model to a multi-country framework and combine it with a modern quantitative Ricardian trade model of Eaton and Kortum (2002).

Acemoglu and Ventura (2002) proposed a model with Armington trade that features an AK-model of growth with a stable distribution of income over the balanced growth path. Eaton and Kortum (2006) developed one of the earliest Ricardian models of trade and growth, in which they proposed a two-country model in which technology endogenously evolved and diffused across countries and trade happened by comparative advantage. In their model, if trade costs were low enough, the most efficient country in innovation specialized in R&D. Baldwin and Robert-Nicoud (2008) built a very general model of heterogeneous firms that nested several of the previous trade and growth models in the literature, including some of the Romer-style expanding variety models.

More recently, Perla et al. (2015) study how trade interacts with domestic technology adoption, shutting down the channels of international technological diffusion, In their model, trade liberalization spurs growth because it induces higher exports and profits and shifts the domestic productivity distribution outwards. Arkolakis (2016) introduced one of the first models of selection and growth, in a dynamic version of the Melitz model with exogenous growth. Sampson (2016) develops a framework with selection and knowledge spill-overs from incumbent firms to entrant firms: in his model, trade induces faster and higher selection, thereby spurring faster growth. Aghion et al. (2018) present a model of heterogeneous firms and use it to explain the impact of innovation on exports using French microdata.

Lucas (2009) and F. E. Alvarez et al. (2013) spearheaded a literature of idea diffusion, with countries learning from some frontier international distribution of knowledge while Buera and Oberfield (2020) linked this literature to trade. Santacreu (2020) extended the model of idea diffusion of Eaton and Kortum (1999) to a multi-sector framework.

Since modeling the complete state space of dynamics and countries is nontrivial, most of this literature has to make compromises. Part of these parts simplify by assuming a world of symmetric countries or a two-country world. Others, by ruling out forward-looking dynamics and modeling growth as some externality or exogenous process. My model departs from most of the literature by having both asymmetric countries and forward-looking dynamics in a theoretical and quantitative framework.

As it will be clear in the next section, it is a "true macro model" combined with a "true trade model". In this sense, it is more similar to the very recent models of Sampson (2023) and

[4]This is in section VII of P. M. Romer (1990).

Kleinman et al. (2023). However, unlike mine, the latter is a model of convergence rather than a model of long-run growth and the former is a model of firm-productivity growth rather than product innovation.

I also contribute to two strands of the empirical literature (a) one that documents the importance of the extensive margin to the growth in trade flows; and (b) another one that emphasizes the causes and effects of product innovation.

Hummels and Klenow (2005) show that the extensive margin account for about 60% of the growth in trade for large countries relative to smaller countries. Bernard et al. (2009) use very detailed U.S. data to decompose the extensive and intensive margin contributions to trade growth in the U.S. and find that year-to-year contributions primarily come from the intensive margin but the extensive margin contributions increase at longer horizons. More recently, Kehoe and Ruhl (2013) document changes in trade patterns around liberalization episodes such as the creation of the North American Free Trade Area or the expansion of the European Economic Community and show that bulk of the expansion of trade in both of those episodes for the majority of the countries involved came from a set *least traded goods*, which they denote as a representative of the extensive margin. Arkolakis et al. (2020) use data from Brazil to document a set of facts at the firm level, showing that differences in the extensive margin come from differences in the distribution of product scope across firms.

On the second group, Goldberg et al. (2010) study an episode of trade liberalization in India and show that *input liberalization* led to a higher number of *export varieties*. Bas (2012) had a similar finding using data from Ecuador. Rachapalli (2021) ties together the knowledge diffusion literature with the product innovation one and uses Indian input-output data to show that upstream and downstream introductions of new products anticipate product diffusion through the production network.

My paper makes two sets of contributions to the empirical literature. First, it documents a collection of facts using production-and-trade data around the enlargement episodes of the European Union. It compares what happens to countries that NMS joined the European Union compared to similar countries that selected into becoming candidate countries but had not yet joined at a given horizon. In a dynamic fashion, the NMS produce more products, invest more in private R&D per capita and trade more. This first part of the analysis is more akin to the first group of papers, which is a noncausal documentation of novel stylized facts.

But the paper also goes beyond that, using plausibly exogenous variation in an event-study design using a very detailed source-destination-product-year dataset. In doing so, it relates more to the second set of papers listed above, which tries to estimate well-identified empirical effects.

2 Theory

Here I present a dynamic multi-country model of the world economy with intertemporal optimization, investment in research and development, and trade in final and intermediate goods. In this economy, time is continuous with $t \in \mathcal{T} \equiv [0, \infty)$ and countries indexed by $s \in \mathbf{K} \equiv \{1, \cdots, N\}$.

Every country has the ability to produce final goods $\omega \in [0,1]$. However, they differ in their ability to produce non-rival intermediate inputs $\nu \in [0, M_s(t)]$, where the upper bound of the interval $M_s(t)$ defines the product space of a particular country. Here, I described intermediate goods as *non-rival* in the same spirit as in the endogenous growth literature: new blueprints can be simultaneously used by multiple producers at the same time, inducing increasing returns to scale[5].

As intermediate goods are invented, trade acts as a mechanism that diffuses new blueprints: producers expand their production function by sourcing newly minted inputs from around the world. Exporters are monopolists in their intermediate varieties and therefore have the incentive to invest in the development of new varieties, thereby propelling growth. Therefore, international trade will work as a vehicle that integrates global research and development stocks and induces growth-rate convergence over the balanced growth path.

My goal is to make this model easily accessible and recognizable for someone who is familiar with either modern trade theory or modern growth theory. This model will recover, as special cases, the Eaton and Kortum (2002) model of trade and the P. M. Romer (1990) model of growth. Some modeling choices and functional form assumptions will be such that this nesting is clear.

2.1 Demand

In each country $s \in \mathbf{K}$, there is a representative household the maximizes its lifetime utility according to:

$$
\begin{aligned}
\max_{C_s(t), c_s(t,\omega)_{\omega \in [0,1]}} \quad & \int_0^\infty \exp\{-\rho t\} \log\left(C_s(t)\right) dt \\
s.t. \quad P_s(t) I_s(t) + P_s(t) C_s(t) &= r_s(t) A_s(t) + w_s(t) L_s \\
C_s(t) &= \left[\int_0^1 c_s(t,\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right]^{\frac{\sigma}{\sigma-1}} \\
P_s(t) C_s(t) &= \int_0^1 p_s(t,\omega) c_s(t,\omega) d\omega
\end{aligned}
$$

where $P_s(t) C_s(t)$ are aggregate consumption good prices and quantities in country s; $c_s(t,\omega)$, $p_s(t,\omega)$ are consumption quantities and prices of variety $\omega \in [0,1]$; $I_s(t)$ are instantaneous investment flows; and $w_s(t), r_s(t)$ are wages and interest rates. At any instant, the state of asset holdings is simply the cumulative investment flows: $A_s(t) \equiv \int_0^t I(s) ds$[6].

[5] See Jones (2005) and Jones (2019) for extensive reviews.

[6] This, of course, implies that one can write investments as $I_s(t) = \dot{A}_s(t)$, which clarifies the optimal control problem at hand.

The solution to this problem comes from a two step procedure. First, households choose a sequence of consumption quantities for the aggregate good, satisfying the Euler Equation:

$$\frac{\dot{C}_s(t)}{C_s(t)} = \frac{r_s(t)}{P_s(t)} - \rho \tag{1}$$

then, at each instant, households choose optimal consumption of varieties $c_s(t,\omega)$, taking optimal expenditure period $P_s(t)C_s(t)$ as given. The solution then corresponds to the following expenditure shares for each good ω:

$$\frac{p_s(t,\omega)c_s(t,\omega)}{P_s(t)C_s(t)} = \left(\frac{p_s(t,\omega)}{P_s(t)}\right)^{1-\sigma}$$

Additionally, given these preferences, the price of the aggregate good $P_s(t)$ is:

$$P_s(t) = \left[\int_0^1 p_s(t,\omega)^{1-\sigma}d\omega\right]^{\frac{1}{1-\sigma}}$$

2.2 Production and Trade in Varieties

There are three kinds of producers in each country: **those who produce varieties of the final good**, **those who produce varieties of intermediate goods**, and **those who invest in research and development**. This section will focus on the two first ones.

Final Goods Producers. In each country, a producer of variety $\omega \in [0,1]$ of the final good is endowed with a constant returns to scale technology that combines labor and intermediate inputs $\nu \in [0, M_s(t)]$ coming from multiple countries $k \in \mathbf{K}$:

$$y_s(t,\omega) = z_s(t,\omega)[\ell_s(t,\omega)]^{1-\alpha}\left(\frac{1}{\alpha}\sum_{k\in\mathbf{K}}\int_0^{M_k(t)}[x_{ks}(t,\omega,\nu)]^{\alpha}d\nu\right)$$

where $z_s(t,\omega)$ is total factor productivity; $\ell_s(t,\omega)$ is factor demand for labor for variety $\omega \in [0,1]$ located in country s; and $x_{ks}(t,\omega,\nu)$ is the demand for a intermediate good of variety $\nu \in [0, M_k(t)]$ sourced from country k for production as an input of a final good in country s.

Non-rival intermediate goods varieties are differentiated across countries: an input $\nu \in [0, M_k(t)]$ is different from $\nu \in [0, M_n(t)]$, even if it is indexed by the same symbol. For instance, the first one may be a twelve-core computer chip from Estonia while the second one may be a large language model from Malta. Additionally, note that countries differ in their ability to produce intermediate goods, which is denoted by the upper bound of the integral $M_k(t)$.

Optimal demand for an intermediate good x_{ks} satisfies:

$$x_{ks}(t)(\omega,\nu) = \left[\frac{p_{ks}(t,\omega,\nu)}{p_{ss}(t,\omega)}\right]^{-\frac{1}{1-\alpha}} \cdot \ell_s(t,\omega) \cdot z_s(t,\omega)^{\frac{1}{1-\alpha}} \tag{2}$$

Intermediate Goods Producers. Each intermediate goods producer in country s has perpetual rights over the production of each variety $\nu \in [0, M_s(t)]$. They are endowed with a linear technology that transforms one unit of the final good into one unit of the intermediate good.

Assumption 1 (Trade Costs)**.** *Trade is subject to iceberg trade costs, which implies that shipping a final or intermediate good variety from source region s to a consumer in region d requires producing $\tau_{sd} \geq 1$, where $\tau_{dd} = 1$ and $\tau_{sd} = \tau_{ds}$ for all $s, d \in \boldsymbol{K}$.*

Given assumption (1), intermediate goods producers face heterogeneous marginal costs and set optimal prices accordingly through market-specific price discrimination. They take marginal costs and demand curves as given and choose optimal prices to maximize profits, with the optimal price being a mark-up over marginal costs for every variety ν and ω:

$$p_{ks}(t,\omega,\nu) = \frac{\tau_{ks} P_k(t)}{\alpha}$$

Note that this is the standard result of profit maximization under monopolistic competition with two variations. First, as in most trade models, prices are differentiated by destination and are inclusive of trade costs τ_{ks}. Second, since intermediate goods use one unit of the final good at the origin country k to produce one unit of the intermediate good, its marginal cost is $P_k(t)$. This prices imply that demand is:

$$\bar{x}_{ks}(t,\omega) \equiv x_{ks}(t,\omega,\nu) = \left[\alpha \cdot z_s(\omega) \cdot \frac{p_{ss}(t,\omega)}{\tau_{ks} P_k(t)}\right]^{\frac{1}{1-\alpha}} \cdot \ell_s(t,\omega),$$

where I dropped the index ν due to the symmetry for each variety $\nu \in [0, M_k(t)]$. Given the result above, I can rewrite the final goods firm maximization problem in the following:

$$\max_{\ell_s(t,\omega)} [p_{ss}(t,\omega) \cdot z_s(t,\omega)]^{\frac{1}{1-\alpha}} \cdot \tilde{M}_s(t) \cdot \ell_s(t,\omega) - \ell_s(t,\omega) w_s(t)$$

which comes from substituting for $\bar{x}_{ks}(t)$ and defining:

$$\tilde{M}_s(t) \equiv \frac{1}{\alpha} \cdot \sum_{k \in \mathbf{K}} \underbrace{M_k(t)}_{\substack{\text{measure of} \\ \text{varieties} \\ \text{in each } k}} \cdot \left(\underbrace{\frac{\tau_{ks} P_k(t)}{\alpha}}_{\substack{\text{optimal monopolist} \\ \text{price from } k \text{ to } s}} \right)^{-\frac{\alpha}{1-\alpha}} \tag{3}$$

The effective measure of input varieties $\tilde{M}_s(t)$ is a key object in this model that captures the diffusion of non-rival intermediate goods to country s. It measures input varieties from each country weighted by marginal cost. The first term $M_k(t)$ captures heterogeneity in the source-country measure of varieties since final goods producers are sourcing intermediate varieties internationally. The second term $\left(\frac{\tau_{ks} P_k(t)}{\alpha}\right)^{-\frac{\alpha}{1-\alpha}}$ captures the fact that these intermediate varieties have different prices and therefore final goods producers will face a decision regarding how to optimally substitute across intermediate goods. The term $-\frac{\alpha}{1-\alpha}$ is the elasticity of substitution across intermediate varieties, down-weighting the relative importance of intermediate goods coming from source countries k with relatively more expensive intermediate inputs.

In the standard P. M. Romer (1990) model, assemblers source intermediate goods exclusively from domestic suppliers. One important implication of that assumption is symmetry: the price of all intermediate goods will be the same. Conversely, in this framework, when sourcing intermediate goods from multiple countries $k \in \mathbf{K}$, the prices of these goods will no longer be necessarily the same. This will induce an elasticity of substitution across varieties sourced from different countries, which is reflected in the effective measure of input varieties above.

As a final remark, one should observe that the final goods producer's technology in this model is related to those in both the Eaton-Kortum and the Romer models. It is equivalent to a simple Eaton-Kortum model that uses the final good as an intermediate input with an added extensive margin shifter $\tilde{M}_s(t)$. It also is equivalent to a Romer model, which is linear in labor (or human capital), except for the fact that the measure of varieties component is a weighted average of inputs coming from domestic and international suppliers.

Trade in final goods. The factory gate price $p_{ss}(t, \omega)$ for a variety has three components: the unit production cost $w_s(t)$, the measure of varieties component $\tilde{M}_s(t)$, and a producer-specific productivity $z_s(t, \omega)$. Destination prices also include iceberg trade costs. Under perfect competition, consumers in country d choose the lowest landed-price variety ω available at the domestic market:

$$p_d(t, \omega) = \min_{s \in \mathbf{K}} \left\{ p_{sd}(t, \omega) \right\} = \min_{s \in \mathbf{K}} \left\{ \tau_{sd} p_{ss}(t, \omega) \right\} = \min_{s \in \mathbf{K}} \left\{ \frac{w_s(t)^{1-\alpha} \tau_{sd}}{\tilde{M}_s(t)^{1-\alpha} z_s(t, \omega)} \right\} \tag{4}$$

Assumption 2 (Productivity draws)**.** *Following Eaton and Kortum (2002), I consider independent productivity draws across countries and periods for final goods producers varieties. I assume that* $z_s(t, \omega)$ *is an iid random variable drawn from a market-specific Fréchet distribution*

$$F_s(t)(z) = \exp\left\{-T_s\, z^{-\theta}\right\}.$$

where T_s *is the the scale parameter and* θ *is the shape parameter.*

Given assumption (2), both prices and demanded quantities (which are functions of productivity draws) are also random variables. By the law of large numbers, the share of varieties sourced from s to d equals[7]:

$$\lambda^F_{sd}(t) \equiv \frac{E^F_{sd}(t)}{E^F_d(t)} = \frac{T_s\left(\tilde{M}_s(t)^{1-\alpha}\right)^{\theta}(w_s(t)^{1-\alpha}\tau_{sd})^{-\theta}}{\sum_{n=1}^{N} T_n\left(\tilde{M}_n(t)^{1-\alpha}\right)^{\theta}(w_n(t)^{1-\alpha}\tau_{nd})^{-\theta}} \tag{5}$$

where $E^F_{sd}(t)$ denotes the expenditure on final goods going from country s to country d; $E^F_d(t)$ denotes total expenditure on final goods in country d.

Note that the productive shifter $\tilde{M}_s(t)$ shows up in the expression for trade shares in the Ricardian Eaton-Kortum model in a way that resembles models with differentiated varieties, such as in M. J. Melitz (2003). Therefore, this formulation effectively incorporates an extensive margin in Ricardian trade models.

2.3 Research and Development

The **research sector** creates new varieties of the intermediate good. One can think of this sector as investing in the invention of new machines, which result in new blueprints. These firms use ψ units of the final good as inputs to research and development (R&D), but success is not guaranteed.

Assumption 3 (Research and Development Process)**.** *The success rate of R&D follows a Poisson process with flow arrival rate equal to* $\psi I_s(t)dt$*, where* $I_s(t)$ *is the research input per time unit —i.e., the units of the final good used for investment research and development at given instant.*

Once researching firms invent a new machine, they hold perpetual monopoly rights over the new variety ν. They can either set up their own shop to produce and enjoy the profits of producing such variety at the market or, alternatively, they can sell the rights to this patent to an intermediate variety producer. In either case, domestic households, that finance the invention of new varieties through capital markets, will collect the profits.

[7] Since there are infinitely many varieties ω and productivities are iid random variables, by the law of large numbers, the share of varieties sourced from s to d converges almost surely to the probability of sourcing a specific variety from s to d.

As shown in Appendix B.4, total profits of producers of variety ν per unit of time are:

$$\pi_k(t,\nu) = \underbrace{(1-\alpha)}_{\text{Intermediate use-share}} \cdot \sum_{s\in\mathbf{K}} \left[\underbrace{\frac{\tau_{ks}P_k(t)}{\alpha}}_{\substack{\text{optimal monopolist}\\\text{price from}\\k \text{ at } s}} \right]^{-\frac{\alpha}{1-\alpha}} \cdot \underbrace{\frac{w_s(t)L_s}{\tilde{M}_s(t)}}_{\substack{\text{demand per effective}\\\text{variety at } s}} \tag{6}$$

which is independent of ν, so we can write the aggregate level of profit in k as: $\Pi_k(t) = M_k(t)\pi_k(t) = M_k(t)\pi_k(t,\nu)$. The economic value of a new variety is the present value of producing the new varieties and selling them as intermediate inputs to final goods producers, which is, at period t:

$$V_k(t,\nu) = \int_t^\infty \exp\left\{-\int_t^\tau r_k(s)ds\right\}\pi_k(\tau,\nu)d\tau \tag{7}$$

Research firms will only invest if the expected return of their investment is positive, that is $\psi V_k(t,\nu)I_k(t,\nu) - P_k(t)I_k(t,\nu) \geq 0$. Due to free entry, in equilibrium, $V_k(t,\nu) = P_k(t)/\psi$. Since the only asset market in this economy is the domestic equity market, domestic households save by funding investments in new varieties through a balanced portfolio of infinitely many small firms, such that they face no idiosyncratic risk.

At the aggregate level, then, $\dot{M}_k(t) = \psi I_k(t)$, where $I_k(t)$ is the level of aggregate investment in the domestic economy. The value of aggregate assets is simply the value of all invented varieties $P_k(t)A_k(t) = M_k(t)V_k(t)$ and, since the arrival rate of ideas is constant, the total stock of assets is a function of the total measure of varieties $A_k(t) = M_k(t)/\psi$[8].

Taking the derivative of both sides of (7) with respect to time and noting that both $V_k(t,\nu)$ and $\pi_k(\tau,\nu)$ are independent of ν pins down the interest rate in this economy[9]. The result is a non-arbitrage condition relating returns on assets to returns on R&D:

$$r_k(t) = \underbrace{\frac{\psi\cdot\pi_k(t)}{P_k(t)}}_{\text{flow dividend rate}} + \underbrace{\frac{\dot{P}_k(t)}{P_k(t)}}_{\text{capital gains}} \tag{8}$$

[8]Noting that $\dot{M}(t) = \psi I(t)$, integrating both sides up to time t, and using the fact that $\int_0^t I(t)ds \equiv A(t)$ results in $M(t) = \psi A(t)$.

[9]After droping the indices ν, the resulting derivative is $\dot{V}_k(t) = -\pi_k(t) + r_k(t)\int_t^\infty \exp\left\{-\int_t^\tau r_k(s)ds\right\}\pi_k(\tau)d\tau = -\pi_k(t) + r_k(t)V_k(t)$

2.4 Market Clearing and Equilibrium

Market Clearing Let $Y_d(t)$ denote the total output of the final good and $X_d(t), I_d(t)$ denote the use of the final good as inputs for the production of intermediate inputs and R&D, respectively. Then total output in the final good for a given country must satisfy:

$$Y_d(t) = C_d(t) + I_d(t) + X_d(t)$$

where $I_d(t)$ and $C_d(t)$ are pinned down by the dynamic problem, described below, and $X_d(t)$ can be expressed as a function of aggregate demand in all destinations.

From the income side, since the final good aggregate output works in a competitive economy, total labor income will be equal to the labor share of total final goods sales:

$$w_s(t)L_s(t) = (1-\alpha)\sum_{d\in\mathbf{K}} \underbrace{\lambda^F_{sd}(t)P_d(t)Y_d(t)}_{\text{functions of wages}} \tag{9}$$

where which is a system of N equations that solve for the equilibrium wages[10]. Similarly, intermediates will be paid their marginal contribution to output[11]:

$$\sum_{k\in\mathbf{K}} \int_0^{M_k(t)} \int_0^1 p_{ks}(t,\nu)x_{ks}(t,\omega,\nu)d\omega d\nu \quad = \quad \alpha \sum_{d\in\mathbf{K}} \lambda^F_{sd}P_d(t)Y_d(t)$$

Trade Balance Since there are no international capital markets in this economy, trade will be balanced at any instant. This means that:

$$\underbrace{\sum_{k\in\mathbf{K}} E^F_{ks}(t)}_{\text{imports in final goods}} + \underbrace{\sum_{k\in\mathbf{K}} E^I_{ks}(t)}_{\text{imports in intermediate goods}} = \underbrace{\sum_{d\in\mathbf{K}} E^F_{sd}(t)}_{\text{exports in final goods}} + \underbrace{\sum_{k\in\mathbf{K}} E^I_{sd}(t)}_{\text{exports in intermediate goods}}$$

where:

$$E^F_{ks}(t) = \lambda^F_{ks}(t)P_s(t)Y_s(t), \qquad E^I_{ks}(t) = M_k(t)\cdot\left[\frac{\tau_{ks}P_k(t)}{\alpha}\right]^{-\frac{\alpha}{1-\alpha}}\cdot\left[\frac{w_s(t)L_s}{\tilde{M}_s(t)}\right]$$

[10] As shown in the Appendix B.7, I can express $Y_d(t)$ fully as a function of wages that does not depend on $C_d(t)$.

[11] In the Appendices, I work out the integrals on the left-hand side, but the economics of this equation is easier to understand if left in this format. In particular, Appendix B.4 will specify a worked-out version of $p_{ks}(t,\nu)x_{ks}(t,\omega,\nu)$.

Dynamic Equilibrium The dynamics in each of the countries of this world economy are governed by the following system of differential equations:

$$\begin{aligned} \dot{C}_s(t) &= \left[\frac{r_s(t)}{P_s(t)} - \rho\right] C_s(t) \\ \dot{M}_s(t) &= \frac{r_s(t)}{P_s(t)} M_s(t) + \psi \frac{w_s(t)}{P_s(t)} L_s - \psi C_s(t) \end{aligned} \tag{10}$$

As it is clear from the system above, the dynamics of the model are essentially neoclassical. However, since openness to trade impacts the cross-sectional distribution of wages and prices, it will also impact the path of consumption product measures over time.

The first equation —the Euler Equation —states that the household in a country $s \in \mathbf{K}$ will choose an upward-sloping consumption path if the real interest rate is greater than the rate of time preference. The higher this gap, the more a household will be willing to defer current consumption and take advantage of higher returns in the asset and R&D markets.

The second equation is less obvious to interpret in its current form, but it states that the growth in the product measure in each country is proportional to the net investment rate. Since expected profits of new varieties are always positive, the net investment rate is also always positive, which means that new varieties are always created, inducing growth in this model.

A more explicit way to observe the net investment way is by writing the second equation in its equivalent asset representation. Since $M_s(t) = \psi A_s(t)$, then:

$$\dot{A}_s(t) = \underbrace{\frac{r_s(t)}{P_s(t)} A_s(t)}_{\text{real capital income}} + \underbrace{\frac{w_s(t)}{P_s(t)} L_s}_{\text{real labor income}} - \underbrace{C_s(t)}_{\text{real consumption}}$$

which, along with the discussion regarding the non-arbitrage condition in the previous section, helps clarify that asset markets and varieties markets are two sides of the same coin.

The assumption of log preferences substantially simplifies the dynamic problem. In Appendix B.2, I show that instantaneous consumption is always well-defined as a constant fraction of lifetime wealth:

$$C_s(t) = \rho \left[\underbrace{A_s(t)}_{\text{wealth at } t} + \underbrace{\int_t^{\infty} \frac{w_s(\tau)}{P_s(\tau)} L_s \cdot \exp\left\{-\bar{r}_s(\tau) \cdot \tau\right\} d\tau}_{\text{PV of future labor income}} \right]$$

where $\bar{r}_s(\tau) \equiv \frac{1}{\tau}\int_t^{\tau} \frac{r_s(\nu)}{P_s(\nu)} d\nu$ is the average real interest rate between periods t and τ. Once there is an explicit solution for consumption at every t, the differential equation for $\dot{M}_s(t)$ becomes autonomous, and also has an explicit solution as a function of the path of prices.

In Appendix B.2, I show that there is a unique initial choice of consumption that is consistent with the optimal choices described by (10) and the transversality condition. Since the other conditions to satisfy the Maximum Principle are satisfied, this is equivalent to showing that the solution to the dynamic problem is unique.

Definition 1 (Dynamic Equilibrium)**.** *The dynamic equilibrium of the world economy is defined by a collection of paths of consumption quantities, assets stocks, and profit flows* $[C_s(t), A_s(t), \Pi_s(t)]$*; paths of final goods varieties output quantities* $[c_s(t,\omega)]$*; paths of intermediate goods varieties output quantities* $[x_{ks}(t,\omega,\nu)]$*; paths of prices* $[w_s(t), r_s(t), P_s(t),\ p_{ss}(t,\omega), p_{sk}(t,\omega,\nu)]$*; and a vector of fundamentals* $(\theta, \sigma, \boldsymbol{T}, \boldsymbol{\tau})'$ *where* $\boldsymbol{T} \equiv \{T_s\}$ *is a collection of location parameters of the Fréchet distribution and* $\boldsymbol{\tau} \equiv [\tau_{sd}]$ *is a matrix of trade costs, such that: (a) households maximize utility given the path for prices; (b) final goods firms maximize profits given the path for prices; (c) intermediate goods firms choose prices to maximize profits given demand functions and final goods prices; (d) trade balances; and (e) factors and goods markets clear.*

Nesting of Romer and Eaton-Kortum In this subsection, I will briefly describe how to recover the canonical P. M. Romer (1990) and Eaton and Kortum (2002) models from the framework described above. A more detailed description of the nesting can be found in Appendix B.10.

Setting $\alpha = 0$ implies that the value of new varieties is zero since the demand for and profits of intermediate varieties is also zero. Therefore, $I_s(t) = 0$ for all t and s. While the Eaton-Kortum model is a static model, here it can be thought of as an infinite sequence of static models with no intertemporal decision, since there are no longer asset markets that permit households to save. Furthermore, since $\alpha = 0$, the intermediate and research and development sectors disappear. The problem of the final goods producer becomes:

$$\max_{\ell_s(t,\omega)} p_{ss}(t,\omega) \cdot z_s(t,\omega) \cdot \ell_s(t,\omega) - \ell_s(t,\omega) w_s(t)$$

which is identical to the one in the standard Eaton-Kortum model. Equilibrium will take the form of a system of labor market determination equations that solve for N wages using trade expenditure shares.

Conversely, setting $\tau_{sd} \to \infty$ for $s \neq d$ implies trade costs are prohibitively high internationally, such that varieties of both final goods and intermediate goods become sold only locally. The dynamic household problem as will look like the standard neoclassical model with the consumption good being a CES aggregator. Furthermore, redefine assumption (2) in the following terms:

Assumption 4 (Productivity draws to recover Romer)**.** *Let $z_s(\omega)$ be a random variable distributed according to the following CDF:*

$$F_s(t)(z) = \begin{cases} 0 \textit{ for } z < T_s \\ 1 \textit{ for } z \geq T_s \end{cases}$$

The final goods assembler technology becomes:

$$y_s(t,\omega) = T_s[\ell_s(t,\omega)]^{1-\alpha}\left(\frac{1}{\alpha}\int_0^{M_s(t)}[x_{ss}(t,\omega,\nu)]^{\alpha}d\nu\right)$$

which is identical to the single-country Romer model. Profits and demand per variety $\nu \in [0, M_s(t)]$ will be constant and growth will be driven by the domestic R&D sector. Equilibrium will take the following form: labor markets will clear; total final goods produced being equal to total final goods used for consumption, intermediate production; and R&D production; and optimized household optimal dynamics will be described by an Euler equation and an asset/measure accumulation equation.

