Econ 110A: Lecture 18

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Final Review

Final is 3hr long and cumulative

Go back and watch the midterm review

Redo your midterm!

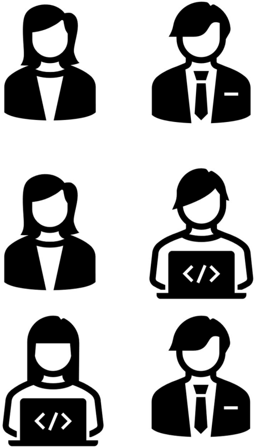
if you have time, work through all the practice midterms and finals!

That all but guarantees you a good grade!

Why can't we have sustained growth in the Solow Model?
$\rightarrow$ Diminishing Marginal Returns

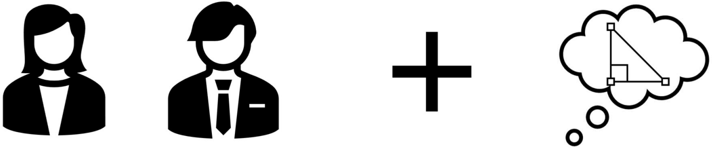
- Depreciation rises one-for-one with capital but output and investment rise less than one-for-one due to diminishing marginal returns
- Eventually, investment is only sufficient to offset depreciation and the model reaches a steady state
- Therefore, we cannot have sustained long-run growth

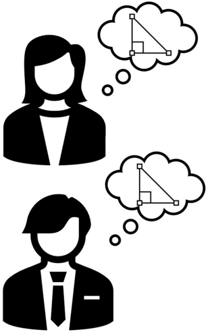
An introduction to the Economics of Ideas



With physical capital, there are diminishing marginal returns. It would be very hard for Maria and Juan to simultaneously use the same laptop.

An introduction to the Economics of Ideas





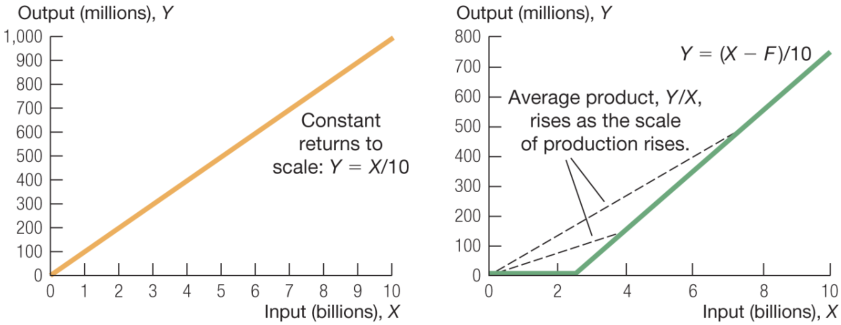
Suppose that instead of a piece of capital, we are considering a new idea, such as the pythagorean theorem. Now Maria and Juan can both that same idea at work <u>at the same time</u>! Adding new workers (or new ideas) do not have diminishing marginal returns but increasing returns to scale!

Ideas can lead to increasing returns

Consider the production of a **new antibiotic**.

- to first to up with the medicine, there is a large **fixed cost investment** F of \$2.5 billion to develop and get approval for the drug

- after the drug is developed and approved, producing new doses can be produced with a constant marginal cost: each 100 doses cost \$10 to produce



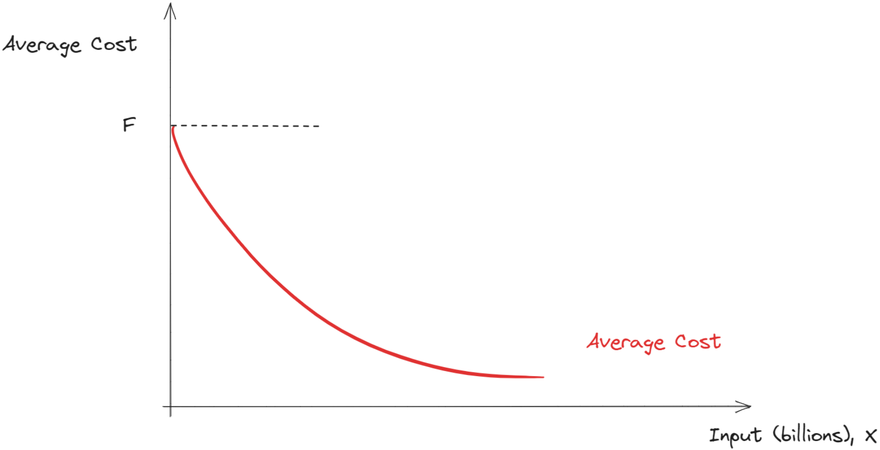
(a) Constant returns to scale: $Y = X/10$

(b) Increasing returns from fixed cost: $\bar{F} = 2.5$ billion

Ideas can lead to increasing returns

Consider the production of a **new antibiotic**.

- Decreasing average cost



Let us go back to our numerical example

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Let us go back to our numerical example

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- **Fixed Cost**: $F = \$2.5\text{billion}$

