Industrial Organization - Homework I

Chalup, Miguel & Gao, Titus

Octuber 9, 2024

1 Logit Model

Consider the homogeneous-consumer logit model:

$$u_{ijt} = x_{jt}\beta + \alpha p_{jt} + \xi_{jt} + \epsilon_{ijt}$$

Here the notation is the same as in Handbook Chapter 2 and the lecture slides. Specifically:

- u_{ijt}: utility of consumer i of product j in market t
- x_{jt}: observed characteristics of product j in market t (note: this should **include a constant** as well as the variable 'sugar')
- p_{jt}: price of product j in market t
- ξ_{jt}: unobserved demand shifter
- ϵ_{ijt}: i.i.d. logit draws

Instructions:

1. Estimate the above model using OLS and report the coefficients (no need to report standard errors).

Table 1: OLS-Logit estimation

	Estimate	SE	t-Stat	p-Value
Constant	-2.9665	0.10855	-27.328	2.901e-142
Price	-10.204	0.87555	-11.654	1.6144e-30
Sugar	0.046306	0.0043923	10.543	2.1205e-25

2. Estimate the above model using 2SLS, instrumenting for price using the instruments: `demand_instruments0, ..., demand_instruments19`. Report the coefficients (no need to report standard errors).

Table 2: IV-Logit estimation

	Estimate	SE	t-Stat	p-Value
Constant	-2.8303	0.10862	-26.056	4.768e-131
Price	-11.385	0.87589	-12.999	2.6442e-37
Sugar	0.047735	0.0043592	10.951	3.1646e-27

The OLS-Logit regression provides a consistent estimate of the parameters under the assumptions that: 1) our utility model is correctly specified (with everything this implies for a random utility logit model), and 2) $E[\xi|X] = 0$. Therefore, a main concern is that price may be set as a function of ξ_{jt}, which is unobservable to the econometrician, but potentially known to the firm.

In the IV-Logit regression, we use instrumental variable to avoid the plausible bias. Results still qualitatively similar, although the effect of prices on utility, and therefore demand, is a little higher in absolute value.

3. Using your estimated coefficients from Part (b), compute own-price elasticities for each product in the market 'C01Q1'. Draw a scatterplot with the computed own-price elasticities on the y-axis and prices on the x-axis (each dot on the scatterplot should correspond to an individual product in the market 'C01Q1'). Discuss the observed relationship between prices and the own-price elasticities predicted by the model.

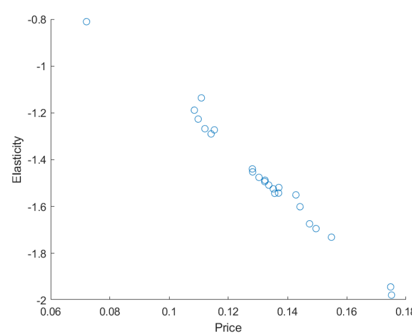

Figure 1: Scatter plot own price elasticity, IV logit model

The scatterplot shows a clear negative relationship between price and own-price elasticities, where higher-priced cereals tend to have more elastic demand, meaning consumers are more responsive to price changes. In contrast, lower-priced cereals exhibit less elastic demand, indicating that price changes have a smaller impact on their quantity demanded. This suggests that higher-priced cereals may face more competition or have more substitutes, while lower-priced cereals may have fewer alternatives or stronger consumer preferences. Overall, the model's predictions align with the observed pattern, confirming the typical inverse relationship between price and elasticity.

2 Mixed Logit/BLP Model

Consider the mixed logit/BLP model:

$$u_{ijt} = x_{jt}\beta_{it} + \alpha_{it}p_{jt} + \xi_{jt} + \epsilon_{ijt}$$

In the above equation the notation is the same as in Handbook Chapter 2 and the lecture slides. The coefficients that comprise the vector β_{it} are a function of income but do not have a random component and are given by:

$$\beta^0_{it} = \beta^0_0 + \beta^0_{\text{income}}[\text{income}_{it}]$$

$$\beta^{\text{sugar}}_{it} = \beta^{\text{sugar}}_{\text{income}}[\text{income}_{it}]$$

The coefficient on price is a function of the consumer's income and a random part (denoted by ν_{it}, where ν_{it} is drawn from a standard normal distribution):

$$\alpha_{it} = \alpha_0 + \alpha_{\text{income}}[\text{income}_{it}] + \alpha_\nu \nu_{it}$$

Therefore, there are six unknown parameters. There are two 'linear' parameters: β^0_0 and α_0. There are four 'nonlinear' parameters: $\beta^0_{\text{income}}, \beta^{\text{sugar}}_{\text{income}}, \alpha_{\text{income}}, \alpha_\nu$.

