International Trade: Lecture 8

Specific Factors Model (i)

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- Corollary: changes induce distributional concerns

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- Trade affects the income distribution within countries.
  Gains from trade are unevenly distributed

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- Key force: Differences in sector specific endowments

Production

- Production with sector-specific factors land T and capital K:

$$\max_{L_{i,M},K_i} \quad P_M \times Z_{i,M} \times K_i^{\beta_i} L_{i,M}^{1-\beta_i} - w_i L_{i,M} - r_{i,M} K_i$$
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  - Viner: Specific factor endowments determine labor productivity
  - Implication: Unit labor requirements change with labor employment

Decreasing Marginal Returns

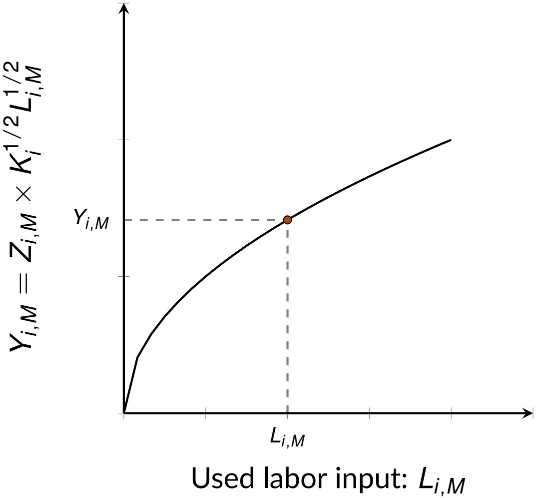

Figure: Decreasing returns to scale in labor

Optimality conditions

- At their optimal points, factor prices equal their marginal (revenue) product for labor...

$$
\begin{aligned}
P_M \times MPL_{i,M} = P_M \times \frac{\partial Y_{i,M}}{\partial L_{i,M}} &= w_i \\
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- ... and for labor and capital, respectively:

$$P_M \times MPK_{i,M} = P_M \times \frac{\partial Y_{i,M}}{\partial K_{i,M}} \quad = \quad r_{i,M}$$

$$P_A \times MPT_{i,A} = P_A \times \frac{\partial Y_{i,A}}{\partial T_{i,A}} \quad = \quad r_{i,A}$$

Marginal Products and Factor Prices

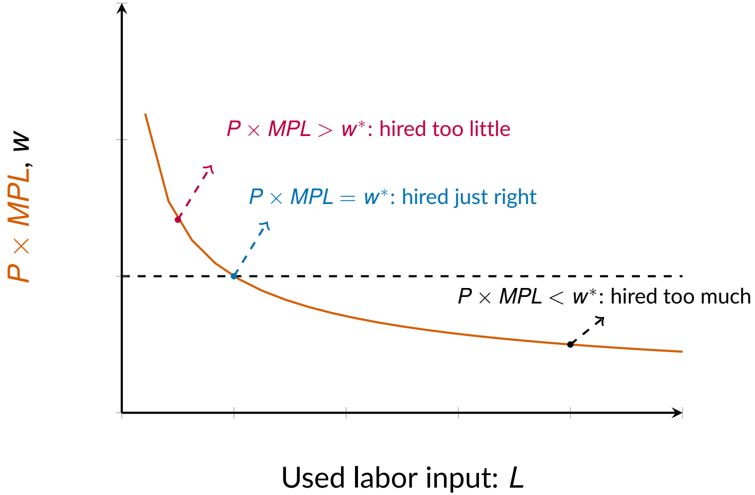


Figure: Labor market equilibrium: intuition

Supply of Factors of Production

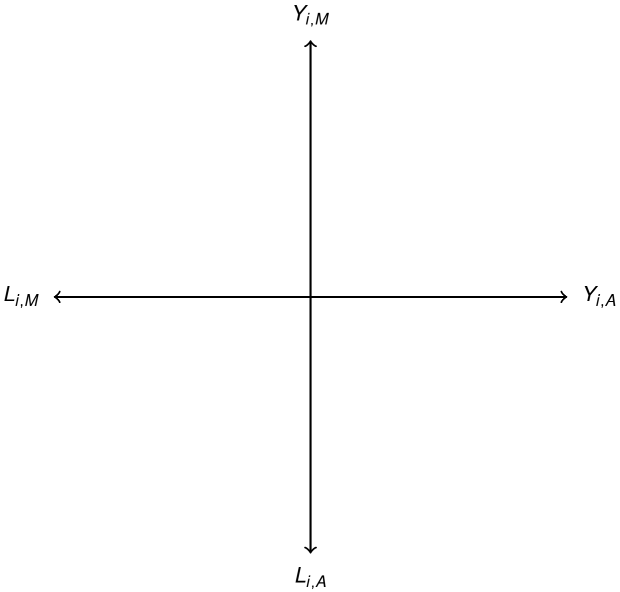
- Total labor can be distributed for the production of either good, such that:

$$\underbrace{L_{i,A}}_{\substack{\text{labor used in} \\ \text{production of } A}} + \underbrace{L_{i,M}}_{\substack{\text{labor used in} \\ \text{production of } M}} \leq \underbrace{L_i}_{\substack{\text{total labor} \\ \text{available in } i}}$$

- Supply of land T and capital is inelastic, so in equilibrium each sectors uses all endowment of specific factor:

$$\underbrace{T_i}_{\substack{\text{land} \\ \text{demand}}} = \underbrace{\bar{T}_i}_{\substack{\text{land} \\ \text{supply}}}, \qquad \underbrace{K_i}_{\substack{\text{capital} \\ \text{demand}}} = \underbrace{\bar{K}_i}_{\substack{\text{capital} \\ \text{supply}}}$$

The economy in one diagram



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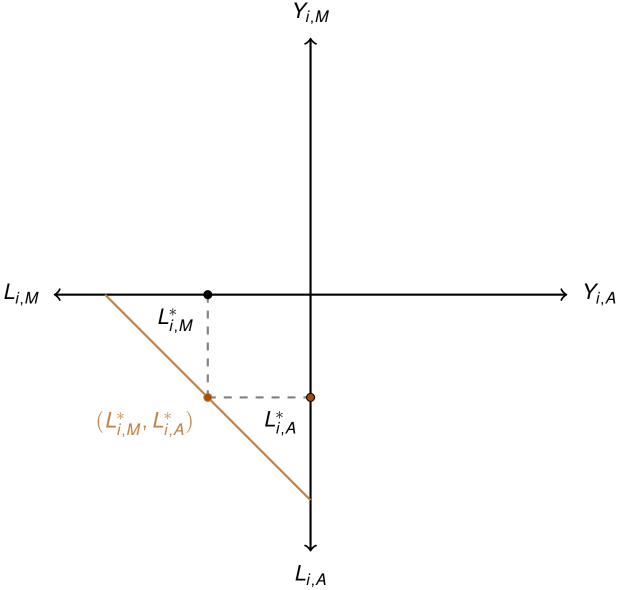

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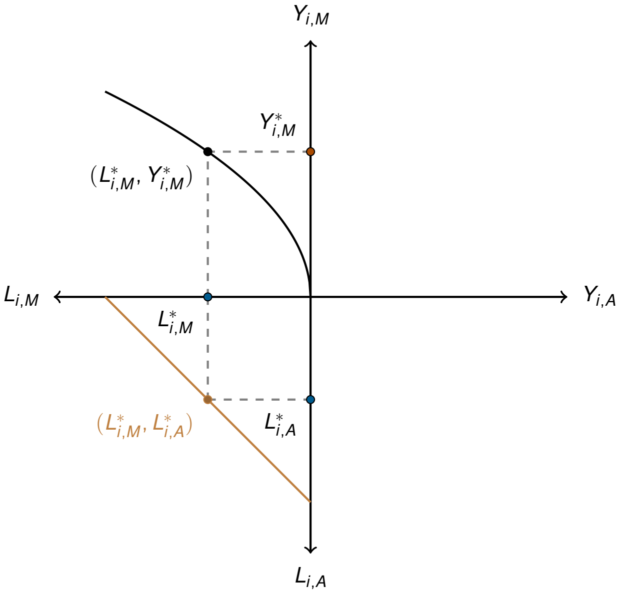

