Econ 110A: Lecture 8

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What are the takeaways from the Solow model?

- Determinants of long-run output per-capita: investment (saving) rate and TFP.
- TFP differences still main factor in per-capita income differences across countries
- Transition Dynamics helps understand differences in growth rates across countries
- It does NOT explain sustained long-run growth
- Differences in investment rates, TFP also not explained

What determines the Investment Rate?

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Should Ford build a new plant?

Ford Plant in Dearborn, MI

What determines the Investment Rate?

- Highly complex decision, many factors involved
- Our approach: use the principle of No-Arbitrage

"At market equilibrium, any two active investments must yield the same return."

No-Arbitrage Equation for Investment

Consider a firm thinking of investing in asset (think of it as a big machine) $\$P_{K,t}$ today.

The firm has two options:

- Deposit in a bank the dollar equivalent of $\$P_{K,t}$ in a bank today and earn the returns; or
- Buy the asset, rent it out, earn ($\bar{r}$), incur in depreciation ($\bar{d}$). Furthermore, the machine might change in price between today and tomorrow, so we need to account for the fact that in the change in returns $\$P_{K,t+1} - \$P_{K,t}$.

By non-arbitrage, these two are equal (we will debate more why in a bit):

$$\underbrace{\$P_{K,t}(1+R) - \$P_{K,t}}_{\text{return on bank deposit}} \underbrace{=}_{\substack{\text{non-arbitrage}\\ \text{condition}}} \underbrace{\bar{r}\$P_{K,t} - \bar{d}\$P_{K,t} + \$P_{K,t+1} - \$P_{K,t}}_{\text{return on physical capital}}$$

Non-Arbitrage Equation for Investment

Let us manipulate this equation a bit to simplify our non-arbitrage condition.

$$\$P_{K,t}(1+R) - \$P_{K,t} = \bar{r}\$P_{K,t} - \bar{d}\$P_{K,t} + \$P_{K,t+1} - \$P_{K,t}$$

Let $\frac{\$P_{K,t+1} - \$P_{K,t}}{\$P_{K,t}} \equiv \frac{\Delta\$P_{K,t+1}}{\$P_{K,t}}$. Then, diving through by $\$P_{K,t}$:

$$R = \bar{r} - \bar{d} + \Delta\$P_{K,t+1} \iff \bar{r} \equiv MPK = \underbrace{R + \bar{d} - \frac{\Delta\$P_{K,t+1}}{\$P_{K,t}}}_{\text{user cost of capital}}$$

- R: opportunity cost of funds
- $\bar{d}$: depreciation cost
- $\frac{\Delta\$P_{K,t+1}}{\$P_{K,t}}$: capital gain (+) or loss (-)

What is the user cost of capital

Intuition:

Minimum return necessary to justify a given investment rather than putting it in the bank; or

Estimate of cost of increasing the firm's capital stock in one unit if the firm **owns** capital (marginal cost of capital) rather than rents it out in the market!

$$\bar{r} \equiv MPK = \underbrace{R + \bar{d} - \Delta\$P_{K,t+1}}_{\text{user cost of capital}}$$

- R: opportunity cost of funds
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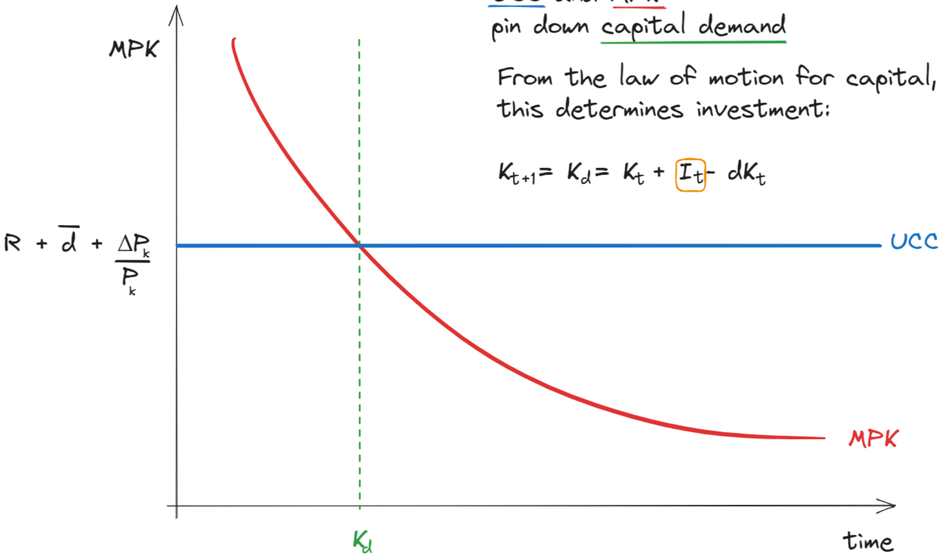
How does this determine investment?