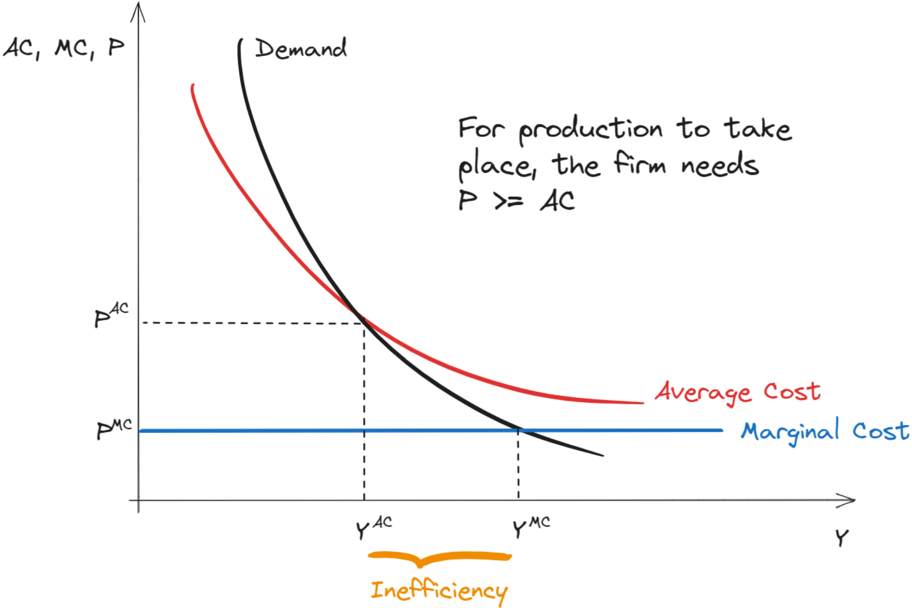
- after the drug is developed and approved, producing new doses can be produced with a constant marginal cost: each 100 doses cost \$10 to produce

- **Variable cost**: $\$0.1$

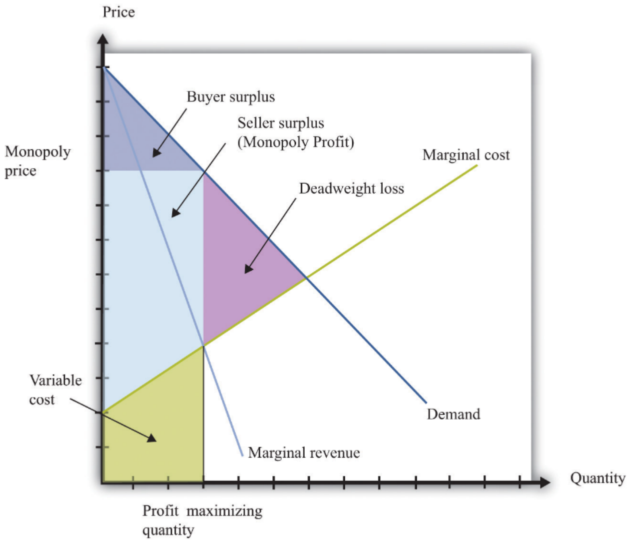
- **Total Cost**: $C(Y) = \$2.5\text{billion} + \$0.1\,Y$

- **Production**: $Y = \begin{cases} 0 & \text{if } C(Y) < \$2.5\text{billion} \\ L = (C - \$2.5B)/(\$0.1) & \text{if } C(Y) \geq \$2.5\text{billion} \end{cases}$

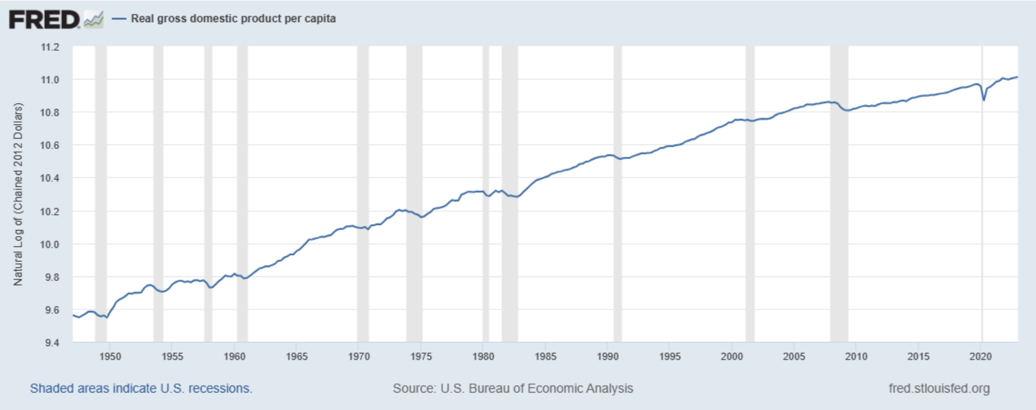
Inefficiency in Markets with Increasing Returns



Micro diversion (iii)



What is the ultimate cause of sustained economic growth?



The Romer Model

"Factor-based" representation of the production function

$$\underbrace{Y_t}_{\substack{\text{output}\\\text{value added}}} = F(\underbrace{A_t}_{\text{ideas}}, \underbrace{K_t}_{\text{capital}}, \underbrace{L_{yt}}_{\text{labor}})$$

$F(A_t, K_t, L_t)$ is your production function. If $F(\cdot, \cdot, \cdot)$ is Cobb-Douglas, then:

$$Y_t = A_t \cdot K_t^{\alpha} L_{yt}^{1-\alpha}, \qquad 0 \leq \alpha \leq 1$$

The Romer Growth Model: Taking Stock

Normalizing the price of the output good $P_t = 1$ each period, for each period $t \in \{0, 1, 2, \cdots\}$, given parameters $\bar{z}, \bar{\ell}, \bar{L}$ and the initial value of the stock of ideas A_0 there are four unknowns $Y_t, A_{t+1}, L_{yt}, L_{at}$ and four equations:

$$\begin{aligned}
Y_t &= A_t L_{yt} &(1)\\
\bar{L} &= L_{yt} + L_{at} &(2)\\
\Delta A_{t+1} &= \bar{z} A_t L_{at} &(3)\\
L_{at} &= \bar{\ell}\bar{L} &(4)
\end{aligned}$$

that characterize the solution to this model.