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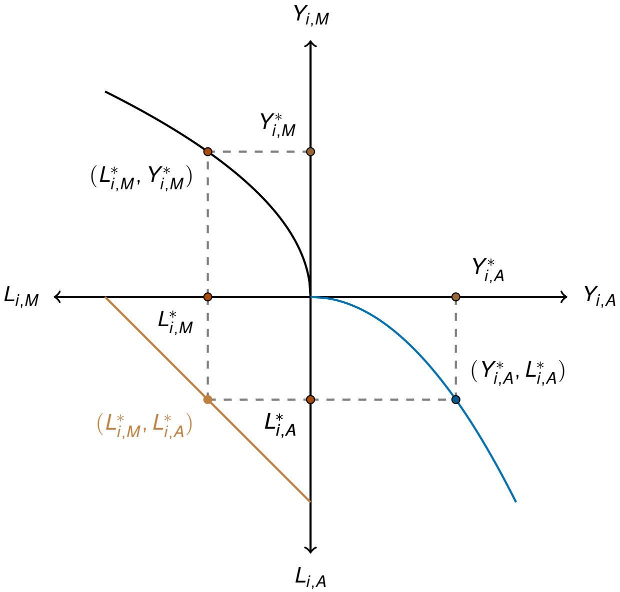

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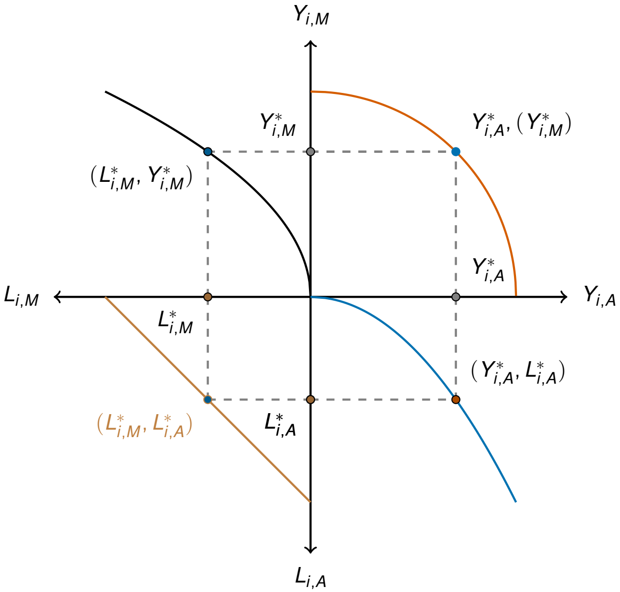

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- Preferences over goods $Q_{i,A}$, $Q_{i,M}$, given prices P_A, P_M, maximizing:

$$\max_{\{Q_{i,A}, Q_{i,M}\}} U_i(Q_{i,A}, Q_{i,M}) \equiv Q_{i,A}^{\alpha_i} Q_{i,M}^{1-\alpha_i} \qquad s.t. \quad P_A Q_{i,A} + P_M Q_{i,M} = I_i$$

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- We can also show that, at optimal choices, prices equal the marginal rate of subsitution?

$$-MRS_{i,AM} = -\frac{\partial U_i / \partial Q_{i,A}}{\partial U_i / \partial Q_{i,M}} = -\frac{\alpha_i}{1-\alpha_i}\frac{Q_{i,M}}{Q_{i,A}} = -\frac{P_A}{P_M} \tag{1}$$

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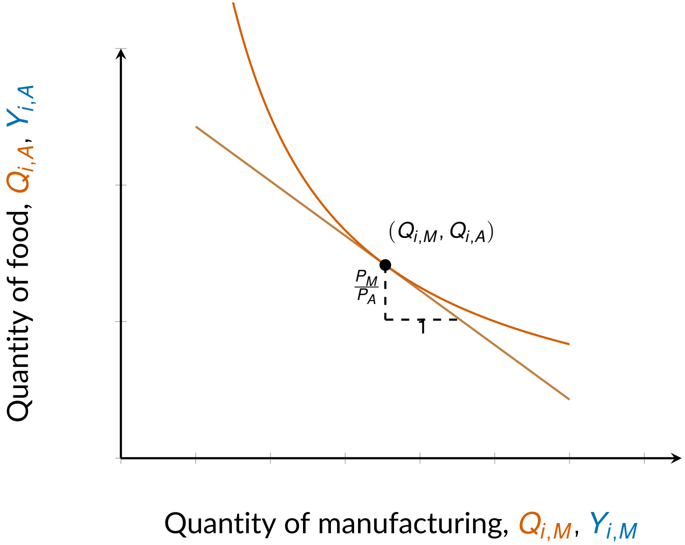
- What is intuition here?