UCC and MPK pin down capital demand

From the law of motion for capital, this determines investment:

$$K_{t+1} = K_d = K_t + \boxed{I_t} - dK_t$$

Midterm Review

Two-Period Neoclassical Growth Model

The consumer maximizes lifetime utility subject to the intertemporal budget constraint

$$\begin{aligned} \max_{\{C_1,C_2\}} \quad & U(C_1) + \beta U(C_2) \\ s.t. \quad & Y_1 + \frac{Y_2}{1+R} = C_1 + \frac{C_2}{1+R} \end{aligned}$$

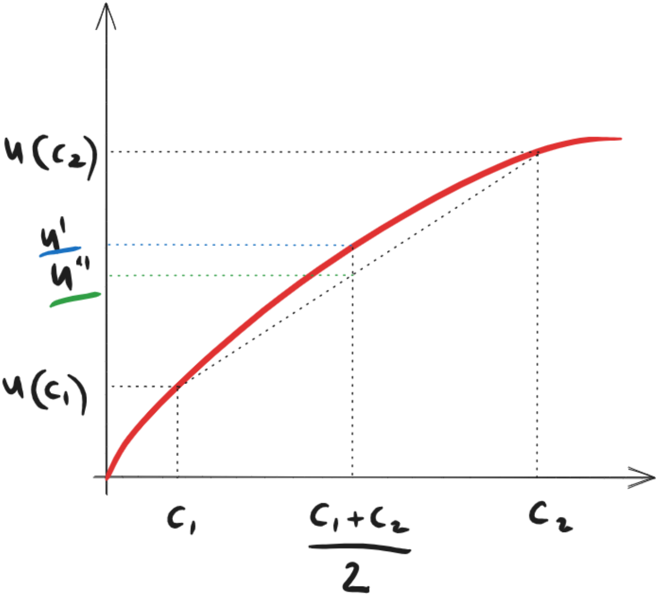
Solution, using Lagrangian :

$$\mathcal{L} = U(C_1) + \beta U(C_2) + \lambda \left[Y_1 + \frac{Y_2}{1+R} - C_1 - \frac{C_2}{1+R} \right]$$

Take first order conditions, combine (will do in discussion section) into the Euler Equation (EE):

$$U'(C_1) = \beta(1+R)U'(C_2)$$

Two-Period Neoclassical Growth Model



$$u' = u\left(\frac{c_1+c_2}{2}\right) > \frac{1}{2}\left(u(c_1) + u(c_2)\right) = u''$$

Gross Domestic Product: market value of the final goods and services produced in an economy over a certain period of time.

Three methods of calculation

Production = Income = Expenditure

- Production: value added produced
- GDP by Value Added (=Sales − Cost of Inputs)
- Income: remuneration to factors of production
- GDP by Incomes (=Wages + Net Taxes + Profits + Depreciation)
- Expenditure: end-use of value added produced
- GDP by Expenditure (=$C + G + I + X - M$)

The Role of Prices: Comparing GDP Across Time

- Nominal GDP:

$$
\begin{aligned}
GDP_t &= \sum_i P_{i,t} \times Q_{i,t} \\
&\text{for } i \in \{\text{food, rent, cars, haircuts, clothes}, \cdots\} \\
&\text{for } t \in \{1951, 1952, \cdots, 2021, 2022, 2023\}
\end{aligned}
$$

- Real GDP:

$$RGDP_t = \sum_i P_{i,X} \times Q_{i,t}$$

  - Initial Price method (Laspeyres): $P_{i,X}$ are earliest date prices
  - Final Price method (Paasche): $P_{i,X}$ are latest date prices
  - Chained-Weighted method: $P_{i,X}$ are "weighted" averages across dates

