International Trade: Lecture 15

Firms and Trade: The Krugman Model

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So far...

- We have seen the versions of the “standard trade model” (or neoclassical trade model)
- What they share in common is that they feature CRS (in all factors combined) and prefect competition
- In that framework, trade happens because we countries different (in tech or endowments)
- If countries were identical in every way $\rightarrow$ no change in prices $\rightarrow$ no trade

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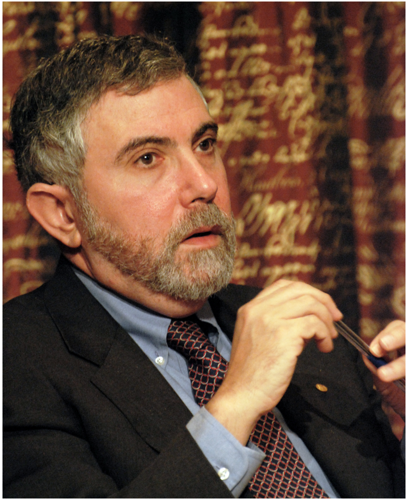
- Producers have a monopoly over their goods: only France can produce Champagne
- But consumers can substitute across goods...
- ... we call this monopolistic competition

Paul Krugman (1953–)

- You probably know Paul Krugman...
- Born: 28 Feb 1953, Albany (NY), USA
- Professor at MIT, Princeton, and now CUNY; pioneer of *New Trade Theory* and *New Economic Geography*
- *New York Times* op-ed columnist (since 2000); influential voice on fiscal, trade & inequality debates
- John Bates Clark Medal (1991); Nobel Prize in Economics (2008)



Paul Krugman (Wikipedia)

The Krugman Model

- Intra-industry Trade Model: trade in goods within the same industry
- Trade because of differentiated goods
- Firms have a monopoly of the type of good φ they produce
- Upon entry, firms maximize operational profits (Stage 2).
- However, free entry erodes profits to zero in equilibrium (Stage 1)

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- Mobility of factors of production: yes
- Number of sectors: one
- Number of goods: (countably) many
- Imperfect competition (transport costs 0)
- Key force: internal increasing returns to scale

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- How do they decide how to substitute across goods depending on the price?
- Elasticity of substitution:

$$\sigma_{ij} \equiv \frac{d \ln(q_i / q_j)}{d \ln(p_i / p_j)} = \frac{\text{\% change in relative quantity demanded}}{\text{\% change in relative price}}$$

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- What does that mean?
  - Suppose relative price of computers increases $\uparrow$ by 1%
  - Then consumers shift relative quantities of computers purchased down $\downarrow$ by 1%
  - Quantities adjust inversely to prices at a one to one ratio
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- Can you substitute away from gasoline? (yes, but difficult on the short run)
- Much easier to substitute Lay's for Ruffles.

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- Key parameter: $\sigma > 1$ is the elasticity of substitution (same for every good)
- Larger σ is:
  - high σ: more responsive to changes in prices (think different brands of bottled water)
  - low σ: less responsive to changes in prices (think steak vs tofu)

Demand under constant elasticity of substitution

- Consumers maximize:

$$\max_{\{q_i(\varphi)\}_{\varphi\in\Phi}} \quad Q_i \equiv \left[q_i(1)^{\frac{\sigma-1}{\sigma}} + \cdots + q_i(N)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

$$s.t. \qquad P_iQ_i = p_i(1)q_i(1) + \cdots + p_i(N)q_i(N) \leq I_i = w_iL_i$$

(don't get scared with the math, I want to you get the intuition)

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- Q_i: *composite consumption basked* defined as an aggregate of all goods
- P_i: "price" of the composite consumption basket (implicitly defined)
- Total expenditure across all goods = cost of the consumption basket $P_i Q_i$

Demand functions

After some algebra, we can solve for demand functions:

$$q_i(\varphi) = \underbrace{\left(\frac{p_i(\varphi)}{P_i}\right)^{-\sigma}}_{\text{relative price}} \times \underbrace{\frac{I_i}{P_i}}_{\text{real income}}$$

(check handout for step by step derivation)

