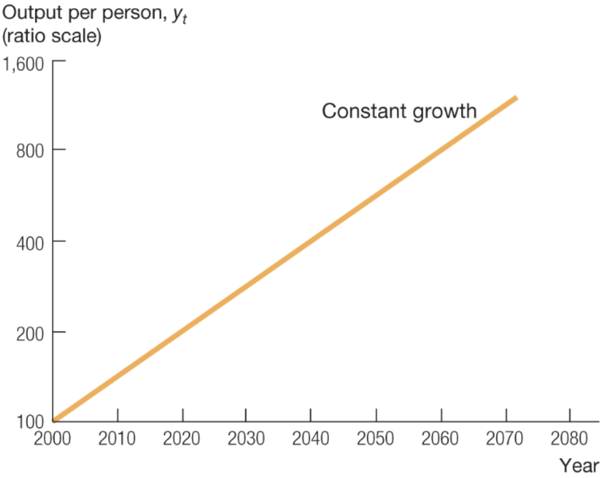
Econ 110A: Lecture 11

Carlos Góes[1]

[1]UC San Diego

UCSD, Summer Session II

The Romer Model



$$\ln y_t = \underbrace{\ln[A_0(1-\bar{\ell})]}_{\text{intercept}} + t \cdot \ln \underbrace{(1+\bar{g})}_{\text{slope}}$$

Moving parts of the model:

$$\begin{aligned} Y_t &= A_t L_{yt} \\ \bar{L} &= L_{yt} + L_{at} \\ \Delta A_{t+1} &= \bar{z} A_t L_{at} \\ L_{at} &= \bar{\ell} \bar{L} \end{aligned}$$

$$\bar{g} = \bar{z}\bar{\ell}\bar{L}$$

Some properties of growth rates

- **Product**: if $x = yz$, then $g_x = g_y + g_z$
- **Quotient**: if $x = y/z$, then $g_x = g_y - g_z$
- **Power**: if $x = y^\alpha$, then $g_x = \alpha g_y$
- **Sum**: if $x = y + z$, then $g_x = s_y g_y + s_z g_z$, where $s_y = y/x$, $s_z = z/x$

The Romer Model: a Closer Look

Are Increasing Returns enough for sustained constant growth?

The Romer Model: a Closer Look

Are Increasing Returns enough for sustained constant growth?

- **Output:** $Y_t = (A_t)^\delta L_{yt}$ where $\delta \in (0, 1)$
- **Dynamics:** $\Delta A_{t+1} = \bar{z} A_t L_{at}$

The Romer Model: a Closer Look

Are Increasing Returns enough for sustained constant growth?

- **Output:** $Y_t = (A_t)^{\delta} L_{yt}$ where $\delta \in (0, 1)$
- **Dynamics:** $\Delta A_{t+1} = \bar{z} A_t L_{at}$

Note:

$$\begin{aligned} \bar{g} &= \frac{A_{t+1}}{A_t} = \bar{z} A_t L_{at} = \bar{z}\bar{\ell}\bar{L} \\ g_y &= \delta \bar{g} + g_{L_y} = \delta \bar{g} \end{aligned}$$

The Romer Model: a Closer Look

Are Increasing Returns enough for sustained constant growth?

- **Output:** $Y_t = (A_t)^\delta L_{yt}$ where $\delta \in (0, 1)$
- **Dynamics:** $\Delta A_{t+1} = \bar{z} A_t L_{at}$

Note:

$$\begin{aligned} \bar{g} &= \frac{A_{t+1}}{A_t} = \bar{z} A_t L_{at} = \bar{z}\bar{\ell}\bar{L} \\ g_y &= \delta\bar{g} + g_{L_y} = \delta\bar{g} \end{aligned}$$

So we still get perpetual constant growth!

The Romer Model: a Closer Look

Are Increasing Returns enough for sustained constant growth?

The Romer Model: a Closer Look

Are Increasing Returns enough for sustained constant growth?

- **Output:** $Y_t = A_t L_{yt}$
- **Dynamics:** $\Delta A_{t+1} = \bar{z}(A_t)^{\gamma} L_{at}$ where $\gamma \in (0, 1)$

The Romer Model: a Closer Look

Are Increasing Returns enough for sustained constant growth?