The Role of Prices: Comparing GDP Across Countries

Now, dropping the \$ and ¥ to accommodate any potential currency in the world:

$$GDP^{CH}_{t,P^{US}} = GDP^{CH}_{t} \times E_t \times \frac{P^{US}_t}{P^{CH}_t}$$

where

- $GDP^{CH}_{t,P^{US}}$: foreign GDP in U.S. dollars and in U.S. prices
- E_t: exchange rate (U.S. dollar per foreign currency)
- $\frac{P^{US}_t}{P^{CH}_t}$: Price Level Ratio GDP Conversion Factor (prices in the U.S. relative to prices in foreign country)

What are the limits to GDP?

- Inequality?
- Environment?
- Home production?
- Health?
- Education?
- Capital stocks?
- Does it still say a lot about all of the about all of the above?

Computing a Compounded Constant Growth Rate

Suppose we know y_0 (initial level) and y_t (current level). How do we compute the compounded constant growth rate g from 0 to t?

$$y_t = y_0 \cdot (1+g)^t \iff g = \left(\frac{y_t}{y_0}\right)^{\frac{1}{t}} - 1$$

In the U.S., take $y_0 = y_{1870}^{US} = \$5,000$, and $y_t = y_{2015}^{US} = \$50,800$. Then:

$$g^{US} = \left(\frac{y_{2015}^{US}}{y_{1870}^{US}}\right)^{\frac{1}{165}} - 1 = \left(\frac{50,800}{5,000}\right)^{\frac{1}{165}} - 1 = 0.0193 \approx 2\%$$

Remember:

$$\ln[y_t] \approx t \cdot g + \ln[y_0]$$

Facts about long-run growth

- Fact 1: Growth is a relatively recent phenomenon
- Fact 2: Continued persistent growth at the “frontier”
- Fact 3: We observe heterogeneous growth experiences
- Fact 4: Average GDP per person diverged until 2000 and has been converging since then
- Fact 5: Initial divergence seems to be as old as the Industrial Revolution
- Fact 6: Conditional Convergence in the West, but it doesn’t generalize: Lack of Conditional Convergence Globally!
- Fact 7: Lower Fraction of World Population Living in Poverty

Production Model: The Production Function

- we are interested in modeling the production of value added

- from the income approach to GDP, we know what factors are ultimately responsible for creation of value e.g.: labor, management, capital, government

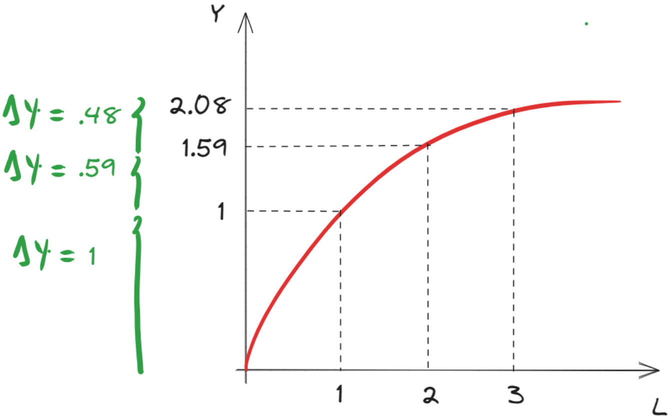
- this suggests a "factor-based" representation of the production function

$$\underbrace{Y}_{\substack{\text{output}\\\text{value added}}} = F(\underbrace{A}_{\substack{\text{technology}\\\text{institutions}\\\text{ideas}}}, \underbrace{K}_{\text{capital}}, \underbrace{L}_{\text{labor}})$$

- $F(A, K, L)$ is your production function. If $F(\cdot, \cdot, \cdot)$ is Cobb-Douglas, then:

$$Y = A \cdot K^{\alpha} L^{1-\alpha}, \qquad 0 < \alpha < 1$$

Diminishing Marginal Product



Diminishing Marginal Product: Extra output produced by increasing one factor while keeping all the other fixed is decreasing in the one increasing factor.

