Econ 110A: Lecture 9 (Mid-term Review)

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The consumer maximizes lifetime utility subject to the intertemporal budget constraint

$$\max_{\{C_1, C_2\}} \quad \log(C_1) + \beta \log(C_2)$$

s.t. $Y_1 + \frac{Y_2}{1+R} = C_1 + \frac{C_2}{1+R}$

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Replace $C_1 = Y_1 + \frac{Y_2}{1+R} - \frac{C_2}{1+R}$, problem becomes:

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Solution:

$$\underbrace{\frac{1}{\underbrace{Y_1 + \frac{Y_2}{1+R} - \frac{C_2}{1+R}}}_{=C_1} \times \left(-\frac{1}{1+R}\right) + \beta \frac{1}{C_2} = 0$$
(1)

$$\operatorname{Let} g(C_{2}) \equiv \log \left(Y_{1} + \frac{Y_{2}}{1+R} - \frac{C_{2}}{1+R} \right) + \beta \log (C_{2})$$

$$g(C_{2})^{\wedge} \qquad g'(C_{2}) = \frac{1}{\underbrace{Y_{1} + \frac{Y_{2}}{1+R} - \frac{C_{2}}{1+R}}_{=C_{1}}} \times \left(-\frac{1}{1+R} \right) + \beta \frac{1}{C_{2}} = 0$$

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- Suppose that we specify $U(C) = 4C^{1/4}$, $\beta = 1$, R = .1, $Y_1 = Y_2 = 100$. Compute the optimal consumption C_1 , C_2 and the amount of income saved (or borrowed) by the household in period 1. Please show all your work. (note that $U'(C) = C^{-3/4}$).

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- Finally:
$$S = Y_1 - C_1 = 100 - 93.93 = 6.07$$
.

An introduction to GDP

Gross Domestic Product: market value of the final goods and services produced in an economy over a certain period of time.

Three methods of calculation

Production = Income = Expenditure

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- Production: value added produced
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- Income: remuneration to factors of production
- GDP by Incomes (=Wages + Net Taxes + Profits + Depreciation)
- Expenditure: end-use of value added produced
- GDP by Expenditure (=C + G + I + X M)

The Role of Prices: Comparing GDP Across Time

- Nominal GDP:

$$egin{array}{rcl} GDP_t &=& \displaystyle\sum_i P_{i,t} imes oldsymbol{Q}_{i,t} \ & ext{for } i \in \{ ext{food, rent, cars, haircuts, clothes, } \cdots \} \ & ext{for } t \in \{ ext{1951, 1952, } \cdots , ext{2021, 2022, 2023} \} \end{array}$$

- Real GDP:

$$RGDP_t = \sum_i P_{i,X} \times Q_{i,t}$$

- Initial Price method (Laspeyres): $P_{i,X}$ are earliest date prices
- Final Price method (Paasche): $P_{i,X}$ are latest date prices
- Chained-Weighted method: $P_{i,X}$ are "weighted" averages across dates

The Role of Prices: Comparing GDP Across Countries

Now, dropping the \$ and ¥ to accommodate any potential currency in the world:

$$GDP_{t,P^{US}}^{CH} = GDP_t^{CH} \times E_t \times \frac{P_t^{US}}{P_t^{CH}}$$

where

- $GDP_{t,PUS}^{CH}$: foreign GDP in U.S. dollars and in U.S. prices
- *E*_t: exchange rate (U.S. dollar per foreign currency)
- $\frac{P_t^{US}}{P_t^{CH}}$: Price Level Ratio GDP Conversion Factor (prices in the U.S. relative to prices in foreign country)

Computing a Compounded Constant Growth Rate

Suppose we know y_0 (initial level) and y_t (current level). How do we compute the compounded constant growth rate g from 0 to t?

$$y_t = y_0 \cdot (1+g)^t \iff g = \left(\frac{y_t}{y_0}\right)^{\frac{1}{t}} - 1$$

In the U.S., take $y_0 = y_{1870}^{US} = \$5,000$, and $y_t = y_{2015}^{US} = \$50,800$. Then:

$$g^{US} = \left(\frac{y_{2018}^{US}}{y_{1870}^{US}}\right)^{\frac{1}{148}} - 1 = \left(\frac{61,000}{3,600}\right)^{\frac{1}{148}} - 1 = 0.0193 \approx 2\%$$