2.5 Balanced Growth Path

Autarky I will start characterizing the Balanced Growth Path (BGP) under autarky, which is a special case in which trade costs are prohibitively high such that countries are isolated as single-country economies.

Proposition 1 (BGP under autarky)**.** *Given a vector of fundamentals $(\theta, \sigma, \boldsymbol{T}, \boldsymbol{\tau})$, if $\tau_{sd} \to \infty$ for all $s \neq d$, then there are unique country specific growth rates $g_s^{autarky}$ such that, in every country $s \in \boldsymbol{K}$, $g_{M_s} = g_{Y_s} = g_{C_s} = g_{w_s} = g_{A_s} = g_s^{autarky}$. Furthermore, these growth rates satisfy:*

$$g_s^{autarky} = \left[(1-\alpha)\cdot\alpha\cdot\psi\cdot\frac{w_s(t^*)L_s}{M_s(t^*)} - \rho\right] \tag{11}$$

for a BGP inclusive of each period $t \geq t^*$. I can also express the autarky growth rate fully in terms of exogenous objects, satisfying:

$$g_s^{autarky} = \left[(1-\alpha)\cdot\psi\cdot L_s\cdot\left(\frac{1-\alpha}{L_s}\left[\int_0^{\infty}\left(z\cdot\ell_s(t^*,z)\right)^{\frac{\sigma-1}{\sigma}}dF_s(z)\right]^{\frac{\sigma}{\sigma-1}}\right)^{\frac{1}{1-\alpha}} - \rho\right]$$

Proof. Appendix B.8. □

Intuitively, Proposition (1) characterizes the BGP in a collection of closed AK economies with expanding varieties each of them as in the original P. M. Romer (1990) model. Growth happens endogenously in each of the countries as households invest in the equity market to fund new intermediate varieties. However, the mass of non-rival goods available for production will be completely different across different countries, since final good producers only have access to domestic intermediate inputs and are therefore less productive than they would be if they were trading internationally. Similarly, in general BGPs will be characterized by different growth rates.

The term in the integral denotes the joint product of productivity and labor allocation across firms. In aggregate terms, since both the distribution of productivity and the population are fixed for every t; and relative wages are fixed along the BGP, this term will be constant. The integral changed from indexing varieties by ω to indexing it by its distribution of productivities z. This follows F. Alvarez and Lucas (2007) in the argument that since all goods enter symmetrically in the definition of the aggregate final good and they differ only by their productivity level, I can express the integral above also in terms of the productivity distribution $F_s(z)$.

Note that none of the terms on the right-hand side depend on endogenous objects. While any particular $\ell_s(t, z)$ is a demand function that depends on a firm's specific productivity and on endogenous objects of the economy, the integral over all of these choices does not, since both the productivity distribution and total labor supply are fixed.

If $F_s(z)$ were a degenerate random variable such that productivity were the same for every firm in this economy —as, it is the case in the Romer model, under Assumption (5) —, then $g_s^{autarky}$ would just collapse to be expressed in terms of scalars. However, since in the general version of this model productivities have strictly positive support, the balanced growth path growth rate will account for the "average" productivity integrating over the distribution $F_s(z)$.

Zero gravity I now move on to characterize the BGP under the polar opposite case: zero gravity. This is one in which trade is costless and even geographical barriers are inexistent. The term comes from Eaton and Kortum (2002).

Before I do so, I have to define the concept of a cross-sectional equilibrium. Intuitively, cross-sectional equilibrium is the static solution for the trade equilibrium. Since along the BGP prices are constant, then the dynamic problem simplifies to a sequence of identical problems in which the dynamic optimization conditions still hold but there are no more adjustments —i.e., effectively, no more transition dynamics.

The intertemporal channel that links the labor market determination equations off of the BGP, when prices are adjusting, shuts off along BGP, such that I can define the concept of a cross-sectional equilibrium along the BGP. Formally, it is defined below.

Definition 2 (Cross-sectional Equilibrium). *A cross-sectional equilibrium is a vector of wages*

$\lambda_w \cdot [w_s(t^*)]_{s\in \boldsymbol{K}}$ *that, up to the choice of a numéraire, solves the non-linear system of equations represented by the labor market determination equation:*

$$w_s(t)L_s(t) = (1-\alpha)\sum_{d\in \boldsymbol{K}} \underbrace{\lambda^F_{sd}(t)P_d(t)Y_d(t)}_{\textit{functions of wages}}$$

along the BGP.

Lemma 1 (Cross-sectional equilibrium uniqueness under zero gravity)**.** *Given a vector of fundamentals* $(\theta, \sigma, \boldsymbol{T}, \boldsymbol{\tau})$*, if* $\tau_{sd} = 1$ *for all* (s, d)*, then the cross- sectional equilibrium is unique.*

Proof. Appendix B.8. □

In the zero gravity case, the cross-sectional equilibrium is relatively simpler. It can be proved to be unique since the excess demand function derived from the labor market determination equation (9) satisfies the gross substitution property. This result is important because the existence and uniqueness of the cross-sectional equilibrium will pin down the BGP under zero gravity. Proposition 2 below uses the fact that the cross-section equilibrium is unique to guarantee the uniqueness of the BGP under zero gravity.

Proposition 2 (BGP under zero gravity)**.** *If* $\tau_{sd} = 1$ *for all* (s, d)*, then there is a unique world equilibrium growth rate* $g^{\textit{zero gravity}}$ *such that, in every country* $s \in \boldsymbol{K}$*,* $g_{M_s} = g_{Y_s} = g_{C_s} = g_{w_s} = g_{A_s} = g^{\textit{zero gravity}}$*. Furthermore, this growth rate satisfies:*

$$g_s^{\textit{zero gravity}} = \left[(1-\alpha)\cdot\alpha\cdot\psi\cdot\left[\frac{\sum_{k\in \boldsymbol{K}} w_k(t^*)L_k}{\sum_{k\in \boldsymbol{K}} M_k(t^*)}\right] - \rho\right] \tag{12}$$

for a BGP inclusive of each period $t \geq t^*$*.*

Proof. Appendix B.8 □

By comparing $g_s^{\text{zero gravity}}$ and $g_s^{\textit{autarky}}$, it is immediately clear that while the latter is proportional to *domestic* value added per variety $\left(\frac{w_s(t^*)L_s}{M_s(t^*)}\right)$, the former is proportional to *global* value added per variety $\left(\frac{\sum_{k\in \mathbf{K}} w_k(t^*)L_k}{\sum_{k\in \mathbf{K}} M_k(t^*)}\right)$. Intuitively, under zero gravity, growth happens as if the world were a single integrated Romer economy.

In the absence of trade costs, the world economy is fully integrated in terms of final goods varieties suppliers and the law of one price holds in the final good. As the final good serves as an input for intermediate varieties, the price of intermediate varieties equalizes globally.

A corollary is that the effective measure of input varieties $\tilde{M}_s(t^*)$ also equalizes globally, indicating that non-rival inputs fully diffuse across the world. Finally, since the final good serves as an input for R&D in every country, the marginal cost of R&D and interest rates equalize across countries, which pushes the growth rate of varieties to be the same.

Note, however, that income levels need not be the same in this world economy. In fact, those countries that have a higher relative wage at the start of the BGP will have a higher wage relative forever. Therefore, under zero gravity, this model features a *stable global distribution of income* as in the model of Armington trade and capital accumulation-driven growth of Acemoglu and Ventura (2002).

Costly but finite trade I now arrive at the more realistic case of a BGP of positive but finite trade costs.

Proposition 3 (Balanced growth with costly trade)**.** *Given a vector of fundamentals $(\theta, \sigma, \boldsymbol{T}, \boldsymbol{\tau})$, in the dynamic world economy presented above, if $\tau_{sd} < \infty$ for all $s \neq d$, there* ***exists a balanced growth path world equilibrium growth rate*** *satisfying:*

$$
\begin{aligned}
g^* &= g_{M_s} = g_{Y_s} = g_{C_s} = g_{w_s} = g_{A_s} \qquad \forall s \in \boldsymbol{K} \\
&= \frac{(1-\alpha)\cdot\alpha\cdot\psi}{P_s(t^*)^2} \cdot \sum_{k \in \boldsymbol{K}} w_k(t^*) L_k \left[\sum_{n \in \boldsymbol{K}} M_n(t^*) \left(\frac{\tau_{nk} P_n(t^*)}{\tau_{sk} P_s(t^*)} \right)^{-\frac{\alpha}{1-\alpha}} \right]^{-1} - \rho
\end{aligned}
\tag{13}
$$

and the world equilibrium growth rate g^* is pinned down by a vector of wages $\lambda_w \cdot [w_s(t^*)]_{s \in \boldsymbol{K}}$ and a vector of measures of varieties $\lambda_M \cdot [M_s(t^*)]_{s \in \boldsymbol{K}}$, up to the choice of scalars λ_w, λ_M, for a BGP inclusive of each period $t \geq t^*$.

Furthermore, if there exists a equilibrium ***within a subset of the parameter space*** *that guarantee that $\frac{\partial P_s(t)}{\partial M_s(t)} < 0$ and the cross-sectional equilibrium is unique,* ***then the BGP growth rate is unique****.*

Proof. Appendix B.8. □

While the result in the Proposition (3) is more general regarding the parameter space of trade costs, it is less general regarding the guarantee of the uniqueness with free parameters for the rest of the parameter space, when compared with the zero gravity case. While a BGP is guaranteed to be unique under zero gravity, I need to restrict the parameter space to rule out the possibility of a multiplicity of solutions in the general case. These come both from the cross-sectional equilibrium and from the growth dynamics but stem from the same origin.

Since my model follows the structure of the lab-equipment version of the Romer model, I assumed that assemblers use one unit of the final good to produce intermediate goods and research goods. In a model with trade, where prices of intermediate goods differ across sources, this assumption is effectively an input-output structure to production à la Caliendo and Parro (2015) or Acemoglu et al. (2012).

It is in general hard to prove the uniqueness of the cross-sectional equilibrium in models with an input-output structure, because the derivatives of the excess demand function will not have a closed-form solution. In a later result, Allen et al. (2023) have shown that both of the aforementioned models have unique equilibria[12].

Here, I do not explicitly characterize the bounds of the parameter space that guarantee the uniqueness of the equilibrium. Rather, I simply characterize the *conditional* uniqueness of the BGP whenever two conditions are met: (a) the uniqueness of the cross-sectional equilibrium; and (b) restrictions on the parameter space that guarantee that $\frac{\partial P_s(t)}{\partial M_s(t)} < 0$.

Whenever that happens, the intuition of the BGP is very similar to what happens under zero gravity. There will be a stable distribution of income, characterized by the vector of wages $\lambda_w \cdot [w_s(t^*)]_{s \in \mathbf{K}}$ and also a stable distribution of measure of varieties $\lambda_M \cdot [M_s(t^*)]_{s \in \mathbf{K}}$, that will both grow that at the same rate g^* over time.

Furthermore, the real interest rate equalizes globally. Even though there are no international equity markets, the fact that households can invest in new varieties through equity markets and earn expected profits that are linked to exports means that trade acts as a vehicle to integrate international R&D and equity markets.

In the proof of Propositions (2) and (3), I introduce the idea of a *excess varieties growth function*, which is a growth rate analog of the excess demand function, to characterize the existence of the BGP. The economic intuition behind it is straightforward: if in a given country the measure of varieties is *above* its long-run equilibrium, then the price of intermediate goods (the marginal returns to innovation) will be *below* its long-run equilibrium and the country will grow *slower* than average. The equilibrium growth rate will be reached when growth rates and returns to R&D equalize internationally.

As seen by the growth rate formula in equation (13), g^* is, homogeneous of degree zero in wages and measures of varieties simultaneously[13]. Therefore, reassuringly, re-scaling all wages and all measures of varieties by some arbitrary constant $\lambda > 0$ does not change the BGP growth rate.

The growth rate clearly shows that growth is proportional to market access and the size of

[12]While they have proved the existence and uniqueness of the equilibrium in those two models, their proof method only works for models with a log-linear structure in the equilibrium equations, which is not the case for my model. They have in a previous version of their paper proved the existence and uniqueness of Caliendo and Parro (2015) using a more general proof method, but that proof required more astringent assumptions of the parameter space, which is the same argument that I am developing here.

[13]To see that, note that the price levels are themselves functions of wages and measures of varieties that are homogeneous of degree zero, so the only relevant wages and measures are those that are shown explicitly in (13). By scaling them simultaneously by some parameter $\lambda > 0$, it is obvious that will cancel out and return the same growth rate g^*.

the effective market, as measured by value-added weighted by a metric of competitiveness of domestic intermediate exports relative to foreign intermediate goods (I delve deeper into this discussion in the next subsection). Therefore, changes in trade policy that increase market access, such as increased market integration in net terms, are predicted to increase growth, through the lens of the model.

Changes in trade costs As a final step, I state two propositions regarding what happens to the equilibrium growth rate once trade costs change. The growth rate is a general equilibrium object that depends on the whole distribution of prices across countries and periods. Therefore, characterizing changes to it is not a trivial task.

Nonetheless, in order to connect the theory to the empirical and quantitative analysis, I first show what happens in a simplified framework of symmetric countries, in which I am able to characterize what happens to the **long-run** equilibrium growth rate after a permanent change in trade costs. In that case, g^* can be shown to unambiguously increase in the long run after an episode of trade liberalization.

Proposition 4 (Effects of changes in trade costs over the long run in symmetric economies)**.** *Suppose there exist a collection of symmetric economies that grow over the BGP with costly trade with trade costs $\tau > 1$. Then $\frac{\partial g^*}{\partial \tau} < 0$.*

Proof. Appendix B.8 □

In this model, the long-run growth rate will change after a permanent change in trade costs if there is a change in the effective market size, represented by how much of the global market exporters can tap into —that is why foreign aggregate demand $\sum_k w_k(t^*)L_k$ is modulated by foreign effective measures of varieties $\sum_k \tilde{M}_k(t^*)$ in the profit formula. In a symmetric world, I can show in closed form that real profits increase when trade costs go down.

While the assumption of symmetry can be quite restrictive, the intuition translates to numerical exercises that show the same results with asymmetric countries. However, in that case, due to the input-output structure embedded in the lab-equipment version of the Romer model, there are no closed-formed solutions, and proving similar results is nontrivial.

I now move to a more realistic case but with more restrictive analytical results. Consider what happens to the growth rates in a particular country s if, along the BGP, there is a change in trade costs. The results below should be interpreted bearing in mind the fact that such a change in trade costs will induce re-optimization —i.e., it will induce countries to shift away from the original BGP and possibly to go through a period of transitional dynamics.

For that reason, the result below **cannot** be taken as to what happens to long-run equilibrium growth rate after a permanent change in trade costs, but rather a **short-term response**. Long-term responses can only be calculated after equilibrium prices adjust. The result below should be interpreted as what happens, at the margin, to the growth rate of a particular country s, relative to the original BGP growth rate g^* after a change in trade costs, instantaneously during a period of adjustment.

Proposition 5 (Effects of increased market access over short run in asymmetric economies)**.** *If $\tau_{sd} \in (1, \infty)$ for all $s \neq d$, along the BGP, the first-order effect of market access liberalization is always positive, that is:*

$$\left.\frac{\partial g_s}{\partial \tau_{sd}}\right|_{\boldsymbol{P}=\boldsymbol{P}(t^*),\boldsymbol{w}=\boldsymbol{w}(t^*),\boldsymbol{M}=\boldsymbol{M}(t^*)} < 0$$

Furthermore, the total general equilibrium effect will depend on the cross-elasticities of price levels, wages, and measures with respect to trade costs and take the following form:

$$\begin{aligned}\frac{\partial g_s}{\partial \tau_{sd}} &= \underbrace{-\frac{\Xi}{\tau_{sd}} \cdot \sum_{n\in\boldsymbol{K}} \mu_n(t^*)\left[1+\epsilon^P_{s,sd}-\epsilon^P_{n,sd}\right]}_{\text{net market access effect}} \\ &+ \frac{(g_s+\rho)}{\tau_{sd}}\left(\underbrace{\sum_{k\in\boldsymbol{K}}\left[\epsilon^w_{k,sd}-\mu_k(t^*)\epsilon^M_{k,sd}\right]}_{\text{global market expansion effect}} - \underbrace{\frac{1}{2}\epsilon^P_{s,sd}}_{\text{input cost effect}}\right)\end{aligned} \tag{14}$$

where $\epsilon^X_{s,sd} \equiv \frac{\partial X_s(t^*)}{\partial \tau_{sd}} \cdot \frac{\tau_{sd}}{X_s(t^*)}$, $\tilde{\Xi}$ is a positive number[14]*, and $\mu_n(t) \equiv \dfrac{M_n(t^*)\left(\frac{\tau_{nd}P_n(t^*)}{\tau_{sd}P_s(t^*)}\right)^{-\frac{\alpha}{1-\alpha}}}{\sum_{k\in\boldsymbol{K}} M_k(t^*)\left(\frac{\tau_{kd}P_k(t^*)}{\tau_{sd}P_s(t^*)}\right)^{-\frac{\alpha}{1-\alpha}}}$.*

Proof. Appendix B.8. □

The expression in (14) is the composition of three effects. The first one is the net market access effect. As costs go down, the price of intermediate goods decreases, making locally produced intermediate varieties more competitive and inducing substitution from foreign varieties towards varieties produced in s and exported to markets n, thereby increasing profits and short-term growth.

There is a first-order effect, captured by the 1 within the brackets —that says that short-term growth locally increases when market access trade costs decrease. But there are also second-order effects, whose direction depends on the sign of the price elasticities with respect to trade costs.

[14]In particular, $\Xi \equiv \frac{\alpha^2}{P_s(t^*)^2} \cdot w_d(t^*)L_d\left[\sum_{n\in\boldsymbol{K}} M_n(t^*)\left(\frac{\tau_{nd}P_n(t^*)}{\tau_{sd}P_s(t^*)}\right)^{-\frac{\alpha}{1-\alpha}}\right]^{-1}$

Typically $\epsilon^P_{s,sd} < 0$, since a foreign trade barrier will increase the availability of goods domestically. It is harder to put bounds on $\epsilon^P_{n,sd}$, but with some spillovers, it is not unreasonable to assume that $\epsilon^P_{n,sd} > 0$, even if small. Note that, if those two assumptions are true, these two elasticities lean **against** the first-order effect. Under those assumptions, a sufficient (but not necessary) condition for the net market effect to be positive after a decrease in trade costs is for $|\epsilon^P_{s,sd}| + |\epsilon^P_{n,sd}| < 1$ for all n.

The second effect is a global market expansion effect. As mentioned in the discussion of the BGP, whenever there is a growth in the market size, there will be an expansion in growth in this model. It is hard to put bounds on these elasticities since these are new objects without parallel in the literature. However, given the theory set forth above, it is reasonable to conjecture that they are positive.

The third and last effect is simply a product of the lab-equipment structure of the model. Since the final good serves and input to R&D, it is itself the "price of research." Therefore, market access trade liberalization, while having a positive direct effect on profits, has a negative indirect effect on R&D by indirectly increasing the price of of inputs.

2.6 Welfare

With log preferences, at any moment, consumption over the BGP is a fraction of assets plus real labor income. Since such consumption flow grows at a constant rate g^* and the measure of products is simply a linear transformation of assets, as shown in Appendix B.9, welfare along the BGP can be decomposed between a product measure component, a real income component, and a growth component.

$$\int_{t^*}^{\infty} \exp\{-\rho(t-t^*)\} \log\left(\exp\{g^* t\} C_s(t^*)\right) dt = \underbrace{\underbrace{\log\left(\frac{1}{\psi} M_s(t^*)\right)}_{\text{product measure}}}_{\text{transitional}} + \underbrace{\underbrace{\frac{1}{\rho}\log\left(\frac{w_s(t^*) L_s}{P_s(t^*)}\right)}_{\text{real income}}}_{\text{static}} + \underbrace{\underbrace{\frac{g^*}{\rho^2}}_{\text{growth}}}_{\text{dynamic}} \tag{15}$$

Consider what happens to welfare after a change in trade costs from $\boldsymbol{\tau}$ to $\boldsymbol{\tau} + d\boldsymbol{\tau}$, as in Arkolakis et al. (2012) —hereinafter ACR. In this dynamic setting, to make a comparison to the static framework, I need to compare what happens across the two BGPs, comparing the preserved value of discounted lifetime utility across the beginning of the two initial equilibria. For that, let me introduce some notation: suppose t^* is the initial period of the original BGP; t^{**} is the first period of the final BGP and let $\hat{x} \equiv x(t^{**})/x(t^*)$.

Then, relative level changes in the first component of welfare across two BGPs can be expressed as $\log\left(\widehat{M}_s\right)$. Changes in the equilibrium product measure will depend on whether the measure of varieties in country s expands or contracts, **relative to the distribution of varieties across countries**, across BGPs. There is no general prediction in the model regarding the direction of this effect.

Countries that have started with a measure of varieties above optimal (relative to other countries) will see a shift in exports (and therefore R&D expenditures) towards other countries and will see their measures of varieties shrink. The opposite is true for countries that started with a measure of varieties below optimal.

Importantly, however, this first component will not compound over time, as highlighted by the fact is not multiplied by the factor ρ^{-1} or ρ^{-2}. This means that it will only change the (relative) income level that a given arrives with at the BGP and it will have no impact going forward. For that reason, I call this first component the **transitional effect** of welfare. For most reasonable calibrations of ρ, the transitional effect will have a very small weight on total welfare changes.

The second component will be familiar to most trade economists. It looks like the traditional **static welfare formula in ACR**. In the same spirit as ACR, I can also write the static welfare component in changes:

$$\frac{1}{\rho}\log\left(\frac{\widehat{w_s}}{P_s}\right) = \frac{1}{\rho}\log\left(\widehat{\lambda_{ss}^{F}}^{-\frac{1}{(1-\alpha)\theta}}\right) + \frac{1}{\rho}\log\left(\sum_{k\in\mathbf{K}}\mu_k \cdot \underbrace{\widehat{M}_k}_{\substack{\text{product}\\ \text{measure in } k}} \cdot \left(\underbrace{\frac{\widehat{\tau}_{ks}\widehat{P}_k}{\widehat{\tau}_{ss}\widehat{P}_s}}_{\substack{\text{relative price of}\\ \text{intermediate}\\ \text{good } k \text{ at } s}}\right)^{-\frac{\alpha}{1-\alpha}}\right) \tag{16}$$

where $\mu_k \equiv \dfrac{M_k(t^*)\cdot\left(\frac{\tau_{ks}P_k(t^*)}{\tau_{ss}P_s(t^*)}\right)^{-\frac{\alpha}{1-\alpha}}}{\sum_{k\in\mathbf{K}} M_k(t^*)\cdot\left(\frac{\tau_{ks}P_k(t^*)}{\tau_{ss}P_s(t^*)}\right)^{-\frac{\alpha}{1-\alpha}}}$.

This component preserves the standard feature that changes in consumer welfare are decreasing in changes in domestic trade share $\widehat{\lambda_{ss}^{F}}$[15]. This captures the Ricardian intuition of the model: at the margin, there are static gains from specialization in this model.

These Ricardian gains are augmented by an extensive margin, represented by the weighted change in the measure of varieties on the right-hand side. Note that this welfare impact from product innovation resembles how the change in the measure of varieties shows up in the ACR formula in Melitz-type models, highlighting once again that the nested structure of production featured in this model effectively adds an extensive margin to the Eaton-Kortum framework.

[15]The elasticity of this effect is $-\frac{1}{(1-\alpha)\theta}$ rather than the standard $-\frac{1}{\theta}$ due to the input-output structure of the model.

While they do not grow over time, both of these effects have an impact in every period over the BGP as it is made clear by it being multiplied by the factor ρ^{-1}. For that reason, I call the combination of these the **level effect** or **static effect** of welfare.

The third and last component is the common growth rate g^*. As made clear in the previous subsection, this rate is pinned down by the distributions of wages and measures of varieties in the beginning of the BGP. Importantly, since it compounds the BGP level of consumption, it is multiplied by a factor ρ^{-2} rather than ρ^{-1} and it will in general have a larger weight on welfare. This is a metric of **dynamic gains from trade** that I term a **growth effect** or **dynamic effect** of welfare.

Comparing the dynamic and static components of welfare yields important insights regarding the economic mechanisms behind this model. In fact, the forces of specialization and innovation are reflected in these two components.

To see that, note that, as made clear by (16), country d's **static welfare** is decreasing in the price of foreign intermediate goods. By contrast, dynamic welfare is *increasing* in the relative price of foreign intermediate goods:

$$g_s \propto \pi_s(t) \quad = \quad (1-\alpha)\cdot\alpha\cdot\sum_{n\in\mathbf{K}} \underbrace{w_n(t)L_n}_{\text{value added in } n} \left[\sum_{k\in\mathbf{K}} \underbrace{M_k(t)}_{\substack{\text{product}\\ \text{measure in } k}} \left[\underbrace{\frac{\tau_{kn}P_k(t)}{\tau_{sn}P_s(t)}}_{\substack{\text{relative price of}\\ \text{intermediate}\\ \text{good } k \text{ at } n}}\right]^{-\frac{\alpha}{1-\alpha}}\right]^{-1}$$

The intuition for this contrast is quite straightforward and underscores the different underlying economic mechanisms of the model. The static welfare formula captures the effect of d as final producers and consumers. As d purchases more foreign intermediate varieties for a cheaper price, it becomes more productive by increasing its effective measure of varieties $\tilde{M}_s(t)$. The other side of the coin is that it decreases the local price index $P_s(t)$, which directly benefits consumers and increases welfare.

Conversely, the growth effect captures the effect of d as forward-looking investors in the R&D market and intermediate good producers. Since the intermediate goods are substitutes, all else equal, demand for intermediate goods from s and maximized profits are higher when the price of intermediate goods of foreign competitors from third-party countries k relative to domestic producers from s at each destination market n is higher.

Intuitively, the *growth effect* captures that, from a **seller's perspective**, the domestically produced and exported intermediate variety s is more attractive and competitive when foreign varieties k are more expensive. Conversely, the *static effect* captures that, from a **buyer's per-**

spective, when foreign varieties k relative to one's domestic purchasing power at d are more expensive, the domestic consumer is worse off. Both channels are economically sensible and the model captures both mechanisms.

Along the BGP, prices, and measures of varieties will adjust to make sure that growth rates equalize such that $g_s = g_{s'}$ for every $s, s' \in \mathbf{K}$. While the economic mechanisms are still operating under the hood and take over if there is any shock that drives the system off the BGP, these different effects will wash out once differences in prices, measures of varieties and wages endogenously adjust towards a BGP.

3 Empirical Evidence

3.1 Data Description

I build a dataset that consists of the measure varieties produced and traded by countries that joined the European Union at different enlargement episodes.

Production data Production data comes from Eurostat's Prodcom (*Production Communautaire*), which is an annual full coverage survey of the European mining, quarry and manufacturing sectors, which reports the value of production of 4,000+ different product-lines of EU members and candidate countries. Prodcom reports, for each product line, country, and year, the value (in euros) and volume (in kg, m^2, number of items, etc.) of production. Product lines follow the Statistical Classification of Products by Activity in the EU (CPA).

The target population of the full coverage sample is every enterprise that manufactures some good in the Prodcom List. Data quality is good for member countries since European Law[16] mandates National Statistical Institutes to collect enterprise-level information on the value and volume of production covering at least 90% of national production in each NACE class, defined as the first four digits of each product code. In practice, reporting goes beyond this minimum threshold and, according to Eurostat, the coverage error is estimated to be below 10%.

Let n, i, p, t index countries, sectors, products, and periods, respectively; and denote Y_{pint} as the market value of production of product p[17]. The set of varieties produced in each sector is $\mathcal{M}_{nit} = \{k : Y_{nikt} > 0\}$. The measure of varieties is simply the cardinality of the set of produced varieties $M_{nit} = |\mathcal{M}_{nit}| = \sum_k \mathbb{1}_{\{k:Y_{nikt}>0\}}$. The overall measure over varieties produced in a country is, then: $M_{nt} = \sum_i M_{nit}$. These measures can be directly calculated

[16]"PRODCOM statistics are compiled under the legal basis provided by Council Regulation (EEC) NO 3924/1991 of 19 December 1991 and by Commission Regulation (EC) No 0912/2004 of 29 April 2004 implementing the Council Regulation (EEC) No 3924/91 on the establishment of a Community survey of industrial production. Additionally, a Commission Regulation updating the PRODCOM classification is available annually since 2003."

[17]To construct sector codes, I use Eurostat concordances to map Prodcom product codes to Harmonized System (HS) product codes. I then used the respective HS-2 division codes as sector codes.

from Prodcom's table.

Oftentimes, the value of production is labeled as confidential information by the National Statistical Institute, particularly in cases in which production is concentrated on a few enterprises. In those cases, while the value and volume are not publicly available, Eurostat reports this number as *confidential*, which still allows one to infer that $Y_{nikt} > 0$ for that particular variety k, implying that the variety is produced.

