Econ 110A: Lecture 5

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Last lecture we saw

- What are returns to scale
- What are diminishing marginal returns
- How to set up and solve a simple model
- In the production model, total output is proportional to technology/ideas/institutions A, total capital K and total labor force L, so countries with better technology, more wealth/capital stock, and larger populations will have larger economies

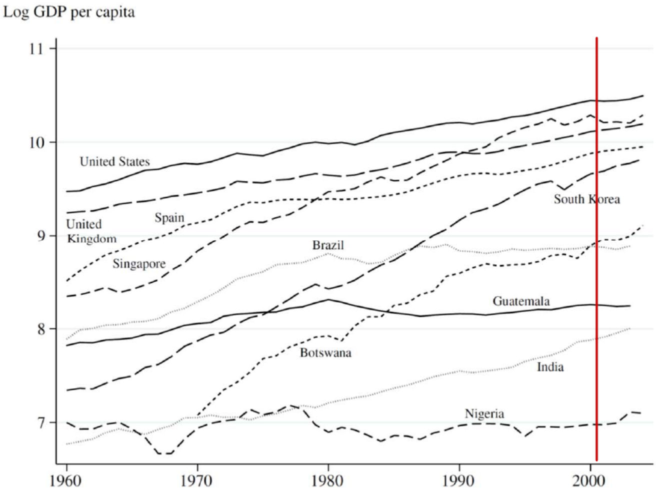
Assumptions of the Production Model

- Firms maximize profits
- Factor markets are competitive: prices are taken as given by firms
- There exists a representative firm (aggregation)
- Factors supply are inelastic to changes in prices
- Prices are determined in equilibrium when supply equals demand

But what can we use this model for?

...and can we explain the large differences in income we observe in the data?

Why do countries have persistent differences in GDP per capita?



Reproduced from Ch. 1 of Daron Acemoglu, “Introduction to Modern Economic Growth,” 2014

Applications of the Production Model

This class we will introduce the use of

Development Accounting

resorting to the Production Model

and using real world data to calibrate the model

GDP per Capita

First, express variables in *per capita terms*:

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so y^* is **strictly concave** in k^*.

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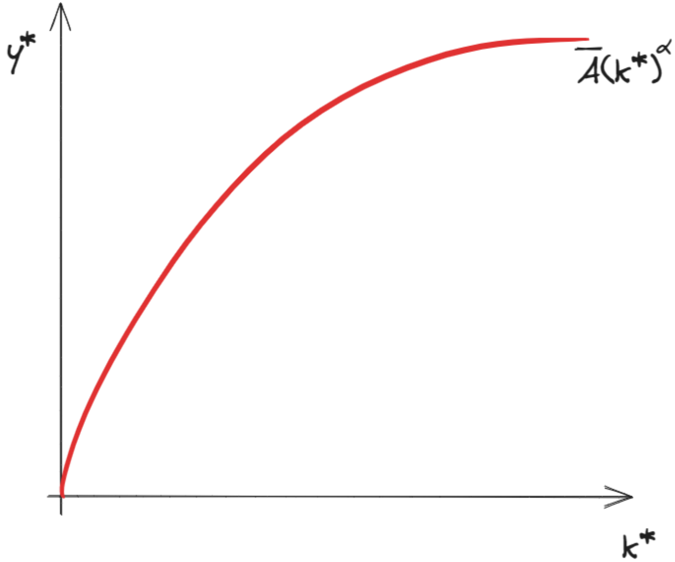

Experiment

Assuming all countries have the same Total Factor of Productivity (TFP), $\bar{A}$, can differences in capital-per-worker, k, explain differences in GDP per-capita, y?

Experiment 1

Strategy:

- set $\bar{A} = 1$;

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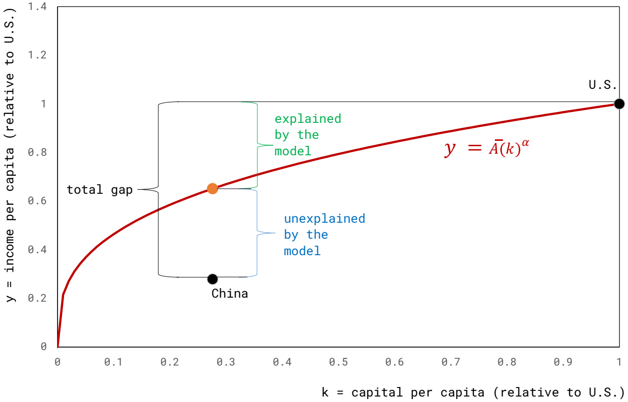
- compare $y_i^{\text{predicted}}$ to y_i and evaluate how the model fares.