Demand Choices in Autarky



Optimality conditions

- Implication: at the optimal point, marginal rate of transformation = relative price:

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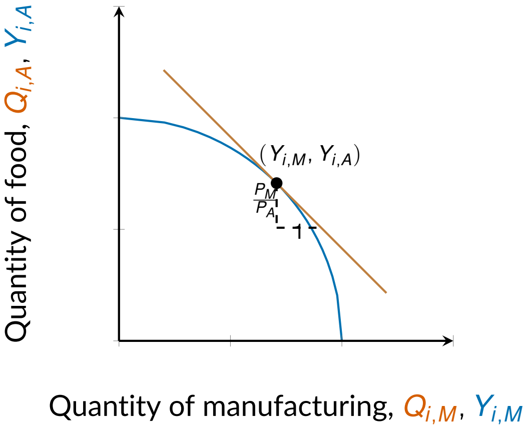

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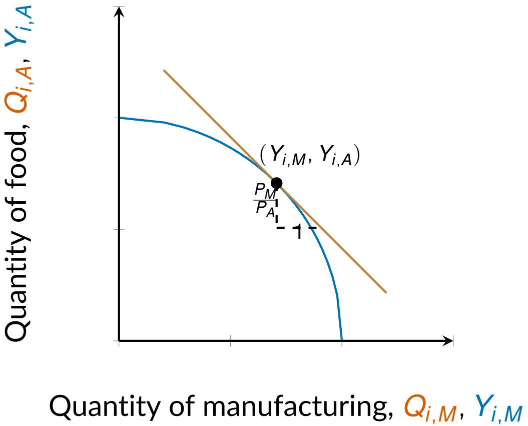
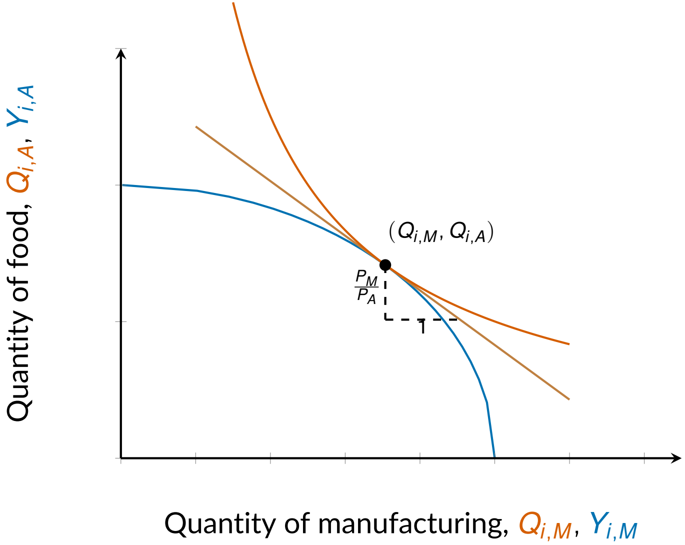
- Why is the PPF belly shaped?



Production + Demand Choices in Autarky



Labor Market Equilibrium: Intuition

- Recall the labor market resource constraint:

$$L_{i,A} + L_{i,M} = \bar{L}_i \iff L_{i,M} = \bar{L}_i - L_{i,A}$$

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- Also, recall the equilibrium condition for labor:

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- Implication: equilibrium wage w_i^* will be the point that equalizes marginal revenue