Typically, production information at the variety level is not available, which pushed researchers to use product-level trade data instead. Some exceptions include Goldberg et al. (2010) and Rachapalli (n.d.), who use firm-product links from the Indian Survey of Manufacturers; Bernard et al. (2011), who use US Manufacturing Censuses firm-product data.

Tariff and trade flow data Bilateral tariff data come from WITS (World Integrated Trade Solution Trade Stats). It consolidates tariff data from the UNCTAD's Trade Analysis Information System (TRAINS) as well as from the WTO.

I constructed effective tariff rates by constructing several baseline tables of most favored nation tariffs at the source-country $\times$ destination-country $\times$ HS6-code $\times$ year in my dataset. I then superimposed every bilateral product level preferential tariff available at the WITS database on each of these tables. Furthermore, whenever there are gaps between two identical bilateral preferential tariffs, I fill in those gaps. The result is a dataset of effectively applied tariff rates.

Bilateral trade flow data comes from UNCOMTRADE. These data, which are widely used in research, come natively in a source-country $\times$ destination-country $\times$ HS-6 product-code $\times$ year format, which makes it readily compatible with the tariff data mentioned above.

Let s, d, i, p, t index source countries, destination countries, sectors, products, and periods, respectively; and denote X_{sdipt} as the market value of bilateral trade of product p.

The set of traded varieties in each sector is $\mathcal{X}_{nit} = \{k : X_{sdikt} > 0\}$. Analogously as with production, I can observe the total number of traded varieties $\sum_k \mathbb{1}_{\{k:X_{nikt}>0\}}$. To make sure these are comparable to PRODCOM's codes, whenever possible, I used concordances and restricted the set of goods to create a dataset that matched both trade and production.

Other data I also collected data on (a) the dates of accession of new member states to the European Union; (b) trade agreements existent and entered into force between the European Union and third parties before 2004; and (c) expenditure in private research & development expenditures per capita. The first two come from hand collecting documents and tables from the European Commission's official websites while the latter comes from Eurostat.

3.2 Qualitative evidence: life among product innovators

As an initial exploratory part of this research, I conducted a qualitative survey of managers in firms of New Member States. I first collected a list of notable firms from publicly available sources, restricted the sample to those who were active for at least two years before the time their respective countries joined the European Union, then crawled through their English-language websites to collect the publicly available contact information. I sent the questionnaire available on Appendix C to 221 firms.

My goal was to assess if the description of the world that macro theorists set forth aligns with the practical intuitions of entrepreneurs. And it turns out that, at among the group of managers that responded to my email, they do. I will highlight two illustrative cases in the text.

For instance, the dynamic mechanism that propels growth, as I have described in the theory section is that increased access to foreign markets increases expected profits, thereby increasing the incentive to invest in research and development. This is entirely consistent with the description of the facts by one Czech biotech entrepreneur:

> "Once we joined the EU [...] this allowed us to increase our exports and fund our own genetic programmes." —**CEO of a Czech Biotech company**

In their comments, they went on to specify the importance of having access of not only to the European market itself, but also third party markets. They mentioned that after the Czech Republic joined the EU, his firm had immediate access to the standards for labeling and certification in existing trade agreements between the EU and third parties, which facilitated their firm's exports. These kinds of non-tariff barriers are typically considered part of trade costs τ in most trade models.

In this firm's particular case, product innovation came through the invention of breeding of new varieties of farm animals, that were then commercialized. But we see a similar story in a very different market: alcoholic beverages:

> "In 2004, we first started producing the ultra-luxury variation of our signature vodka, which became a popular export product [...] and later started production of 18 new products." —**Spokesperson of a Latvian liquor manufacturer**

In this case, the firm reported having used the European market's exports as a platform for global expansion. For context, 2004 marks the year Latvia accessed the EU —and also the year that this manufacturer decided to expand its product line by introducing the ultra-luxury versions of its signature product, which they claimed was adequate to the Western European market.

Once again, this is qualitatively consistent with the theoretical mechanism proposed in the model, with market access likely inducing product innovation. Of course, these individual experiences are not necessarily representative of a large universe of firms, which is why in the next two sections I will perform a detailed quantitative exploration of the data, first detailing some stylized facts, then going into causal inference. Nonetheless, the type of qualitative evidence presented here is useful to show to that the big picture is consistent with the individual experiences.

3.3 Stylized Facts

I calculate the increase in product varieties in countries that became in EU new members *relative to those candidate countries that were not members at a given time horizon*, exploiting the staggered nature of the enlargement of the European Union. Here, one can think of industries in countries that became EU members as individual members of a "treatment group" and industries in candidate countries that applied for EU membership but had not yet become members by that time as individual members of a "control group."

Of course, since treatment assignment, in this case, is not random, this is not actually a true experiment. I am using this technique to summarize the data and compare what happened to new member states (NMS) relative to similar candidate countries that have not yet joined the European Union.

If there were only one treatment and one control group, and the treatment assignment were random, one could calculate the treatment effect of EU membership on product change in the measure of varieties produced with a classical difference-in-differences approach. When there are multiple treatment dates across groups, applied economists have traditionally resorted to two-way fixed-effects (TWFE) event study models —i.e., a model with dummy variables for leads and lags around the group-specific treatment date as well as time and group fixed-effects. However, a recent wave of econometric papers has shown that TWFE models are not a generalization of the difference-in-differences estimator. In fact, they will be biased, except under very strong assumptions.

Goodman-Bacon (2021) shows that the TWFE estimator is a weighted average of all possible two period-two group comparisons and that, as emphasized by Borusyak et al. (2022), it is biased if treatment effects are heterogeneous. Sun and Abraham (2021) proposed a new estimator that accommodates treatment effect heterogeneity, which was later generalized by Callaway and Sant'Anna (2021).

In this paper, to avoid the potential biases of TWFE in summarizing the data, I adopt the Callaway-Sant'Anna (CS) estimator. In a nutshell, CS calculates group-specific treatment effects by: (a) comparing the treated group with either the not-yet-treated groups or the never-treated groups; and then (b) aggregating them into an average treatment effect given a specific set of weights. This estimator is consistent even if true treatment effects are heterogeneous.

Therefore, even if my objective is to simply summarize the data rather than to make causal claims, I still want to avoid making "forbidden comparisons." The CS estimator, in this case, will simply recover the average difference in outcomes for NMS relative to countries that are candidate countries but are not yet members, at different horizons around EU enlargement events.

Formally, let a "treatment" group g be defined as being treated for all periods $t \geq g$. Note that, since the EU enlargement happened simultaneously for more than one country, there is more than one country n for each $g_n = g$. If some industry-country cluster is in group g, then $G_{nit} = g$ $(\forall t)$. If it is never treated, it is in the control group, and then $G_{nit} = \infty$ $(\forall t)$.

The parameter of interest is the average treatment on the treated for a given treatment group g and horizon t, i.e.:

$$ATT(g,t) = \mathbb{E}[M_{nit}(g) - M_{nit}(0)|G_{nit} = g] \tag{17}$$

where $M_{nit}(g)$ is the potential outcome of industry i in country n at period t if treated at period g; $M_{nit}(0)$ is the potential outcome of industry i in country n at period t if untreated; X_{ing-1} are pre-treatment time-invariant covariates; and $G_{nit} = g$ is a group indicator.

Note that the $ATT(g,t)$ is group and period specific. It can be recovered under assumptions similar to the standard difference-in-differences framework: parallel trends and no-anticipation[18]. The next step is to summarize the ATT across groups by appropriately weighting the results as:

$$\theta(t) = \sum_{g} \mathbb{1}\{g \leq t\} w_{gt} ATT(g,t) \tag{18}$$

for some weights w_{gt}. Callaway and Sant'Anna (2021) propose the weights $w_{gt} = P(G_{nit} = g|G_{nit} \leq t)$, which is the share of industry-country clusters from group $g \geq t$ out of all industry-country clusters being treated at time t.

In the estimates reported below, this means that the event that will have the largest weight will be the 2004 EU expansion, which enlarged membership by nine countries. The other episodes of expansion – Bulgaria and Romania, in 2007; and Croatia, in 2013 – influence the estimates with proportional weights for the horizons in which data is available. It is important to highlight that throughout the sample, there is readily available data for candidate countries that never became EU members, which serve as a natural comparison group.

Here, I run the staggered difference-in-differences event study regressions for a set of variables. First, I use the **measure of produced varieties** as the dependent variable. I also run similar models with the **log of real private research and development expenditures** and the **log of real value of yearly trade** as dependent variables.

[18]Formally, parallel trends is the assumption that potential outcomes evolve almost surely equally to the untreated group: $\mathbb{E}[M_{nit}(0) - M_{int-1}(0)|G_{nit} = g] = \mathbb{E}[M_{nit}(0) - M_{int-1}(0)|G_{nit} > g]$ for all $t \geq g$. No anticipation means that potential outcomes for a treated group are equal to the untreated group for any date before the treatment —i.e., for all $t < g$, $\mathbb{E}[M_{nit}(g)|G_{nit} = 1] = \mathbb{E}[M_{nit}(0)|G_{nit} = 1]$ almost surely.

I take these variables as aggregate **macro moments** that the theoretical model makes predictions about. Relative to a candidate country that did not fully integrate its economy with the European Union and did not have preferential access to the trade partners of the EU, what happens, *on average*, to these variables in New Member States?

As shown in Figure 2, fifteen years after membership, the expected differential increase in varieties is 306, or about 17% relative to the year of membership[19]. The differential effect seems to cumulatively increase after the year of membership.

The effects regarding private R&D, shown in Figure 3 show a clear break in trend in the differential averages around the year of membership. Fifteen years after the expected differential growth in private R&D expenditures is about 60%.

Finally, the results relative to trade also show a differential growth, as illustrated by Figure 4. There are no signs of pre-treatment trends and, seven years after membership, the expected differential growth in the value of yearly trade is about 50%.

The theory laid down in this paper suggests that economic integration should lead to faster product innovation, higher R&D expenditures, and more trade. In total, I take these macro moments as an *initial validation* of general predictions of the model. The broader correlations are consistent with the predictions of the model.

However, since the process of EU accession is not random, there are no claims of causality and a more rigorous test of the mechanism of the model is necessary. This is what I hope to do in the next subsection of the paper.

[19]The average treatment effect is 306.23 and the conditional average number of produced varieties in treatment year zero is 1804.6.

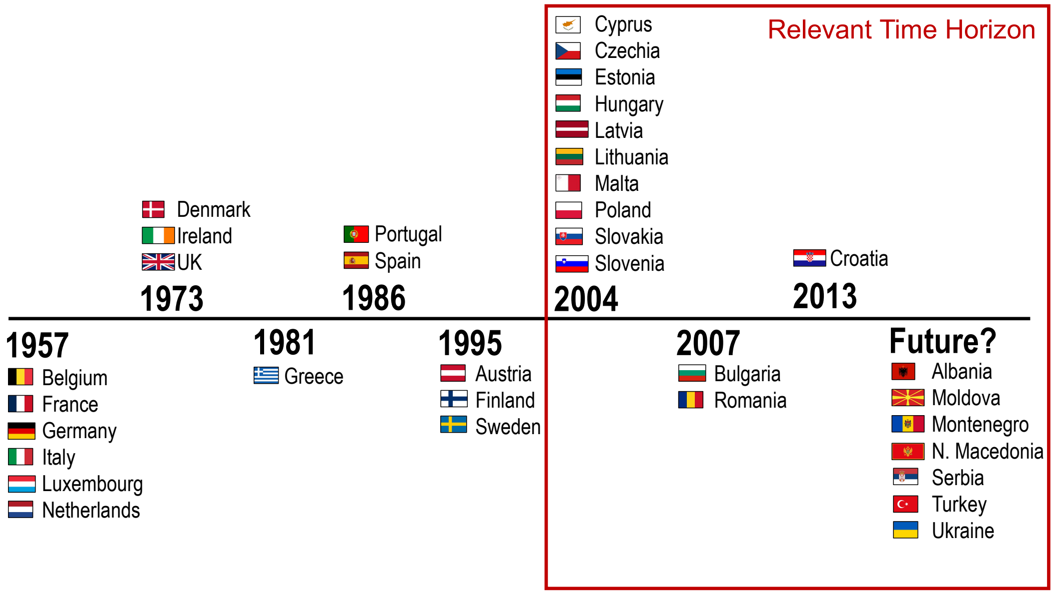

Figure 1: **Institutional Context: Timeline of European Union Enlargement.** Since the EU enlargement comes in waves, the future cohorts serve as comparison groups, for some time, to previous cohorts. Importantly, there are candidate countries that never join throughout the relevant time horizon, such that they can stay in the comparison group.

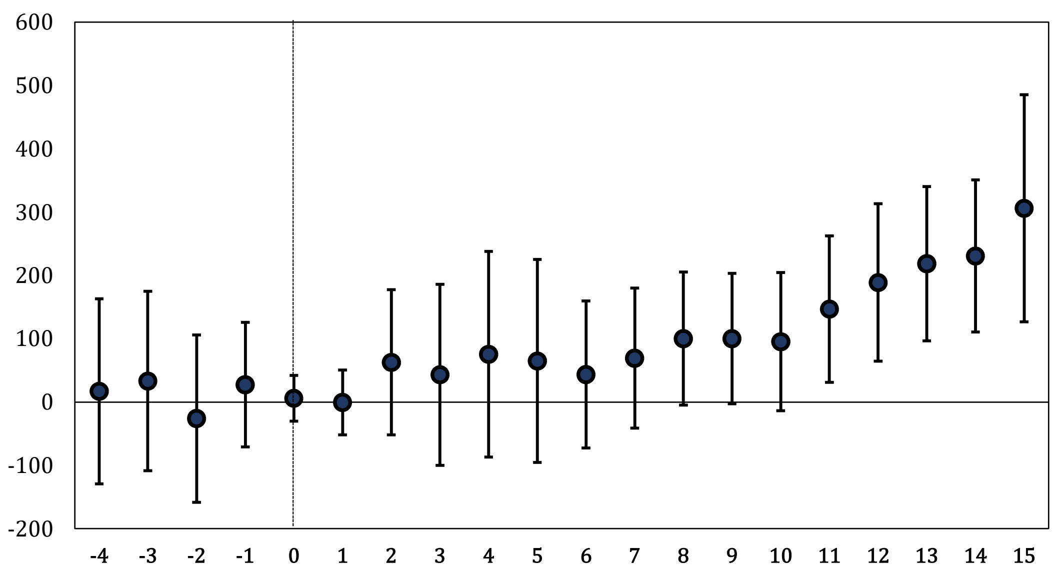

Figure 2: **Staggered difference-in-differences: Measure of Varieties.** This plot shows the estimated coefficients $\theta(t)$ time-specific average treatment on the treated coefficient described by equation (18) at the industry and aggregate level, respectively. The bars around the red line denote 95% bootstrapped standard errors.

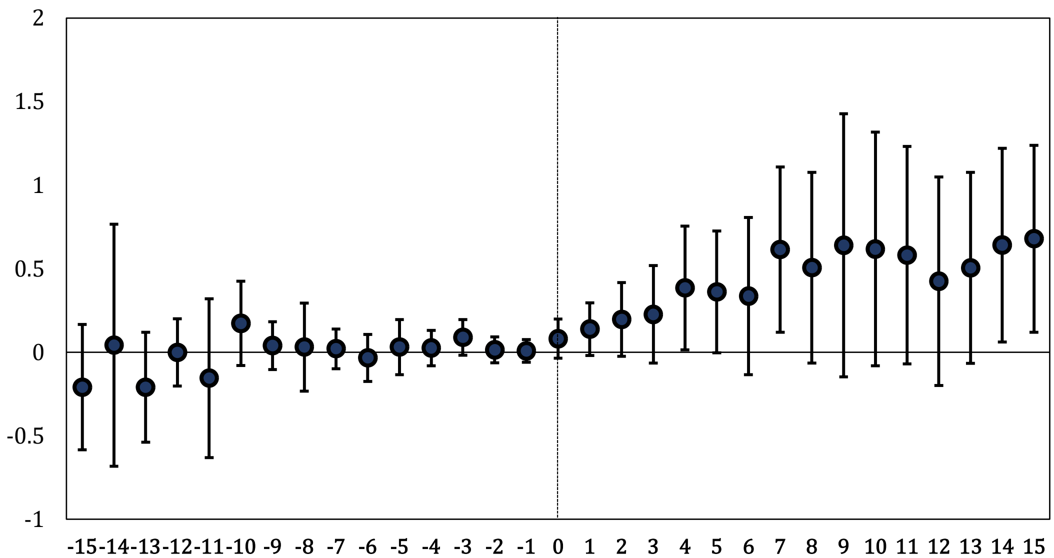

Figure 3: **Staggered difference-in-differences: Log of Private Research and Development Expenditures.** This plot shows the estimated coefficients $\theta(t)$ time-specific average treatment on the treated coefficient described by equation (18) at the industry and aggregate level, respectively. The bars around the red line denote 95% bootstrapped standard errors.

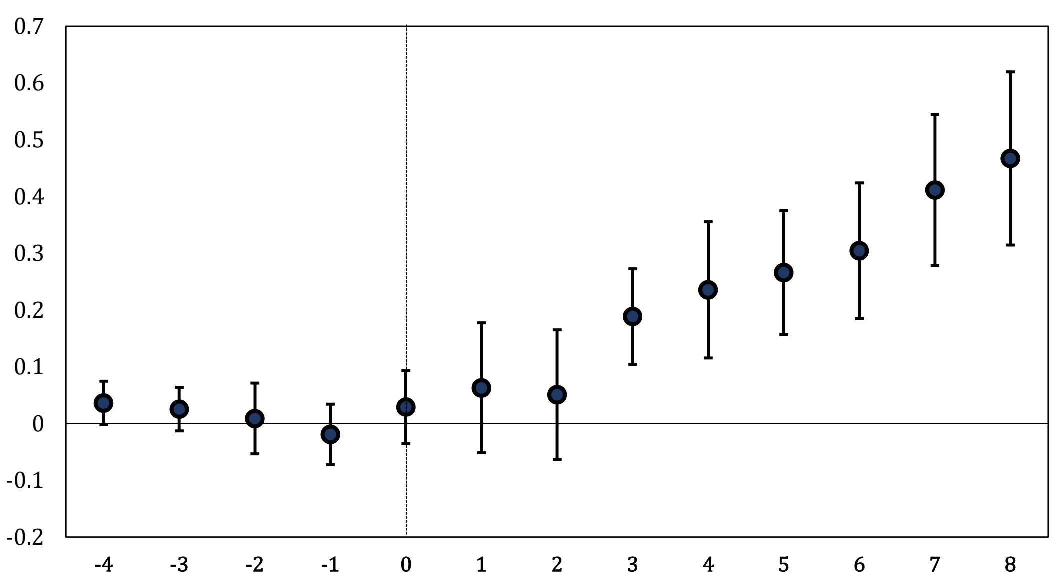

Figure 4: **Staggered difference-in-differences: Log of Real Value of Yearly Trade.** This plot shows the estimated coefficients $\theta(t)$ time-specific average treatment on the treated coefficient described by equation (18) at the industry and aggregate level, respectively. The bars around the red line denote 95% bootstrapped standard errors.

3.4 Causal Evidence

In order to go beyond correlational analysis, I need to exploit some plausibly exogenous variation around the EU expansion of the European Union. One potential avenue to do so is to use the fact that once NMS join the EU, they not only have preferential access to the European market, but they also have to adhere to the common trade policy of the European Union.

This means that these countries have immediate preferential access to third-party markets via previously existing trade agreements between the EU and these third-parties. Furthermore, since these trade agreements are previously existing, while the NMS have access to them, they did not get to negotiate the tariff variation that they face —they are only a byproduct of the EU accession process.

Figure 5 illustrates how this happened in a specific example: the Free Trade Agreement between the EU and Mexico. The EU joined a FTA with Mexico in 2000, but the NMS only joined the EU in 2004, so in 2004 the latter immediately adhered to these previously negotiated tariff schedule. The product-level bilateral tariff variation $\Delta\tau_{sdip,2004}$ which was a by-product of the EU accession process is my measure of the market accession shock.

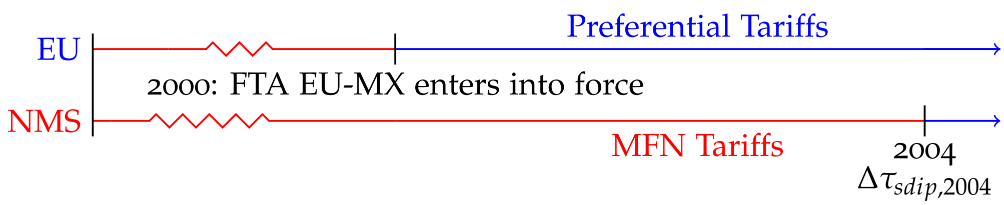

Figure 5: **Event Study Design, Constructing the Trade Shock**: I use the fact that when the NMS joined the EU in 2004, they had immediate preferential access to third-party markets via previously signed EU trade agreements which the NMS did not get to negotiate. The product-level bilateral tariff variation $\Delta\tau_{sdip,2004}$ which was a by-product of the EU accession process is my measure of the market accession shock. In the example above, the EU joined a FTA with Mexico in 2000, but the NMS only joined the EU in 2004, so in 2004 they immediately adhered to these previously negotiated tariff schedule.

Since the largest wave of enlargement was in 2004, in this analysis I will focus exclusively on that wave. The source of variation is at the source-country $\times$ destination-country $\times$ HS-code product level. In each year, there are about 300 thousand observations. Figure 6 shows the interquartile range of bilateral product-level tariff rates between NMS and the set of countries that had concluded trade agreements with the EU prior to 2004.

It shows that there is not much change in tariffs leading up to membership and then a median drop of about 2.5 percentage points between 2003 and 2004. In the years immediately after membership, there is also not a large change in the distribution of bilateral tariff rates. There are some changes after 2007, possibly because some future provisions in trade agreements kick in.

I use as my metric of the tariff shock change simply $\Delta\tau_{sdip,2004} \equiv \tau_{sdip,2004} - \tau_{sdip,2003}$, which is the change in the level of effectively applied bilateral tariffs at the product level between

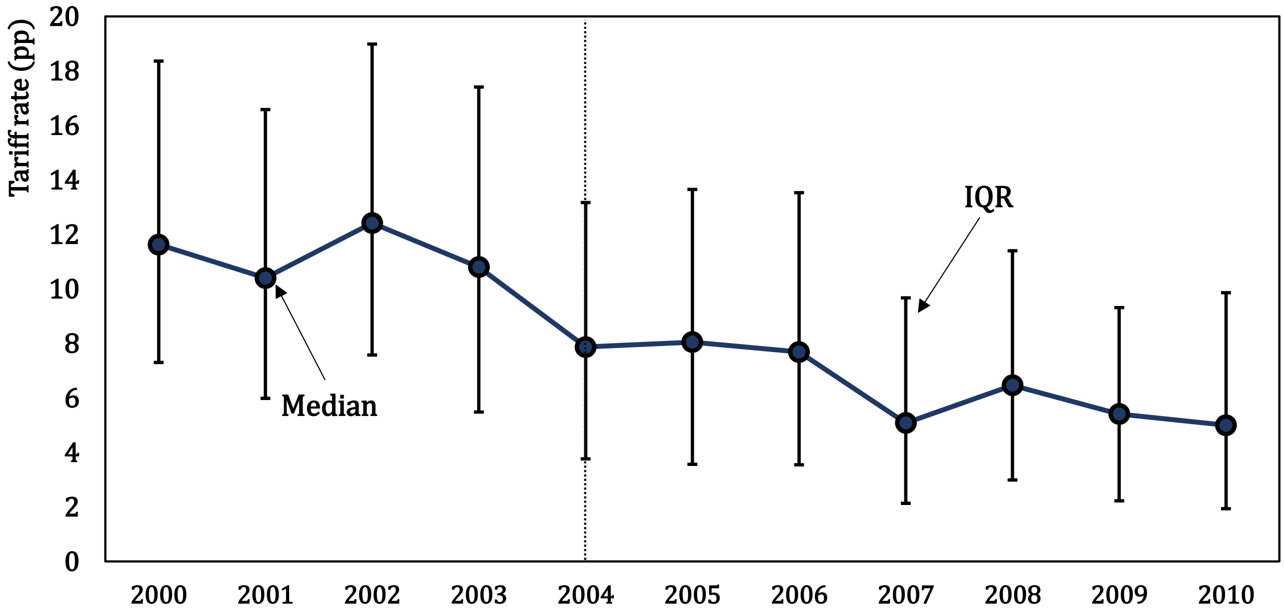

Figure 6: **Distribution of Tariff Changes Over Time**: Interquartile Range Bilateral HS6-Product-Level Tariff Rates Between New Member States (2004 EU Enlargement) and Set of Countries that Concluded Trade Agreements with EU prior to 2004. Data were constructed from WITS Preferential and MFN databases.

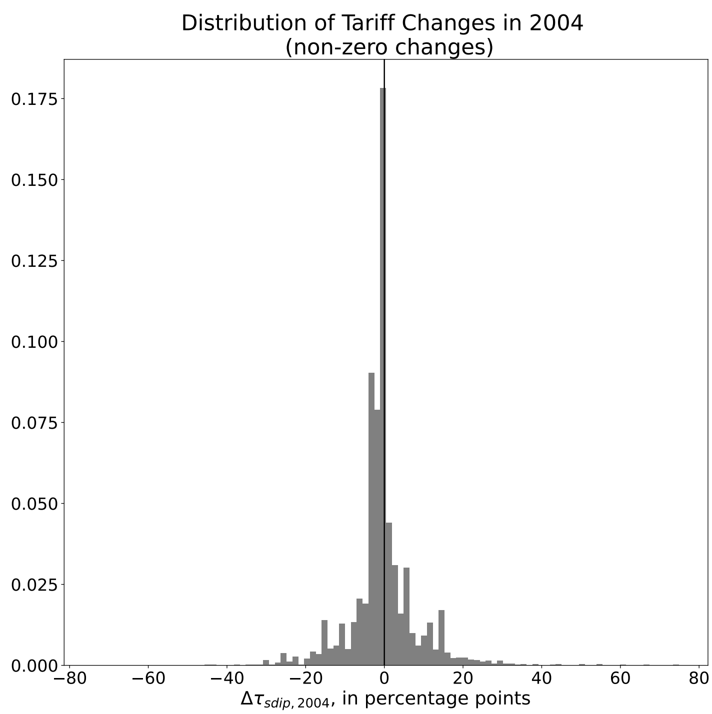

Figure 7: **Tariff Shock:** Distribution of the (non-zero) observations of the changes in Bilateral HS6-Product-Level Tariff Rates Between New Member States (2004 EU Enlargement) and Set of Countries that Concluded Trade Agreements with EU prior to 2004. Data were constructed from WITS Preferential and MFN databases.

2003 and 2004. Figure 7 plots the distribution of $\Delta\tau_{sdip,2004}$, excluding the zero-valued observations. The average $\Delta\tau_{sdip,2004}$ is -0.14% and the standard deviation is 12%.

The main causal mechanism of the theoretical model purports that trade liberalization induces higher expected exports and profits, which in its turn induces higher investment in R&D and product innovation. In order to test this causal chain in reduced form, I test the relationship between the first and last links of this chain.

I estimate a sequence of cross-sectional local-projection linear probability models, which estimate what is the marginal effect of an *increase* in the tariffs on exports of a given product *p, conditional on that country s not producing that particular product before joining the EU in 2003*. The fact the data is highly granular permits me to exploit within *industry* × *source* × *destination* × *horizon* (across product) variation.

Formally, I estimate the following equation:

$$\begin{aligned} P\left(X_{sdip,h} > 0 \middle| Y_{s\cdot ip,2003} = 0\right) &= \alpha_h + \beta_h \cdot \Delta\tau_{sdip,2004} + \gamma_{sdi,h} + \nu_{sdip,h} \\ &\quad \text{for } h \in \{2000, \cdots, 2010\} \end{aligned} \tag{19}$$

where $X_{sdip,h}$ is the market value of exports between country s and country d of product p of industry i at horizon h; $Y_{s\cdot ip,2003}$ is the market value of production in country s of product p of industry i in 2003; α_h are horizon (time) fixed-effects; $\gamma_{sdi,h}$ are *source* × *destination* × *industry* interactions fixed-effects for each h.

Note that, since these are local projections, the right-hand side coefficients, the regressor $\tau_{sdip,2004}$ is fixed for all horizons, and the coefficients β_h change. As initially argued by Chodorow-Reich (2020) and later formalized by Dube et al. (2023), these types of cross-sectional event studies with local projections can be interpreted as differences in differences with continuous treatments. If consistently estimated, the estimated coefficients β_h, then, are simply the average treatment on treated compared to the potential outcomes of not being treated, normalized to a treatment of intensity of one unit.

I take the assertion in Baier and Bergstrand (2007) (henceforth B&B) that countries engage endogenously in free trade agreements (FTAs) and one needs to look for a plausibly exogenous source of variation to check whether or not FTA "actually increase members' international trade" very seriously. Here, I rely on their strategy of running dynamic panels with fixed effects to control for unobserved heterogeneity.