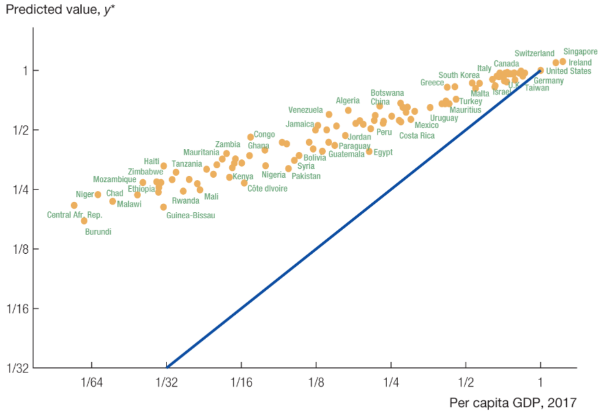
Experiment 1

Country	k_i in data	$y_i^{\text{predicted}} = \bar{A}(k_i)^{\alpha}$	y_i in data
USA	1	1	1
China	0.276	0.651	0.279



Experiment 1 for every country

It gets the trend right but overestimates predicted GDP per capita... why?



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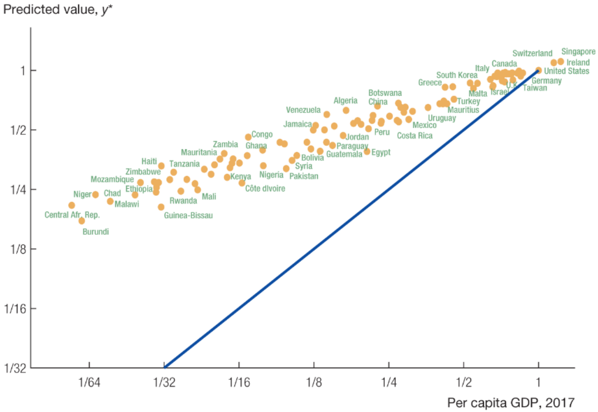

Is it reasonable to assume $\bar{A}$ (technology, institutions, rule of law) is the same in the U.S. and in China, Ethiopia, Brazil? If we do so, could we overestimate GDP per capita in the latter group?

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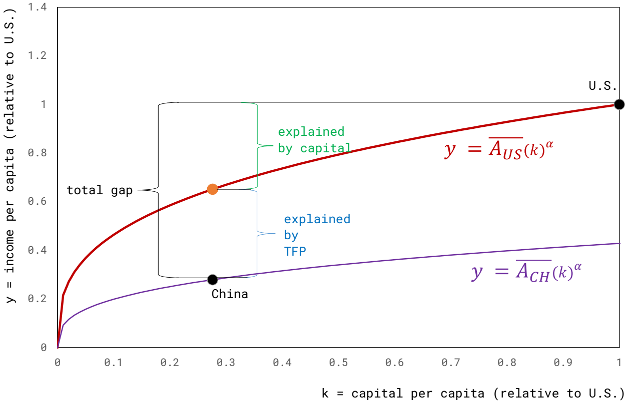
- decompose differences in income in contributions from TFP and contributions from capital differences.

Experiment 2

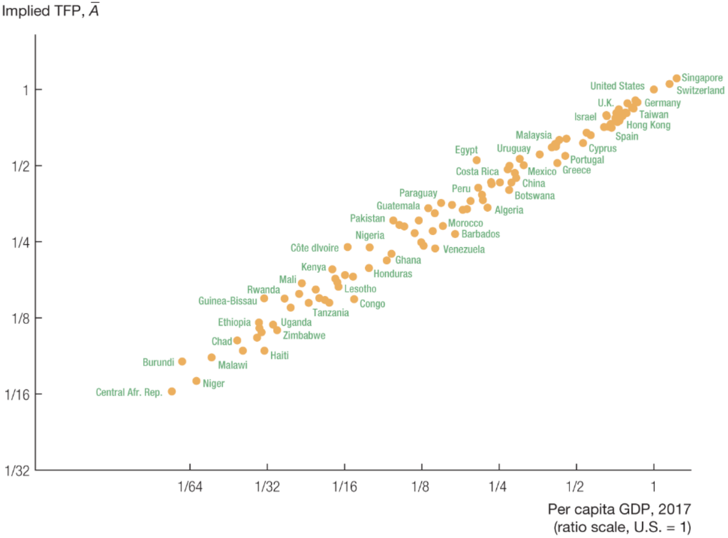
Country	k_i in data	y_i in data	$\bar{A}_i = \frac{y_i}{k_i^{\frac{1}{3}}}$
USA	1	1	1
China	0.276	0.279	0.428



Open Excel File

Experiment 2 with every country

Richer countries have higher productivity



Insight from the Model

$$\underbrace{\frac{y^*_{rich}}{y^*_{poor}}}_{64} = \underbrace{\frac{\bar{A}^*_{rich}}{\bar{A}^*_{poor}}}_{12.8 \approx 13} \times \underbrace{\left(\frac{\bar{k}^*_{rich}}{\bar{k}^*_{poor}}\right)^{\frac{1}{3}}}_{5}$$

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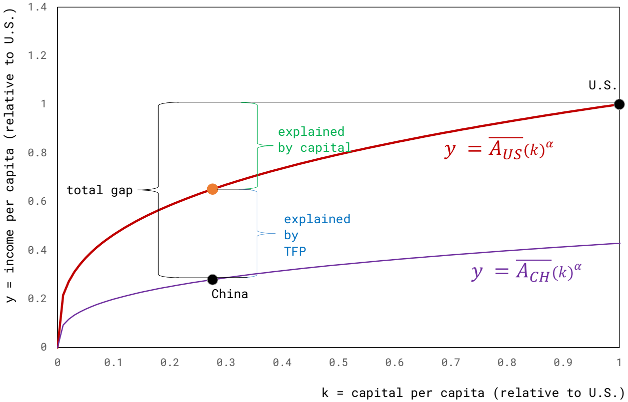
- Why do I write "explained"?