Production Model: General Equilibrium

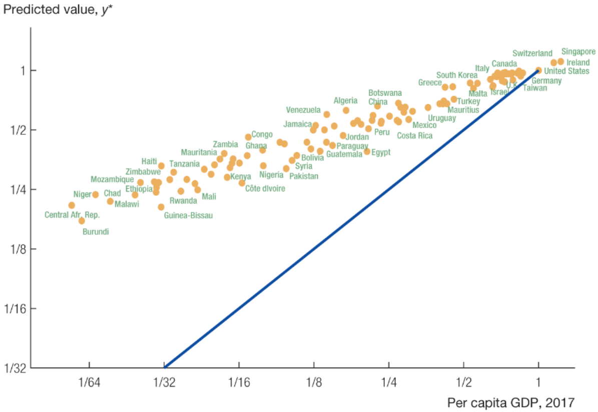
Endogeneous Variables: Y, K, L, w, r

Five equations for five unknowns

$$\begin{aligned} Y &= \bar{A}(K)^{\alpha}(L)^{1-\alpha} && (1) \\ P(1-\alpha)\bar{A}\left(\frac{K}{L}\right)^{\alpha} &= w && (2) \\ P\alpha\bar{A}\left(\frac{L}{K}\right)^{1-\alpha} &= r && (3) \\ L &= \bar{L} = L^{s} && (4) \\ K &= \bar{K} = K^{s} && (5) \end{aligned}$$

Production Model: Experiment 1 for every country

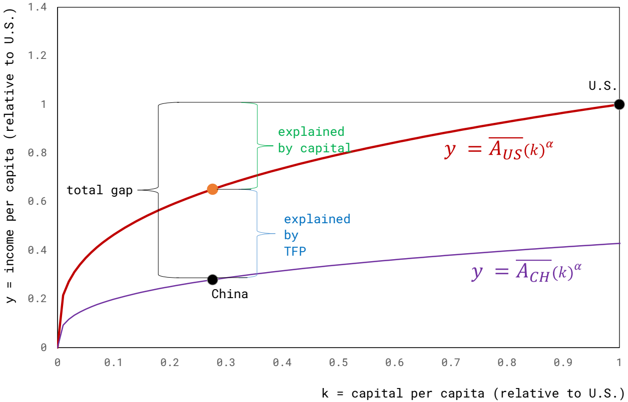
It gets the trend right but overestimates predicted GDP per capita... why?



Is it reasonable to assume $\bar{A}$ (technology, institutions, rule of law) is the same in the U.S. and in China, Ethiopia, Brazil? If we do so, could we overestimate GDP per capita in the latter group?

Production Model: Experiment 2

Country	k_i in data	y_i in data	$\bar{A}_i = \frac{y_i}{k_i^{\frac{1}{3}}}$
USA	1	1	1
China	0.276	0.279	0.428



Insight from the Production Model

$$\underbrace{\frac{y^*_{rich}}{y^*_{poor}}}_{64} = \underbrace{\frac{\bar{A}^*_{rich}}{\bar{A}^*_{poor}}}_{13} \times \underbrace{\left(\frac{\bar{k}^*_{rich}}{\bar{k}^*_{poor}}\right)^{\frac{1}{3}}}_{5}$$

- Per Capita GDP of 5 richest countries is 64 times that of 5 poorest
- Capital per person explains a factor of about 5 of this difference
- The rest, a factor of 13, is "explained" by differences in TFP
- Why do I write "explained"?

The Solow Growth Model: Taking Stock

Normalizing the price of the output good $P_t = 1$ each period, for each period $t \in \{0, 1, 2, \cdots\}$, given parameters $\bar{d}$, $\bar{s}$, $\bar{A}$, $\bar{L}$, α and the initial value of capital K_0 there are five unknowns Y_t, K_{t+1}, L_t, C_t, I_t and five equations:

$$
\begin{aligned}
Y_t &= \bar{A} K_t^{\alpha} L_t^{1-\alpha} && (6)\\
Y_t &= C_t + I_t && (7)\\
\Delta K_{t+1} &= I_t - \bar{d} \cdot K_t && (8)\\
L_t &= \bar{L} && (9)\\
I_t &= \bar{s} Y_t && (10)
\end{aligned}
$$

that characterize the solution to this model.