Importantly, while they estimate their models at the aggregate country level with *source* × *destination* × *period*, I have enough variability and data availability to estimate it at the product-level adding *industry* × *source* × *destination* × *period* fixed effects. Hence, my approach adds granularity to B&B's strategy, thereby controlling for more unobserved heterogeneity.

The identification assumption is that conditional on the very saturated fixed effects that this model includes, the unobserved components $\nu_{sdip,h}$ are uncorrelated with the change in tariffs $\Delta\tau_{sdip,2004}$. Intuitively, the identification is robust to a NMS (say, Poland's) policymakers endogenously targeting EU accession to have preferential access to a third-party's (say, Mexico's) car industry (relative to other industries and countries), but not if they want to have preferential access to compact cars relative to SUVs in Mexico.

The identification strategy is plausible. In general, neither lobbyists of industry trade groups nor trade negotiations work in such a disaggregated product-level setting. Typically, lobbyists consolidate the interests of the producers of many products under the same umbrella and try to influence negotiations. Similarly, even when governments are negotiating tariffs schedule changes —which was not the case in this particular case! —these negotiations typically also happen in blocs, with governments exchanging positions in some products for others. Hence, the fact that I have a highly disaggregated dataset at the product level adds a lot of strength to the identification strategy.

As shown in Figure 8, an increase in market access by 1 percentage point increases the probability of starting to produce and export a given product by about 1 percent by 2010. To benchmark this result, it is about one-third of the conditional mean $\mathbb{E}[X_{sdip,h} > 0 | X_{s\cdot ip,h} > 0, h > 2003] = 2.9\%$. There are no signs of a pre-existing trend before 2004: both the magnitude of the coefficients and the standard errors are very small before the treatment date.

I then run a set of continuation regressions, which are very similar to the model estimated in equation (19), except that now I condition in initial production being active:

$$\begin{aligned} P\left(X_{sdip,h} > 0 \middle| Y_{s\cdot ip,2003} = 1\right) &= \alpha_h + \beta_h \cdot \Delta\tau_{sdip,2004} + \gamma_{sdi,h} + \nu_{sdip,h} \\ &\quad \text{for } h \in \{2000, \cdots, 2010\} \end{aligned} \tag{20}$$

In this case, there are no effects observed on the extensive margin. When countries already have the ability to produce a given product, additional market access produces very noisy results in the extensive margin. The coefficients are large and bounce between positive and negative and the confidence bands are even larger. One potential explanation is that the countries possibly already had market access before 2004, as illustrated by the positive (albeit insignificant results) for 2000-03, since they already had the production capacity. It is possible that most of the effects concentrate on the intensive margin, something that futures iteration of this paper would need to check.

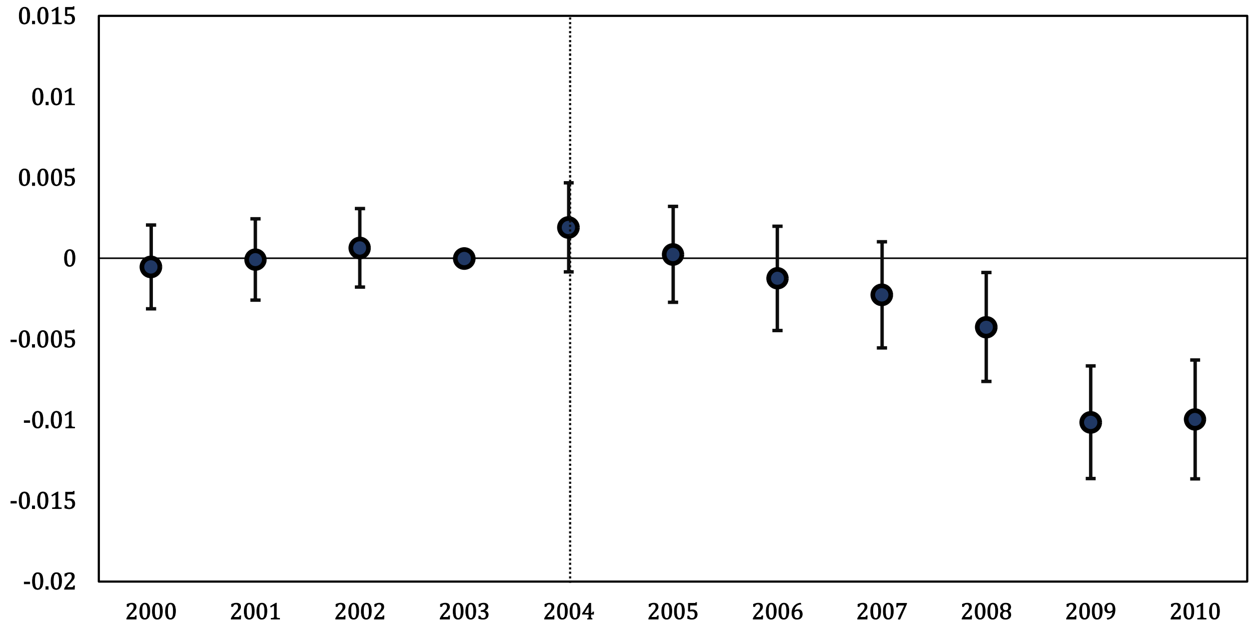

Figure 8: **Entry Regressions.** This plot shows the coefficients β_h of the local projection linear probability models specified in equation (19). Each year is a different cross-sectional regression with approximately 300 thousand observations. The whiskers show 95% confidence intervals with robust standard errors clustered at the source-destination-industry level.

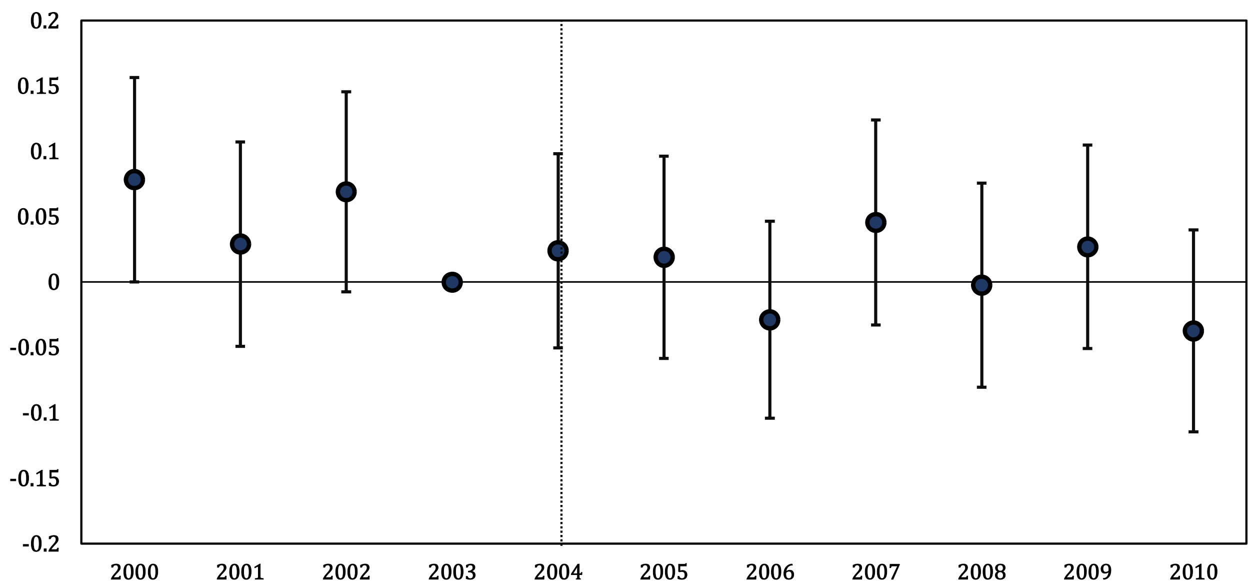

Figure 9: **Continuation Regressions.** This plot shows the coefficients β_h of the local projection linear probability models specified in equation (20). Each year is a different cross-sectional regression with approximately 300 thousand observations. The whiskers show 95% confidence intervals with robust standard errors clustered at the source-destination-industry level.

4 Quantification and Policy Exercise

In this section, I describe a numerical quantification of the model, which solves for three endogenous objects along the Balanced Growth Path: (a) the distribution of wages; (b) the distribution of Measures of Varieties; and (c) the common equilibrium growth rate. I calibrate the model to EU-15 countries and the New Member States (NMS) that joined in the 2004 expansion.

To simplify the exercise, I group these countries into four sets: three groups of Western European countries and a set of NMS[20]. The groups are: (a) the New Member States (NMS) of the 2004 enlargement wave; (b) Portugal, Italy, Spain, Greece, and Ireland; (c) France, Belgium, the Netherlands, and Luxembourg (the last three are also known as the Benelux countries); and (d) Germany, Austria plus the Scandinavians (Denmark, Sweden, and Finland). The country groups are asymmetric both in terms of labor force and productivity.

The solution method is straightforward. I calibrate the model to a baseline scenario and then change iceberg trade costs to induce a trade liberalization shock. By comparing the endogenous equilibria along the Balanced Growth Path of these two scenarios, which include distributions of the measures of varieties $[M_s(t^*)]_{s\in\mathbf{K}}$ and wages $[w_s(t^*)]_{s\in\mathbf{K}}$ as well as a common equilibrium growth rate g^*, I can infer the welfare consequences of a change in this parameter along the BGP.

While this exercise does not incorporate the welfare effects along the transition path, it still allows us to talk indirectly about what happens along the transition by comparing the distributions of wages and measures of varieties across equilibria.

Model Calibration My estimates of the short-term (σ) and long-term (θ) elasticities of trade come from Boehm et al. (2020), which are $\sigma = 0.76$ and $\theta = 2.12$, respectively. The results are not very sensitive to using a $\theta = 4.0$. The vector of labor force $\{L_s\}$ comes from Penn World Tables. The share of intermediate goods $\alpha = 0.36$ is set to equal the average share of intermediate goods in the sample of countries between 2000-2003 from the World Input-Output Database.

I use observed trade flows to infer trade costs. The strategy goes back to Head and Ries (2001). According to the handbook chapter by Head and Mayer (2014), the index is called the Head-Ries Index (HRI) since 2011 (when the working paper version of Eaton et al. (2016) was published). As shown in Appendix E, I can write trade costs as:

[20]This simplification is just a matter of computational tractability. As described in Appendix E, each guess of my solution algorithm solves for a static version of an Eaton-Kortum model with input-output linkages, which itself has multiple steps for solutions. So the problem grows quite fast in complexity in the number of countries. Improving the solution algorithms for this new class of dynamic models is a fruitful avenue of future research.

$$\tau_{sd} = \left(\frac{E^F_{sd}(t^*)}{E^F_{dd}(t^*)} \cdot \frac{E^F_{ds}(t^*)}{E^F_{ss}(t^*)} \right)^{-\frac{1}{2\theta(1-\alpha)}} \tag{21}$$

where each flow $E^F_{sd}(t^*)$ defined to be an average between $2000 - 2003$. The data on bilateral expenditure values $E^F_{sd}(t^*)$ comes from the World Input-Output Database.

Figure 10 plots the change in trade costs before and after the 2004 enlargement of the European Union, calculated from an average of for the years 2000-2003 for the "before" period and an average for the years 2004-2007 for the "after" period. Reassuringly, the calculations confirm that (a) there were large changes in trade costs between NMS and the Western European countries during this period; and (b) changes in trade costs among Western European countries have been minimal (or even positive) compared to those that happened between NMS and Western European countries.

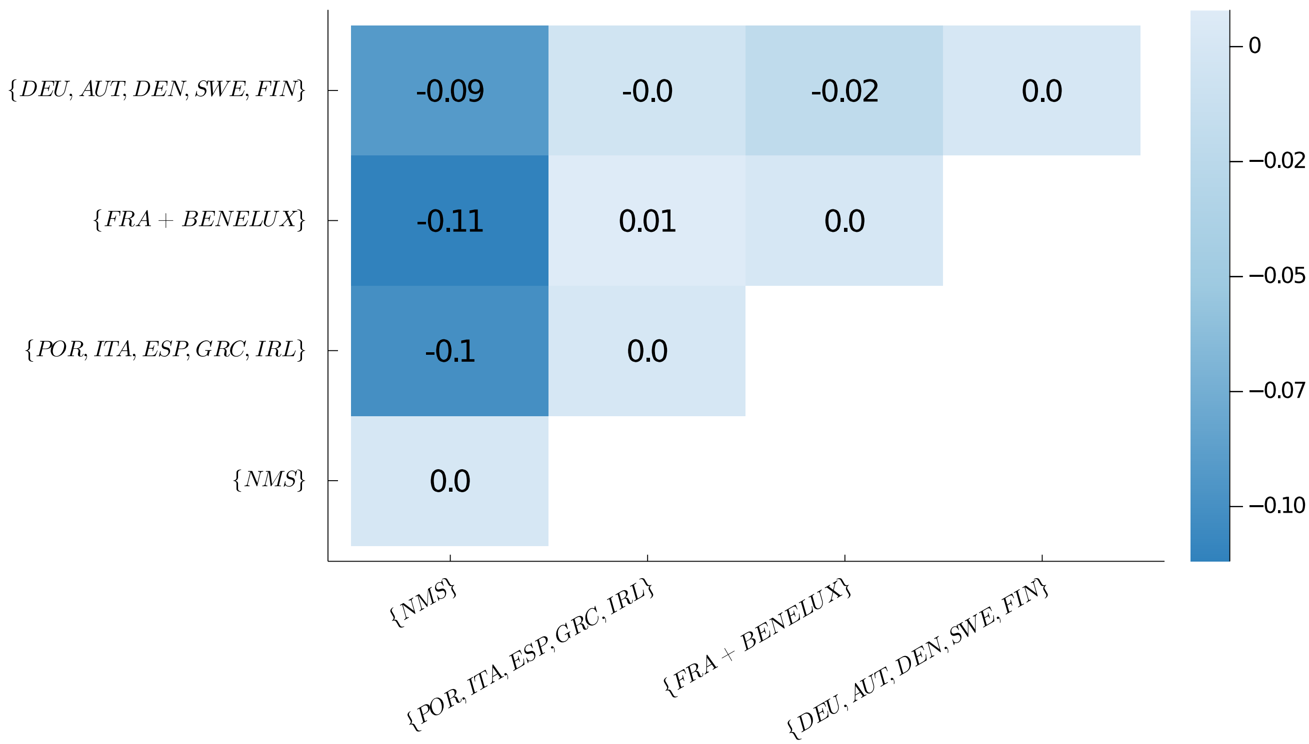

Figure 10: **Changes in Trade Costs Before and After 2004 EU Enlargement (in percentage terms).** This matrix shows the bilateral changes in trade costs, calculated using the method inferred from equation (21), before and after the 2004 EU Enlargement. The before period is an average for the years 2000-2003 and the after period is an average for the years 2004-2007. Underlying data comes from the World Input-Output Database.

This is important because these changes in trade costs will act as the main shock across calibrations of BGPs in my numerical exercise. It is relevant that the key driver of changes across equilibria is the enlargement of the EU.

The location parameter of the Fréchet distribution $\{T_s\}$ and ψ are free parameters that I vary to match the distribution of wages and the average growth rate of the EU-15 countries in the $1989-2003$ period —i.e., fifteen years prior to the 2004 expansion of the European Union. The rationale is that I am calibrating this model to BGP growth rate and the EU-15 countries were very likely closer to the BGP than the transition economies of Eastern Europe, so it is reasonable to match the model to their growth rate.

Model Validation To validate the model, I look at two untargeted moments. First, I compare the (endogenous) distribution of the number of produced varieties in the model to the distribution of the number of produced varieties in the data for the 2000-2003 period. These comparisons are in Figure 11. Even though the empirical distribution is mostly uniform, which prevents us from doing more rigid tests of the model, qualitatively, the model seems to do a good job of reproducing the data.

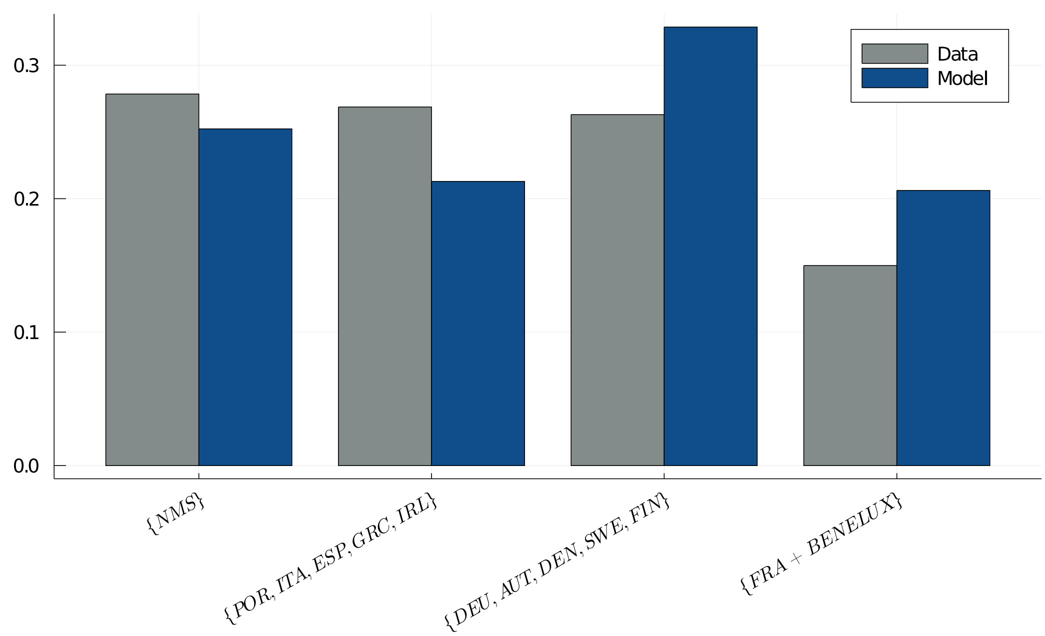

Figure 11: **Model Validation: Distribution in the Number of Produced Varieties Across Regions.** In the model, the distribution of measures of varieties $\lambda_M \cdot [M_s(t^*)]_{s\in\mathbf{K}}$ is normalized with a choice of λ_M such that $\sum_{s\in\mathbf{K}} M_s(t^*) = 1$. For consistency in the comparison, what I show in the data bars are the relative shares of each country group in the total universe of the product measure, or: $M_s(t) / \sum_{s'\in\mathbf{K}} M_{s'}(t)$. This assumes, as in the model, that product varieties in the data are differentiated across countries, so the global product space is $\sum_{s'\in\mathbf{K}} M_{s'}(t)$. Data comes from Prodcom (Eurostat) and are averages for the 2000-2003 period.

A more stark test of the model is a look in the relative change in real wages. I calculated the predicted changes in wages across equilibria in the model after the introduction of the trade shock and compared it to the relative changes in real wages calculated from the data.

To compare the data with the model, I calculated GDP per capita and then divided it by the average of the group, which gives me a wage that is normalized for the periods of 2000-2003 and 2005-2008. The way to interpret the data is to see whether or not each group's income per capita grew faster (slower) than average across these periods.

Here, one can see that the model in fact matches the data quite correctly. It predicts relative a catch-up of the New Member States. The model predicts that real wages in NMS would grow about 37% faster than the average of the Western European countries. While the data was a bit would than that (about 26%), qualitatively the results hold. Importantly, for all other country groups the model predicted that average income would be close to the average, which also holds true in the data.

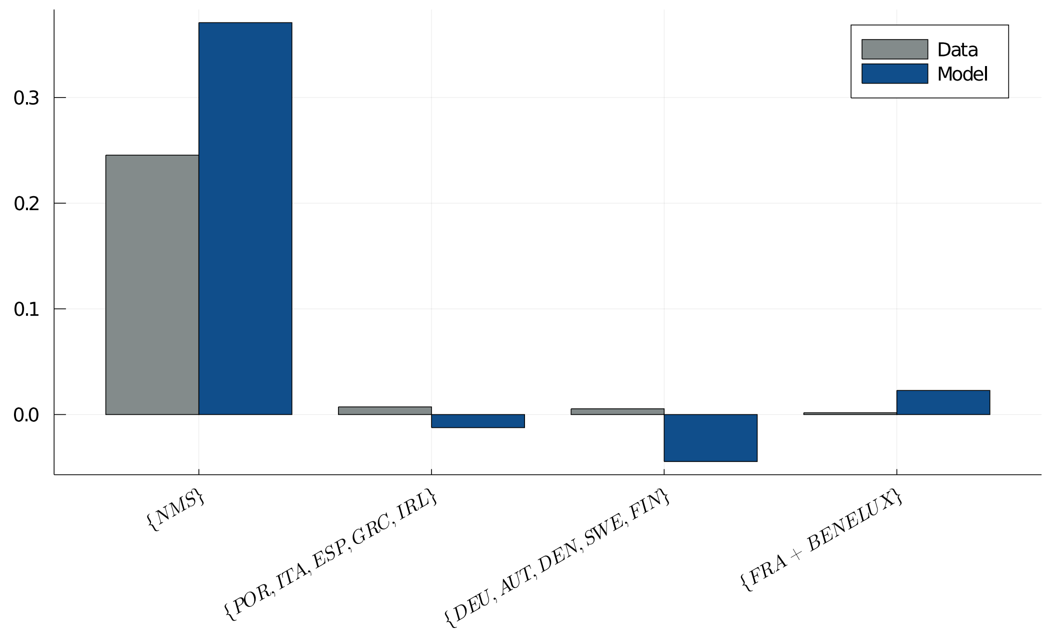

Figure 12: **Model Validation: Changes in Real Wages, Relative to the Average.** In the model, the distribution of wages $\lambda_w \cdot [w_s(t^*)]_{s\in\mathbf{K}}$ is normalized with a choice of λ_w such that $\sum_{s\in\mathbf{K}} w_s(t^*)L_s = 1$. What is shown in the chart is the percentage change across equilibria $\frac{w_s(t^{**})-w_s(t^*)}{w_s(t^*)}$, where L_s is assumed to be fixed. In the data, for consistency, I calculated GDP per capita $w_s(t)$ then divided it by the average of the group $\bar{w}_s(t)$, which gives me a wage that is normalized for the periods of 2000-2003 and 2005-2008. I then calculated changes and plotted the data. Data comes from the Penn World Tables 10.01.

Results The main result of this exercise relates to the theoretical welfare decomposition in equation (15). One can compare two paths of consumption along the BGP and decompose them into:

$$\int_\tau^\infty \exp\{-\rho(t-\tau)\}\log\left(\exp\{g^{**}t\}C_s(t^{**},\tau)\right) - \log\left(\exp\{g^{*}t\}C_s(t^{*},\tau)\right)dt =$$
$$\underbrace{\log\left(\hat{M}_s\right)}_{\text{transitional}} + \underbrace{\frac{1}{\rho}\log\left(\frac{\widehat{w_s}}{P_s}\right)}_{\text{static}} + \underbrace{\frac{g^{**}-g^{*}}{\rho^2}}_{\text{dynamic}}$$

where $C_s(t^*, \tau), C_s(t^{**}, \tau)$ are the paths of consumption along the original and new BGPs, respectively.

For all countries, the transitional component is negligible. They never contribute with more than 0.03% of total absolute value of welfare, in the largest case.

Static gains from trade can be as large as 37% of domestic income in the case of NMS or even *negative* or close to zero in the case of the Northern and Central European countries. In either case, they are only large and relevant for NMS. However, even in that case, they pale in comparison to dynamic gains from trade. For NMS, static gains from trade account for a total of 6% of total welfare.

Finally, the main numerical outcome of the exercise is the differences in growth rates across BGPs $g^{**} - g^*$. In the current calibration, **the trade liberalization embedded in the 2004 enlargement of the European Union induced the EU long-run yearly growth rate to increase** 0.16%. One implication is that the dynamic part of welfare accounts for the bulk of gains from trade and not accounting for this channel ignores the vast majority of gains from trade. In fact, in this parametrization, dynamic gains from trade are **fifteen times larger than static gains from trade.**

In monetary terms, a back-of-the-envelope calculation suggests an additional 0.16% yearly growth rate to the aggregate GDP of the Western European plus the New Member States since the year of accession —that is, between 2004 and 2023 —would have induced an additional current production level of approximately \$517 billion in the continent, which accounts for 3.15% of the total level of production of the European Union.

5 Conclusion

I focus on the long-lasting question of the relationship between trade and growth and, in particular, trade and product innovation. I make several contributions: theoretical, empirical, and quantitative.

On the theoretical front, my main contribution is a new framework that reconciles the forces of specialization and market size, rationalizes foreign market access as a rationale for growth in a dynamic framework, and provides an analytical formula for dynamic gains from trade. In all of those points, I maintain active dialogues with the literature, such as nesting the Eaton-Kortum model of trade and Romer growth model as special cases of my model and subsuming the ACR static welfare formula in my dynamic welfare formula.

In my empirical work, I rely on the eastward expansion of the European Union and document several new facts that are consistent with the mechanisms of my model. Compared to countries that selected into becoming candidates but had not joined at given horizon, countries started producing more product varieties, investing more in R&D, and trading more.

I go beyond these facts and exploit plausibly exogenous variation to show that a plausibly exogenous increase in market access leads to a higher probability of initiating production and exporting a given product, which is consistent with the main mechanism of the theoretical model.

Finally, I solve for a quantitative model and replicate the 2004 expansion of the European Union in the computer. The results of the simulation imply that: (a) the EU expansion increased its long-run yearly growth rate by about 0.16%; and (b) dynamic gains from trade account for more than 90% of welfare gains from trade.

This paper points to the fact that dynamic gains from trade are likely too large to be ignored. Advancing on this agenda is a fruitful avenue of future research.