Experiment 2

Wait a minute... this looks familiar...



Experiments 1 and 2

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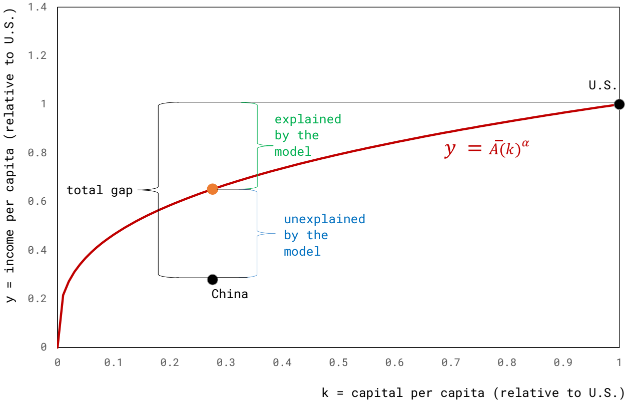

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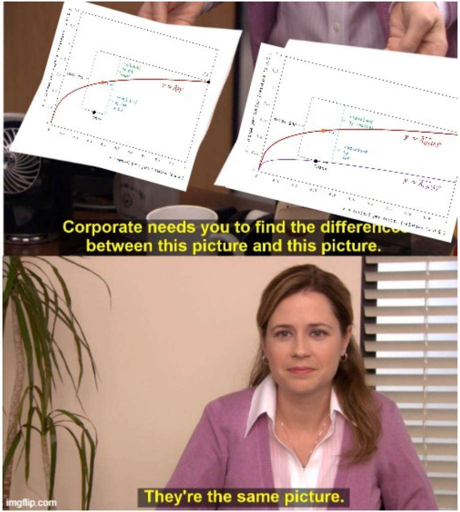

Experiments 1 and 2

- By definition, this is a model that explains income differences by differences in capital stocks...
- TFP is a fancy word for what the model cannot explain... it is a measure of our ignorance.
- But it nonetheless has an economic significance!
- If two countries have the same factors of production (capital and labor), one of them can only have higher output if its factors are **more productive** —hence the name **total factor productivity**.
- But this model does not have a theory for TFP. It is outside of the model.

Why are some countries more efficient in using capital than others?

Differences in per-capita GDP largely due to Total Factor of Productivity (TFP)

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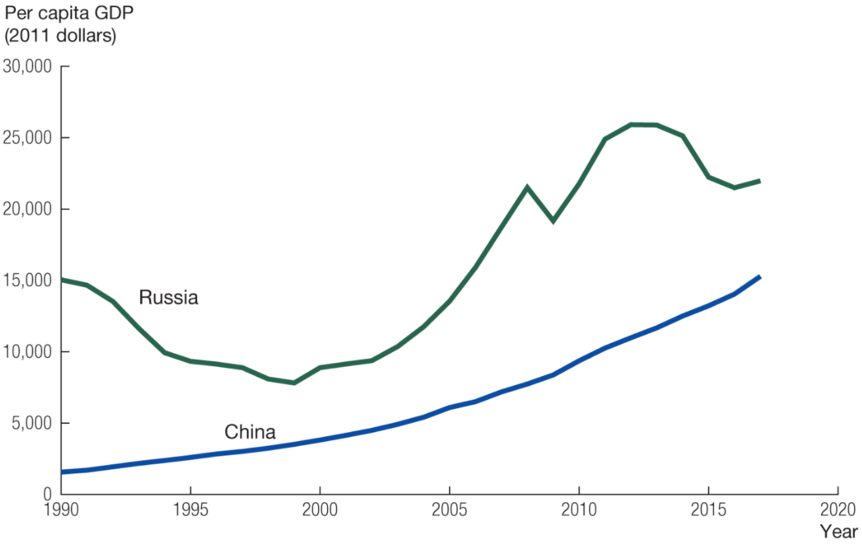
- Institutions
- Human Capital
- Climate
- Innovation/Technology
- Misallocation

Importance of Institutions/Government is Clear...

...but complicated



Source: Penn World Tables, Version 9.1.

What have we learned?

- There are large differences in income across countries
- These differences can be partially explained by differences in capital stocks per capita
- The Production Function model can therefore partially explain income differences
- However, an even more important consideration is the fact that these countries are more effective in using their factors of production – something that is outside this model.