Solving the Romer Model

- Output per capita is:

$$\frac{Y_t}{\bar{L}} \equiv y_t = A_t L_{yt} = \frac{A_t(1-\bar{\ell})\bar{L}}{\bar{L}} = A_t(1-\bar{\ell})$$

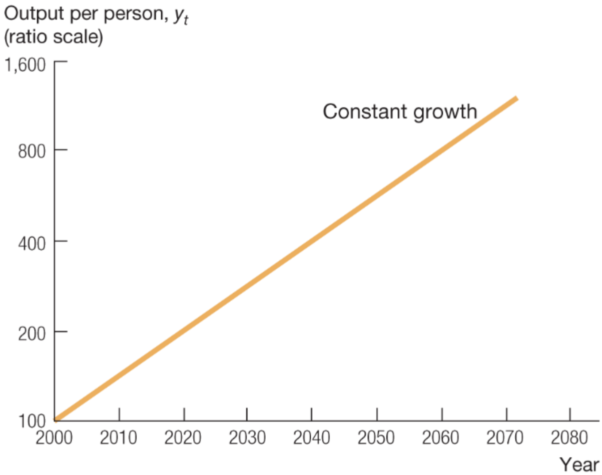
- Ideas are produced by allocating new researchers (labor) to it:

$$\Delta A_{t+1} = \bar{z} A_t L_{at} \iff \frac{\Delta A_{t+1}}{A_t} \equiv \bar{g} = \bar{z}\ell\bar{L}$$

- Using the properties of **compound growth** (remember those) can then write A_t, y_t:

$$A_t = A_0(1+\bar{g})^t, \qquad y_t = A_0(1+\bar{g})^t(1-\bar{\ell})$$

The Balanced Growth Path

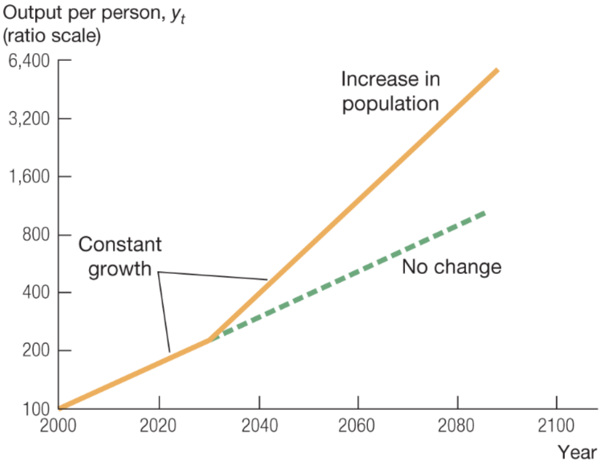


$$\ln y_t = \underbrace{\ln[A_0(1-\bar{\ell})]}_{\text{intercept}} + t \cdot \underbrace{\ln(1+\bar{g})}_{\text{slope}}$$

The economy is in a balanced growth path (BGP) when **all endogenous variables grow at the same constant rate.**

The Romer model has **no transition dynamics.** It jumps directly to the BGP.

Experiment 1: Increase in Population

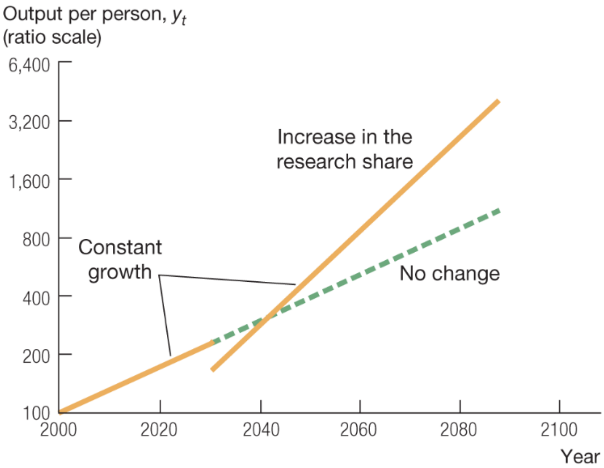


$$\underbrace{(1+\bar{z}\bar{\ell}\bar{L}')^{t-t'}}_{\text{new slope}} > \underbrace{(1+\bar{z}\bar{\ell}\bar{L})^{t-t'}}_{\text{old slope}} \text{ for all } t \geq t'$$

Experiment 1: Increase in Population

- Production Model: y falls when $\bar{L}$ is increased.
- Solow Model: y falls at first when $\bar{L}$ is increased, then returns to initial level.
- Romer Model: y does not fall when $\bar{L}$ is increased, it grows faster instead due to scale effects and nonrivalry of ideas.

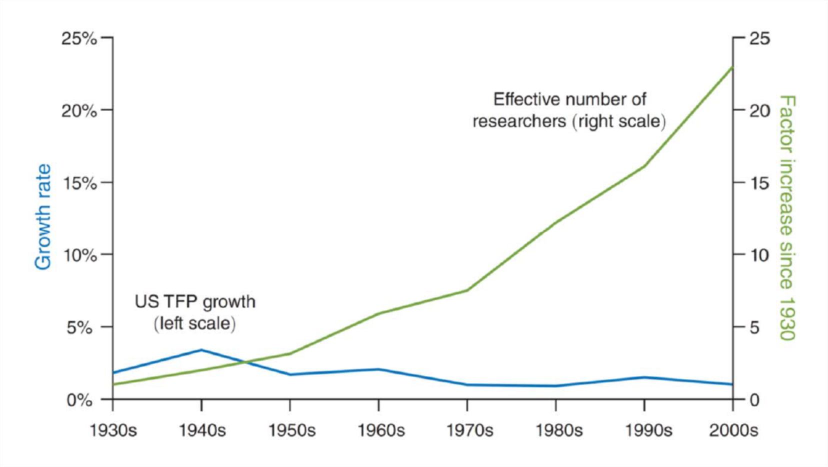
Experiment 2: Increase in the share of researchers in the population



$$y_t = A_0 \underbrace{(1 - \bar{\ell})}_{\text{level effect} \downarrow} \underbrace{(1 + \bar{z}\bar{\ell}\bar{L}')^t}_{\text{growth effect} \uparrow}$$

Experiment 2: Increase in the share of researchers in the population

The Romer model predicts that more researchers (higher $\bar{\ell}\bar{L}$) imply a higher sustained TFP growth rate. Is there evidence of this in the data?