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A Timeline of EU Trade Agreements

Partner	Signed	Provisional application	Full entry into force
Switzerland	1972		1973
Iceland	1992		1994
Norway	1992		1994
Turkey	1995		1995
Tunisia	1995		1998
Israel	1995	1996	2000
Mexico	1997		2000
Morocco	1996		2000
Jordan	1997		2002
Egypt	2001		2004
North Macedonia	2001	2001	2004
South Africa	1999	2000	2004
Chile	2002	2003	2005

B Mathematical derivations

B.1 Optimal control problem

In the dynamic optimal control problem, the household chooses an optimal path of $C_s(t)$ at every instant, taking as given prices. The problem of choosing varieties $c_s(t,\omega)$ is separable and can be solved conditional on a path for $C_s(t)$, such that only aggregates matter for the dynamic path. Therefore, the current-value Hamiltonian for this problem is:

$$H(t,C,L,\mu) = \log\big(C_s(t)\big) + \mu_s(t)\left[\frac{r_s(t)}{P_s(t)}A_s(t) + \frac{w_s(t)}{P_s(t)}L_s - C_s(t)\right]$$

with optimality conditions satisfying:

$$\begin{aligned}\frac{1}{C_s(t)} &= \mu_s(t) \\ \frac{\dot{\mu}_s(t)}{\mu_s(t)} &= \rho - \frac{r_s(t)}{P_s(t)}\end{aligned}$$

and a transversality condition:

$$\lim_{t\to\infty}\left[\exp\{-\int_0^t \frac{r_s(\nu)}{P_s(\nu)}d\nu\}P_s(t)A_s(t)\right] = 0$$

Taking time derivatives of the first optimality condition and then replacing for $\frac{\dot{\mu}(t)}{\mu(t)}$ yields the Euler equation:

$$\frac{\dot{C}_s(t)}{C_s(t)} = \left[\frac{r_s(t)}{P_s(t)} - \rho\right]$$

B.2 Solution to the dynamic problem

Growth in each of the $s \in \mathbf{K}$ of the national economies evolve according to the following system of differential equations:

$$\begin{aligned}\dot{C}_s(t) &= \left[\frac{r_s(t)}{P_s(t)} - \rho\right]C_s(t) \\ \dot{M}_s(t) &= \frac{r_s(t)}{P_s(t)}M_s(t) + \psi\frac{w_s(t)}{P_s(t)}L_s - \psi C_s(t)\end{aligned}$$

In this section, I will first derive this system of equations, then solve it. First, one sees that consumption evolves according to a first-order differential equation. Let $a(t) \equiv \left[\frac{r_s(t)}{P_s(t)} - \rho\right]$ and write the Euler equation as:

$$\dot{C}_s(t) = a(t)C_s(t)$$

Multiplying both sides by the integration factor $\exp\{-\int_0^t a(\tau)d\tau\}$:

$$\dot{C}_s(t)\exp\{-\int_0^t a(\tau)d\tau\} - a(t)C_s(t)\exp\{-\int_0^t a(\tau)d\tau\} = 0$$

Now, using Leibnitz lemma, note that the time derivative of $\exp\{-\int_0^t a(\tau)d\tau\}C_s(t)$ is $\dot{C}_s(t)\exp\{-\int_0^t a(\tau)d\tau\} - a(t)C_s(t)\exp\{-\int_0^t a(\tau)d\tau\}$. Therefore, integrating both sides with respect to time:

$$\exp\{-\int_0^t a(\tau)d\tau\}C_s(t) = C(0)$$

where $C(0)$ is the constant of integration. Dividing both sides by $\exp\{-\int_0^t a(s)ds\}$ and replacing for $a(t)$ yields the solution for the consumption path:

$$C_s(t) = C(0)\exp\left\{\int_0^t \left[\frac{r_s(\tau)}{P_s(\tau)} - \rho\right] d\tau\right\}$$

which can be rewritten as:

$$C_s(t) = C_s(0)\exp\left\{\left[\bar{r}_s(t) - \rho\right] t\right\}$$

where $\bar{r}_s(t) \equiv \frac{1}{t}\int_0^t \frac{r_s(\nu)}{P_s(\nu)}d\nu$ is the average real interest rate between periods 0 and t. Now recall that the budget constraint is:

$$P_s(t)I_s(t) + P_s(t)C_s(t) = r_s(t)A_s(t) + w_s(t)L_s \tag{B.1}$$

and that $\psi I_s(t) = \dot{M}_s(t)$ and $\psi A_s(t) = M_s(t)$. Replacing above and solving for $\dot{M}_s(t)$ results in:

$$\dot{M}_s(t) = \frac{r_s(t)}{P_s(t)}M_s(t) + \psi\frac{w_s(t)}{P_s(t)}L_s - \psi C_s(t)$$

which, after replacement, yields the following equation:

$$\dot{M}_s(t) = \frac{r_s(t)}{P_s(t)} M_s(t) + \psi \frac{w_s(t)}{P_s(t)} L_s - \psi C_s(0) \exp\left\{ \left[\bar{r}_s(t) - \rho \right] t \right\}$$

In turn, this equation has a solution satisfying:

$$\begin{aligned}
M_s(t) &= M_s(0) \cdot \exp\left\{ \int_0^t \frac{r_s(\nu)}{P_s(\nu)} d\nu \right\} \\
&+ \int_0^t \psi \frac{w_s(\xi)}{P_s(\xi)} L_s \cdot \exp\left\{ - \int_0^\xi \frac{r_s(v)}{P_s(v)} dv \right\} d\xi \cdot \exp\left\{ \int_0^t \frac{r_s(\nu)}{P_s(\nu)} d\nu \right\} \\
&- \int_0^t \psi C_s(0) \exp\left\{ \left[\bar{r}_s(\xi) - \rho \right] \xi \right\} \cdot \exp\left\{ - \int_0^\xi \frac{r_s(v)}{P_s(v)} dv \right\} d\xi \cdot \exp\left\{ \int_0^t \frac{r_s(\nu)}{P_s(\nu)} d\nu \right\}
\end{aligned}$$

which, using the definition of $\bar{r}(t)$, becomes:

$$\begin{aligned}
M_s(t) &= M_s(0) \cdot \exp\left\{ \bar{r}(t) \cdot t \right\} \\
&+ \int_0^t \psi \frac{w_s(\xi)}{P_s(\xi)} L_s \cdot \exp\left\{ -\bar{r}(\xi) \cdot \xi \right\} d\xi \cdot \exp\left\{ \bar{r}(t) \cdot t \right\} \\
&- \psi C_s(0) \cdot \int_0^t \exp\left\{ \left[\bar{r}_s(\xi) - \rho \right] \xi \right\} \cdot \exp\left\{ -\bar{r}(\xi) \cdot \xi \right\} d\xi \cdot \exp\left\{ \bar{r}(t) \cdot t \right\}
\end{aligned}$$

simplifying the last integral:

$$\begin{aligned}
M_s(t) &= M_s(0) \cdot \exp\left\{ \bar{r}(t) \cdot t \right\} \\
&+ \int_0^t \psi \frac{w_s(\xi)}{P_s(\xi)} L_s \cdot \exp\left\{ -\bar{r}(\xi) \cdot \xi \right\} d\xi \cdot \exp\left\{ \bar{r}(t) \cdot t \right\} \\
&- \psi C_s(0) \cdot \int_0^t \exp\left\{ -\rho \xi \right\} d\xi \cdot \exp\left\{ \bar{r}(t) \cdot t \right\}
\end{aligned}$$

Finally, note that both $P_s(t)$ and $r_s(t)$ are functions of wages. Therefore, given the initial measure of varieties $M_s(0)$ and the wages for all countries, which are defined at every instance through the trade equilibrium, paths for consumption $C_s(t)$, varieties $M_s(t)$ and assets $A_s(t) = 1/\psi M_s(t)$ follow the equations above.

As a final step, one needs to pin down the starting values. $M_s(0)$ is given and calibrated to reflect the technological level of country s. Choice of $C_s(0)$, by contrast, is an endogenous object that guarantees that, given lifetime income and the initial level of assets, consumption as governed by the euler equation will be optimal. Start from the equation above, multiply both sides by $\exp\left\{-\bar{r}(t)\cdot t\right\}$:

$$\begin{aligned}\exp\left\{-\bar{r}(t)\cdot t\right\}M_s(t) &= M_s(0)+\int_0^t \psi\frac{w_s(\xi)}{P_s(\xi)}L_s\cdot\exp\left\{-\bar{r}(\xi)\cdot\xi\right\}d\xi\\ &- \psi C_s(0)\cdot\int_0^t \exp\left\{-\rho\xi\right\}d\xi\end{aligned}$$

Now evaluate this equation taking the limit $t\to\infty$.

$$\begin{aligned}\lim_{t\to\infty}\Big(\exp\left\{-\bar{r}(t)\cdot t\right\}M_s(t)\Big) &= M_s(0)+\int_0^\infty \psi\frac{w_s(t)}{P_s(t)}L_s\cdot\exp\left\{-\bar{r}(t)\cdot t\right\}dt\\ &- \psi C_s(0)\cdot\int_0^\infty \exp\left\{-\rho t\right\}dt\end{aligned}$$

Recall that the transversality condition is:

$$\lim_{t\to\infty}\left[\exp\{-\int_0^t\frac{r_s(\nu)}{P_s(\nu)}d\nu\}P_s(t)A_s(t)\right]=0$$

which states that the value of assets cannot grow faster than the interest rate, the standard no-Ponzi scheme condition. Using the fact that $\psi A_s(t)=M_s(t)$, noting that prices $P_s(t)$ are always positive and finite, and dividing both sides by $P_s(t)/\psi$, we can rewrite this as:

$$\lim_{t\to\infty}\left[\exp\{-\bar{r}_s(t)t\}M_s(t)\right]=0$$

Using the fact that $\lim_{t\to\infty}\Big(\exp\left\{-\bar{r}(t)\cdot t\right\}M_s(t)\Big)=0$, we can then solve for $C_s(0)$ as:

$$C_s(0)=\left[\frac{1}{\psi}M_s(0)+\int_0^\infty\frac{w_s(t)}{P_s(t)}L_s\cdot\exp\left\{-\bar{r}_s(t)\cdot t\right\}dt\right]\cdot\left[\int_0^\infty\exp\left\{-\rho t\right\}dt\right]^{-1}$$

Using the fact that $\int_0^\infty\exp\left\{-\rho t\right\}dt=\frac{1}{\rho}$, then:

$$
\begin{aligned}
C_s(0) &= \rho \left[\frac{1}{\psi} M_s(0) + \int_0^\infty \frac{w_s(t)}{P_s(t)} L_s \cdot \exp\left\{-\bar{r}_s(t)\cdot t\right\} dt \right] \qquad \text{(B.2)} \\
&= \rho \left[\underbrace{A_s(0)}_{\text{initial wealth}} + \underbrace{\int_0^\infty \frac{w_s(t)}{P_s(t)} L_s \cdot \exp\left\{-\bar{r}_s(t)\cdot t\right\} dt}_{\text{PV of real labor income}} \right]
\end{aligned}
$$

Therefore, at any instant t, consumption is proportional to lifetime wealth:

$$
\begin{aligned}
C_s(t) &= \rho \left[A_s(0) + \int_0^\infty \frac{w_s(\tau)}{P_s(\tau)} L_s \cdot \exp\left\{-\bar{r}_s(\tau)\cdot \tau\right\} d\tau \right] \cdot \exp\left\{ \left[\bar{r}_s(t) - \rho\right] t \right\} \qquad \text{(B.3)} \\
&= \rho \left[A_s(t) + \int_t^\infty \frac{w_s(\tau)}{P_s(\tau)} L_s \cdot \exp\left\{-\bar{r}_s(\tau)\cdot \tau\right\} d\tau \right]
\end{aligned}
$$

B.3 Final varieties producers problem

Each final goods producer chooses intermediate inputs and labor to maximize profits according to:

$$
\begin{aligned}
\max_{\ell_s(t,\omega),\{x_{ks}(t,\omega,\nu)\}} \quad & p_{ss}(t,\omega) z_s(t,\omega) [\ell_s(t,\omega)]^{1-\alpha} \left(\frac{1}{\alpha} \sum_{k\in\mathbf{K}} \int_0^{M_k(t)} [x_{ks}(t,\omega,\nu)]^\alpha d\nu \right) \\
- \quad & w_s(t)\ell_s(t,\omega) - \sum_{k\in\mathbf{K}} \int_0^{M_k(t)} p_{ks}(t,\nu) x_{ks}(t,\omega,\nu) d\nu
\end{aligned}
$$

There are infinitely many first order conditions for this problem: one for each variety ν and one for labor. These satisfy:

$$
\begin{aligned}
w_s(t)\ell_s(t,\omega) &= (1-\alpha)\cdot p_{ss}(t,\omega) z_s(t,\omega) [\ell_s(t,\omega)]^{1-\alpha} \left(\frac{1}{\alpha} \sum_{k\in\mathbf{K}} \int_0^{M_k(t)} [x_{ks}(t,\omega,\nu)]^\alpha d\nu \right) \\
p_{ks}(t,\nu) x_{ks}(t,\omega,\nu) &= \alpha \cdot p_{ss}(t,\omega) z_s(t,\omega) [\ell_s(t,\omega)]^{1-\alpha} \left(\frac{1}{\alpha} [x_{ks}(t,\omega,\nu)]^\alpha \right)
\end{aligned}
$$

Solving for $x_{ks}(t,\omega,\nu)$ yields equation (2):

$$x_{ks}(t,\omega,\nu) = \left[\frac{p_{ks}(t,\omega,\nu)}{p_{ss}(t,\omega)}\right]^{-\frac{1}{1-\alpha}} \cdot \ell_s(t,\omega) \cdot z_s(t,\omega)^{\frac{1}{1-\alpha}}$$

B.4 Intermediate varieties producers problem

Each intermediate varieties producer holds perpetual rights over variety ν, which they sell to final goods varieties in every country $d \in \mathbf{K}$. For each destination, they take demand as given and choose prices to maximize profits at every moment:

$$\max_{p_{ks}(t,\omega,\nu)} \frac{1}{\tau_{ks}} p_{ks}(t,\omega,\nu) x_{ks}(t,\omega,\nu) - P_k(t) x_{ks}(t,\omega,\nu)$$

Replacing for $x_{ks}(t,\omega,\nu)$:

$$\max_{p_{ks}(t,\omega,\nu)} \left[p_{ks}(t,\omega,\nu) - \tau_{ks} P_k(t)\right] \left[\frac{p_{ks}(t,\omega,\nu)}{p_{ss}(t,\omega)}\right]^{-\frac{1}{1-\alpha}} \cdot \ell_s(t,\omega) \cdot z_s(t,\omega)^{\frac{1}{1-\alpha}}$$

which, after taking the FOC and solving for $p_{ks}(t,\omega,\nu)$ yields the optimal price as a mark-up over marginal price, which is independent of ω or ν:

$$p_{ks}(t,\omega,\nu) = \frac{\tau_{ks} P_k}{\alpha} \qquad \forall \omega \in [0,1], \quad \forall \nu \in [0, M_s(t)]$$

Replacing the optimal price in the objective function yields the profit per variety ν exported from country k per each producer of each final goods variety ω for use in country s:

$$\begin{aligned}
\pi_{ks}(t,\nu,\omega) &= \left[p_{ks}(t,\omega,\nu) - \tau_{ks} P_k(t)\right] \left[\frac{p_{ks}(t,\omega,\nu)}{p_{ss}(t,\omega)}\right]^{-\frac{1}{1-\alpha}} \cdot \ell_s(t,\omega) \cdot z_s(t,\omega)^{\frac{1}{1-\alpha}} \\
&= \left[\frac{\tau_{ks} P_k(t)}{\alpha} - \tau_{ks} P_k(t)\right] \left[\frac{\tau_{kd} P_k(t)}{\alpha}\right]^{-\frac{1}{1-\alpha}} \cdot \left[z_s(t,\omega) p_{ss}(t,\omega)\right]^{\frac{1}{1-\alpha}} \cdot \ell_s(t,\omega) \\
&= \frac{1-\alpha}{\alpha} \left[\tau_{ks} P_k(t)\right]^{-\frac{\alpha}{1-\alpha}} \cdot \left[\alpha z_s(t,\omega) p_{ss}(t,\omega)\right]^{\frac{1}{1-\alpha}} \cdot \ell_s(t,\omega) \\
&= \frac{1-\alpha}{\alpha} \left[\tau_{ks} P_k(t)\right]^{-\frac{\alpha}{1-\alpha}} \left[\alpha \frac{w_s(t)^{1-\alpha}}{\tilde{M}_s(t)^{1-\alpha} z_s(t,\omega)} z_s(t,\omega)\right]^{\frac{1}{1-\alpha}} \ell_s(t,\omega) \\
&= (1-\alpha) \left[\frac{\tau_{ks} P_k(t)}{\alpha}\right]^{-\frac{\alpha}{1-\alpha}} \cdot \frac{w_s(t)}{\tilde{M}_s(t)} \cdot \ell_s(t,\omega) \qquad \forall \omega \in [0,1], \quad \forall \nu \in [0, M_s(t)]
\end{aligned}$$

Total profits for the producer of variety ν sums over all destinations and varieties:

$$
\begin{aligned}
\pi_k(t,\nu) &= \sum_{s\in\mathbf{K}} \int_0^1 \pi_{ks}(t,\nu,\omega)d\omega \\
&= \sum_{s\in\mathbf{K}} (1-\alpha) \left[\frac{\tau_{ks}P_k(t)}{\alpha}\right]^{-\frac{\alpha}{1-\alpha}} \cdot \frac{w_s(t)}{\tilde{M}_s(t)} \cdot \underbrace{\int_0^1 \ell_s(t,\omega)d\omega}_{=L_s} \\
&= \underbrace{(1-\alpha)}_{\text{Intermediate use-share}} \cdot \sum_{s\in\mathbf{K}} \underbrace{\left[\frac{\tau_{ks}P_k(t)}{\alpha}\right]^{-\frac{\alpha}{1-\alpha}}}_{\text{Optimal mark-up}} \cdot \underbrace{\left[\frac{w_s(t)L_s}{\tilde{M}_s(t)}\right]}_{\text{Trade-cost-adjusted demand at } s} \qquad \forall \nu \in [0, M_k(t)]
\end{aligned}
$$

which is independent of ν, so I can drop the indices: $\pi_k(\nu,t) \equiv \pi_k(t)$.

By expanding $\tilde{M}_s(t)$, I can also give another interpretation to profits, which depend on the relative price of intermediates across sources:

$$
\begin{aligned}
\pi_k(t) &= (1-\alpha) \cdot \sum_{s\in\mathbf{K}} \left[\frac{\tau_{ks}P_k(t)}{\alpha}\right]^{-\frac{\alpha}{1-\alpha}} \cdot \frac{w_s(t)L_s}{\sum_{n\in\mathbf{K}} \frac{1}{\alpha} M_n(t) \left[\frac{\tau_{ns}P_n(t)}{\alpha}\right]^{-\frac{\alpha}{1-\alpha}}} \\
&= (1-\alpha)\cdot\alpha\cdot \sum_{s\in\mathbf{K}} \underbrace{w_s(t)L_s}_{\text{value added in } s} \left[\sum_{n\in\mathbf{K}} \underbrace{M_n(t)}_{\text{product measure in } n} \left[\underbrace{\frac{\tau_{ns}P_n(t)}{\tau_{ks}P_k(t)}}_{\substack{\text{relative price} \\ \text{of intermediate good}}} \right]^{-\frac{\alpha}{1-\alpha}} \right]^{-1}
\end{aligned}
$$

Aggregate profits simply integrate over all intermediate varieties: $\Pi_k(t) = \int_0^{M_k(t)} \pi_k(t)d\nu = M_k(t)\cdot\pi_k(t)$. Using similar steps, one can show that the gross revenue per costumer that a given supplier of variety ν has:

$$
\begin{aligned}
p_{ks}(t,\nu)x_{ks}(t,\nu,\omega) &= \left[p_{ks}(t,\omega,\nu)\right]\left[\frac{p_{ks}(t,\omega,\nu)}{p_{ss}(t,\omega)}\right]^{-\frac{1}{1-\alpha}}\cdot \ell_s(t,\omega)\cdot z_s(t,\omega)^{\frac{1}{1-\alpha}} \\
&= \left[\frac{\tau_{ks}P_k(t)}{\alpha}\right]\left[\frac{\tau_{kd}P_k(t)}{\alpha}\right]^{-\frac{1}{1-\alpha}}\cdot\left[z_s(t,\omega)p_{ss}(t,\omega)\right]^{\frac{1}{1-\alpha}}\cdot \ell_s(t,\omega) \\
&= \left[\frac{\tau_{ks}P_k(t)}{\alpha}\right]^{-\frac{\alpha}{1-\alpha}}\cdot\left[z_s(t,\omega)p_{ss}(t,\omega)\right]^{\frac{1}{1-\alpha}}\cdot \ell_s(t,\omega) \\
&= \left[\tau_{ks}P_k(t)\right]^{-\frac{\alpha}{1-\alpha}}\left[\alpha\frac{w_s(t)^{1-\alpha}}{\tilde{M}_s(t)^{1-\alpha}z_s(t,\omega)}z_s(t,\omega)\right]^{\frac{1}{1-\alpha}}\ell_s(t,\omega) \\
&= \left[\frac{\tau_{ks}P_k(t)}{\alpha}\right]^{-\frac{\alpha}{1-\alpha}}\cdot\frac{w_s(t)}{\tilde{M}_s(t)}\cdot \ell_s(t,\omega) \qquad \forall \nu\in[0,M_k(t)]
\end{aligned}
$$

Integrating over varieties and summing over destinations, yields the total expenditure by firms in country s on variety ν inputs from country k:

$$
e^I_{ks}(t,\nu)=\int_0^1 p_{ks}(t,\nu)x_{ks}(t,\omega,\nu)d\omega=\left[\frac{\tau_{ks}P_k(t)}{\alpha}\right]^{-\frac{\alpha}{1-\alpha}}\cdot\frac{w_s(t)}{\tilde{M}_s(t)}\cdot L_s
$$

which is independent of ν, so I can drop indices and call it $e^I_{ks}(t)\equiv e^I_{ks}(t,\nu)$. Aggregate profits simply integrate over all intermediate varieties: $E^I_{ks}(t)=\int_0^{M_k(t)} e^I_{ks}(t)d\nu = M_k(t)\cdot e^I_{ks}(t)$.

Finally, I can solve for the optimal demand for each intermediate good ν from a final goods producer:

$$
\begin{aligned}
x_{ks}(t,\omega,\nu) &= \left[\frac{p_{ks}(t,\omega,\nu)}{p_{ss}(t,\omega)}\right]^{-\frac{1}{1-\alpha}}\cdot \ell_s(t,\omega)\cdot z_s(t,\omega)^{\frac{1}{1-\alpha}} \\
&= \left[\frac{\tau_{ks}P_k(t)}{\alpha}\right]^{-\frac{1}{1-\alpha}}\cdot \ell_s(t,\omega)\cdot\left[p_{ss}(t,\omega)z_s(t,\omega)\right]^{\frac{1}{1-\alpha}} \\
&= \left[\frac{\tau_{ks}P_k(t)}{\alpha}\right]^{-\frac{1}{1-\alpha}}\cdot\frac{w_s(t)}{\tilde{M}_s(t)}\cdot \ell_s(t,\omega) \qquad \forall\omega\in[0,1],\quad \forall\nu\in[0,M_s(t)]
\end{aligned}
$$

Using similar steps, the aggregate demand for intermediates coming from k is:

$$
\begin{aligned}
X_k(t) &= \int_0^{M_k(t)} \sum_{s\in\mathbf{K}} \int_0^1 x_{ks}(t,\omega,\nu)d\omega d\nu \\
&= \int_0^{M_k(t)} \sum_{s\in\mathbf{K}} \int_0^1 \left[\frac{\tau_{ks}P_k(t)}{\alpha}\right]^{-\frac{1}{1-\alpha}} \cdot \frac{w_s(t)}{\tilde{M}_s(t)} \cdot \ell_s(t,\omega)d\omega d\nu \\
&= \int_0^{M_k(t)} \sum_{s\in\mathbf{K}} \left[\frac{\tau_{ks}P_k(t)}{\alpha}\right]^{-\frac{1}{1-\alpha}} \cdot \frac{w_s(t)}{\tilde{M}_s(t)} \cdot L_s d\nu \\
&= M_k(t) \cdot \sum_{s\in\mathbf{K}} \left[\frac{\tau_{ks}P_k(t)}{\alpha}\right]^{-\frac{1}{1-\alpha}} \cdot \frac{w_s(t)}{\tilde{M}_s(t)} \cdot L_s
\end{aligned}
$$

B.5 Trade in Final Goods

Trade shares In this model, since there are infinitely many varieties in the unit interval, the expenditure share of destination region $d \in \mathbf{K}$ on goods coming from source country $s \in \mathbf{K}$ converge to their expected values. Let $\lambda_{sd}(t,\omega)$ denote the probability that consumers in region $d \in \mathcal{D}$ source variety ω from region $s \in \mathcal{D}$. For each each n, let $A_n^{-1}(t,\omega) \equiv \frac{\tilde{x}_{sd}(t)}{\tilde{x}_{nd}(t)}$, with $x_{sd}(t) \equiv \frac{w_s(t)^{1-\alpha}\tau_{sd}}{\tilde{M}_s(t)^{1-\alpha}}$. This probability will satisfy:

$$
\begin{aligned}
\lambda_{sd}(t,\omega) &= Pr\Big(s \text{ is the lowest cost supplier of } \omega \text{ to } d\Big) \\
&= Pr\left(\frac{\tilde{x}_{sd}(t)}{z_s(t,\omega)} < \min_{(n\neq s)}\left\{\frac{\tilde{x}_{nd}(t)}{z_n(t,\omega)}\right\}\right) \\
&= \int_0^\infty Pr(z_s(t,\omega)=z)Pr(z_n(t,\omega) < zA_n(t))dz \\
&= \int_0^\infty f_s(t)(z)\Pi_{(n\neq s)}F_n(t)(A_n z)dz \\
&= \int_0^\infty \theta T_s z^{-(1+\theta)} \exp\left\{-\left(\sum_{n\in\mathbf{K}} T_n A_n(t)^{-\theta}\right) z^{-\theta}\right\} dz \\
&= \frac{T_s\left(\tilde{x}_{sd}(t)\right)^{-\theta}}{\sum_{n\in\mathbf{K}} T_n\left(\tilde{x}_{nd}(t)\right)^{-\theta}} \\
&= \frac{T_s\left(\tilde{x}_{sd}(t)\right)^{-\theta}}{\sum_{n\in\mathbf{K}} T_n\left(\tilde{x}_{nd}(t)\right)^{-\theta}} \\
&= \frac{T_s\left(\tilde{M}_s(t)^{1-\alpha}\right)^{\theta}(w_s(t)^{1-\alpha}\tau_{sd})^{-\theta}}{\sum_{n=1}^{N} T_n\left(\tilde{M}_n(t)^{1-\alpha}\right)^{\theta}(w_n(t)^{1-\alpha}\tau_{nd})^{-\theta}} \qquad \text{(B.4)}
\end{aligned}
$$

Now note that $\lambda_{sd}(t,\omega)$ is independent of ω, so the probability of sourcing each variety from s to d is identical. A corollary is that aggregate expenditure trade shares of final goods from s in d will be equal to the probability of sourcing an arbitrary variety from s in d.

Price distributions and ideal price index Recall that, under the assumption of perfect competition, prices equal their marginal costs, such that the price of a variety ω produced in country s and shipped to d satisfies $p_{sd}(t,\omega) = \frac{\tau_{sd} w_s(t)^{1-\alpha}}{\tilde{M}_s(t)^{1-\alpha} z_s(t,\omega)}$.

Since $z_s(t,\omega)$ is a random variable, $p_{sd}(t,\omega)$ is also a random variable. We can derive the distribution of prices through the following steps. First, note that $z_s(t,\omega) = \frac{\tau_{sd} w_s(t)^{1-\alpha}}{\tilde{M}_s(t)^{1-\alpha} p_{sd}(\omega)}$. Then, note that:

$$p_{sd}(t,\omega) < p = \frac{\tau_{sd} w_s^{1-\alpha}}{M_s(t)^{1-\alpha} z} \iff z_s(t,\omega) > z = \frac{\tau_{sd} w_s^{1-\alpha}}{M_s(t)^{1-\alpha} p}$$

Therefore:

$$\begin{aligned} G_{sd}(t,\omega)(p) &= Pr(p_{sd}(t,\omega) < p) \\ &= Pr\left(z_s(t,\omega) > \frac{\tau_{sd} w_s^{1-\alpha}}{M_s(t)^{1-\alpha} p}\right) \\ &= 1 - \exp\{-T_s(\tau_{sd} w_s(t))^{-\theta}(M_s(t))^{(1-\alpha)\theta} p^{\theta}\} \qquad \forall \omega \in [0,1] \end{aligned}$$

which is the distribution of prices of any variety ω conditional on s being the lowest cost supplier of such a variety to d. To derive the unconditional distribution of prices at d, realize that:

$$\begin{aligned} G_n(t,\omega) &\equiv Pr(p_s(t,\omega) < p) \\ &= Pr((\exists s) \text{ for which } p_s d(t,\omega) < p) \\ &= 1 - Pr((\nexists s) \text{ for which } p_s d(t,\omega) < p) \\ &= 1 - \prod_{s \in \mathbf{K}} Pr(p_s d(t,\omega) > p) \\ &= 1 - \prod_{s \in \mathbf{K}} \exp\{-T_s(\tau_{sd} w_s(t))^{-\theta}(M_s(t))^{(1-\alpha)\theta} p^{\theta} \\ &= 1 - \exp\{-\sum_{s \in \mathbf{K}} T_s(\tau_{sd} w_s(t))^{-\theta}(M_s(t))^{(1-\alpha)\theta} p^{\theta}\} \end{aligned}$$

Recall that the price index is defined as:

$$
\begin{aligned}
P_d(t) &= \left[\int_0^1 p_d(t,\omega)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}} \\
&= \left[\int_0^\infty p^{1-\sigma} dG_n(t,p)\right]^{\frac{1}{1-\sigma}} \\
&= \left[\int_0^\infty p^{1-\sigma}\theta p^{\theta-1} \exp\left\{-\sum_{s\in\mathbf{K}} T_s(\tau_{sd} w_s(t)^{1-\alpha})^{-\theta}\tilde{M}_s(t)^{(1-\alpha)\theta} p^\theta\right\} dp\right]^{\frac{1}{1-\sigma}}
\end{aligned}
$$

Using a change of variables, let $\nu \equiv \sum_{s\in\mathbf{K}} T_s(\tau_{sd} w_s(t)^{1-\alpha})^{-\theta}\tilde{M}_s(t)^{(1-\alpha)\theta} p^\theta$ and note that $d\nu = \theta p^{\theta-1}\sum_{s\in\mathbf{K}} T_s(\tau_{sd} w_s(t)^{1-\alpha})^{-\theta}\tilde{M}_s(t)^{(1-\alpha)\theta} p^\theta dp$. Then:

$$
\begin{aligned}
P_d(t) &= \left[\int_0^\infty \left(\frac{\nu}{\sum_{s\in\mathbf{K}} T_s(\tau_{sd} w_s(t)^{1-\alpha})^{-\theta}\tilde{M}_s(t)^{(1-\alpha)\theta}}\right)^{\frac{1-\sigma}{\theta}} \exp\{-\nu\}\, d\nu\right]^{\frac{1}{1-\sigma}} \\
&= \Gamma\left(\frac{\theta+1-\sigma}{\theta}\right)^{\frac{1}{1-\sigma}} \left(\sum_{s\in\mathbf{K}} T_s(\tau_{sd} w_s(t)^{1-\alpha})^{-\theta}\tilde{M}_s(t)^{(1-\alpha)\theta}\right)^{-\frac{1}{\theta}} \qquad \text{(B.5)} \\
&= \Gamma\left(\frac{\theta+1-\sigma}{\theta}\right)^{\frac{1}{1-\sigma}} \left(\sum_{s\in\mathbf{K}} T_s(\tau_{sd} w_s(t)^{1-\alpha})^{-\theta} \left(\frac{1}{\alpha}\sum_{n\in\mathbf{K}} M_n(t)\left[\frac{\tau_{ns}P_n(t)}{\alpha}\right]^{-\frac{\alpha}{1-\alpha}}\right)^{(1-\alpha)\theta}\right)^{-\frac{1}{\theta}}.
\end{aligned}
$$

which shows that, given parameters $T_s.\tau_{sd}$ and the vector of state variables $\mathbf{M}_s(t) = [M_1(t),\cdots,M_N(t)]$ the closed form solution for the ideal price index $P_d(t)$ is a function of the vector of wages $\mathbf{w}(t) = [w_1(t),\cdots,w_N(t)]'$.