- **Output:** $Y_t = A_t L_{yt}$
- **Dynamics:** $\Delta A_{t+1} = \bar{z}(A_t)^\gamma L_{at}$ where $\gamma \in (0,1)$

Note:

$$
\begin{aligned}
g_{A,t} &= \frac{A_{t+1}}{A_t} = \bar{z} L_{at} A_t^{\gamma-1} = \frac{\bar{z}\bar{\ell}\bar{L}}{A_t^{1-\gamma}}, \qquad 1-\gamma > 0 \\
g_{y,t} &= g_{A,t} + g_{L_y,t} = g_{A,t}
\end{aligned}
$$

The Romer Model: a Closer Look

Are Increasing Returns enough for sustained constant growth?

- **Output**: $Y_t = A_t L_{yt}$
- **Dynamics**: $\Delta A_{t+1} = \bar{z}(A_t)^{\gamma} L_{at}$ where $\gamma \in (0, 1)$

Note:

$$
\begin{aligned}
g_{A,t} &= \frac{A_{t+1}}{A_t} = \bar{z} L_{at} A_t^{\gamma - 1} = \frac{\bar{z}\bar{\ell}\bar{L}}{A_t^{1-\gamma}}, \qquad 1 - \gamma > 0 \\
g_{y,t} &= g_{A,t} + g_{L_y,t} = g_{A,t}
\end{aligned}
$$

Now growth rates are not constant, they decrease over time!

The Romer Model: a Closer Look

Are Increasing Returns enough for sustained constant growth?

- **No**. The Romer Model features a sustained constant growth only when the returns in producing ideas are constant in the scale of ideas (increasing returns are strong enough).
- **General principle**: diminishing returns of an accumulating factor of production eventually prevent sustained growth.

Evidence on Research Productivity

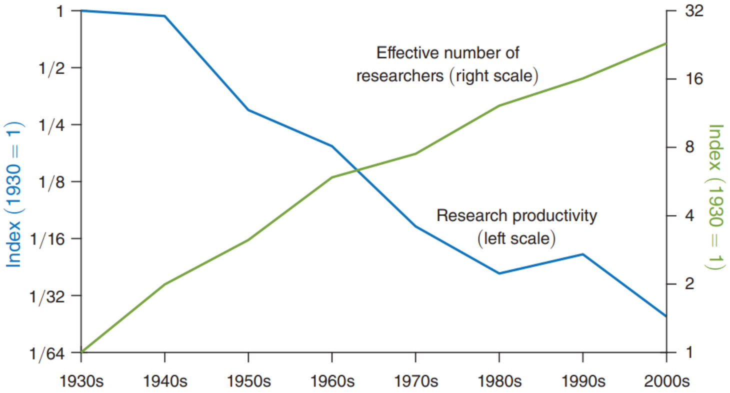


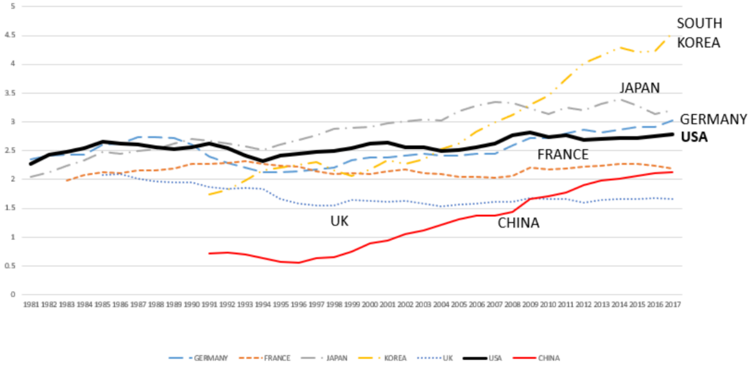
FIGURE 2. AGGREGATE EVIDENCE ON RESEARCH PRODUCTIVITY

$\frac{\Delta A_{t+1}}{L_{at}}$: research productivity $\quad L_{at}$: effective number of researchers