“**Are Ideas Getting Harder to Find?**” Bloom, Jones, Van Reenen, Webb (AER, 2020)

Takeaways

- While Solow divides the world between capital and labor, Romer divides the world into ideas and objects (labor in this simple version)
- Ideas are nonrival, which induces increasing returns to scale in ideas and objects taken together
- But growth also comes with some inefficiencies: increasing returns to scale (fixed costs) imply that $P > MC$ in some places, so there is some deadweight loss.
- Growth ceases in the Solow model because of diminishing returns —which also explains transition dynamics.
- Because of nonrivalry, ideas do not run into diminishing returns and this allows ideas to be sustained.

The Romer Model: a Closer Look

Are Increasing Returns enough for sustained constant growth?

- **Output:** $Y_t = \underbrace{A_t L_{yt}}_{\text{this still is increasing returns in } A_t \text{ and } L_{yt} \text{ combined}}$
- **Dynamics:** $\Delta A_{t+1} = \bar{z}(A_t)^{\gamma} L_{at}$ where $\gamma \in (0, 1)$

Note:

$$
\begin{aligned}
g_{A,t} &= \frac{A_{t+1}}{A_t} = \bar{z} L_{at} A_t^{\gamma-1} = \frac{\bar{z}\bar{\ell}\bar{L}}{A_t^{1-\gamma}}, \qquad 1-\gamma > 0 \\
g_{y,t} &= g_{A,t} + g_{L_y,t} = g_{A,t}
\end{aligned}
$$

Now growth rates are not constant, they decrease over time!

The Romer Model: a Closer Look

Are Increasing Returns enough for sustained constant growth?

- **No**. The Romer Model features a sustained constant growth only when the returns in producing ideas are constant in the scale of ideas (increasing returns are strong enough).
- **General principle**: diminishing returns of an accumulating factor of production eventually prevent sustained growth.

The Combined Romer and Solow Growth Model

Normalizing the price of the output good $P_t = 1$ each period, for each period $t \in \{0, 1, 2, \cdots\}$, given parameters $\bar{z}, \bar{\ell}, \bar{d}, \bar{L}$ and the initial values of the stock of ideas and capital $\{A_0, K_0\}$ there are five unknowns $Y_t, K_{t+1}, A_{t+1}, L_{yt}, L_{at}$ and five equations:

$$
\begin{aligned}
Y_t &= A_t K_t^{\alpha} L_{yt}^{1-\alpha} \\
\Delta K_{t+1} &= \bar{s} Y_t - \bar{d} \cdot K_t \\
\bar{L} &= L_{yt} + L_{at} = L_{yt} + \bar{\ell}\bar{L} \\
\Delta A_{t+1} &= \bar{z} A_t L_{at} = \bar{z} A_t \bar{\ell}\bar{L} \\
Y_t &= C_t + I_t = C_t + \bar{s} Y_t
\end{aligned}
$$

that characterize the solution to this model.

Balanced Growth Path in Solow + Romer

At the BGP, growth rates $g_Y, g_A, g_K, g_{L_y}, g_{L_a}$ must be constant (by definition)

- Output:

$$g_Y = g_A + \alpha \cdot g_K + (1-\alpha) \cdot g_{L_y} = g_A + \alpha \cdot g_K$$

- Ideas:

$$g_A = \frac{\Delta A_{t+1}}{A_t} = \bar{z} L_{at} = \bar{z}\bar{\ell}\bar{L} \text{ (as in Romer)}$$

- Capital:

$$\underbrace{g_K}_{\text{constant}} = \frac{\Delta K_{t+1}}{K_t} = \underbrace{\bar{s}}_{\text{constant}} \cdot \underbrace{\frac{Y_t}{K_t}}_{\text{must be a constant}} - \underbrace{\bar{d}}_{\text{constant}}$$

- $\implies g_k = g_y$ why? if $g_k > g_y$, $\frac{Y_{t+1}}{K_{t+1}} < \frac{Y_t}{K_t}$, if $g_k < g_y$, $\frac{Y_{t+1}}{K_{t+1}} > \frac{Y_t}{K_t}$

Balanced Growth Path in Solow + Romer

Therefore:

$$\begin{aligned} g_Y &= g_A + \alpha \cdot g_K = g_A + \alpha \cdot g_y \\ \iff (1-\alpha) g_Y &= g_A \\ g_Y &= \frac{1}{(1-\alpha)} \cdot g_A = \frac{1}{(1-\alpha)} \cdot \bar{z}\bar{\ell}\bar{L} \end{aligned}$$

Note:

$$g_Y^{Solow+Romer} = \frac{\bar{z}\bar{\ell}\bar{L}}{1-\alpha} > \bar{z}\bar{\ell}\bar{L} = g_Y^{Romer}$$

While capital itself is not an engine of economic growth, it amplifies the effect of the underlying growth in knowledge.

Solow + Romer: Solving the model

We will look for the level of Output per Capita $y_t^* = \frac{Y_t^*}{\bar{L}}$

- Third step, divide it through and show it in terms of parameters:

$$\frac{Y_t^*}{\bar{L}} = y_t^* = A_0^{\frac{1}{1-\alpha}} \cdot (1 + g_A)^{\frac{t}{1-\alpha}} \cdot \left(\frac{\bar{s}}{g_Y + \bar{d}}\right)^{\frac{\alpha}{1-\alpha}} \cdot (1 - \bar{\ell})$$

or, in terms of parameters, replacing for g_A, g_Y:

$$y_t^* = `A_0^{\frac{1}{1-\alpha}} \cdot (1 + \bar{z}\bar{\ell}\bar{L})^{\frac{t}{1-\alpha}} \cdot \left(\frac{\bar{s}(1-\alpha)}{\bar{z}\bar{\ell}\bar{L} + \bar{d}(1-\alpha)}\right)^{\frac{\alpha}{1-\alpha}} \cdot (1 - \bar{\ell})$$