Remember:

$$\ln(y_t) - \ln(y_0) = t \ln(1+g) \approx t \cdot g$$
 for small g

Facts about long-run growth

- Fact 1: Growth is a relatively recent phenomenon
- Fact 2: Continued persistent growth at the "frontier"
- Fact 3: We observe heterogeneous growth experiences
- Fact 4: Average GDP per person diverged until 2000 and has been converging since then
- Fact 5: Initial divergence seems to be as old as the Industrial Revolution
- Fact 6: Conditional Convergence in the West, but it doesn't generalize: Lack of Conditional Convergence Globally!
- Fact 7: Lower Fraction of World Population Living in Poverty

Production Model: The Production Function

- we are interested in modeling the production of value added
- from the income approach to GDP, we know what factors are ultimately responsible for creation of value e.g.: labor, management, capital, government
- this suggests a "factor-based" representation of the production function

$$\underbrace{Y}_{\text{output}} = F(\underbrace{A}_{\text{technology capital labor}}, \underbrace{K}_{\text{technology capital labor}})$$

value added institutions ideas

- F(A, K, L) is your production function. If $F(\cdot, \cdot, \cdot)$ is Cobb-Douglas, then:

$$Y = A \cdot K^{\alpha} L^{1-\alpha}, \qquad 0 < \alpha < 1$$

Diminishing Marginal Product



Diminishing Marginal Product: Extra output produced by increasing one factor while keeping all the other fixed is decreasing in the one increasing factor. Production Model: General Equilibrium

Endogeneous Variables: Y, K, L, w, r

Five equations for five unknowns

$$Y = \bar{A}(K)^{\alpha}(L)^{1-\alpha}$$
(2)

$$P(1-\alpha)\bar{A}\left(\frac{K}{L}\right)^{\alpha} = w$$
(3)

$$P\alpha\bar{A}\left(\frac{L}{K}\right)^{1-\alpha} = r$$
(4)

$$L = \bar{L} = L^{s}$$
(5)

$$K = \bar{K} = K^{s}$$
(6)

Production Model: Experiment 1 for every country

It gets the trend right but overestimates predicted GDP per capita... why?



Is it reasonable to assume \bar{A} (technology, institutions, rule of law) is the same in the U.S. and in China, Ethiopia, Brazil? If we do so, could we overestimate GDP per capita in the latter group?

Production Model: Experiment 2

Country	<i>k_i</i> in data	<i>y</i> _i in data	$ar{A}_i = rac{y_i}{k_i^{rac{1}{3}}}$
USA	1	1	1
China	0.276	0.279	0.428



Insight from the Production Model



- Per Capita GDP of 5 richest countries is 64 times that of 5 poorest
- Capital per person explains a factor of about 5 of this difference
- The rest, a factor of 13, is "explained" by differences in TFP
- Why do I write "explained"?

The Solow Growth Model: Taking Stock

Normalizing the price of the output good $P_t = 1$ each period, for each period $t \in \{0, 1, 2, \dots\}$, given parameters \overline{d} , \overline{s} , \overline{A} , \overline{L} , α and the initial value of capital K_0 there are five unknowns Y_t , K_{t+1} , L_t , C_t , I_t and five equations:

$$Y_t = \bar{A} K_t^{\alpha} L_t^{1-\alpha}$$
(7)

$$Y_t = C_t + I_t \tag{8}$$

$$\Delta K_{t+1} = I_t - \bar{d} \cdot K_t \tag{9}$$

$$L_t = \bar{L} \tag{10}$$

$$I_t = \bar{s}Y_t \tag{11}$$

that characterize the solution to this model.

Solving the Solow Growth Model

Strategy: Reduce system of equations from five to two

- Equations (2) and (5) are redundant, not independent "Walras' Law" (reduces the system to 4)
- Plug-in (4) into (1), becomes $Y_t = \bar{A}K_t^{\alpha}\bar{L}^{1-\alpha}$ (reduces the system to 3)
- Plug-in (5) into (3), using above, becomes $\Delta K_{t+1} = \bar{s}\bar{A}K_t^{\alpha}\bar{L}^{1-\alpha} \bar{d}\cdot K_t$ (reduces the system to 2)

Final system:

$$Y_t = \bar{A} K_t^{\alpha} \bar{L}^{1-\alpha}$$

$$\Delta K_{t+1} = \bar{s} \bar{A} K_t^{\alpha} \bar{L}^{1-\alpha} - \bar{d} \cdot K_t$$