B.6 Research and Development

B.7 Market Clearing and Trade Balance

Market Clearing Let $Y_d(t)$ denote the total output of the final good and $X_d(t), I_d(t)$ denote the use of the final good as inputs for the production of intermediate inputs and R&D, respectively. Then total output in the final good for a given country must satisfy:

$$
Y_d(t) = C_d(t) + I_d(t) + X_d(t)
$$

where $I_d(t)$ and $C_d(t)$ are pinned down by the dynamic problem, described below, and $X_d(t)$ can be expressed as a function of aggregate demand in all destinations:

$$X_d(t) \equiv M_d(t) \cdot \sum_{k \in \mathbf{K}} \left[\frac{\tau_{dk} P_d(t)}{\alpha} \right]^{-\frac{1}{1-\alpha}} \cdot \frac{w_k(t)}{\tilde{M}_k(t)} \cdot L_k$$

The only asset market in each economy is the equity market for funding R&D. Savings will always be positive because expected returns to R&D are always positive. Replacing this in the budget constraint results in[21]:

$$I_d(t) = \frac{1}{\psi} \dot{M}_d(t) = \frac{1}{\psi} \frac{r_d(t)}{P_d(t)} M_d(t) + \frac{w_d(t)}{P_d(t)} L_d - C_d(t)$$

Combining the equations, one can express aggregate output as a function of the state variable $M_d(t)$, parameters, and wages (both $r_d(t)$ and $P_d(t)$ are functions of wages in every country):

$$Y_d(t) = \frac{1}{\psi} \frac{r_d(t)}{P_d(t)} M_d(t) + \frac{w_d(t)}{P_d(t)} \cdot L_d + M_d(t) \cdot \sum_{k \in \mathbf{K}} \left[\frac{\tau_{dk} P_d(t)}{\alpha} \right]^{-\frac{1}{1-\alpha}} \cdot \frac{w_k(t)}{\tilde{M}_k(t)} \cdot L_k$$

From the production side, total output is equal, then to total real capital plus total real labor income in addition to total demand for inputs of intermediate varieties. From the income side, since the final good aggregate output works in a competitive economy, total labor income will be equal to the labor share of total final goods sales:

$$w_s(t) L_s(t) \quad = \quad (1-\alpha) \sum_{d \in \mathbf{K}} \underbrace{\lambda^F_{sd}(t) P_d(t) Y_d(t)}_{\text{functions of wages}}$$

where which is a system of $|\mathbf{K}|$ equations that solve for the equilibrium wages. Similarly, intermediates will be paid their marginal contribution to output[22]:

$$\sum_{k \in \mathbf{K}} \int_0^{M_k(t)} \int_0^1 p_{ks}(t,\nu) x_{ks}(t,\omega,\nu) d\omega d\nu \quad = \quad \alpha \sum_{d \in \mathbf{K}} \lambda^F_{sd} P_d(t) Y_d(t)$$

[21]The equation below uses the fact that $\dot{M}_d(t) = \psi I_d(t) = \psi \dot{A}_d(t)$ and $M_d(t) = \psi A_d(t)$, which comes from the free-entry condition

[22]In the Appendices, I work out the integrals on the left-hand side, but the economics of this equation is easier to understand if left in this format. In particular, Appendix A.4. will specify a worked-out version of $p_{ks}(t,\nu) x_{ks}(t,\omega,\nu)$.

Trade Balance Since there are no international capital markets in this economy, trade will be balanced at any instant. This means that:

$$\underbrace{\sum_{k\in\mathbf{K}} E^F_{ks}(t)}_{\text{imports in final goods}} + \underbrace{\sum_{k\in\mathbf{K}} E^I_{ks}(t)}_{\text{imports in intermediate goods}} = \underbrace{\sum_{d\in\mathbf{K}} E^F_{sd}(t)}_{\text{exports in final goods}} + \underbrace{\sum_{k\in\mathbf{K}} E^I_{sd}(t)}_{\text{exports in intermediate goods}} \tag{B.6}$$

where:

$$E^F_{ks}(t) = \lambda^F_{ks} P_s(t) Y_s(t), \qquad E^I_{ks}(t) = M_k(t) \cdot \left[\frac{\tau_{ks} P_k(t)}{\alpha}\right]^{-\frac{\alpha}{1-\alpha}} \cdot \left[\frac{w_s(t) L_s}{\tilde{M}_s(t)}\right]$$

Recall that prices indices are functions of wages in every country. Note that E^I_{ks} is a function of wages and prices indices. Finally, $Y_s(t)$ is also a function of wages.

B.8 Balanced Growth Path

Autarky

Proof of Proposition (1)

Proof. Without loss of generality, choose an arbitrary country $s \in \mathbf{K}$. Since this world economy is under autarky, evaluate (1) replacing for the real interest rate using equations (8) and (6) and taking the limit $\tau_{sd} \to \infty (\forall s \neq d)$. By assumption (1), $\tau_{ss} = 1 (\forall s)$. Therefore, (1) collapses to:

$$g_s^{\text{autarky}} = \left[\frac{(1-\alpha)\cdot\alpha\cdot\psi}{P_s(t^*)} \cdot \left[\frac{w_s(t^*) L_s}{M_s(t^*)}\right] - \rho\right] \tag{B.7}$$

for a BGP inclusive of each period $t \geq t^*$. Choosing $P_s(t^*)$ to be numéraire of this economy shows that the growth rate will follow the stated equation.

To show that it is consistent with a BGP under autarky, I need to solve for the growth rate in terms of parameters. In order to do so, I need a few intermediate steps. First, note that I can express the demand for intermediates as:

$$\bar{x}_{ss}(t,\omega) \equiv x_{ss}(t,\omega,\nu) = \left[\alpha z_s(\omega) p_{ss}(t,\omega)\right]^{\frac{1}{1-\alpha}} \cdot \ell_s(t,\omega)$$

which, in turn, implies that the optimal price of intermediate varieties is $p_{ss}(t,\omega,\nu) = \frac{1}{\alpha}$ and I can rewrite the production function of the final goods producer as:

$$\begin{aligned} y_s(\omega) &= z_s(\omega)\ell_s(t,\omega)^{1-\alpha}\left(\frac{1}{\alpha}\int_0^{M_s(t)}[\bar{x}_{ss}(t,\omega)]^{\alpha}d\nu\right) \\ &= z_s(\omega)\ell_s(t,\omega)^{1-\alpha}\left(\frac{1}{\alpha}\int_0^{M_s(t)}\left[\left[\alpha z_s(\omega)p_{ss}(t,\omega)\right]^{\frac{1}{1-\alpha}}\cdot\ell_s(t,\omega)\right]^{\alpha}d\nu\right) \\ &= \left[z_s(\omega)\right]^{\frac{1}{1-\alpha}}\cdot\left[\alpha\cdot p_{ss}(t,\omega)\right]^{\frac{\alpha}{1-\alpha}}\cdot\ell_s(t,\omega)\cdot\frac{1}{\alpha}\cdot M_s(t) \end{aligned}$$

Replacing for $p_{ss}(t,\omega)$ using the assumption of pricing under perfect competition:

$$\begin{aligned} y_s(\omega) &= \left[z_s(\omega)\right]^{\frac{1}{1-\alpha}}\cdot\left[\alpha\cdot\left(\frac{\alpha w_s(t)}{M_s(t)}\right)^{1-\alpha}\cdot\frac{1}{\alpha\cdot z_s(\omega)}\right]^{\frac{\alpha}{1-\alpha}}\cdot\ell_s(t,\omega)\cdot\frac{1}{\alpha}\cdot M_s(t) \\ &= \alpha^{-(1-\alpha)}\cdot z_s(\omega)\cdot w_s(t)^{\alpha}\cdot M_s(t)^{1-\alpha}\cdot\ell_s(t,\omega) \end{aligned}$$

Using the first order condition for labor:

$$(1-\alpha)p_{ss}(t,\omega)y_s(\omega) = \ell_s(t,\omega)w_s(t)$$

Integrating over ω gives us the labor market clearing equation under autkary:

$$(1-\alpha)\int_0^1 p_{ss}(t,\omega)y_s(\omega)d\omega = (1-\alpha)P_s(t)Y_s(t) = (1-\alpha)Y_s(t) = \int_0^1 \ell_s(t,\omega)w_s(t)d\omega = L_s w_s(t)$$

where the second equation uses the fact that $P_s(t) = 1$ is the numéraire of this economy under autarky. In aggregate terms, then: $(1-\alpha)Y_s(t) = L_s w_s(t)$. Now, using the definition of the aggregate final good:

$$\begin{aligned} Y_s(t) &= \left[\int_0^1 y_s(t,\omega)^{\frac{\sigma-1}{\sigma}}d\omega\right]^{\frac{\sigma}{\sigma-1}} \\ &= \left[\int_0^1 \left(z_s(\omega)\cdot\ell_s(t,\omega)\right)^{\frac{\sigma-1}{\sigma}}d\omega\right]^{\frac{\sigma}{\sigma-1}}\alpha^{-(1-\alpha)}\cdot w_s(t)^{\alpha}\cdot M_s(t)^{1-\alpha} \end{aligned}$$

The term in the integral denotes the joint product of productivity and labor allocation across firms. In aggregate terms, since both the distribution of productivity and the population are fixed for every t; and relative wages are fixed along the BGP, this term will be constant.

Following F. Alvarez and Lucas (2007), note that all goods enter symmetrically in the definition of the aggregate final good and they differ only by their productivity level. Therefore, I can change from indexing varieties by ω by indexing it by its distribution of productivities z, expressing it in terms of distribution $F_s(z)$:

$$(1-\alpha)Y_s(t) = (1-\alpha)\left[\int_0^{\infty}\left(z \cdot \ell_s(t,z)\right)^{\frac{\sigma-1}{\sigma}} dF_s(z)\right]^{\frac{\sigma}{\sigma-1}} \alpha^{-(1-\alpha)} \cdot w_s(t)^{\alpha} \cdot M_s(t)^{1-\alpha} = L_s w_s(t)$$

and solving for $\frac{w_s(t)}{M_s(t)}$:

$$\frac{w_s(t)}{M_s(t)} = \frac{1}{\alpha} \cdot \left(\frac{1-\alpha}{L_s}\left[\int_0^{\infty}\left(z \cdot \ell_s(t,z)\right)^{\frac{\sigma-1}{\sigma}} dF_s(z)\right]^{\frac{\sigma}{\sigma-1}}\right)^{\frac{1}{1-\alpha}} \tag{B.8}$$

Note that none of the terms on the right-hand side depend on endogenous objects. While any particular $\ell_s(t,z)$ is a demand function that depends on a firm's specific productivity and on endogenous objects of the economy, the integral over all of these choices do not, since both the productivity distribution and total labor supply are fixed.

Therefore, I can express the BGP growth rate of the economy fully in terms of exogenous objects:

$$g_s^{\text{autarky}} = \left[(1-\alpha) \cdot \psi \cdot L_s \cdot \left(\frac{1-\alpha}{L_s}\left[\int_0^{\infty}\left(z \cdot \ell_s(t^*,z)\right)^{\frac{\sigma-1}{\sigma}} dF_s(z)\right]^{\frac{\sigma}{\sigma-1}}\right)^{\frac{1}{1-\alpha}} - \rho\right] \tag{B.9}$$

Since neither the productivity distribution $F_s(z)$ nor the demand functions $\ell_s(t^*,z)$ will change along the BGP and all other terms in the growth rate are parameters, this pins down the uniqueness of the BGP under autarky.

The next step in the proof is to show that $g_{M_s} = g_{Y_s} = g_{C_s} = g_{w_s} = g_{A_s} = g_s^{\text{autarky}}$. Using the Euler Equation, the definition of the BGP growth rate of consumption is:

$$\frac{\dot{C}_s(t^*)}{C_s(t^*)} \equiv \underbrace{g_{C_s}}_{\text{constant}} = \underbrace{r_s(t^*)}_{\text{constant}} - \rho$$

In autarky, $P_s(t^*) = 1(\forall t)$ is the numéraire of the domestic economy and therefore constant in every period. Hence, $g_{P_s} = 0$ by construction. Therefore, $r_s(t^*)$ is also constant along the BGP and $g_{r_s} = 0$.

I can also write the law of motion for varieties in growth form:

$$\frac{\dot{M}_s(t^*)}{M_s(t^*)} \equiv \underbrace{g_{M_s}}_{\text{constant}} = \underbrace{r_s(t^*)}_{\text{constant}} + \psi \underbrace{\left[\frac{w_s(t^*)L_s}{M_s(t^*)} - \frac{C_s(t^*)}{M_s(t^*)}\right]}_{\text{constant}}$$

I have already established that $r_s(t^*)$ is a constant using the Euler Equation. Since g_{M_s} grows at a constant rate at every period along the BGP, it must be the case that the term within the brackets is constant for every $t^* \geq t$.

There are only two possibilities. Either the term in the brackets is zero, implying that $g_{C_s} = g_{w_s}$ and $g_{M_s} = r_s(t^*) = g_{C_s} + \rho$. Or the term in the brackets can take any arbitrary finite value, as long as $g_{w_s} = g_{M_s}$ and $g_{C_s} = g_{M_s}$. Only the latter is consistent with a BGP.

Recall that, as shown above, the BGP growth rate requires that

$$g_s^{\text{autarky}} = \left[(1-\alpha) \cdot \alpha \cdot \psi \cdot \frac{w_s(t^*)L_s}{M_s(t^*)} - \rho\right]$$

for some constant g_s^{autarky}. This rate is only a constant if $g_{w_s} = g_{M_s}$, which, in turn, would require that $g_{w_s} = g_{M_s} = g_{C_s}$.

Re-stating the aggregate labor market clearing equation under autarky: $(1-\alpha)Y_s(t^*) = L_s w_s(t^*)$. Taking logs of the aggregate version of the equation and derivating with respect to time shows that the growth rate of the aggregate final good is equal to the growth rate of wages

$$\frac{\dot{Y}_s(t^*)}{Y_s(t^*)} = g_{Y_s} = g_{w_s} = \frac{\dot{w}_s(t^*)}{w_s(t^*)}$$

Finally, using the free-entry condition, $\dot{M}_s(t) = \psi I_s(t)$, which implies $\int_0^t \dot{M}_s(\tau)d\tau = \psi \int_0^t I_s(\tau)d\tau$ and $M_s(t) = \psi A_s(t)$. Taking logs and time derivatives of this shows:

$$\frac{\dot{A}_s(t^*)}{A_s(t^*)} = g_{A_s} = g_{M_s} = \frac{\dot{M}_s(t^*)}{M_s(t^*)}$$

Therefore, I have shown that:

$$
\begin{aligned}
g_s^{\text{autarky}} &= g_{M_s} = g_{Y_s} = g_{C_s} = g_{w_s} = g_{A_s} = \left[(1-\alpha)\cdot\alpha\cdot\psi\cdot\frac{w_s(t^*)L_s}{M_s(t^*)} - \rho\right] \\
&= \left[(1-\alpha)\cdot\psi\cdot L_s\cdot\left(\frac{1-\alpha}{L_s}\left[\int_0^\infty \left(z\cdot\ell_s(t^*,z)\right)^{\frac{\sigma-1}{\sigma}} dF_s(z)\right]^{\frac{\sigma}{\sigma-1}}\right)^{\frac{1}{1-\alpha}} - \rho\right]
\end{aligned}
$$

which completes the proof. □

Zero gravity

Proof of Lemma (1)

Proof. Recall that the labor market determination equation is:

$$
w_s(t)L_s(t) = (1-\alpha)\sum_{d\in\mathbf{K}} \underbrace{\lambda_{sd}^F(t)P_d(t)Y_d(t)}_{\text{functions of wages}}
$$

while total output of the final good must satisfy:

$$
Y_d(t) = \underbrace{C_d(t)}_{\text{consumption}} + \underbrace{I_d(t)}_{\text{investment in R\&D}} + \underbrace{X_d(t)}_{\text{production of intermediates}}
$$

I can calculate the use of intermediates directly as a function of the measure of varieties in every country and wages as shown in Appendix (B.4):

$$
X_d(t) \equiv M_d(t)\cdot\sum_{k\in\mathbf{K}}\left[\frac{\tau_{dk}P_d(t)}{\alpha}\right]^{-\frac{1}{1-\alpha}}\cdot\frac{w_k(t)}{\tilde{M}_k(t)}\cdot L_k
$$

The only asset market in each economy is the equity market for funding R&D. Savings will always be positive because expected returns to R&D are always positive. Replacing this in the budget constraint results in[23]:

$$
I_d(t) = \frac{1}{\psi}\dot{M}_d(t) = \frac{r_d(t)}{P_d(t)}M_d(t) + \frac{w_d(t)}{P_d(t)} - C_d(t)
$$

[23]The equation below uses the fact that $\dot{M}_d(t) = \psi I_d(t) = \psi\dot{A}_d(t)$ and $M_d(t) = \psi A_d(t)$, which comes from the free-entry condition

which means I can express $Y_d(t)$ without explicitly solving for $C_d(t)$. Combining these equations, one can express aggregate output as a function of the state variable $M_d(t)$ and prices:

$$Y_d(t) = \frac{1}{\psi}\frac{r_d(t)}{P_d(t)}M_d(t) + \frac{w_d(t)}{P_d(t)} \cdot L_d + M_d(t) \cdot \sum_{k \in \mathbf{K}} \left[\frac{\tau_{dk}P_d(t)}{\alpha}\right]^{-\frac{1}{1-\alpha}} \cdot \frac{w_k(t)}{\tilde{M}_k(t)} \cdot L_k$$

which reflects the combined demand for aggregate final goods for the production of both investment varieties or consumption —funded by the two first terms, which reflect real capital and labor income, respectively —, as well as demand for inputs in the production of intermediate varieties. Finally, since the final good aggregate output works in a competitive economy, factors will be paid their marginal contributions to output:

$$\begin{aligned}
w_s(t)L_s &= (1-\alpha)\sum_{d \in \mathbf{K}} \lambda^F_{sd}(t)P_d(t)Y_d(t) \\
&= (1-\alpha)\sum_{d \in \mathbf{K}} \lambda^F_{sd}P_d(t)\left(\frac{1}{\psi}\frac{r_d(t)}{P_d(t)}M_d(t) + \frac{w_d(t)}{P_d(t)} \cdot L_d \right. \\
&\qquad \left. + M_d(t) \cdot \sum_{k \in \mathbf{K}} \left[\frac{\tau_{dk}P_d(t)}{\alpha}\right]^{-\frac{1}{1-\alpha}} \cdot \frac{w_k(t)}{\tilde{M}_k(t)} \cdot L_k\right)
\end{aligned}$$

which is a system of $|\mathbf{K}|$ equations that solve for the equilibrium wages. Recall that, through the non-arbitrage conditions, interest rates are:

$$r_d(t) = \frac{\pi_d(t)}{P_d(t)} + \frac{\dot{P}_d(t)}{P_d(t)}$$

but, along the BGP, $\frac{\dot{P}_d(t)}{P_d(t)} = 0$. Therefore, expanding $\pi_d(t)$, I can write real interest rates along the BGP as:

$$\frac{r_d(t)}{P_d(t)} = \frac{\pi_d(t)}{P_d(t)^2} = \frac{(1-\alpha)}{P_d(t)^2} \cdot \sum_{k \in \mathbf{K}} \left[\frac{\tau_{dk}P_d(t)}{\alpha}\right]^{-\frac{\alpha}{1-\alpha}} \cdot \frac{w_k(t)L_k}{\tilde{M}_k(t)}$$

such that we can write the wage determination equation as:

$$
\begin{aligned}
w_s(t)L_s &= (1-\alpha)\sum_{d\in\mathbf{K}}\frac{T_s\left(\tilde{M}_s(t)^{1-\alpha}\right)^{\theta}(w_s(t)^{1-\alpha}\tau_{sd})^{-\theta}}{\sum_{n=1}^{N}T_n\left(\tilde{M}_n(t)^{1-\alpha}\right)^{\theta}(w_n(t)^{1-\alpha}\tau_{nd})^{-\theta}}\Bigg(w_d(t)L_d(t)+ \\
&\quad M_d(t)\sum_{k\in\mathbf{K}}\frac{w_k(t)L_k}{\tilde{M}_k(t)}\cdot\left[\frac{1-\alpha}{P_d(t)\cdot\psi}\left[\frac{\tau_{dk}P_d(t)}{\alpha}\right]^{-\frac{\alpha}{1-\alpha}}+P_d(t)\cdot\left[\frac{\tau_{dk}P_d(t)}{\alpha}\right]^{-\frac{1}{1-\alpha}}\right]\Bigg)
\end{aligned}
$$

which, as it now explicit, given the vector of state variables $\mathbf{M}(t)$, is simply a function of parameters, wages and prices levels (which is itself a function of wages and states).

Evaluating this equation under zero gravity yields:

$$
w_s(t)L_s = (1-\alpha)\sum_{d\in\mathbf{K}}\frac{T_s(w_s(t)^{1-\alpha})^{-\theta}}{\sum_{n=1}^{N}T_n(w_n(t)^{1-\alpha})^{-\theta}}\left(w_d(t)L_d(t)+M_d(t)\cdot\frac{\sum_{k\in\mathbf{K}}w_k(t)L_k}{\sum_{k\in\mathbf{K}}M_k(t)}\cdot\left[\frac{1-\alpha+\alpha\psi}{\alpha\psi}\right]\right)
$$

because the law of one price holds which implies that $P_s(t) = P_{s'}(t) = P(t) = 1$ for any $s, s' \in \mathbf{K}$ in any period t, choosing $P(t)$ as the numéraire of this world economy. Define the excess demand function:

$$
\begin{aligned}
Z_s(\mathbf{M},\mathbf{w},t) &\equiv \frac{1}{w_s(t)}\Bigg\{(1-\alpha)\sum_{d\in\mathbf{K}}\frac{T_s(w_s(t)^{1-\alpha})^{-\theta}}{\sum_{n=1}^{N}T_n(w_n(t)^{1-\alpha})^{-\theta}}\Bigg(w_d(t)L_d(t) \\
&+ M_d(t)\cdot\frac{\sum_{k\in\mathbf{K}}w_k(t)L_k}{\sum_{k\in\mathbf{K}}M_k(t)}\cdot\left[\frac{1-\alpha+\alpha\psi}{\alpha\psi}\right]\Bigg)-w_sL_s\Bigg\}
\end{aligned}
$$

which is homogenous of degree zero in both wages and measures. Then, it is easy to show $\frac{\partial Z_s(\mathbf{M},\mathbf{w},t)}{\partial w_d} > 0$, satisfying the gross substitute property for all $s \in \mathbf{K}$:

$$
\begin{aligned}
\frac{\partial Z_s(\mathbf{M},\mathbf{w},t)}{\partial w_d} &= \frac{1}{w_s(t)}(1-\alpha)\lambda_{sd}(t)\left(L_d+M_d(t)\cdot\frac{L_d}{\sum_{k\in\mathbf{K}}w_k(t)L_k}\cdot\left[\frac{1-\alpha+\alpha\psi}{\alpha\psi}\right]\right) \\
&+ (1-\alpha)\theta\frac{\lambda_{sd}(t)}{\sum_{n=1}^{N}T_n(w_n(t)^{1-\alpha})^{-\theta}}\cdot T_d w_d(t)^{-\alpha\theta} > 0
\end{aligned}
$$

This completes the proof.

□

I will now use an intermediate result that will show that if a balanced path with trade exists under zero gravity and the cross-sectional equilibrium is unique, then the balanced growth path is unique. This Lemma will be useful in the proofs of the proposition of the BGP under zero gravity below.

Lemma 2 (Uniqueness under zero gravity)**.** *Suppose there balanced growth path such that the world equilibrium growth rate g satisfies, in every country $s \in \mathbf{K}$, $g_{M_s} = g_{Y_s} = g_{C_s} = g_{w_s} = g_{A_s} = g$, $g_{r_s} = g_{P_s} = 0$ for a BGP inclusive of each period $t \geq t^*$ and $\tau_{sd} = 1$ for all (s, d). Then, if the cross-sectional equilibrium is unique, the world equilibrium growth rate g is also unique.*

Proof. Since growth rates equalize across the world then real interest rates must also equalize across the world:

$$\frac{\dot{C}_s(t)}{C_s(t)} = \frac{\dot{C}_{s'}(t)}{C_{s'}(t)} = g \implies \frac{r_s(t)}{P_s(t)} = \frac{r_{s'}(t)}{P_s(t)} = \frac{1}{N}\sum_{s' \in \mathbf{K}} \frac{r_{s'}(t)}{P_{s'}(t)} \quad \forall s, s' \in \mathbf{K}$$

Under zero gravity, the law of one price holds, which over the BGP, implies that $P_s(t) = P_{s'}(t) = P(t) = 1$ for any $s, s' \in \mathbf{K}$ in any period t, choosing $P(t)$ as the numéraire of this world economy. Furthermore, since the growth rate in the measure of varieties also equalizes, over the BGP, then:

$$\frac{\dot{M}_s(t^*)}{M_s(t^*)} - \frac{1}{N}\sum_{s' \in \mathbf{K}} \frac{\dot{M}_{s'}(t^*)}{M_{s'}(t^*)} = \frac{\dot{C}_s(t^*)}{C_s(t^*)} - \frac{1}{N}\sum_{s' \in \mathbf{K}} \frac{\dot{C}_{s'}(t^*)}{C_{s'}(t^*)} = r_s(t^*) - \frac{1}{N}\sum_{s' \in \mathbf{K}} r_{s'}(t^*) = 0$$

where the first equation uses the fact that $g_{C_s} = g_{M_s}$ for all s and the second equation simply uses the definition of g_{C_s} and the fact that $P(t)$ is the numéraire of the world economy. Recall that, along the zero gravity BGP, $r_s(t^*) = \pi_s(t^*)$ and note that, as shown in Appendix B.4, I can write profits as a function of relative prices of intermediate goods:

$$\pi_s(t) = (1-\alpha) \cdot \alpha \cdot \sum_{k \in \mathbf{K}} \underbrace{w_k(t) L_k}_{\text{value added in } k} \left[\sum_{n \in \mathbf{K}} \underbrace{M_n(t)}_{\text{product measure in } n} \left[\underbrace{\frac{\tau_{nk} P_n(t)}{\tau_{sk} P_s(t)}}_{\substack{\text{relative price} \\ \text{of intermediate good}}} \right]^{-\frac{\alpha}{1-\alpha}} \right]^{-1}$$

Define the excess variety growth function as:

$$
\begin{aligned}
Z_s^M(\mathbf{M},\mathbf{w},t) &= r_s(t^*) - \frac{1}{N}\sum_{s'\in\mathbf{K}} r_{s'} \\
&= (1-\alpha)\cdot\alpha\cdot\sum_{k\in\mathbf{K}} w_k(t)L_k\Bigg(\left[\sum_{n\in\mathbf{K}} M_n(t)\left[\frac{\tau_{nk}P_n(t)}{\tau_{sk}P_s(t)}\right]^{-\frac{\alpha}{1-\alpha}}\right]^{-1} \\
&- N\cdot\left[\sum_{n\in\mathbf{K}} M_n(t)\sum_{s'\in\mathbf{K}}\left[\frac{\tau_{nk}P_k(t)}{\tau_{s'k}P_{s'}(t)}\right]^{-\frac{\alpha}{1-\alpha}}\right]^{-1}\Bigg)
\end{aligned}
$$

The function $Z_s^M(\mathbf{M},\mathbf{w},t)$ is a growth rate analog of the excess demand function. To show that this is a fixed point problem, it suffices to show that if a country s has a measure of varieties $M_s(t)$ *above* its long-run equilibrium rate (relative to other countries), then it will have a price of final good *below* average and it will grow *slower* than average. Since this function is *strictly decreasing* in the marginal cost of the price of intermediate goods in the country s $\tau_{sk}P_s(t)$, any if a solution exists satisfying $Z_s^M(\mathbf{M},\mathbf{w},t)=0$, then it will be unique, as long the cross-sectional equilibrium is unique.

I will table this discussion to the general case, since, as it will become clear in the next paragraph, it will not be necessary to delve into it in the zero gravity case. Evaluating function $Z_s^M(\mathbf{M},\mathbf{w})$ under zero gravity means imposing $P_s(t) = P_{s'}(t) = P(t) = 1$ and $\tau_{sd}=1$ for all (s,d).

In doing so, it is clear that $Z_s^M(\mathbf{M},\mathbf{w},t) = 0$ for any zero gravity equilibrium evaluated at any period t. Note that I did not have to impose that the system were at a BGP to recover zero excess variety growth under zero gravity. Since the choice of s was arbitrary, this holds for any $s\in\mathbf{K}$.