$$P_M \times MPL_{i,M} = P_A \times MPL_{i,A}$$

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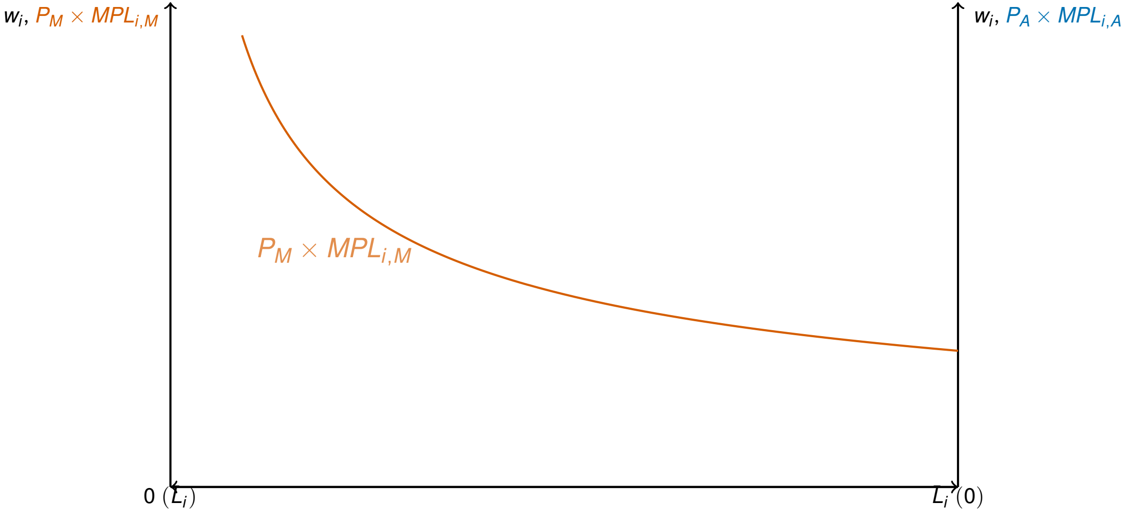

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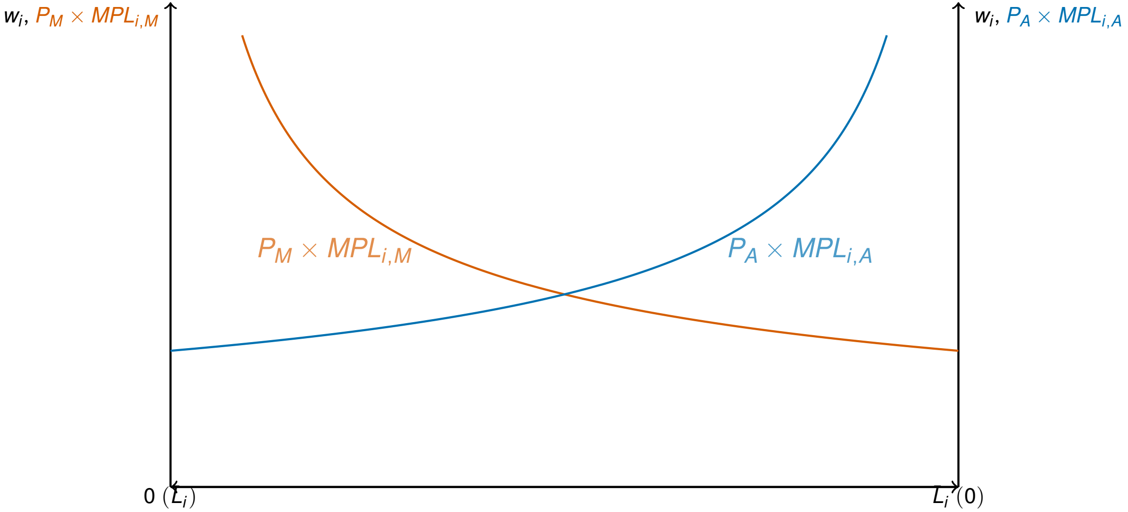

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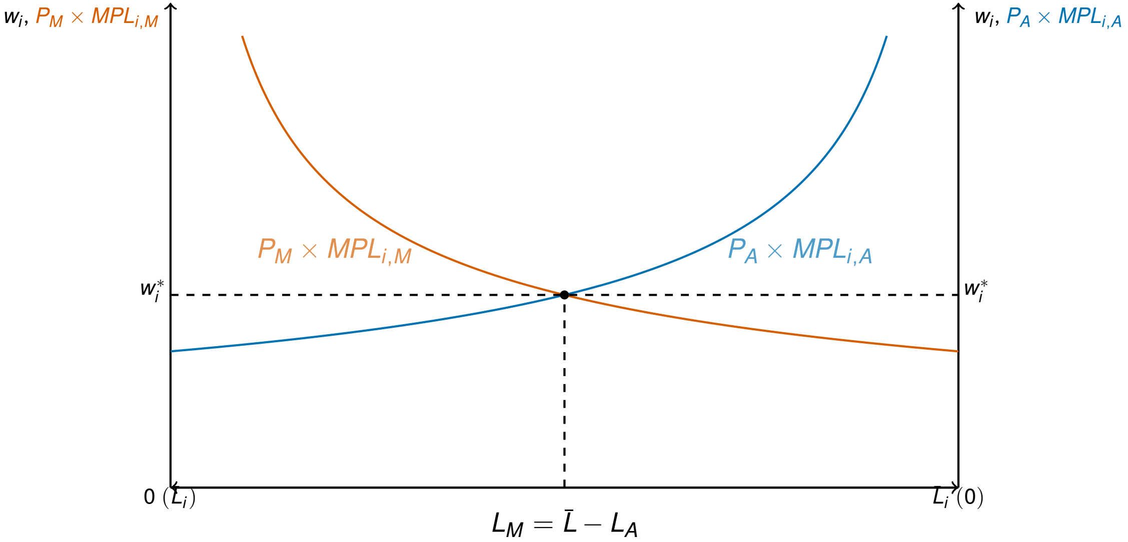