Solving the Solow Growth Model

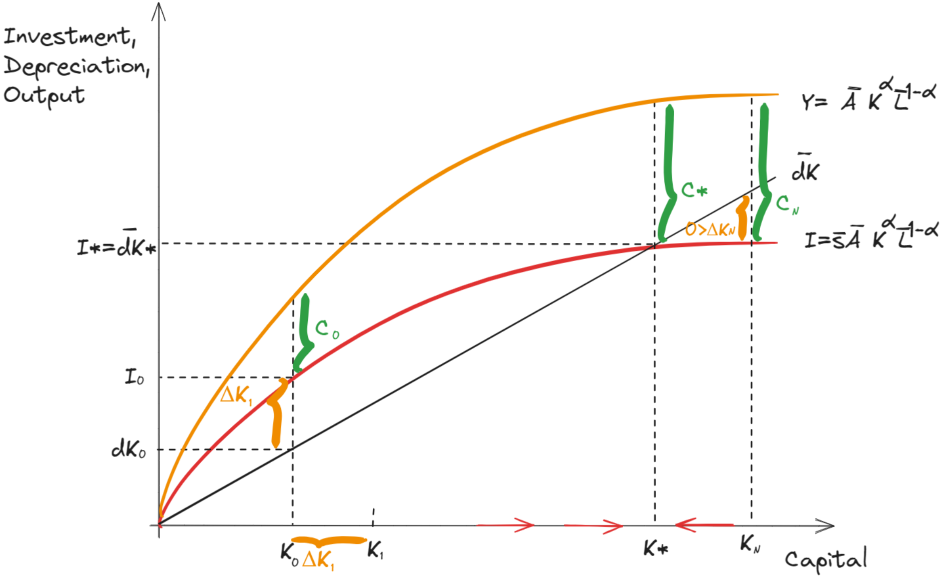
Strategy: Reduce system of equations from five to two

- Equations (2) and (5) are redundant, not independent "Walras' Law" (reduces the system to 4)
- Plug-in (4) into (1), becomes $Y_t = \bar{A}K_t^{\alpha}\bar{L}^{1-\alpha}$ (reduces the system to 3)
- Plug-in (5) into (3), using above, becomes $\Delta K_{t+1} = \bar{s}\bar{A}K_t^{\alpha}\bar{L}^{1-\alpha} - \bar{d} \cdot K_t$ (reduces the system to 2)

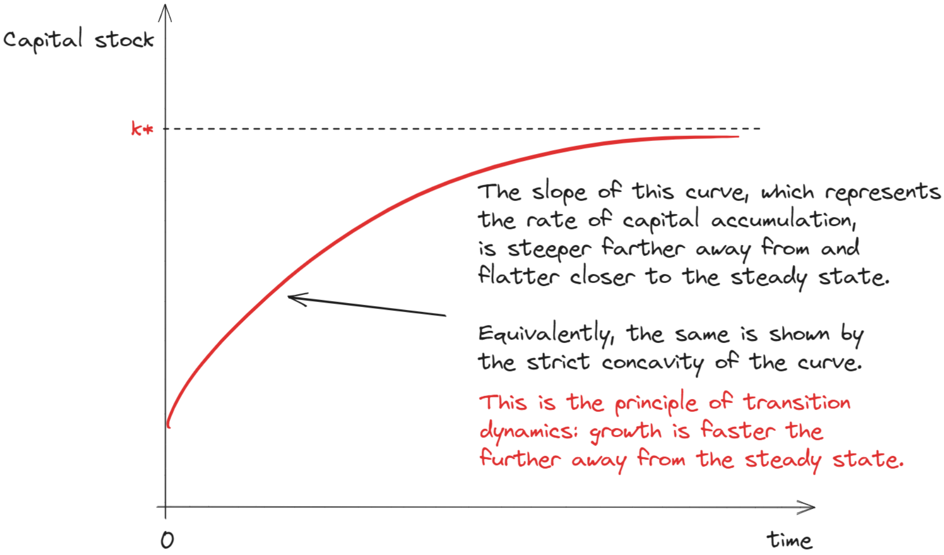
Final system:

$$\begin{aligned} Y_t &= \bar{A}K_t^{\alpha}\bar{L}^{1-\alpha} \\ \Delta K_{t+1} &= \bar{s}\bar{A}K_t^{\alpha}\bar{L}^{1-\alpha} - \bar{d} \cdot K_t \end{aligned}$$