Analyzing the solved model

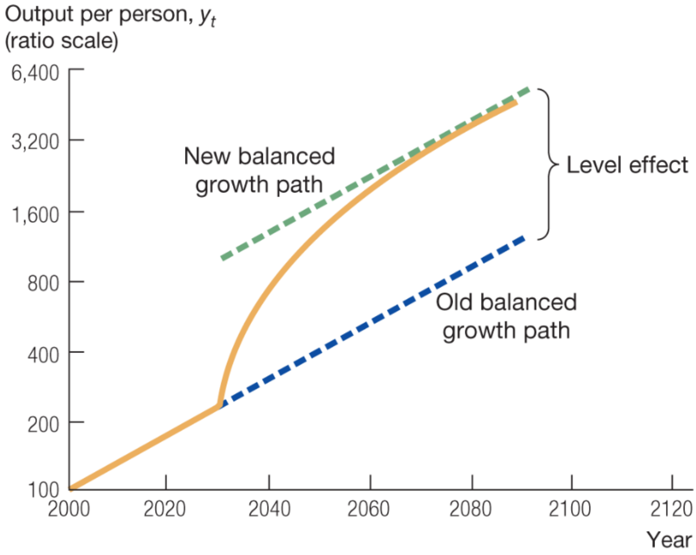
$$y_t^* = \underbrace{A_0^{\frac{1}{1-\alpha}} \cdot (1 + \bar{z}\bar{\ell}\bar{L})^{\frac{t}{1-\alpha}}}_{\substack{\text{growth effects} \\ \text{from ideas}}} \cdot \underbrace{\left(\frac{\bar{s}(1-\alpha)}{\bar{z}\bar{\ell}\bar{L} + \bar{d}(1-\alpha)}\right)^{\frac{\alpha}{1-\alpha}}}_{\substack{\text{level effects} \\ \text{from capital}}} \cdot \underbrace{(1-\bar{\ell})}_{\substack{\text{level effects} \\ \text{from labor}}}$$

Some comments:

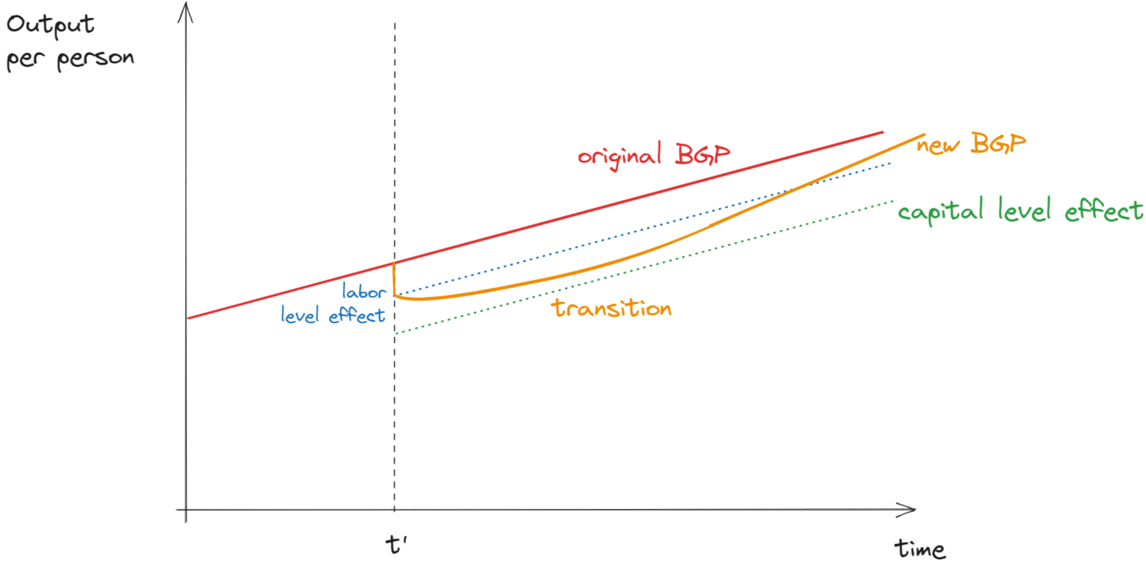
- Changes in $\bar{s}$ and $\bar{d}$ will induce a level effect shift in income per capita, with transition dynamics **across BGPs**

- Changes in $\bar{\ell}$, $\bar{z}$, $\bar{L}$ will both level and growth effects, with transition dynamics **across BGPs**

Experiment 1: Increase in Savings Rate



Experiment 2: Increase in the Share of Researchers



A stylized model of the Labor Market

Labor Demand

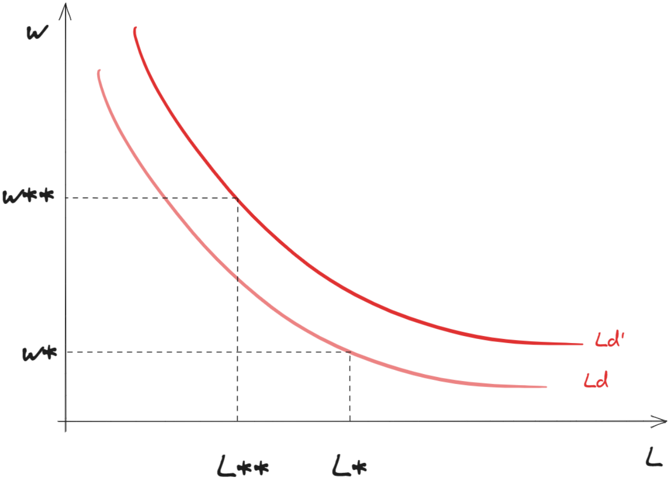
Firm is choosing labor to maximize profits

$$\max_{\{L,K\}} \pi = AK^{\alpha}L^{1-\alpha} - rK - wL$$

Solution for L:

$$w = MPL = (1-\alpha)A\left(\frac{K}{L}\right)^{\alpha}$$

What happens if $A' > A$?