From Bloom, Jones, Van Reenen, and Webb (2020). "Are Ideas Getting Harder to Find?" American Economic Review 2020, 110(4): 1104–1144

Some facts about innovation

Figure 1: R&D as a Proportion of GDP in Selected Countries, 1981-2017

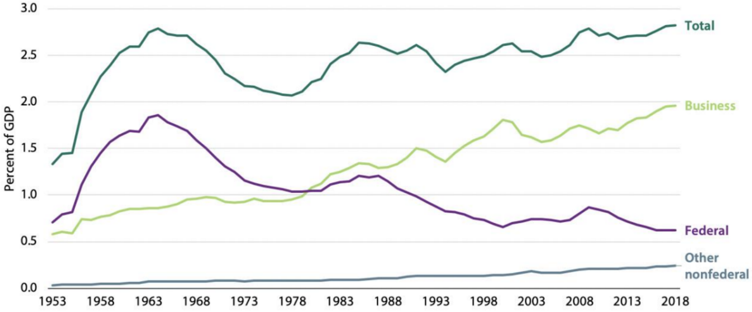


Source: OECD (2019).

From Van Reenen (2021). "Innovation and Human Capital Policy" NBER Working Paper No. 28713 April 2021

Some facts about innovation

Figure 2: US R&D, by source of funds



Source: National Science Board 2018.
Note: R&D spending is categorized by funder rather than performer. Other non-federal funders include, but are not limited to higher education, non-federal government, and other non-profit organizations.

From Van Reenen (2021). "Innovation and Human Capital Policy" NBER Working Paper No. 28713 April 2021

Some facts about innovation

Table 1: Number of researchers per 1,000 employees, Selected Countries

	United States	China	France	Germany	Korea	Japan	United Kingdom
1981	5.28		3.78	4.65		5.23	5.25
2001	7.29	1.02	6.83	6.63	6.32	9.87	6.57
2018	9.23	2.41	10.9	9.67	15.33	9.88	9.43

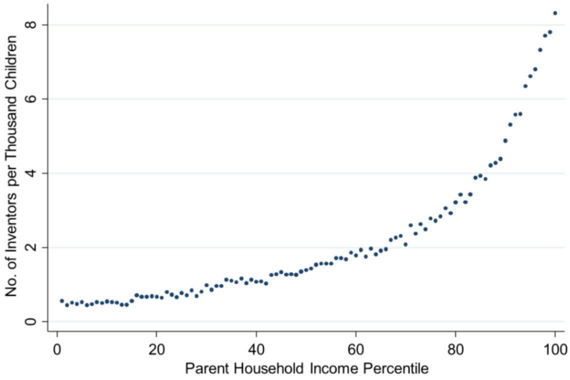
Source: OECD MSTI https://stats.oecd.org/Index.aspx?DataSetCode=MSTI_PUB# downloaded 11.21.20;

Note: US figure is for 2017.

From Van Reenen (2021). "Innovation and Human Capital Policy" NBER Working Paper No. 28713 April 2021

Some facts about innovation

Figure 3: Probability of growing up to be an inventor as a function of parental income

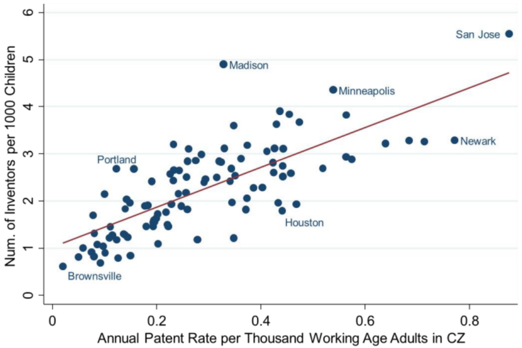


Notes: Sample of children is 1980-84 birth cohorts. Parent Income is mean household income

From Van Reenen (2021). "Innovation and Human Capital Policy" NBER Working Paper No. 28713 April 2021

Some facts about innovation

Figure 5: Growing up in a high innovation area, makes it much more likely you will become an inventor as an adult