Solow Model: The Complete Diagram



Solow Model: principle of transition dynamics



Solow Model: The Steady State

$$\begin{aligned} Y_t &= \bar{A}K_t^{\alpha}\bar{L}^{1-\alpha} \\ \Delta K_{t+1} &= \bar{s}\bar{A}K_t^{\alpha}\bar{L}^{1-\alpha} - \bar{d}\cdot K_t \end{aligned}$$

At the Steady-State (SS), $\Delta K_{t+1} = 0$ and $K_{t+1} = K_t$, so we might as well call it K^*. The same is true for Y, so we call it Y^*. Let us look for K^*, Y^* that satisfy the definition of a SS in the system above:

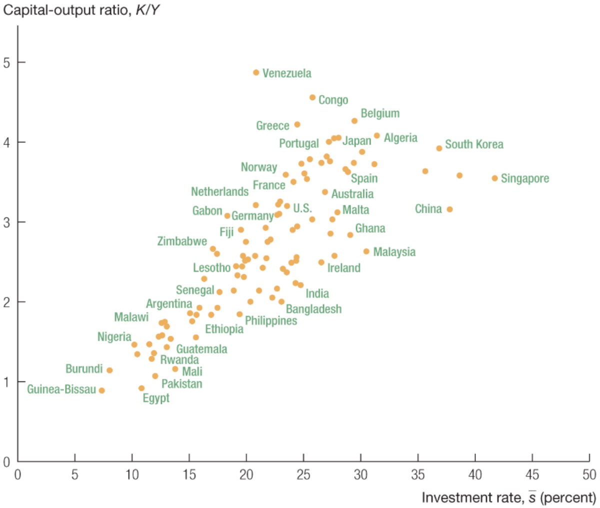
$$\begin{aligned} \bar{s}\bar{A}(K^*)^{\alpha}\bar{L}^{1-\alpha} - \bar{d}\cdot K^* = 0 \iff K^* &= \left(\frac{\bar{s}\bar{A}}{\bar{d}}\right)^{\frac{1}{1-\alpha}}\bar{L} \\ \implies Y^* &= \bar{A}^{\frac{1}{1-\alpha}}\left(\frac{\bar{s}}{\bar{d}}\right)^{\frac{\alpha}{1-\alpha}}\bar{L} \end{aligned}$$

Solow Model: The Steady State

Note that the model predicts that the capital-to-output ratio is increasing in the investment rate:

$$\frac{K^*}{Y^*} = \frac{\bar{s}}{\bar{d}}$$

In the data, there is indeed a positive correlation between those variables.



Source: Penn World Tables, Version 9.1. The capital-output ratio is measured in the year 2017, while the investment rate is averaged over the period 1990 to 2017.

Solow Model: The Steady State

Other predictions of the model do not have a great fit...

$$y^* \equiv \frac{Y^*}{\bar{L}} = \bar{A}^{\frac{1}{1-\alpha}} \left(\frac{\bar{s}}{\bar{d}}\right)^{\frac{\alpha}{1-\alpha}}$$

Now assume $\bar{d}_{rich} = \bar{d}_{poor}$ and let us make a similar decomposition as we did with the production model:

$$\underbrace{\frac{y^*_{rich}}{y^*_{poor}}}_{64} = \underbrace{\left(\frac{\bar{A}_{rich}}{\bar{A}_{poor}}\right)^{\frac{3}{2}}}_{32} \times \underbrace{\left(\frac{\bar{s}_{rich}}{\bar{s}_{poor}}\right)^{\frac{1}{2}}}_{2}$$

while in the production model

$$\underbrace{\frac{y^*_{rich}}{y^*_{poor}}}_{64} = \underbrace{\frac{\bar{A}_{rich}}{\bar{A}_{poor}}}_{13} \times \underbrace{\left(\frac{\bar{k}^*_{rich}}{\bar{k}^*_{poor}}\right)^{\frac{1}{3}}}_{5}$$