Therefore, this proves that under zero gravity the system *will always be at a common BGP* and such *BGP is uniquely pinned down by the initial cross-sectional equilibrium*. Hence, as long as the cross-sectional is unique, then the BGP growth rate is also unique. By the result of Lemma (1), the cross-sectional equilibrium is unique. Therefore, the if a BGP exists under zero-gravity it is unique. □

Proof of Proposition (2)

Proof. Without loss of generality, choose an arbitrary country $s\in\mathbf{K}$. Since this world economy is under zero gravity, evaluate (1) replacing for the real interest rate using equations (8) and (6) and evaluating $\tau_{sd}=1(\forall s,d)$.Therefore, (1) collapses to:

$$
g_s^{\text{zero gravity}} = \left[\frac{(1-\alpha)\cdot\alpha\cdot\psi}{P_s(t^*)}\cdot\left[\frac{\sum_{k\in\mathbf{K}} w_k(t^*)L_k}{\sum_{k\in\mathbf{K}} M_k(t^*)}\right]-\rho\right] \tag{B.10}
$$

for a BGP inclusive of each period $t \geq t^*$. Since there are no trade costs, the law of one price holds, and $P_s(t^*) = P_d(t^*) \equiv P(t^*)$ for every $s, d \in \mathbf{K}$. Choosing $P(t^*)$ to be numéraire of this economy shows that the growth rate will follow the stated equation. Since the choice of the s of arbitrary and the expression in the right-hand side of the equation is equal for every $s \in \mathbf{K}$, it follows that the $g_s^{\text{zero gravity}} = g^{\text{zero gravity}}$ for all $s \in \mathbf{K}$, which shows that the growth rate must be common across all countries.

The next step in the proof is to show that $g_{M_s} = g_{Y_s} = g_{C_s} = g_{w_s} = g_{A_s} = g_s^{\text{zero gravity}}$. Using the Euler Equation, the definition of the BGP growth rate of consumption is:

$$\frac{\dot{C}_s(t^*)}{C_s(t^*)} \equiv \underbrace{g_{C_s}}_{\text{constant}} = \underbrace{r_s(t^*)}_{\text{constant}} - \rho$$

Under zero gravity, the law of one price holds and $P_s(t^*) = P_d(t^*) \equiv P(t^*) = 1$ for every $s, d \in \mathbf{K}, (\forall t)$ is the numéraire of the world economy. Hence, $g_{P_s} = 0$ by construction in every $s \in \mathbf{K}$. Therefore, $r_s(t^*)$ is also constant along the BGP and $g_{r_s} = 0$.

I can also write the law of motion for varieties in growth form:

$$\frac{\dot{M}_s(t^*)}{M_s(t^*)} \equiv \underbrace{g_{M_s}}_{\text{constant}} = \underbrace{r_s(t^*)}_{\text{constant}} + \psi \underbrace{\left[\frac{w_s(t^*)L_s}{M_s(t^*)} - \frac{C_s(t^*)}{M_s(t^*)}\right]}_{\text{constant}} \tag{B.11}$$

I have already established that $r_s(t^*)$ is a constant using the Euler Equation. Since g_{M_s} grows at a constant rate at every period along the BGP, it must be the case that the term within the brackets is constant for every $t^* \geq t$.

There are only two possibilities. Either the term in the brackets is zero, implying that $g_{C_s} = g_{w_s}$ and $g_{M_s} = r_s(t^*) = g_{C_s} + \rho$. Or the term in the brackets can take any arbitrary finite value, as long as $g_{w_s} = g_{M_s}$ and $g_{C_s} = g_{M_s}$. Only the latter is consistent with a BGP.

First, I need to establish that all wages grow at the same rate. Recall that, under zero gravity, the labor market clearing condition is:

$$w_s(t^*)L_s = (1-\alpha) \sum_{d \in \mathbf{K}} \lambda_{sd}(t^*) Y_d(t^*)$$

Taking logs and time derivatives, I can write this equation as:

$$\frac{\dot{w}_s(t^*)}{w_s(t^*)} = g_{w_s} \quad = \quad \sum_{d\in\mathbf{K}} \frac{\lambda_{sd}(t^*)Y_d(t^*)}{\sum_{k\in\mathbf{K}} \lambda_{kd}(t^*)Y_k(t^*)} \left(\frac{\dot{\lambda}_{sd}(t^*)}{\lambda_{sd}(t^*)} + \frac{\dot{Y}_d(t^*)}{Y_d(t^*)} \right)$$
$$= \sum_{d\in\mathbf{K}} \gamma_{sd}(t^*) \left(\frac{\dot{\lambda}_{sd}(t^*)}{\lambda_{sd}(t^*)} + \frac{\dot{Y}_d(t^*)}{Y_d(t^*)} \right)$$
$$= \sum_{d\in\mathbf{K}} \gamma_{sd}(t^*) \left(g_{\lambda_{sd}} + g_{Y_d} \right)$$

where $\gamma_{sd}(t^*) \equiv \frac{\lambda_{sd}(t^*)Y_d(t^*)}{\sum_{k\in\mathbf{K}} \lambda_{kd}(t^*)Y_k(t^*)}$.

Since for all t, $\lambda_{sd}(t) \in (0,1)$, for an equilibrium to exist, it must be the case that $g_{\lambda_{sd}} = 0$. For a proof by a contradiction, suppose $g_{\lambda_{sd}} > 0$. Then $\lim_{t\to\infty} \lambda_{sd}(t) = \infty$, which is a contradiction. Conversely, suppose that $g_{\lambda_{sd}} < 0$ for some s. This implies that for some other s', $g_{\lambda_{s'd}} > 0$, which is a contradiction. Therefore, it must be the case that $g_{\lambda_{sd}} = 0$ for all $s, d \in \mathbf{K}$.

Therefore, the equation above reduces to:

$$g_{w_s} = \sum_{d\in\mathbf{K}} \gamma_{sd}(t^*) g_{Y_d}$$

The next step is showing that, since g_{Y_d}, g_{w_s} are all constant, the weights $\gamma_{sd}(t) = \gamma_{sd}$ must be fixed along the BGP and $(\forall s, d) g_{Y_d} = g_{Y_s} = g_{w_s} = g_{w_d}$. Suppose $g_{Y_{d'}} > g_{Y_d}$ for some d' compared to others d. In that case, $\gamma_{sd'}(t)$ will grow faster relative to other d. However, that implies that g_{w_s} is not a constant, which is a contradiction.

Since $g_{Y_s} = g_{Y_d} = g_Y$ for all $s, d \in \mathbf{K}$, the shares $\gamma_{sd}(t^*)$ are constant for all $t \geq t^*$ along the BGP, and the shares sum to one, it follows trivially that:

$$g_{w_s} = \underbrace{\sum_{d\in\mathbf{K}} \gamma_{sd}(t^*)}_{=1} g_Y = g_Y = g_{Y_s}$$

This establishes that the wages growth rates $g_{w_s} = g_{w_d} = g_{Y_s} = g_{Y_d}$ are equal in every country and also that they are equal to their respective aggregate output growth rates. Recall that, as shown above, the BGP growth rate requires that

$$g_s^{\text{zero gravity}} = \left[(1-\alpha) \cdot \alpha \cdot \psi \cdot \left[\frac{\sum_{k\in\mathbf{K}} w_k(t^*) L_k}{\sum_{k\in\mathbf{K}} M_k(t^*)} \right] - \rho \right]$$

for some constant $g_s^{\text{zero gravity}}$. This rate is only constant if $\frac{\sum_{k\in\mathbf{K}} w_k(t^*)}{\sum_{k\in\mathbf{K}} M_k(t^*)}$ is constant. This requires that the numerator and the denominator grow at equal rates. In turn, per condition (B.11), the product measures grow at constant rates in each country, which are either: (a) equal to wages $g_{M_s} = g_{w_s}$; or (b) equal to an affine transformation of wages $g_{M_s} = g_{C_s} + \rho = g_{w_s} + \rho$.

Furthermore, wages grow at equal rates in every country $s \in \mathbf{K}$. Hence, it follows that the product measure also grows at the same rate in every country. To satisfy a constant $g_s^{\text{zero gravity}}$, it must be the case that $g_{w_s} = g_{M_s}$, which, in turn, would require that $g_{w_s} = g_{M_s} = g_{C_s} = g_{Y_s}$.

Finally, using the free-entry condition, $\dot{M}_s(t) = \psi I_s(t)$, which implies $\int_0^t \dot{M}_s(\tau)d\tau = \psi \int_0^t I_s(\tau)d\tau$ and $M_s(t) = \psi A_s(t)$. Taking logs and time derivatives of this shows:

$$\frac{\dot{A}_s(t^*)}{A_s(t^*)} = g_{A_s} = g_{M_s} = \frac{\dot{M}_s(t^*)}{M_s(t^*)}$$

Therefore, I have shown that:

$$g^{\text{zero gravity}} \quad = \quad g_{M_s} = g_{Y_s} = g_{C_s} = g_{w_s} = g_{A_s} = \left[(1-\alpha)\cdot\alpha\cdot\psi\cdot\left[\frac{\sum_{k\in\mathbf{K}} w_k(t^*)L_k}{\sum_{k\in\mathbf{K}} M_k(t^*)}\right] - \rho\right]$$

Since the choice of s was arbitrary, this holds for any $s \in \mathbf{K}$. Furthermore, using the results of Lemma (2), under zero gravity the system *will always be at a common BGP* and such *BGP is uniquely pinned down by the initial cross-sectional equilibrium*, which is unique.

□

General case As I did in the zero gravity case, I will first prove an intermediate result that shows that if the balanced growth path exists and the cross-sectional equilibrium is unique for the relevant parameter space, then the balanced growth path is unique. The strategy of the proof is to show that the excess varieties growth function has a fixed point.

Lemma 3 (Uniqueness, general case)**.** *Suppose there balanced growth path such that the world equilibrium growth rate g satisfies, in every country $s \in \mathbf{K}$, $g_{M_s} = g_{Y_s} = g_{C_s} = g_{w_s} = g_{A_s} = g$ and $g_{r_s} = g_{P_s} = 0$ for a BGP inclusive of each period $t \geq t^*$. If the cross-sectional equilibrium is unique, then, for restrictions of the parameter space that guarantee that $\frac{\partial P_s(t)}{\partial M_s(t)} < 0$, the world equilibrium growth rate g is unique.*

Proof. Since growth rates equalize across the world then real interest rates must also equalize across the world:

$$\frac{\dot{C}_s(t)}{C_s(t)} = \frac{\dot{C}_{s'}(t)}{C_{s'}(t)} = g \implies \frac{r_s(t)}{P_s(t)} = \frac{r_{s'}(t)}{P_s(t)} = \frac{1}{N}\sum_{s'\in\mathbf{K}}\frac{r_{s'}(t)}{P_{s'}(t)} \quad \forall s,s' \in \mathbf{K}$$

Furthermore, since the growth rate in the measure of varieties also equalizes, over the BGP, then:

$$\frac{\dot{M}_s(t^*)}{M_s(t^*)} - \frac{1}{N}\sum_{s'\in\mathbf{K}}\frac{\dot{M}_{s'}(t^*)}{M_{s'}(t^*)} = \frac{\dot{C}_s(t^*)}{C_s(t^*)} - \frac{1}{N}\sum_{s'\in\mathbf{K}}\frac{\dot{C}_{s'}(t^*)}{C_{s'}(t^*)} = \frac{r_s(t^*)}{P_s(t^*)} - \frac{1}{N}\sum_{s'\in\mathbf{K}}\frac{r_{s'}(t^*)}{P_{s'}(t^*)} = 0$$

where the first equation uses the fact that $g_{C_s} = g_{M_s}$ for all s and the second equation simply uses the definition of g_{C_s}. Recall that, along the BGP, $r_s(t) = \frac{\pi_s(t)}{P_s(t)}$ and note that, as shown in Appendix B.4, I can write profits as a function of relative prices of intermediate goods:

$$\pi_s(t) = (1-\alpha)\cdot\alpha\cdot\sum_{k\in\mathbf{K}}\underbrace{w_k(t)L_k}_{\text{value added in } k}\left[\sum_{n\in\mathbf{K}}\underbrace{M_n(t)}_{\text{product measure in } n}\left[\underbrace{\frac{\tau_{nk}P_n(t)}{\tau_{sk}P_s(t)}}_{\substack{\text{relative price}\\ \text{of the intermediate}\\ \text{good sold in } k}}\right]^{-\frac{\alpha}{1-\alpha}}\right]^{-1}$$

Define the excess variety growth function as:

$$\begin{aligned}
Z_s^M(\mathbf{M},\mathbf{w},t) &= \frac{r_s(t^*)}{P_s(t)} - \frac{1}{N}\sum_{s'\in\mathbf{K}}\frac{r_{s'}}{P_{s'}(t)} \\
&= (1-\alpha)\cdot\alpha\cdot\sum_{k\in\mathbf{K}} w_k(t)L_k\left(\left[\sum_{n\in\mathbf{K}} M_n(t)\cdot P_s(t)^2\cdot\left[\frac{\tau_{nk}P_n(t)}{\tau_{sk}P_s(t)}\right]^{-\frac{\alpha}{1-\alpha}}\right]^{-1}\right. \\
&\quad \left. - N\cdot\left[\sum_{n\in\mathbf{K}} M_n(t)\sum_{s'\in\mathbf{K}} P_{s'}(t)^2\cdot\left[\frac{\tau_{nk}P_n(t)}{\tau_{s'k}P_{s'}(t)}\right]^{-\frac{\alpha}{1-\alpha}}\right]^{-1}\right)
\end{aligned}$$

As mentioned in the description of Lemma (2), the function $Z_s^M(\mathbf{M},\mathbf{w},t)$ is a growth rate analog of the excess demand function. It relates the price of the final good $P_s(t)$ will the growth excess growth rate of varieties. If the measure of varieties is *above* its long-run equilibrium rate (relative to other countries), then it will have a price of final good *below* average and it will grow *slower* than average[24].

To show that this is a fixed point problem, it suffices to show that if a country s has a measure of varieties $M_s(t)$ *above* its long-run equilibrium rate (relative to other countries), then it will have a price of final good *below* average and it will grow *slower* than average (and vice versa). That is, it suffices to show that, $\frac{\partial P_s(t)}{\partial M_s(t)}$ for some relevant neighbourhood around the balanced growth path.

Let $\Gamma \equiv \Gamma\left(\frac{\theta+1-\sigma}{\theta}\right)^{\frac{1}{1-\sigma}} \cdot \alpha^{1-2\alpha}$. Then, per Appendix (B.5), $P_s(t)$ can be expressed as:

$$\begin{aligned} P_s(t) &= \Gamma\left(\frac{\theta+1-\sigma}{\theta}\right)^{\frac{1}{1-\sigma}} \cdot \left(\sum_{k\in\mathbf{K}} T_k(\tau_{kd} w_k(t)^{1-\alpha})^{-\theta} \tilde{M}_k(t)^{(1-\alpha)\theta}\right)^{-\frac{1}{\theta}} \\ &= \Gamma\left(\frac{\theta+1-\sigma}{\theta}\right)^{\frac{1}{1-\sigma}} \cdot \left(\sum_{k\in\mathbf{K}} T_k(\tau_{kd} w_k(t)^{1-\alpha})^{-\theta} \left[\frac{1}{\alpha}\sum_{n\in\mathbf{K}} M_n(t)\left(\frac{\tau_{nk}P_n(t)}{\alpha}\right)^{-\frac{\alpha}{1-\alpha}}\right]^{(1-\alpha)\theta}\right)^{-\frac{1}{\theta}} \\ &= \Gamma \cdot \left(\sum_{k\in\mathbf{K}} T_k(\tau_{kd} w_k(t)^{1-\alpha})^{-\theta} \left[\sum_{n\in\mathbf{K}} M_n(t)\left(\tau_{nk}P_n(t)\right)^{-\frac{\alpha}{1-\alpha}}\right]^{(1-\alpha)\theta}\right)^{-\frac{1}{\theta}} \end{aligned}$$

To evaluate the marginal impact of product measures on the price of the final good, calculate the derivative of this object with respect to $M_s(t)$:

$$\begin{aligned} \frac{\partial P_s(t)}{\partial M_s(t)} = & -\frac{1}{\theta} \cdot \Gamma^{\frac{1}{1-\theta}} \cdot P_s(t)^{-\frac{\theta}{1-\theta}} \sum_{k\in\mathbf{K}} T_k(\tau_{kd} w_k(t)^{1-\alpha})^{-\theta} \cdot \\ & \left(P_s(t)^{-\frac{\alpha}{1-\alpha}} - \frac{\alpha}{1-\alpha}\sum_{n\in\mathbf{K}} M_n(t)\left(\tau_{nk}P_n(t)\right)^{-\frac{\alpha}{1-\alpha}} \cdot \frac{1}{P_n(t)} \cdot \frac{\partial P_n(t)}{\partial M_s(t)}\right) \end{aligned}$$

Hence, the total marginal effect of $M_s(t)$ on $P_s(t)$ depends on a first order effect, which decreases the price, and a sum of second order effects, which may each be positive or negative. Therefore, to ensure that $\frac{\partial P_s(t)}{\partial M_s(t)} < 0$, one needs to place some restrictions on the parameter space to ensure that the second order effects do not dominate over the first order effect.

[24]Economically, this is relating the price of the of the intermediate good $\tau_{nk}P_n(t)/\alpha$ to the measure of varieties M_n, but since τ_{nk}, α are constants, due to the input output structure of the model, this reduces to a relationship between the price of the final good and the measure of varieties

Furthermore, since $Z_s^M(\mathbf{M}, \mathbf{w}, t)$ is strictly decreasing in the price of the final good, if $Z_s^M(\mathbf{M}, \mathbf{w}, t) = 0$ for some equilibrium, and the cross-sectional equilibria are unique for the relevant parameter space, then the BGP is unique.

□

Proof of Proposition (3)

Proof. Without loss of generality, choose an arbitrary country $s \in \mathbf{K}$. Using the Euler Equation, the definition of the BGP growth rate of consumption is:

$$\frac{\dot{C}_s(t^*)}{C_s(t^*)} \equiv \underbrace{g_{C_s}}_{\text{constant}} = \underbrace{\frac{r_s(t^*)}{P_s(t^*)}}_{\text{constant}} - \rho$$

Under the definition of a BGP, $g_{P_s} = 0$ in every $s \in \mathbf{K}$. Therefore, $r_s(t^*)$ is also constant along the BGP and $g_{r_s} = 0$. I can also write the law of motion for varieties in growth form:

$$\frac{\dot{M}_s(t^*)}{M_s(t^*)} \equiv \underbrace{g_{M_s}}_{\text{constant}} = \underbrace{\frac{r_s(t^*)}{P_s(t^*)}}_{\text{constant}} + \psi \underbrace{\left[\frac{w_s(t^*)L_s}{P_s(t^*)M_s(t^*)} - \frac{C_s(t^*)}{M_s(t^*)}\right]}_{\text{constant}} \tag{B.12}$$

I have already established that $r_s(t^*)$ is a constant using the Euler Equation. Since g_{M_s} grows at a constant rate at every period along the BGP, it must be the case that the term within the brackets is constant for every $t^* \geq t$.

There are only two possibilities. Either the term in the brackets is zero, implying that $g_{C_s} = g_{w_s}$ and $g_{M_s} = r_s(t^*) = g_{C_s} + \rho$. Or the term in the brackets can take any arbitrary finite value, as long as $g_{w_s} = g_{M_s}$ and $g_{C_s} = g_{M_s}$. Only the latter is consistent with a BGP.

First, I need to establish that all wages grow at the same rate. Recall that the labor market clearing condition is:

$$w_s(t^*)L_s = (1 - \alpha) \sum_{d \in \mathbf{K}} \lambda_{sd}(t^*) P_d(t^*) Y_d(t^*)$$

Taking logs and time derivatives, I can write this equation as:

$$\frac{\dot{w}_s(t^*)}{w_s(t^*)} = g_{w_s} = \sum_{d\in\mathbf{K}} \frac{\lambda_{sd}(t^*)P_d(t^*)Y_d(t^*)}{\sum_{k\in\mathbf{K}}\lambda_{kd}(t^*)P_k(t^*)Y_k(t^*)}\left(\frac{\dot{\lambda}_{sd}(t^*)}{\lambda_{sd}(t^*)}+\frac{\dot{P}_d(t^*)}{P_d(t^*)}+\frac{\dot{Y}_d(t^*)}{Y_d(t^*)}\right)$$
$$= \sum_{d\in\mathbf{K}} \gamma_{sd}(t^*)\left(\frac{\dot{\lambda}_{sd}(t^*)}{\lambda_{sd}(t^*)}+\frac{\dot{P}_d(t^*)}{P_d(t^*)}+\frac{\dot{Y}_d(t^*)}{Y_d(t^*)}\right)$$
$$= \sum_{d\in\mathbf{K}} \gamma_{sd}(t^*)\left(g_{\lambda_{sd}}+g_{P_d}+g_{Y_d}\right)$$

where $\gamma_{sd}(t^*) \equiv \frac{\lambda_{sd}(t^*)P_d(t^*)Y_d(t^*)}{\sum_{k\in\mathbf{K}}\lambda_{kd}(t^*)P_k(t^*)Y_k(t^*)}$. Since the law of one price holds, then, clearly, $g_{P_d} = 0$.

Since for all t, $\lambda_{sd}(t) \in (0,1)$, for an equilibrium to exist, it must be the case that $g_{\lambda_{sd}} = 0$. For a proof by a contradiction, suppose $g_{\lambda_{sd}} > 0$. Then $\lim_{t\to\infty}\lambda_{sd}(t) = \infty$, which is a contradiction. Conversely, suppose that $g_{\lambda_{sd}} < 0$ for some s. This implies that for some other s', $g_{\lambda_{s'd}} > 0$, which is a contradiction. Therefore, it must be the case that $g_{\lambda_{sd}} = 0$ for all $s, d \in \mathbf{K}$.

Therefore, the equation above reduces to:

$$g_{w_s} = \sum_{d\in\mathbf{K}} \gamma_{sd}(t^*) g_{Y_d}$$

The next step is showing that, since g_{Y_d}, g_{w_s} are all constant, the weights $\gamma_{sd}(t) = \gamma_{sd}$ must be fixed along the BGP and $(\forall s, d) g_{Y_d} = g_{Y_s} = g_{w_s} = g_{w_d}$. Suppose $g_{Y_{d'}} > g_{Y_d}$ for some d' compared to others d. In that case, $\gamma_{sd'}(t)$ will grow faster relative to other d. However, that implies that g_{w_s} is not a constant, which is a contradiction.

Since $g_{Y_s} = g_{Y_d} = g_Y$ for all $s, d \in \mathbf{K}$, the shares $\gamma_{sd}(t^*)$ are constant for all $t \geq t^*$ along the BGP, and the shares sum to one, it follows trivially that:

$$g_{w_s} = \underbrace{\sum_{d\in\mathbf{K}} \gamma_{sd}(t^*)}_{=1} g_Y = g_Y = g_{Y_s}$$

This establishes that the wages growth rates $g_{w_s} = g_{w_d} = g_{Y_s} = g_{Y_d}$ are equal in every country and also that they are equal to their respective aggregate output growth rates.

Given these identities, I can now explicitly solve for the growth rate.

Evaluate (1) replacing for the real interest rate using equations (8) and (6):

$$
\begin{aligned}
g_{C_s} &\equiv \frac{\dot{C}_s(t^*)}{C_s(t^*)} = \frac{r_s(t^*)}{P_s(t^*)} - \rho \\
&= \frac{\psi \pi_s(t^*)}{P_s(t^*)^2} - \rho \\
&= \frac{(1-\alpha)\psi}{P_s(t^*)^2} \cdot \sum_{k\in\mathbf{K}} \left(\frac{\tau_{sk}P_s(t^*)}{\alpha}\right)^{-\frac{\alpha}{1-\alpha}} \cdot \frac{w_k(t^*)L_k}{\tilde{M}_k(t^*)} - \rho \\
&= \frac{(1-\alpha)\psi}{P_s(t^*)^2} \cdot \sum_{k\in\mathbf{K}} \left(\frac{\tau_{sk}P_s(t^*)}{\alpha}\right)^{-\frac{\alpha}{1-\alpha}} \cdot \frac{w_k(t^*)L_k}{\frac{1}{\alpha}\sum_{n\in\mathbf{K}} M_n(t^*)\left(\frac{\tau_{nk}P_n(t^*)}{\alpha}\right)^{-\frac{\alpha}{1-\alpha}}} - \rho \\
&= \frac{(1-\alpha)\cdot\alpha\cdot\psi}{P_s(t^*)^2} \cdot \sum_{k\in\mathbf{K}} w_k(t^*)L_k \left[\sum_{n\in\mathbf{K}} M_n(t^*)\left(\frac{\tau_{nk}P_n(t^*)}{\tau_{sk}P_s(t^*)}\right)^{-\frac{\alpha}{1-\alpha}}\right]^{-1} - \rho
\end{aligned}
$$

for some constant g_{C_s}. This rate is only constant if $\sum_{k\in\mathbf{K}} w_k(t^*)L_k \left[\sum_{n\in\mathbf{K}} M_n(t^*)\left(\frac{\tau_{nk}P_n(t^*)}{\tau_{sk}P_s(t^*)}\right)^{-\frac{\alpha}{1-\alpha}}\right]^{-1}$ is constant. This requires that the global value added and global measure of varieties grow at the same rates. In turn, per condition (B.12), the product measures grow at constant rates in each country, which are either: (a) equal to wages $g_{M_s} = g_{w_s}$; or (b) equal to an affine transformation of wages $g_{M_s} = g_{C_s} + \rho = g_{w_s} + \rho$.

Furthermore, wages grow at equal rates in every country $s \in \mathbf{K}$ and prices are constant along the BGP. Hence, to satisfy a constant g_{C_s}, it follows that the product measure also grows at the same rate in every country. By the same token, to satisfy a constant g_{C_s}, it must be the case that $g_{w_s} = g_{M_s}$, which, in turn, would require that $g_{w_s} = g_{M_s} = g_{C_s} = g_{Y_s}$.

Finally, using the free-entry condition, $\dot{M}_s(t) = \psi I_s(t)$, which implies $\int_0^t \dot{M}_s(\tau)d\tau = \psi \int_0^t I_s(\tau)d\tau$ and $M_s(t) = \psi A_s(t)$. Taking logs and time derivatives of this shows:

$$
\frac{\dot{A}_s(t^*)}{A_s(t^*)} = g_{A_s} = g_{M_s} = \frac{\dot{M}_s(t^*)}{M_s(t^*)}
$$

Therefore, I have shown that:

$$
g^* \;=\; g_{M_s} = g_{Y_s} = g_{C_s} = g_{w_s} = g_{A_s} = \frac{(1-\alpha)\cdot\alpha\cdot\psi}{P_s(t^*)^2} \cdot \sum_{k\in\mathbf{K}} w_k(t^*)L_k \left[\sum_{n\in\mathbf{K}} M_n(t^*)\left(\frac{\tau_{nk}P_n(t^*)}{\tau_{sk}P_s(t^*)}\right)^{-\frac{\alpha}{1-\alpha}}\right]^{-1}
$$

Since the choice of s was arbitrary, this holds for any $s \in \mathbf{K}$. The BGP is characterized by a vector of $[w_s(t^*)]_{s\in\mathbf{K}}$ and a vector of measures of varieties $\lambda_M \cdot [M_s(t^*)]_{s\in\mathbf{K}}$ that, given parameters, pin down the growth rate g^*.

By Lemma (3), the BGP exists. Furthermore, if there exists a equilibrium **within a subset of the parameter space** that guarantee that $\frac{\partial P_s(t)}{\partial M_s(t)} < 0$ and the cross-sectional equilibrium is unique, **then the BGP growth rate is unique**.