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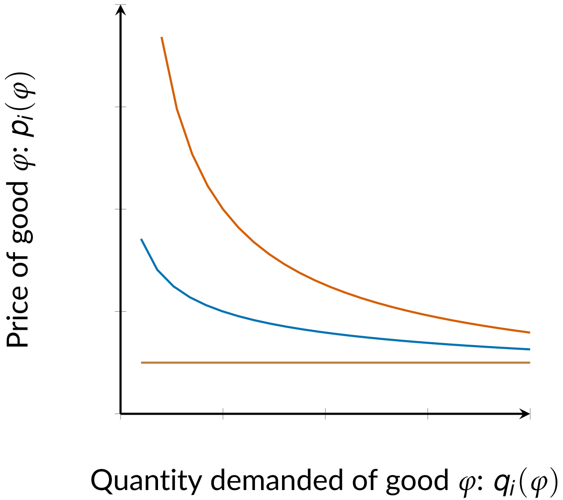


Figure: Demand curve with different elasticities: $\sigma = 1.5$, $\sigma = 3$, $\sigma = \infty$

Production

- Producers of each good φ have a monopoly over the production of their good

- They also will have to pay a fixed cost $\bar{f}$ to set up shop (only if they enter the market)

- To produce a given quantity $q_i(\varphi)$, labor used is:

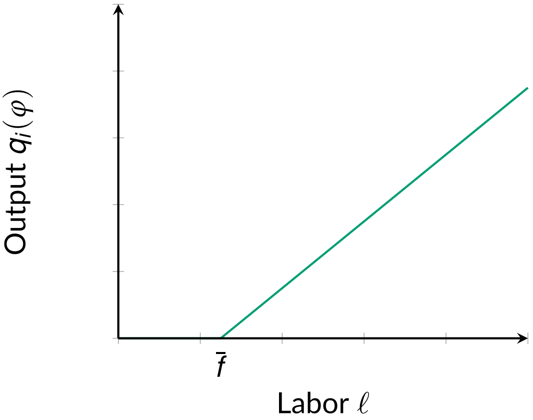
$$\ell = \bar{f} + a^* q_i(\varphi) \iff q_i(\varphi) = \frac{1}{a^*}(\ell - \bar{f})$$

- a^*: denotes how many workers are necessary to produce a single unit of good φ.
  (unit labor requirements, as we have seen in the Ricardian model)

- Only labor $\ell > \bar{f}$ contributes to extra production

- What does this mean for economies of scale?

Fixed costs and increasing return



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- Intuition: $\bar{f}$: “cost of entering the market”
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- After that, producing an extra unit only costs labor at a constant marginal cost.

Average and marginal cost

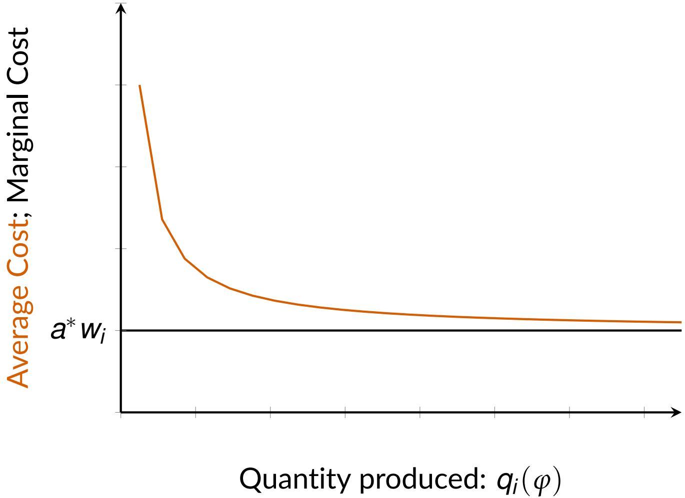

Figure: Average cost as a function of output

Monopolist's maximization problem

- In competitive markets, producers take prices as given
- Monopolists incorporate into their problem the fact that their choice of prices changes demand
- They take demand functions as given and pick price to maximize profits, maximizing:

$$\max_{p} \pi_i = (p - MC)q(p) - w_i\bar{f}$$

Solution (using chain rule):