1. Estimate the above model using the BLP method and report the parameters (please write your own code; no need to report standard errors). To build the moments in the GMM objective function, include the product characteristic 'sugar' as an instrument as well as the precomputed demand instruments. For the weighting matrix, choose the 'optimal weight matrix' $W = (Z'Z)^{-1}$, where Z is the vector of instruments.

Table 3: Mixed logit estimation

variable	mean	sigma	income
constant	-3.8112	0	4.8562
price	-3.0674	0.95641	-37.2
sugar	0	0	0.15517

Table 3 presents the results of the mixed regression. We notice that the mean coefficient for price is very similar to the one estimated using the

IV-logit, which suggests that the assumption that consumer heterogeneity enters the model only through the separable additive random shock — that is, all the nonlinear coefficients were set to 0 — may be a reasonable simplification in this case.

2. Using your estimated parameters from Part (a), compute own-price elasticities for each product in the market 'C01Q1'. Draw a scatterplot with the computed own-price elasticities on the y-axis and prices on the x-axis (each dot on the scatterplot should correspond to an individual product in the market 'C01Q1'). Discuss the observed relationship between prices and the own-price elasticities predicted by the model. Also, discuss differences with Part 1(c).

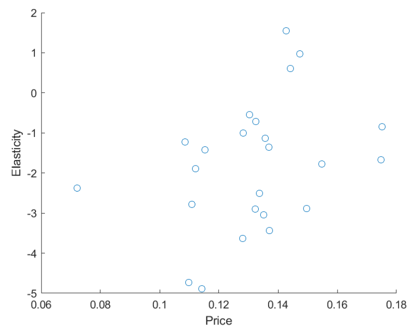

Figure 2: Scatter plot own price elasticity, mixed logit model

The scatterplot does not show a clear negative relationship between price and own-price elasticities. This suggests that the finding from Part 1, where higher-priced cereals tend to have more elastic demand, may have been driven by a restriction imposed by the model rather than an inherent relationship. This observation suggest that the model's assumption of consumer heterogeneity entering solely through the separable additive random shock, ϵ_{ijt}, might be overly restrictive.

3. (Optional/challenge question) Using the ASU Sol cluster, submit a job that computes the model optimization at 10 different initial starting points. To choose the initial points, pick one of the parameters to vary (say, α_{income}) and fix the other parameters. The batch job should submit 10 sub-jobs, with each job running the optimization for one of the starting points. Does the model optimization find the same coefficients regardless of the starting point, or does it hit local minima?

   When we estimated the model by changing the initial values of α_{income} from -33.5 to -32.6, we obtained slightly different results. For instance, the estimated mean coefficient for price exhibited some variation, as shown in Figure 3.

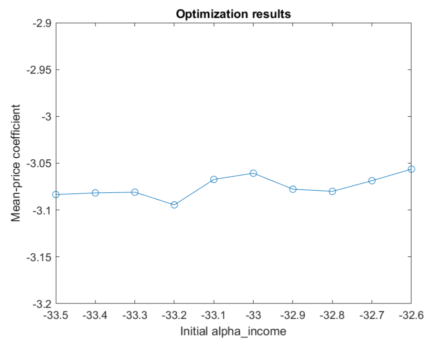

Figure 3: Line graph of sub-jobs, α_{income}

This may be due to the fact that we used a relatively lax tolerance value of 1e-3 in the optimization, which we did to accelerate convergence. For a more accurate estimation, a tighter tolerance value, typically around 1e-12, should be used.

To use the supercomputer, we first created an account. Then, we opened the terminal (in our case, Windows PowerShell) and connected to the cluster with the following command:

```
ssh account@sol.asu.edu
```

We can either directly upload the necessary files to SOL.asu.edu or use the following commands to navigate and transfer files:

```
ssh mchalupc@sol.asu.edu

cd ~/blp_project

ls ~/blp_project/

mkdir -p logs

mkdir -p results

scp path/optimize_model_SOL.m \
path/gmmobjg_SOL.m \
path/product_data.csv \
path/agent_data.csv \
path/submit_job_SOL.sh \
account@sol.asu.edu:~/blp_project/

dos2unix submit_job_SOL.sh
```

```
chmod +x submit_job_SOL.sh
```

The next step is to submit the batch job using the following command:

```
sbatch submit_job_SOL.sh
```

The job submission should be fast, and the results will be stored in:

```
account@sol.asu.edu:~/blp_project/results
```

You can monitor the status of the batch job using the following commands:

```
squeue -u account
sacct -j batchjobnumber
```

If necessary, you can also cancel any active batch jobs with the command:

```
scancel -u mchalupc
```