Source: Bell et al (2019a). 100 most populous Commuting Zones

From Van Reenen (2021). "Innovation and Human Capital Policy" NBER Working Paper No. 28713 April 2021

Takeaways

- While Solow divides the world between capital and labor, Romer divides the world into ideas and objects (labor in this simple version)
- Ideas are nonrival, which induces increasing returns to scale in ideas and objects taken together
- But growth also comes with some inefficiencies: increasing returns to scale (fixed costs) imply that $P > MC$ in some places, so there is some deadweight loss.
- Growth ceases in the Solow model because of diminishing returns —which also explains transition dynamics.
- Because of nonrivalry, ideas do not run into diminishing returns and this allows ideas to be sustained.
- Empirically, productivity of research has been falling
- **Ultimate insight**: empowering people to full potential benefits everyone – e.g. the "missing Einsteins"

Solow + Romer

The Solow Growth Model: Taking Stock

Normalizing the price of the output good $P_t = 1$ each period, for each period $t \in \{0, 1, 2, \cdots\}$, given parameters $\bar{d}$, $\bar{s}$, $\bar{A}$, $\bar{L}$, α and the initial value of capital K_0 there are four unknowns Y_t, K_{t+1}, L_t, C_t, I_t and four equations:

$$\begin{aligned} Y_t &= \bar{A}K_t^{\alpha}L_t^{1-\alpha} \\ Y_t &= C_t + I_t \\ \Delta K_{t+1} &= \bar{s}Y_t - \bar{d}\cdot K_t \\ L_t &= \bar{L} \end{aligned}$$

that characterize the solution to this model.

The Romer Growth Model: Taking Stock

Normalizing the price of the output good $P_t = 1$ each period, for each period $t \in \{0, 1, 2, \cdots\}$, given parameters $\bar{z}, \bar{\ell}, \bar{L}$ and the initial value of the stock of ideas A_0 there are four unknowns $Y_t, A_{t+1}, L_{yt}, L_{at}$ and four equations:

$$
\begin{aligned}
Y_t &= A_t L_{yt} \\
\bar{L} &= L_{yt} + L_{at} \\
\Delta A_{t+1} &= \bar{z} A_t L_{at} \\
L_{at} &= \bar{\ell} \bar{L}
\end{aligned}
$$

that characterize the solution to this model.

The Combined Romer and Solow Growth Model

Normalizing the price of the output good $P_t = 1$ each period, for each period $t \in \{0, 1, 2, \cdots\}$, given parameters $\bar{z}, \bar{\ell}, \bar{d}, \bar{L}$ and the initial values of the stock of ideas and capital $\{A_0, K_0\}$ there are five unknowns $Y_t, K_{t+1}, A_{t+1}, L_{yt}, L_{at}$ and five equations:

$$
\begin{aligned}
Y_t &= A_t K_t^{\alpha} L_{yt}^{1-\alpha} \\
\Delta K_{t+1} &= \bar{s} Y_t - \bar{d} \cdot K_t \\
\bar{L} &= L_{yt} + L_{at} = L_{yt} + \bar{\ell}\bar{L} \\
\Delta A_{t+1} &= \bar{z} A_t L_{at} = \bar{z} A_t \bar{\ell}\bar{L} \\
Y_t &= C_t + I_t = C_t + \bar{s} Y_t
\end{aligned}
$$

that characterize the solution to this model.

Balanced Growth Path in Solow + Romer

At the BGP, growth rates $g_Y, g_A, g_K, g_{L_y}, g_{L_a}$ must be constant (by definition)

- Output:

$$g_Y = g_A + \alpha \cdot g_K + (1-\alpha) \cdot g_{L_y} = g_A + \alpha \cdot g_K$$

- Ideas:

$$g_A = \frac{\Delta A_{t+1}}{A_t} = \bar{z} L_{at} = \bar{z}\bar{\ell}\bar{L} \text{ (as in Romer)}$$

- Capital:

$$\underbrace{g_K}_{\text{constant}} = \frac{\Delta K_{t+1}}{K_t} = \underbrace{\bar{s}}_{\text{constant}} \cdot \underbrace{\frac{Y_t}{K_t}}_{\text{must be a constant}} - \underbrace{\bar{d}}_{\text{constant}}$$