□

Changes in trade costs

Proof of Proposition 4

Proof. The equilibrium growth rate is:

$$g^* = \frac{(1-\alpha)\cdot\alpha}{P_s(t^*)^2} \cdot \sum_{k\in\mathbf{K}} w_k(t^*) L_k \left[\sum_{n\in\mathbf{K}} M_n(t^*) \left(\frac{\tau_{nk} P_n(t^*)}{\tau_{sk} P_s(t^*)}\right)^{-\frac{\alpha}{1-\alpha}}\right]^{-1} - \rho$$

Since these economies are symmetric, then: $P_s(t^*) = P_{s'}(t^*)$, $w_s(t^*) = w_{s'}(t^*)$, $M_s(t^*) = M_{s'}(t^*)$ for every s, s' and $\tau_{sd} = \tau$ for every sd. Therefore, denoting $P_s(t^*) = P(t^*)$, $w_s(t^*) = P(t^*)$, $M_s(t^*) = P(t^*)$, $L_s = L$ the expression above reduces to:

$$g^* \quad = \quad \frac{(1-\alpha)\cdot\alpha}{P(t^*)^2} \cdot \frac{w(t^*)L}{M(t^*)} - \rho$$

According to Proposition 3, $w(t^*), P(t^*)$ are only defined up to a scalars that pin down their respective distributions (in this case, due to symmetry, they will be degenerate distributions). Therefore, without loss of generality, set $w(t^*) = 1$ and $M(t^*) = 1$. This reduces the growth rate to:

$$g^* \quad = \quad \frac{(1-\alpha)\cdot\alpha}{P(t^*)^2} \cdot L - \rho$$

so the proof hinges on showing whether or not $P(t^*)$ is increasing in τ. In equation (B.5), I have shown price levels can be expressed as:

$$P_d(t) = \gamma \cdot \left(\sum_{s \in \mathbf{K}} T_s (\tau_{sd} w_s(t)^{1-\alpha})^{-\theta} \left(\frac{1}{\alpha} \sum_{n \in \mathbf{K}} M_n(t) \left[\frac{\tau_{ns} P_n(t)}{\alpha} \right]^{-\frac{\alpha}{1-\alpha}} \right)^{(1-\alpha)\theta} \right)^{-\frac{1}{\theta}}$$

where $\gamma \equiv \Gamma\left(\frac{\theta+1-\sigma}{\theta}\right)^{\frac{1}{1-\sigma}}$. Imposing symmetry, this reduces to:

$$\begin{aligned} P(t^*) &= \gamma \cdot \left(T w(t^*)^{-(1-\alpha)\theta} \left(1 + (N-1)\tau^{-\theta}\right) N^{(1-\alpha)\theta} \left(\frac{1}{\alpha} M(t^*) \left[\frac{\tau P(t^*)}{\alpha} \right]^{-\frac{\alpha}{1-\alpha}} \right)^{(1-\alpha)\theta} \right)^{-\frac{1}{\theta}} \\ P(t^*)^{1-\alpha} &= \gamma \cdot \left(T w(t^*)^{-(1-\alpha)\theta} \left(1 + (N-1)\tau^{-\theta}\right) N^{(1-\alpha)\theta} \cdot \alpha^{\alpha-(1-\alpha)\theta} \cdot M(t^*)^{(1-\alpha)\theta} \cdot \tau^{-\alpha\theta} \right)^{-\frac{1}{\theta}} \end{aligned}$$

Finally, used the normalization $w(t^*) = 1$ and $M(t^*) = 1$, this reduces to:

$$P(t^*)^{1-\alpha} = \gamma \cdot \left(T \left(1 + (N-1)\tau^{-\theta}\right) \tau^{-\alpha\theta} N^{(1-\alpha)\theta} \cdot \alpha^{\alpha-(1-\alpha)\theta} \right)^{-\frac{1}{\theta}}$$

which shows that $\frac{\partial P(t^*)}{\partial \tau} > 0$.

Since $\frac{\partial P(t^*)}{\partial \tau} > 0$ and g^* is inversely related to the price level, $\frac{\partial g^*}{\partial \tau} > 0$. This completes the proof. □

Proof of Proposition 5

Proof. First write the growth rate at the steady state:

$$g^* = \frac{(1-\alpha) \cdot \alpha}{P_s(t^*)^2} \cdot \sum_{k \in \mathbf{K}} w_k(t^*) L_k \left[\sum_{n \in \mathbf{K}} M_n(t^*) \left(\frac{\tau_{nk} P_n(t^*)}{\tau_{sk} P_s(t^*)} \right)^{-\frac{\alpha}{1-\alpha}} \right]^{-1} - \rho$$

Denote $\Xi \equiv \frac{\alpha^2}{P_s(t^*)^2} \cdot w_d(t^*) L_d \left[\sum_{n \in \mathbf{K}} M_n(t^*) \left(\frac{\tau_{nd} P_n(t^*)}{\tau_{sd} P_s(t^*)} \right)^{-\frac{\alpha}{1-\alpha}} \right]^{-1}$. Now calculate:

$$\left.\frac{\partial g_s}{\partial \tau_{sd}}\right|_{\mathbf{P}=\mathbf{P}(t^*),\mathbf{w}=\mathbf{w}(t^*),\mathbf{M}=\mathbf{M}(t^*)} = -\Xi \cdot \left[\sum_{n\in\mathbf{K}} M_n(t^*)\left(\frac{\tau_{nd}P_n(t^*)}{\tau_{sd}P_s(t^*)}\right)^{-\frac{\alpha}{1-\alpha}}\right]^{-1} \cdot \sum_{n\in\mathbf{K}} M_n(t^*)\left(\frac{\tau_{nd}P_n(t^*)}{\tau_{sd}P_s(t^*)}\right)^{-\frac{\alpha}{1-\alpha}} \cdot \frac{1}{\tau_{sd}} < 0$$

This proves the first part of the proposition. Now calculate the full effect:

$$\begin{aligned}
\frac{\partial g_s}{\partial \tau_{sd}} &= -\Xi \cdot \left[\sum_{k\in\mathbf{K}} M_k(t^*)\left(\frac{\tau_{kd}P_k(t^*)}{\tau_{sd}P_s(t^*)}\right)^{-\frac{\alpha}{1-\alpha}}\right]^{-1} \cdot \sum_{n\in\mathbf{K}} \Bigg\{ M_n(t^*)\left(\frac{\tau_{nd}P_n(t^*)}{\tau_{sd}P_s(t^*)}\right)^{-\frac{\alpha}{1-\alpha}} \\
&\cdot \frac{1}{\tau_{sd}}\left[1+\frac{\tau_{sd}}{P_s(t^*)}\frac{\partial P_s(t^*)}{\partial \tau_{sd}}+\frac{\tau_{sd}}{P_n(t^*)}\frac{\partial P_n(t^*)}{\partial \tau_{sd}}\right]\Bigg\} \\
&+ \frac{(g_s+\rho)}{\tau_{sd}}\sum_{k\in\mathbf{K}}\frac{\tau_{sd}}{w_k(t^*)}\frac{\partial w_k(t^*)}{\partial \tau_{sd}} - \frac{1}{2}\frac{(g_s+\rho)}{\tau_{sd}}\frac{\tau_{sd}}{P_s(t^*)}\frac{\partial P_s(t^*)}{\partial \tau_{sd}} \\
&- \frac{(g_s+\rho)}{\tau_{sd}}\left[\sum_{k\in\mathbf{K}} M_k(t^*)\left(\frac{\tau_{kd}P_k(t^*)}{\tau_{sd}P_s(t^*)}\right)^{-\frac{\alpha}{1-\alpha}}\right]^{-1}\left(\sum_{n\in\mathbf{K}}\frac{\tau_{sd}}{M_n(t^*)}\frac{\partial M_n(t^*)}{\partial \tau_{sd}}M_n(t^*)\left(\frac{\tau_{nd}P_n(t^*)}{\tau_{sd}P_s(t^*)}\right)^{-\frac{\alpha}{1-\alpha}}\right)
\end{aligned}$$

Define $\epsilon^X_{s,sd} \equiv \frac{\partial X_s(t^*)}{\partial \tau_{sd}} \cdot \frac{\tau_{sd}}{X_s(t^*)}$ as the cross-elasticity of variable X with respect to a change in trade cost τ_{ns}. Denote $\mu_n(t) \equiv \frac{M_n(t^*)\left(\frac{\tau_{nd}P_n(t^*)}{\tau_{sd}P_s(t^*)}\right)^{-\frac{\alpha}{1-\alpha}}}{\sum_{k\in\mathbf{K}} M_k(t^*)\left(\frac{\tau_{kd}P_k(t^*)}{\tau_{sd}P_s(t^*)}\right)^{-\frac{\alpha}{1-\alpha}}}$. Then I can express the equation above as:

$$\frac{\partial g_s}{\partial \tau_{sd}} = -\frac{\Xi}{\tau_{sd}} \cdot \sum_{n\in\mathbf{K}} \mu_n(t^*)\left[1+\epsilon^P_{s,sd}-\epsilon^P_{n,sd}\right] + \frac{(g_s+\rho)}{\tau_{sd}}\left(\sum_{k\in\mathbf{K}}\left[\epsilon^w_{k,sd}-\mu_k(t^*)\epsilon^M_{k,sd}\right]-\frac{1}{2}\epsilon^P_{s,sd}\right)$$

□

B.9 Welfare

Recall that $C_s(t^*)$ can be expressed as a constant fraction of total lifetime wealth:

$$C_s(t^*) = \rho \left[A_s(t^*) + \int_{t^*}^{\infty} \frac{w_s(\tau)}{P_s(\tau)} L_s \cdot \exp\left\{ -\bar{r}_s(\tau) \cdot \tau \right\} d\tau \right]$$

where $\bar{r}_s = \frac{1}{\tau} \int_{t^*}^{\tau} r_s(t) dt$ is the average interest rate between t^* and τ. Since this holds along the BGP, $\frac{w_s(\tau)}{P_s(\tau)} = \frac{\exp\{(\tau - t^*) g_{w_s}\} w_s(t^*)}{P_s(t^*)}$. Furthermore, since $\frac{r_s(t^*)}{P_s(t^*)}$ is constant along the BGP, $\bar{r}_s(\tau) = \frac{r_s(t^*)}{P_s(t^*)}$ for all $\tau \geq t^*$. Replacing those above results in:

$$\begin{aligned}
C_s(t^*) &= \rho \left[A_s(t^*) + \frac{w_s(t^*)}{P_s(t^*)} L_s \int_{t^*}^{\infty} \cdot \exp\left\{ - \left(\frac{r_s(t^*)}{P_s(t^*)} - g_{w_s} \right) \cdot (\tau - t^*) \right\} d\tau \right] \\
&= \rho \left[A_s(t^*) + \frac{w_s(t^*)}{P_s(t^*)} \frac{L_s}{\frac{r_s(t^*)}{P_s(t^*)} - g_{w_s}} \right] \\
&= \rho A_s(t^*) + \rho \frac{w_s(t^*)}{P_s(t^*)} \frac{L_s}{\frac{r_s(t^*)}{P_s(t^*)} - g_{w_s}}
\end{aligned}$$

Since $g_{w_s} = g_{C_s}$ and $g_{C_s} = \frac{r_s(t^*)}{P_s(t^*)} - \rho$, $\frac{r_s(t^*)}{P_s(t^*)} - g_{w_s} = \rho$. Hence, over the BGP, real consumption is a fraction of assets plus real labor income:

$$C_s(t^*) = \rho A_s(t^*) + \frac{w_s(t^*) L_s}{P_s(t^*)}$$

Welfare over the BGP is:

$$\begin{aligned}
\int_{t^*}^{\infty} \exp\{-\rho(t - t^*)\} \log\left(\exp\{g^* t\} C_s(t^*) \right) dt &= \int_{t^*}^{\infty} \exp\{-\rho(t - t^*)\} \log\left(C_s(t^*) \right) dt \\
&+ \int_{t^*}^{\infty} \exp\{-\rho(t - t^*)\} g^* t dt \\
&= \frac{\log\left(C_s(t^*) \right)}{\rho} + \frac{g^*}{\rho^2} \\
&= \log\left(A_s(t^*) \right) + \frac{1}{\rho} \log\left(\frac{w_s(t^*) L_s}{P_s(t^*)} \right) + \frac{g^*}{\rho^2}
\end{aligned}$$

Finally, using the fact that $\psi A_s(t^*) = M_s(t^*)$, I can write:

$$\int_{t^*}^{\infty} \exp\{-\rho(t-t^*)\} \log\left(\exp\{g^* t\} C_s(t^*)\right) dt = \log\left(\frac{1}{\psi} M_s(t^*)\right) + \frac{1}{\rho}\log\left(\frac{w_s(t^*)L_s}{P_s(t^*)}\right) + \frac{g^*}{\rho^2}$$

Static welfare For real labor income, start from equation (5) evaluated at $s = d$ and use the fact that, as shown in equation (B.5) of Appendix B.5,

$$P_s(t) = \gamma \cdot \left[\sum_{n\in\mathbf{K}} T_n \left(\tilde{M}_n(t)^{1-\alpha}\right)^{\theta} \left(w_n(t)^{1-\alpha}\tau_{nd}\right)^{-\theta}\right]^{-\frac{1}{\theta}}$$

where $\gamma \equiv \Gamma\left(\frac{\theta+1-\sigma}{\theta}\right)^{\frac{1}{1-\sigma}}$. Then, own trade share in a given country can be represented by:

$$\lambda^F_{dd}(t) = \gamma^{\theta} \cdot \frac{T_d\left(\tilde{M}_d(t)^{1-\alpha}\right)^{\theta}\left(w_d(t)^{1-\alpha}\right)^{-\theta}}{[P_d(t)]^{-\theta}}$$

Replacing $\tilde{M}_d(t) \equiv \frac{1}{\alpha} \cdot \sum_{k\in\mathbf{K}} M_k(t) \cdot \left(\frac{\tau_{kd}P_k(t)}{\alpha}\right)^{-\frac{\alpha}{1-\alpha}}$ delivers:

$$\begin{aligned}
\lambda^F_{dd}(t) &= \gamma^{\theta} \cdot T_d \cdot P_d(t)^{\theta} \cdot w_d(t)^{-(1-\alpha)\theta} \cdot \left(\frac{1}{\alpha} \cdot \sum_{k\in\mathbf{K}} M_k(t) \cdot \left(\frac{\tau_{kd}P_k(t)}{\alpha}\right)^{-\frac{\alpha}{1-\alpha}}\right)^{(1-\alpha)\theta} \\
&= \alpha^{-(1-2\alpha)\theta} \cdot \gamma^{\theta} \cdot T_d \cdot \left(\frac{w_d(t)}{P_d(t)}\right)^{-(1-\alpha)\theta} \cdot \left(\sum_{k\in\mathbf{K}} M_k(t) \cdot \left(\frac{\tau_{kd}P_k(t)}{\tau_{dd}P_d(t)}\right)^{-\frac{\alpha}{1-\alpha}}\right)^{(1-\alpha)\theta}
\end{aligned}$$

which means we can write real wages as:

$$\frac{w_d(t)}{P_d(t)} = \alpha^{-(1-2\alpha)\theta} \cdot \gamma^{\frac{1}{1-\alpha}} \cdot T_d^{\frac{1}{(1-\alpha)\theta}} \cdot \left(\lambda^F_{dd}(t)\right)^{-\frac{1}{(1-\alpha)\theta}} \cdot \sum_{k\in\mathbf{K}} M_k(t) \cdot \left(\frac{\tau_{kd}P_k(t)}{\tau_{dd}P_d(t)}\right)^{-\frac{\alpha}{1-\alpha}}$$

Consider what happens to welfare after a change in trade costs from $\boldsymbol{\tau}$ to $\boldsymbol{\tau} + d\boldsymbol{\tau}$, as in Arkolakis et al. (2012). In this dynamic setting, to compare the static component of welfare, I need to compare what happens across the two BGPs, comparing the two initial equilibria. Suppose t^* is the initial period of the original BGP while t^** is the first period of the final BGP. To fit this framework to the general trade literature, I will compare the static component of these BGP as if they happened in the same period, and compound the difference over time.

Let $\hat{x} \equiv x(t^{*}*)/x(t^{*})$. Then cumulative changes in static welfare are:

$$\frac{1}{\rho}\log\left(\widehat{\frac{w_s(t^{**})}{P_s(t^{**})}}\right) = \frac{1}{\rho}\log\left(\widehat{\lambda_{dd}^{F}}(t^{**})^{-\frac{1}{(1-\alpha)\theta}}\right) + \frac{1}{\rho}\log\left(\sum_{k\in\mathbf{K}}\mu_k(t^{*})\widehat{M}_k(t^{**})\cdot\left(\frac{\widehat{\tau}_{kd}\widehat{P}_k(t^{**})}{\widehat{P}_d(t^{**})}\right)^{-\frac{\alpha}{1-\alpha}}\right)$$

where $\mu_k(t) \equiv \frac{M_k(t)\cdot\left(\frac{\tau_{kd}P_k(t)}{\tau_{dd}P_d(t)}\right)^{-\frac{\alpha}{1-\alpha}}}{\sum_{k\in\mathbf{K}} M_k(t)\cdot\left(\frac{\tau_{kd}P_k(t)}{\tau_{dd}P_d(t)}\right)^{-\frac{\alpha}{1-\alpha}}}$

B.10 Nesting of Romer and Eaton-Kortum

In this subsection, I will briefly describe how to recover the canonical P. M. Romer (1990) and Eaton and Kortum (2002) models from the framework described above.

Eaton-Kortum Setting $\alpha = 0$ implies that the value of new varieties is zero since the demand for and profits of varieties is also zero. Therefore, $I_s(t) = 0$ and $A_s(t) = 0$ for all t and s. While the Eaton-Kortum model is a static model, here it can be thought of as an infinite sequence of static models with no intertemporal decision, since there are no longer asset markets that permit households to save:

$$\begin{aligned}
\max_{C_s(t),c_s(t,\omega)_{\omega\in[0,1]}} &\quad \int_0^\infty \exp\{-\rho t\}\log\left(C_s(t)\right)dt \\
s.t.\quad P_s(t)C_s(t) &= w_s(t)L_s \\
C_s(t) &= \left[\int_0^1 c_s(t,\omega)^{\frac{\sigma-1}{\sigma}}d\omega\right]^{\frac{\sigma}{\sigma-1}} \\
P_s(t)C_s(t) &= \int_0^1 p_s(t,\omega)c_s(t,\omega)d\omega
\end{aligned}$$

Furthermore, since $\alpha = 0$, the intermediate and research and development sectors disappear. The problem of the final goods producer becomes:

$$\max_{\ell_s(t,\omega)} p_{ss}(t,\omega)\cdot z_s(t,\omega)\cdot\ell_s(t,\omega) - \ell_s(t,\omega)w_s(t)$$

which is identical to the one in the standard Eaton-Kortum model. Equilibrium will take the form of a system of labor market determination equations that solve for N wages using trade expenditure shares.

Romer Setting $\tau_{sd} \to \infty$ for $s \neq d$ implies trade costs are prohibitively high internationally, such that varieties of both final goods and intermediate goods become sold only locally. Normalizing the price of the domestic final good to be the numéraire in each country, I write the dynamic household problem as:

$$\begin{aligned} \max_{C_s(t), c_s(t,\omega)_{\omega\in[0,1]}} & \int_0^\infty \exp\{-\rho t\} \log\left(C_s(t)\right) dt \\ s.t. \quad I_s(t) = \dot{A}(t) &= r_s(t)A_s(t) + w_s(t)L_s - C_s(t) \\ C_s(t) &= \left[\int_0^1 c_s(t,\omega)^{\frac{\sigma-1}{\sigma}} d\omega\right]^{\frac{\sigma}{\sigma-1}} \end{aligned}$$

Furthermore, redefine assumption (2) in the following terms:

Assumption 5 (Productivity draws to recover Romer)**.** *To recover the Romer model as a special case of the general model, I need to specify productivity terms $z_s(\omega)$ which are homogeneous across firms in each country. In order to do so, redefine the cumulative distribution function $F_s(t)(z)$ of the baseline case to be one of a degenerate random variable with a point mass concentrated at a certain scalar for each country. Formally:*

$$F_s(t)(z) = \begin{cases} 0 \text{ for } z < T_s \\ 1 \text{ for } z \geq T_s \end{cases}$$

Using the symmetry assumption above, the numéraire normalization and the unavailability of foreign intermediate goods in the domestic market, the final goods assembler technology becomes:

$$y_s(t,\omega) = T_s[\ell_s(t,\omega)]^{1-\alpha} \left(\frac{1}{\alpha}\int_0^{M_s(t)} [x_{ss}(t,\omega,\nu)]^\alpha d\nu\right)$$

which is identical to the single-country Romer model. Profits and demand per variety $\nu \in [0, M_s(t)]$ will be constant and growth will be driven by the domestic R&D sector. Equilibrium will take the following form: labor markets will clear; total final goods produced being equal to total final goods used for consumption, intermediate production; and R&D production; and optimized household optimal dynamics will be described by an Euler equation and an asset/measure accumulation equation.

C Qualitative Questionnaire

1. After your country joined the European Union, did your company:
   - start producing more products/services or varieties;
   - start producing fewer products/services or varieties; or
   - keep producing about the same number of products/services or varieties?
2. If your company changed the number of products/services or varieties after EU accession, how was the change implemented and what were the results? Please include any important information or relevant anecdotes.
3. If your company changed the number of products/services or product/service varieties after EU accession, was the decision primarily motivated by access to new technologies/imports, access to new markets/exports, or both? Explain.
4. After your country joined the European Union, did your company:
   - stay in the same industry;
   - expanded to another industry; or
   - move completely to a new industry?
5. If your company expanded to another industry or moved to a new industry. Please explain whether the change was related to your country's EU accession.

D Data Appendix

E Computational Appendix

This computation appendix explains how I solve for the BGP growth rate.

E.1 Description of Algorithms

1. **Inner loop (Prices of Final Goods)**. Given parameters $\{\theta, \psi, \alpha, \mathbf{L}, \mathbf{T}, \boldsymbol{\tau}\}$ and guesses for wages $\mathbf{w}$ a measures of varieties $\mathbf{M}$, I use the input-output structure of the model to solve for the prices of the final goods. Note that I can write the price of final goods as:

$$\begin{aligned} P_s(t) &= \gamma \cdot \sum_{n \in \mathbf{K}} \left(w_n(t)^{1-\alpha} \tau_{ns} \right)^{-\theta} \left(\tilde{M}_n(t) \right)^{\theta(1-\alpha)} \\ P_s(t) &= \gamma \cdot \sum_{n \in \mathbf{K}} \left(w_n(t)^{1-\alpha} \tau_{ns} \right)^{-\theta} \left(\frac{1}{\alpha} \sum_{k \in \mathbf{K}} M_k(t) \left(\frac{\tau_{kn} P_k(t)}{\alpha} \right)^{-\frac{\alpha}{1-\alpha}} \right)^{\theta(1-\alpha)} \end{aligned}$$

   with $\gamma \equiv \Gamma \left(\frac{\theta+1-\sigma}{\theta} \right)^{\frac{1}{1-\sigma}}$. The last equation makes it explicit that, given parameters, wages, and measures of varieties, this is a system of $|N|$ equations and $|N|$ unknowns in final goods prices. A simple grid search algorithm finds a fixed point for final goods prices.

2. **Intermediate loop (Alvarez-Lucas Algorithm)**. Given parameters $\{\theta, \psi, \alpha, \mathbf{L}, \mathbf{T}, \boldsymbol{\tau}\}$ and guesses the measures of varieties $\mathbf{M}$, I use a variant of the F. Alvarez and Lucas (2007) algorithm to find the solution to the cross-sectional equilibrium (see Definition 2).

   Define the excess demand function as:

$$\begin{aligned} Z_s(\mathbf{M}, \mathbf{w}, t) &\equiv \frac{1}{w_s(t)} \left(\sum_{d \in \mathbf{K}} \lambda^F_{sd}(t) P_d(t) Y_d(t) - w_s L_s \right) \\ &\equiv \frac{1}{w_s(t)} \sum_{d \in \mathbf{K}} \lambda^F_{sd}(t) P_d(t) \left(\frac{1}{\psi} \frac{r_d(t)}{P_d(t)} M_d(t) + \frac{w_d(t)}{P_d(t)} \cdot L_d \right. \\ &+ \left. M_d(t) \cdot \sum_{k \in \mathbf{K}} \left[\frac{\tau_{dk} P_d(t)}{\alpha} \right]^{-\frac{1}{1-\alpha}} \cdot \frac{w_k(t)}{\tilde{M}_k(t)} \cdot L_k - w_s L_s \right) \end{aligned}$$

   Then, define the operator:

$$O_s(\mathbf{M},\mathbf{w},t) = w_s\left(1+\kappa\cdot\frac{Z_s(\mathbf{M},\mathbf{w},t)}{L_s}\right)$$

for some $\kappa \in (0,1)$. To guarantee that $O_s(\mathbf{M},\mathbf{w},t)$ is an operator, in every iteration I always normalize wages such that such as $\sum_{s\in\mathbf{K}} w_s L_s = 1$, such that $O_s(\mathbf{M},\mathbf{w},t)$ maps onto itself. Then, as shown by F. Alvarez and Lucas (2007), by the contraction mapping theorem, the algorithm will converge to a (conditional) solution, given the guess of the measure of varieties $\mathbf{M}$.

3. **Outer loop (Growth rates)**. Given parameters $\{\theta,\psi,\alpha,\mathbf{L},\mathbf{T},\boldsymbol{\tau}\}$, I use the fact that along the BGP the real interest rate must equalize to find convergence and update the measures of varieties. Given wages and guesses of $\mathbf{M}$, I can calculate real interest rates and update measures of varieties. Those with real measures too high will have real returns below average (and vice versa).

   Define the excess measure function:

$$\begin{aligned}
Z_s^M(\mathbf{M},\mathbf{w},t) &= \frac{r_s(t^*)}{P_s(t)} - \frac{1}{N}\sum_{s'\in\mathbf{K}}\frac{r_{s'}}{P_{s'}(t)} \\
&= (1-\alpha)\cdot\alpha\cdot\sum_{k\in\mathbf{K}} w_k(t)L_k\left(\left[\sum_{n\in\mathbf{K}} M_n(t)\cdot P_s(t)^2\cdot\left[\frac{\tau_{nk}P_n(t)}{\tau_{sk}P_s(t)}\right]^{-\frac{\alpha}{1-\alpha}}\right]^{-1}\right. \\
&- \left. N\cdot\left[\sum_{n\in\mathbf{K}} M_n(t)\sum_{s'\in\mathbf{K}} P_{s'}(t)^2\cdot\left[\frac{\tau_{nk}P_n(t)}{\tau_{s'k}P_{s'}(t)}\right]^{-\frac{\alpha}{1-\alpha}}\right]^{-1}\right)
\end{aligned}$$

   which we can calculate given the parameters $\{\theta,\psi,\alpha,\mathbf{L},\mathbf{T},\boldsymbol{\tau}\}$, prices $P_s(t)$ (calculated in the inner loop), and wages $w_s(t)$ (calculated in the intermediate loop), and the current guess for $\mathbf{M}$. To update $\mathbf{M}$, I use the fact that, over the BGP, the law of motion for measures of varieties satisfies:

$$\frac{\dot{M}_s(t)}{M_s(t)} = \frac{r_s(t)}{P_s(t)} - \rho$$

   I follow in steps in a convex combination between the old and the new guess until all of the the real interest rates are numerically close enough to a predetermined tolerance level.

E.2 Calibration of Trade Shares

I use observed trade flows to infer trade costs. The strategy goes back to Head and Ries (2001). According to the handbook chapter by Head and Mayer (2014), the index is called the Head-Ries Index (HRI) since 2011 (when the working paper version of Eaton et al. (2016) was published).

Expenditure in final goods is defined as:

$$E_{sd}^F(t) = \lambda_{sd}^F(t)P_d(t)Y_d(t) = \frac{T_s\left(\tilde{M}_s(t)^{1-\alpha}\right)^{\theta}\left(w_s(t)^{1-\alpha}\tau_{sd}\right)^{-\theta}}{\sum_{n=1}^{N} T_n\left(\tilde{M}_n(t)^{1-\alpha}\right)^{\theta}\left(w_n(t)^{1-\alpha}\tau_{nd}\right)^{-\theta}} \cdot P_d(t)Y_d(t)$$

The ratio between $E_{sd}^F(t)$ and $E_{dd}^F(t)$ is, then:

$$\frac{E_{sd}^F(t)}{E_{dd}^F(t)} = \frac{T_s\left(\tilde{M}_s(t)^{1-\alpha}\right)^{\theta}\left(w_s(t)^{1-\alpha}\tau_{sd}\right)^{-\theta}}{T_d\left(\tilde{M}_d(t)^{1-\alpha}\right)^{\theta}\left(w_d(t)^{1-\alpha}\tau_{dd}\right)^{-\theta}}$$

Analogously, the ratio between $E_{ds}^F(t)$ and $E_{ss}^F(t)$ is:

$$\frac{E_{ds}^F(t)}{E_{ss}^F(t)} = \frac{T_s\left(\tilde{M}_d(t)^{1-\alpha}\right)^{\theta}\left(w_d(t)^{1-\alpha}\tau_{ds}\right)^{-\theta}}{T_d\left(\tilde{M}_s(t)^{1-\alpha}\right)^{\theta}\left(w_s(t)^{1-\alpha}\tau_{ss}\right)^{-\theta}}$$

Therefore:

$$\frac{E_{sd}^F(t)}{E_{dd}^F(t)} \cdot \frac{E_{ds}^F(t)}{E_{ss}^F(t)} = \left(\frac{\tau_{sd}\tau_{ds}}{\tau_{ss}\tau_{dd}}\right)^{-(1-\alpha)\theta}$$

Using Assumption (1), $\tau_{ss} = \tau_{dd} = 1$ and $\tau_{sd} = \tau_{ds}$. Hence, I can express the trade cost τ_{sd} as:

$$\tau_{sd} = \left(\frac{E_{sd}^F(t)}{E_{dd}^F(t)} \cdot \frac{E_{ds}^F(t)}{E_{ss}^F(t)}\right)^{-\frac{1}{2\theta(1-\alpha)}} \tag{B.13}$$