Labor Supply

Household is choosing time to supply to the labor market. Time supplied is remunerated by wage, which is taxed at rate τ, but it also provides disutility in the form of less leisure.

$$\max_{\{C,L\}} U(C,L) = C - \frac{1}{2}\gamma L^2 \qquad s.t. \quad C = wL(1-\tau)$$

Substituting it in:

$$\max_{\{L\}} wL(1-\tau) - \frac{1}{2}\gamma L^2$$

First order condition:

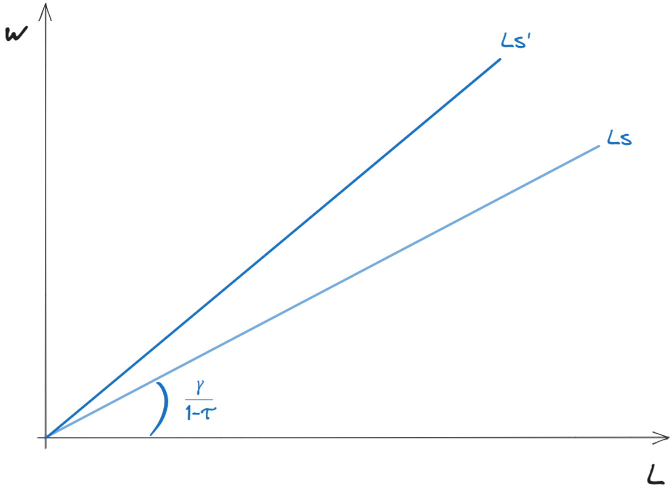
$$w(1-\tau) - \gamma L = 0 \implies w = \frac{\gamma}{1-\tau}L$$

Labor Supply

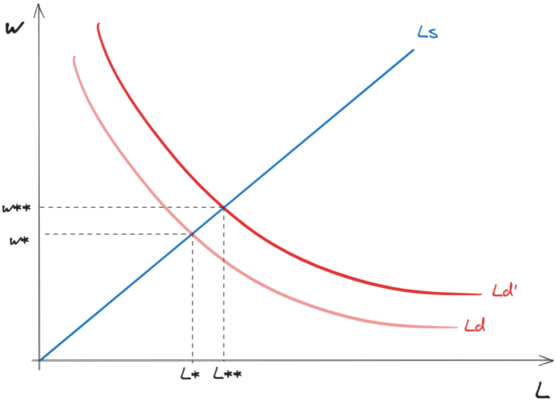
Optimal household labor supply

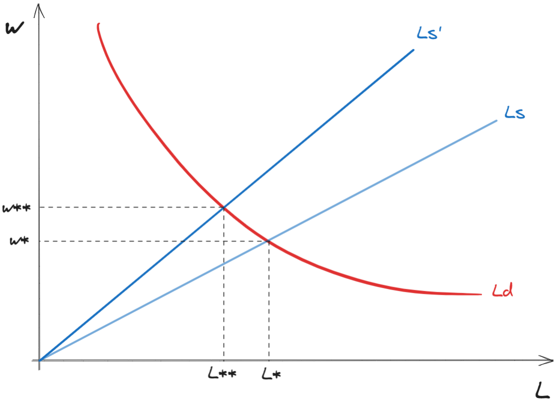
$$w(1-\tau) - \gamma L = 0 \implies w = \frac{\gamma}{1-\tau} L$$

What happens if $\tau' > \tau$?



Labor Market Equilibrium





Logic of a monopsonist

- Take labor supply curve as given

- Then choose labor level that maximizes profits. Given $w = \frac{\gamma}{1-\tau} L$

$$\max_{\{K,L\}} \pi = AK^{\alpha} L^{1-\alpha} - rK - \underbrace{\frac{\gamma}{1-\tau} L}_{=w} \cdot L$$

- Note that this is the same as **choosing wages** that maximize profits, given the labor supply. Given $L = \frac{1-\tau}{\gamma} w$

$$\max_{\{K,w\}} \pi = AK^{\alpha} \left(\underbrace{\frac{1-\tau}{\gamma} w}_{=L} \right)^{1-\alpha} - rK - w \cdot \underbrace{\frac{1-\tau}{\gamma} w}_{=L}$$

Monopsony in the Labor Market

- Take the first case

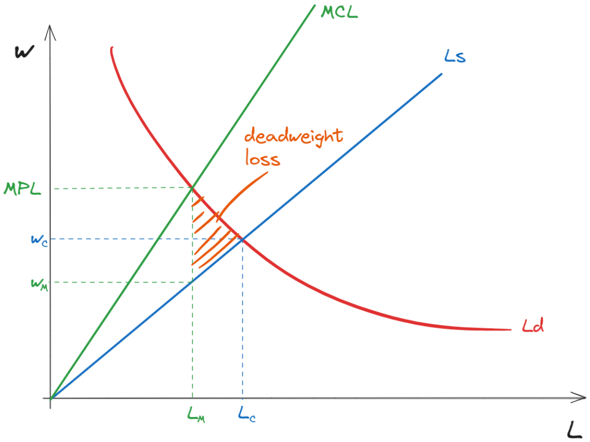
$$\max_{\{K,L\}} \pi = AK^{\alpha}L^{1-\alpha} - rK - \frac{\gamma}{1-\tau}L \cdot L$$

First order conditions:

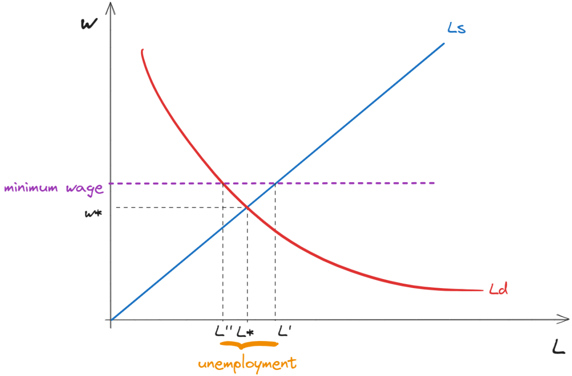
$$r = MPK$$

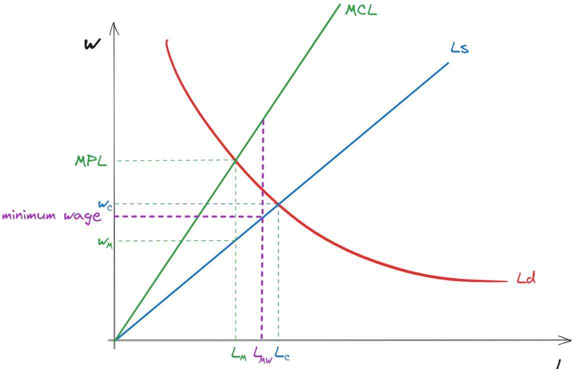
$$0 = \underbrace{(1-\alpha)A\left(\frac{K}{L}\right)^{\alpha}}_{=MPL} - 2\frac{\gamma}{1-\tau}L$$

$$MPL = 2\frac{\gamma}{1-\tau}L = \underbrace{MCL}_{\text{marginal cost of labor}}$$



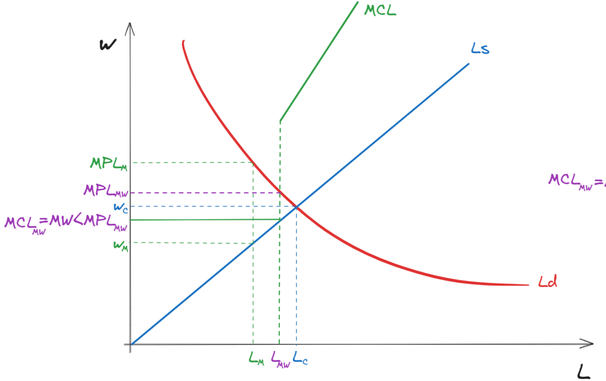
The Minimum Wage and Labor Market Structure



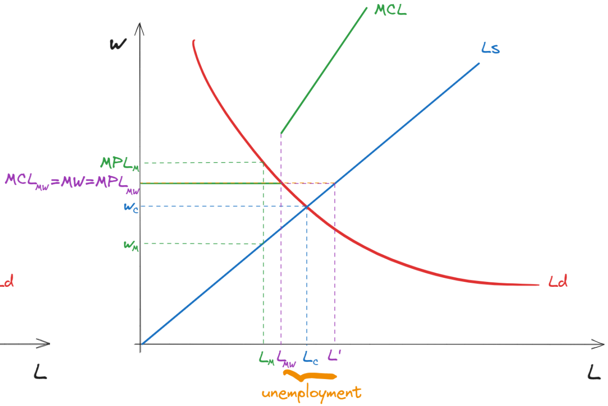


The Minimum Wage and Labor Market Structure

Minimum wage BELOW competitive equilibrium wage increases employment relative to the competitive equilibrium



Minimum wage ABOVE competitive equilibrium wage induces unemployment relative to the competitive equilibrium



The “Bathtub” Model of Unemployment

Strategy: find the long-run equilibrium where $\Delta U_{t+1} = 0$

$$\begin{aligned}
\Delta U^* = \bar{s} \cdot E^* - \bar{f} \cdot U^* &= 0 \\
\iff \bar{s} \cdot (\bar{L} - U^*) - \bar{f} \cdot U^* &= 0 \\
\iff U^* &= \frac{\bar{s}}{\bar{s} + \bar{f}} \bar{L} \\
\iff \frac{U^*}{\bar{L}} = u^* &= \frac{\bar{s}}{\bar{s} + \bar{f}}
\end{aligned}$$

The "Bathtub" Model of Unemployment

$$u_{US} = \frac{0.01}{0.01 + 0.2} = 4.8\%, \qquad u_{US}^{A} = \frac{0.01}{0.01 + 0.4} = 2.5\%$$

Changes in labor market policies affect both separation rate and job finding rate, which makes it difficult to use this model for policy analysis

Example: suppose the government introduces a law that makes it very difficult to layoff workers, what do you think would happen to the long-run unemployment?

Human Capital

Human Capital

Human capital is the stock of skills of people in the labor force.
e.g.: education, training, experience, health...

- some returns to human capital are private
- but strong externalities exist

but how much is human capital worth?

What is the PDV of Lifetime Income of US Workers

- The average American worker enters the labor force at 22 and retires at 67, so she works for 45 years.

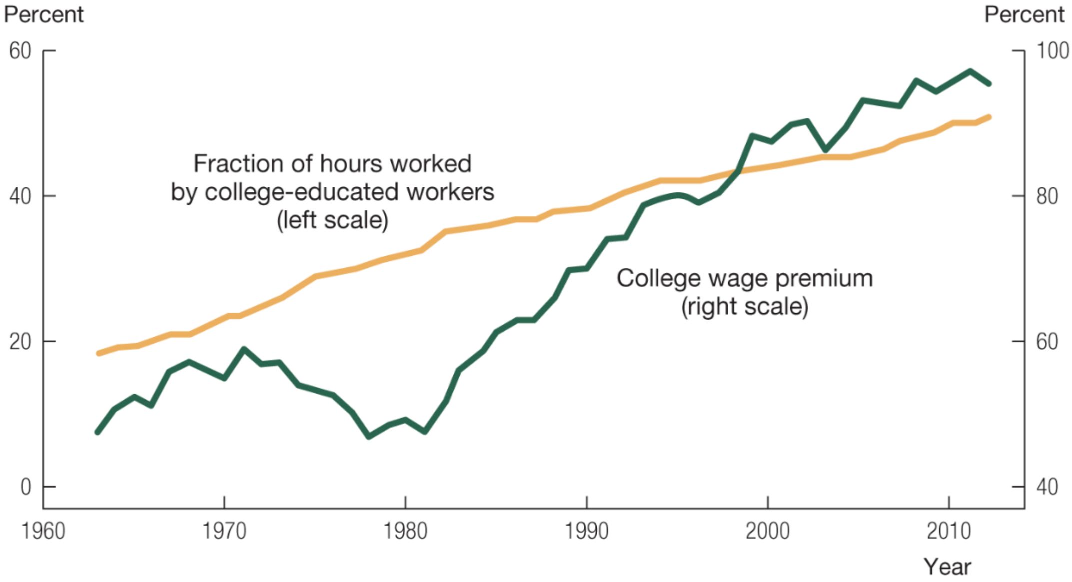
- Average income is $\$63,000$ per year.