Solow Model: The Complete Diagram



Solow Model: principle of transition dynamics



Solow Model: The Steady State

$$Y_t = \bar{A} K_t^{\alpha} \bar{L}^{1-\alpha}$$

$$\Delta K_{t+1} = \bar{s} \bar{A} K_t^{\alpha} \bar{L}^{1-\alpha} - \bar{d} \cdot K_t$$

At the Steady-State (SS), $\Delta K_{t+1} = 0$ and $K_{t+1} = K_t$, so we might as well call it K^* . The same is true for Y, so we call it Y^* . Let us look for K^* , Y^* that satisfy the definition of a SS in the system above:

$$\bar{s}\bar{A}(K^*)^{\alpha}\bar{L}^{1-\alpha} - \bar{d}\cdot K^* = 0 \iff K^* = \left(\frac{\bar{s}\bar{A}}{\bar{d}}\right)^{\frac{1}{1-\alpha}}\bar{L}$$
$$\implies Y^* = \bar{A}^{\frac{1}{1-\alpha}}\left(\frac{\bar{s}}{\bar{d}}\right)^{\frac{\alpha}{1-\alpha}}\bar{L}$$

Solow Model: The Steady State

Note that the model predicts that the capital-to-output ratio is increasing in the investment rate:

 $\frac{K^*}{Y^*} = \frac{\bar{s}}{\bar{d}}$

In the data, there is indeed a positive correlation between those variables.



Solow Model: The Steady State

Other predictions of the model do not have a great fit...

$$\mathbf{y}^{*}\equivrac{\mathbf{Y}^{*}}{ar{L}}=ar{\mathbf{A}}^{rac{1}{1-lpha}}\left(rac{ar{\mathbf{s}}}{ar{\mathbf{d}}}
ight)^{rac{lpha}{1-lpha}}$$

Now assume $\bar{d}_{rich} = \bar{d}_{poor}$ and let us make a similar decomposition as we did with the production model:

$$\frac{\underline{y_{rich}^{*}}}{\underbrace{y_{poor}^{*}}_{64}} = \underbrace{\left(\frac{\bar{A}_{rich}}{\bar{A}_{poor}}\right)^{\frac{3}{2}}}_{32} \times \underbrace{\left(\frac{\bar{s}_{rich}}{\bar{s}_{poor}}\right)^{\frac{1}{2}}}_{2}$$

while in the production model

$$\frac{\underline{y_{\textit{rich}}^*}}{\underbrace{y_{\textit{poor}}^*}_{64}} = \underbrace{\frac{\bar{A}_{\textit{rich}}}{\bar{A}_{\textit{poor}}}}_{12.8 \approx 13} \times \underbrace{\left(\frac{\bar{k}_{\textit{rich}}^*}{\bar{k}_{\textit{poor}}^*}\right)^{\frac{1}{3}}}_{5}$$

Experiment 2: Increase in Depreciation Rate

- Suppose the economy starts at the steady-state K^* , Y^* , I^*
- But then, the depreciation rate increases to $\bar{d}' > \bar{d}$
- What happens?
- The depreciation curve shifts up $\bar{d}'K > \bar{d}K$
- Everything else stays the same because while the other variables will change, they will be changes along the curves, not changes of the curves.

Experiment 2: Increase in Depreciation Rate



Experiment 2: Increase in Depreciation Rate

Predictions:

- Depreciation rate \uparrow
- Total depreciation increases \uparrow immediately
- Capital stock \downarrow over time (it is now optimal to have a smaller capital stock)
- Investment drops \downarrow over time (you do so with smaller investment)
- Output \downarrow over time (as a consequence you have smaller output)

Experiment 2: Increase in Depreciation Rate, over time



What is the user cost of capital Intuition:

Minimum return necessary to justify a given investment rather than putting it in the bank; or

Estimate of cost of increasing the firm's capital stock in one unit if the firm **owns** capital (marginal cost of capital) rather than rents it out in the market!

$$\bar{r} \equiv MPK = \underbrace{R + \bar{d} - \frac{\Delta \$ P_{K,t+1}}{\$ P_{K,t}}}_{\text{upper part of capital}}$$

user cost of capital

- *R*: opportunity cost of funds
- \bar{d} : depreciation cost
- $\frac{\Delta \$ P_{K,t+1}}{\$ P_{K,t}}$: capital gain (+) or loss (-)

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- Two parts for the intuition:
 - Financial cost: it is more expensive to borrow money to finance investments
 - Opportunity cost: returns on financial investments are higher, so funds move away from real investments into financial investments up to the point that they are equal (nonarbitrage)