$$MR = p + \frac{q(p)}{q'(p)} = MC$$

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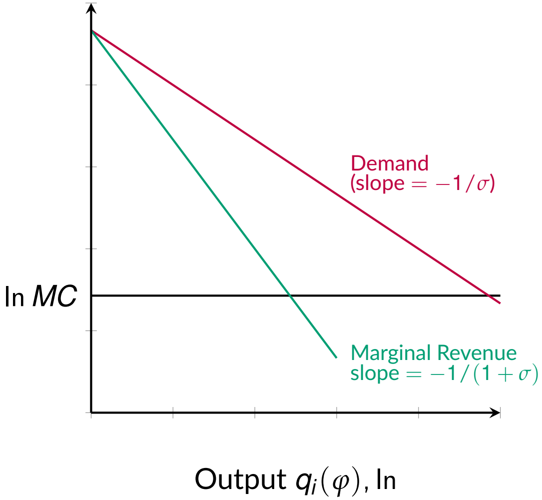
- Optimal price = mark-up $\times$ marginal cost.

Monopolistic competition



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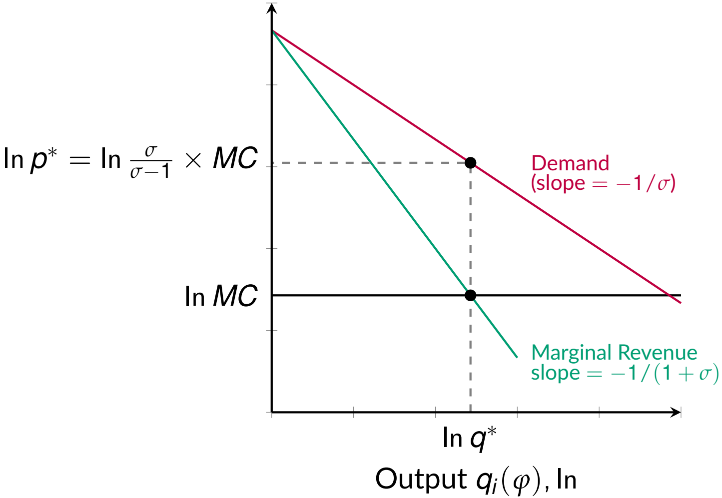

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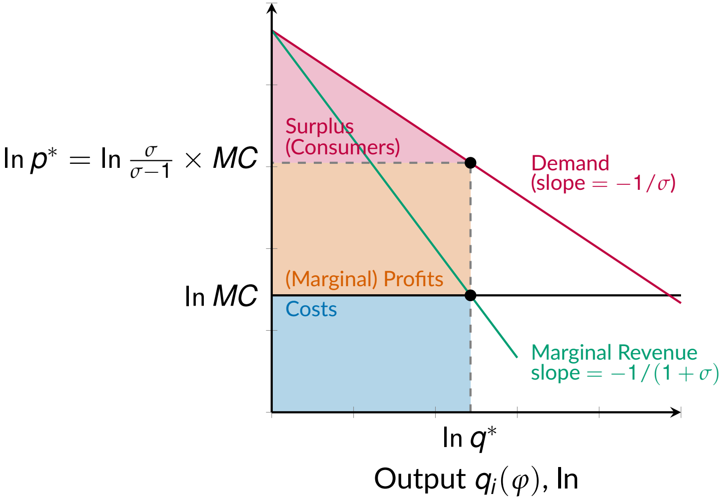

Takeaways (so far)

- Demand for differentiated goods
- Elasticity of substitution (σ)
- For all good, optimal choice implies MRS = relative prices
- Production under monopolistic competition, fixed costs, and IRS = profit opportunities
- Monopoly power implies $p^* > MC$
- In our framework, prices = mark up over MC.

Next class

- How to solve for the equilibrium of this model?
- What are the prices $\{p^*, P_i\}$, quantities demanded $\{q^*, Q_i\}$, goods in eqm $\{N\}$?
- What happens once we open up to trade?

Appendix

Calculus crash course

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- Set up a Lagrangian and take first order conditions:

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- λ's job is to keep track of how valuable the last dollar in the consumer's pocket is.

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- Set up a Lagrangian and take first order conditions:

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- What is λ?

- λ's job is to keep track of how valuable the last dollar in the consumer's pocket is.

- If you slip one more \$ into the consumer's wallet, utility rises by λ:

$$\frac{\partial \mathcal{L}}{\partial I_i} = \lambda$$

(we also call λ the shadow price)

Calculus crash course

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- N first order conditions, one for each $q_i(\varphi)$, satisfying:

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- Setting them equal forces every good to deliver the same utility per dollar spent.

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- Note:

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