- Ignoring growth in wages for now and considering an interest rate of $R = 3\%$, we can calculate the PDV of lifetime income for the US worker.

$$\text{present discounted value} = \$63,000 \times \frac{1 - \left(\frac{1}{1+3\%}\right)^{46}}{1 - \left(\frac{1}{1+3\%}\right)} = \$1.6 \text{ million}$$

- It is actually more than that if you consider that the economy (and wages) is growing over those 45 years!

The Market for College Graduates



The Lucas Model

- "Effective" labor: $(1-\bar{e})h_t\bar{L}$, where:
  - h_t: average level of human capital in society
  - $\bar{e}$: share of time (out of 1) spend accumulating human capital
- Output: $Y_t = AK^\alpha \left([1-\bar{e}]h_t\bar{L}\right)^{1-\alpha}$
- Human capital accumulation: $\Delta h_{t+1} = (h_t)^\gamma \bar{h}\bar{e}$
  - if we take $\gamma < 1$ there is diminishing returns to the accumulation of human capital
  - we will assume $\gamma = 1$, such that there are **no decreasing returns**

The Lucas Model

- Human capital growth:

$$\frac{\Delta h_{t+1}}{h_t} \equiv g_h = \bar{h}\bar{e}, \qquad h_t = h_0(1 + g_h)^t$$

The Lucas Model

- Human capital growth:

$$\frac{\Delta h_{t+1}}{h_t} \equiv g_h = \bar{h}\bar{e}, \qquad h_t = h_0(1+g_h)^t$$

- Output growth:
  Note only h_t is not a constant here...

$$Y_t = AK^{\alpha}\left([1-\bar{e}]h_t\bar{L}\right)^{1-\alpha} \implies g_Y = (1-\alpha)g_h$$

The Lucas Model

- Human capital growth:

$$\frac{\Delta h_{t+1}}{h_t} \equiv g_h = \bar{h}\bar{e}, \qquad h_t = h_0(1+g_h)^t$$

- Output growth:
  Note only h_t is not a constant here...

$$Y_t = AK^{\alpha}\left([1-\bar{e}]h_t\bar{L}\right)^{1-\alpha} \implies g_Y = (1-\alpha)g_h$$

So long-run growth can be sustained!

The Lucas Model

- Wages grow in the long run:

$$w = MPL = (1-\alpha)A\left(\frac{K}{L}\right)^{\alpha}\left([1-\bar{e}]h_0(1+g_h)^t\right)^{1-\alpha}$$

- Intuition:
  - human capital works as improving the **efficiency of labor**
  - while labor itself still has diminishing returns, human capital (under these assumptions) does not
  - as you can see, it basically operates as a growing productivity term!

How Much is Your Human Capital Worth?

- Average wage of HS graduates $\$40,000$.
- Average wage of college graduates $\$70,000$.
- College Premium $= \$30,000$.

$$\text{present discounted value} = \$30,000 \times \frac{1 - \left(\frac{1}{1+3\%}\right)^{46}}{1 - \left(\frac{1}{1+3\%}\right)} = \$765,561$$

Development accounting: solution

- Hendricks and Shoellman find the following solution:

- when worker i migrates from India to the U.S., their human capital stays $\approx$ the same, but the country contribution changes

- If you observe the changes in their wages, you can infer the country contribution!

$$\frac{w_{i,US}}{w_{i,India}} = \frac{(1-\alpha)}{(1-\alpha)} \times \underbrace{\frac{Z_{US}}{Z_{India}}}_{\text{country contribution}} \times \frac{h_i}{h_i} = \underbrace{\frac{Z_{US}}{Z_{India}}}_{\text{country contribution}}$$

Table 3: Human Capital Share in Development Accounting by Subgroups

Robustness Check	Human Capital Share	95% Confidence Interval	N
Panel A: Baseline			
Baseline	0.60	(0.55, 0.64)	907
Panel B: Decomposition by Country			
Ethiopia	0.77	(0.67, 0.86)	41
India	0.63	(0.58, 0.69)	167
Philippines	0.47	(0.39, 0.55)	111
China	0.70	(0.57, 0.83)	63
Panel C: Decomposition by Visa Status			
Employment visa	0.52	(0.46, 0.59)	196
Family visa	0.64	(0.53, 0.74)	148
Diversity visa	0.58	(0.49, 0.67)	186
Other visa	0.58	(0.47, 0.68)	121

Table note: Each column shows the implied human capital share in development accounting (one minus the wage gain at migration relative to the GDP per worker gap); the 95 percent confidence interval for that statistic; and the number of immigrants in the corresponding sample. Each row gives the result from constructing these statistics for a different sample or using different measures of pre-migration wages, post-migration wages, or the GDP per worker gap.

Inflation

percentage change in an economy's overall price level

$$\pi_t \equiv \frac{P_t - P_{t-1}}{P_{t-1}}$$

How to measure prices over time?

- Suppose you know 1 gallon of gas was $\$0.29$ in 1950
- You also know that the consumer price index for 2018 is $P_{2018} = 100$ and for 1950 is $P_{1950} = 9.59$.
  - This means that, on average, prices grew $\frac{P_{2018} - P_{1950}}{P_{1950}} = 942\%$ between those years!
- This is enough information to calculate the price of gasoline in 2018 dollars:

$$P_{gasoline,2018} = P_{gasoline,1950} \frac{P_{2018}}{P_{1950}} = \$_{1950}0.29 \times \frac{\$_{2018}100}{\$_{1950}9.59} = \$_{2018}2.82$$