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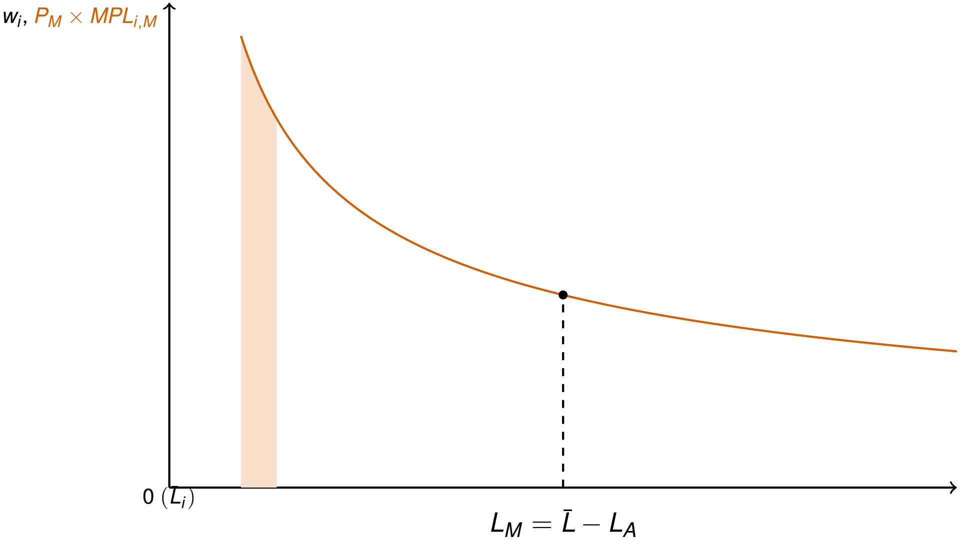
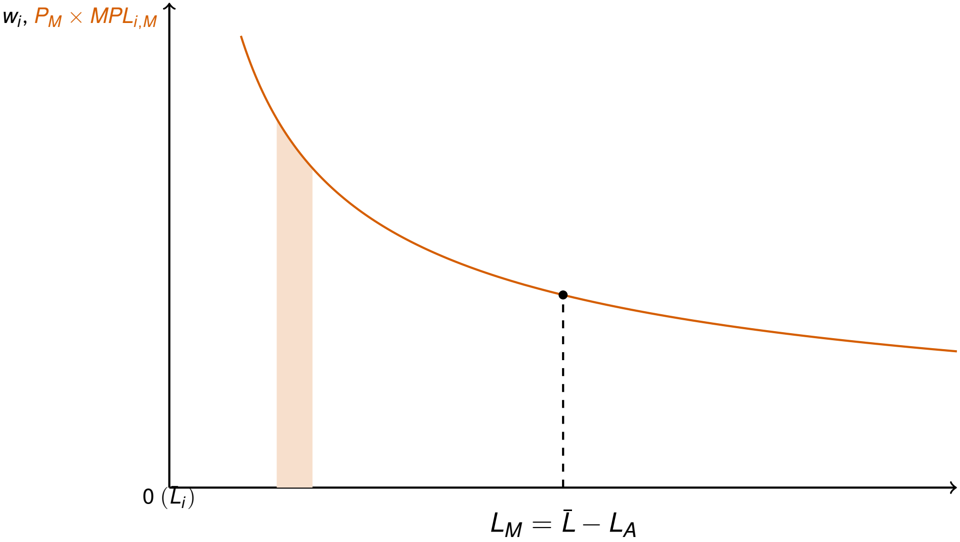
- If you sum over the marginal product of every worker, step by step, you get...