- $\implies g_k = g_y$ why? if $g_k > g_y$, $\frac{Y_{t+1}}{K_{t+1}} < \frac{Y_t}{K_t}$, if $g_k < g_y$, $\frac{Y_{t+1}}{K_{t+1}} > \frac{Y_t}{K_t}$

Balanced Growth Path in Solow + Romer

Therefore:

$$
\begin{aligned}
g_Y &= g_A + \alpha \cdot g_K = g_A + \alpha \cdot g_y \\
\iff (1-\alpha) g_Y &= g_A \\
g_Y &= \frac{1}{(1-\alpha)} \cdot g_A = \frac{1}{(1-\alpha)} \cdot \bar{z}\bar{\ell}\bar{L}
\end{aligned}
$$

Balanced Growth Path in Solow + Romer

Therefore:

$$
\begin{aligned}
g_Y &= g_A + \alpha \cdot g_K = g_A + \alpha \cdot g_y \\
\iff (1-\alpha) g_Y &= g_A \\
g_Y &= \frac{1}{(1-\alpha)} \cdot g_A = \frac{1}{(1-\alpha)} \cdot \bar{z}\bar{\ell}\bar{L}
\end{aligned}
$$

Note:

$$
g_Y^{Solow+Romer} = \frac{\bar{z}\bar{\ell}\bar{L}}{1-\alpha} > \bar{z}\bar{\ell}\bar{L} = g_Y^{Romer}
$$

Balanced Growth Path in Solow + Romer

Therefore:

$$\begin{aligned} g_Y &= g_A + \alpha \cdot g_K = g_A + \alpha \cdot g_y \\ \Longleftrightarrow \quad (1-\alpha) g_Y &= g_A \\ g_Y &= \frac{1}{(1-\alpha)} \cdot g_A = \frac{1}{(1-\alpha)} \cdot \bar{z}\bar{\ell}\bar{L} \end{aligned}$$

Note:

$$g_Y^{Solow+Romer} = \frac{\bar{z}\bar{\ell}\bar{L}}{1-\alpha} > \bar{z}\bar{\ell}\bar{L} = g_Y^{Romer}$$

While capital itself is not an engine of economic growth, it amplifies the effect of the underlying growth in knowledge.

Solow + Romer: Solving the model

We will look for the level of Output per Capita $y_t^* = \frac{Y_t^*}{\bar{L}}$

- First step, use capital growth equation:

$$
\begin{aligned}
g_Y &= g_K = \bar{s} \cdot \frac{Y_t^*}{K_t^*} - \bar{d} \\
\iff \bar{s} \cdot \frac{Y_t^*}{K_t^*} &= g_Y + \bar{d} \\
\iff K_t^* &= \left(\frac{\bar{s}}{g_Y + \bar{d}} \right) \cdot Y_t^*
\end{aligned}
$$

Solow + Romer: Solving the model

We will look for the level of Output per Capita $y_t^* = \frac{Y_t^*}{\bar{L}}$

- Second step, replace this in the production function:

$$\begin{aligned}
Y_t^* &= A_t \cdot K_t^{\alpha} L_{yt}^{1-\alpha} \\
Y_t^* &= A_t \cdot \left[\left(\frac{\bar{s}}{g_Y + \bar{d}}\right) \cdot Y_t^*\right]^{\alpha} L_{yt}^{1-\alpha} \\
(Y_t^*)^{1-\alpha} &= A_0(1+g_A)^t \cdot \left(\frac{\bar{s}}{g_Y + \bar{d}}\right)^{\alpha} ((1-\bar{\ell})\bar{L})^{1-\alpha} \\
Y_t^* &= A_0^{\frac{1}{1-\alpha}} \cdot (1+g_A)^{\frac{t}{1-\alpha}} \cdot \left(\frac{\bar{s}}{g_Y + \bar{d}}\right)^{\frac{\alpha}{1-\alpha}} \cdot (1-\bar{\ell})\bar{L}
\end{aligned}$$

Solow + Romer: Solving the model

We will look for the level of Output per Capita $y_t^* = \frac{Y_t^*}{\bar{L}}$

- Third step, divide it through and show it in terms of parameters:

$$\frac{Y_t^*}{\bar{L}} = y_t^* = A_0^{\frac{1}{1-\alpha}} \cdot (1+g_A)^{\frac{t}{1-\alpha}} \cdot \left(\frac{\bar{s}}{g_Y + \bar{d}}\right)^{\frac{\alpha}{1-\alpha}} \cdot (1-\bar{\ell})$$

or, in terms of parameters, replacing for g_A, g_Y:

$$y_t^* = \text{`}A_0^{\frac{1}{1-\alpha}} \cdot (1+\bar{z}\bar{\ell}\bar{L})^{\frac{t}{1-\alpha}} \cdot \left(\frac{\bar{s}(1-\alpha)}{\bar{z}\bar{\ell}\bar{L} + \bar{d}(1-\alpha)}\right)^{\frac{\alpha}{1-\alpha}} \cdot (1-\bar{\ell})$$

Analyzing the solved model

$$y_t^* = \underbrace{A_0^{\frac{1}{1-\alpha}} \cdot (1 + \bar{z}\bar{\ell}\bar{L})^{\frac{t}{1-\alpha}}}_{\substack{\text{growth effects} \\ \text{from ideas}}} \cdot \underbrace{\left(\frac{\bar{s}(1-\alpha)}{\bar{z}\bar{\ell}\bar{L} + \bar{d}(1-\alpha)}\right)^{\frac{\alpha}{1-\alpha}}}_{\substack{\text{level effects} \\ \text{from capital}}} \cdot \underbrace{(1-\bar{\ell})}_{\substack{\text{level effects} \\ \text{from labor}}}$$

Some comments:

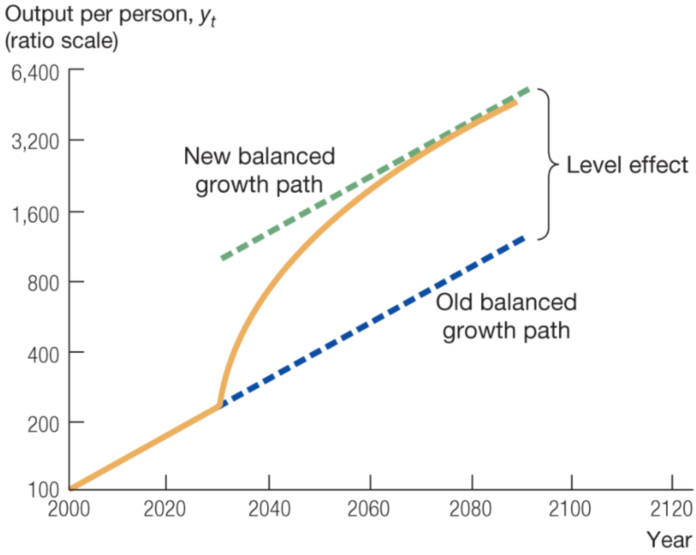
- Changes in $\bar{s}$ and $\bar{d}$ will induce a level effect shift in income per capita, with transition dynamics **across BGPs**
- Changes in $\bar{\ell}$, $\bar{z}$, $\bar{L}$ will both level and growth effects, with transition dynamics **across BGPs**

Experiment 1: Increase in the Savings Rate

Suppose the economy is in the balanced growth path of the Solow + Romer model.

Unexpectedly, there is a permanent increase in the savings rate, from $\bar{s}$ to $\bar{s}' > \bar{s}$ for all $t \geq t'$. What happens to output per person?

Experiment 1: Increase in Savings Rate



Experiment 2: Increase in the Share of Researchers

Suppose the economy is in the balanced growth path of the Solow + Romer model.

Unexpectedly, there is a permanent increase in the savings rate, from $\bar{s}$ to $\bar{s}' > \bar{s}$ for all $t \geq t'$. What happens to output per person?

Experiment 2: Increase in the Share of Researchers