  Total Output = MP of first worker + MP of second worker + ... + MP of last worker

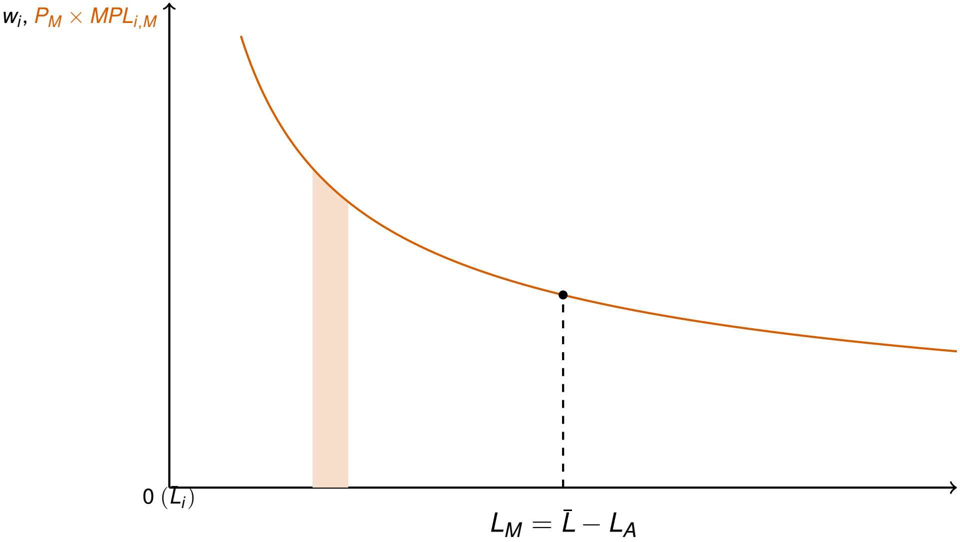
First worker's contribution to total revenue....



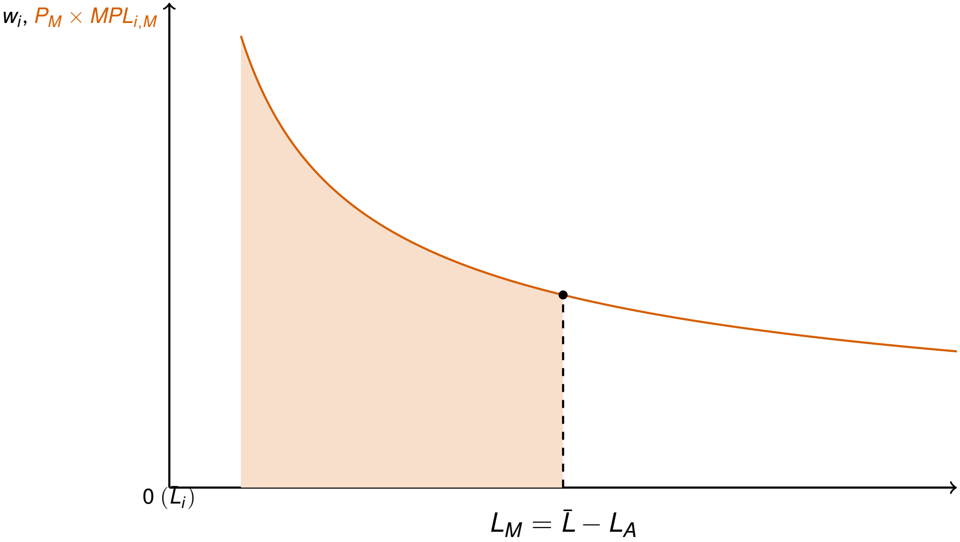
Second worker's contribution to total revenue....



Third worker's contribution to total revenue....



Sum of L_M workers contributions to total revenue....



Distribution of income: Preliminaries

- Workers are paid: $L_{i,M}w_i + L_{i,A}w_i = \bar{L}_i w_i$
- Capitalists are paid: $r_{i,M}K_i = P_M Y_{i,M} - L_{i,M}w_i$
- Landowners are paid: $r_{i,A}T_i = P_A Y_{i,A} - L_{i,A}w_i$

Distribution of income: Diagram