Discrete Probability Distribution

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Probability Distribution
Following are the two types of probability distributions:

1. Discrete probability distribution
2. Continuous probability distribution

Discrete Probability Distribution
“A **discrete distribution** describes the probability of occurrence of each value of a discrete random variable.”

- It is also known as Probability mass function(pmf)
- A discrete probability distribution is made up of discrete variables.

With a discrete probability distribution, each possible value of the discrete random variable can be associated with a non-zero probability. Thus, a discrete probability distribution is often presented in tabular form.

Random Variable
“A variable whose value is associated with the outcomes of sample space of random experiment.”

- If a random variable is discrete, then it will have a discrete probability distribution.

Example
The probability of Head or Tale of coin can be described by using Discrete Probability distribution

Random Experiment
Tossing of 3 coins

Sample space
{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}
Here, the sample space is: **n(S)=8**.

Random Variable
Let,

X: Number of Heads

X = 0,1,2,3

So, the numbers of can either be zero, one, two or three.

Probability distribution

X	0	1	2	3
P(x)	1/8	3/8	3/8	1/8

Conditions

The probabilities in the probability distribution of a random variable X must satisfy the following two conditions:

1. Each probability P(x), P(x) must be between 0 and 1.
   i.e **0<=P(x)<=1**
2. For a pmf, summation of all these probabilities is equal to one.
   i.e **ΣP(x) = 1**

If both these conditions are satisfied then the function can be regarded as pmf.

Cumulative Distribution Function

The cumulative distribution function (c.d.f.) of a discrete random variable X is the function F(x) which tells you the probability that X is less than or equal to x

F(x)= P (X $\leq \boldsymbol{x} = \sum_{x \leq x_i} \boldsymbol{P}(\boldsymbol{x_i})$

Mean Variance and Standard Deviation of pmf

Mean

It is also known as expected value or average.

The mean of a discrete random variable with probability function is given by

Mean= E(x) = Σx P(x)

Variance

"The variance is a measure of how much the probability mass is spread out around this center."

- It measures how far each number in the set is from the mean and thus from every other number in the set.

Variance is the expectation of the squared deviation of a random variable from its mean.

$V(x) = \Sigma x^2 P(x) – [\Sigma x P(x)]^2$

We can also write it as:

$E(x) = \Sigma x.P(x)$

$E(x^2) = \Sigma x^2 P(x)$

$V(x)= E(x^2) – [E(x)]^2$

Standard Deviation

"The standard deviation is a statistic that measures the dispersion of a dataset relative to its mean and is calculated as the square root of the variance."

$\sqrt{V(x)}$

Example

Two balanced dice are rolled. Let X be the sum of the two dice.
a) Obtain the probability distribution of X.
b) Find the mean and standard deviation of X.

When the two balanced dice are rolled, there are 36 equally likely possible outcomes

$$=\begin{Bmatrix} (1,1), (1,2), (1,3), (1,4), (1,5), (1,6) \\ (2,1), (2,2), (2,3), (2,4), (2,5), (2,6) \\ (3,1), (3,2), (3,3), (3,4), (3,5), (3,6) \\ (4,1), (4,2), (4,3), (4,4), (4,5), (4,6) \\ (5,1), (5,2), (5,3), (5,4), (5,5), (5,6) \\ (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \end{Bmatrix}$$

The possible values of X are: 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12.

The possible outcomes are equally likely hence the probabilities P(X) are given by

P(2) = P(1,1) = 1 / 36

P(3) = P(1,2) + P(2,1) = 2 / 36 = 1 / 18

P(4) = P(1,3) + P(2,2) + P(3,1) = 3 / 36 = 1 / 12

P(5) = P(1,4) + P(2,3) + P(3,2) + P(4,1) = 4 / 36 = 1 / 9

P(6) = P(1,5) + P(2,4) + P(3,3) + P(4,2) + P(5,1)= 5 / 36

P(7) = P(1,6) + P(2,5) + P(3,4) + P(4,3) + P(5,2) + P(6,1)

= 6 / 36 = 1 / 6

P(8) = P(2,6) + P(3,5) + P(4,4) + P(5,3) + P(6,2) = 5 / 36

P(9) = P(3,6) + P(4,5) + P(5,4) + P(6,3) = 4 / 36 = 1 / 9

P(10) = P(4,6) + P(5,5) + P(6,4) = 3 / 36 = 1 / 12

P(11) = P(5,6) + P(6,5) 2 / 36 = 1 / 18

P(12) = P(6,6) = 1 / 36

X	2	3	4	5	6	7	8	9	10	11	12
P(X)	1/36	1/18	1/12	1/9	5/36	1/6	5/36	1/9	1/12	1/18	1/36

The mean of X is given by:

E(X) = ∑ X P(X)

= 2*(1/36) +3*(1/18) +4*(1/12) +5*(1/9) +6*(5/36) +7*(1/6) +8*(5/36) +9*(1/9)

+10*(1/12) +11*(1/18) +12*(1/36)

= 7

The standard deviation of is given by:

Standard Deviation =$\sqrt{V(x)}$

Since,

V(x) = Σx^2 P(x) – [Σx P(x)]^2

Standard deviation = √ Σx^2 P(x) – [Σx P(x)]^2

=√ [$(2-7)^2$*(1/36) + $(3-7)^2$*(1/18) + $(4-7)^2$*(1/12) + $(5-7)^2$*(1/9) +$(6-7)^2$*(5/36) + $(7-7)^2$*(1/6) + $(8-7)^2$*(5/36) + $(9-7)^2$*(1/9) + $(10-7)^2$*(1/12) + $(11-7)^2$*(1/18) + $(12-7)^2$*(1/36)]

=2.41

Types

Following are the types of Discrete Probability distribution

- Poisson.
- Bernoulli.
- Binomial.
- Multinomial.

Poisson Distribution

The Poisson distribution, named after the French mathematician **Sime'on Denis Poisson** (**1781-1840**) who published its derivation in **1837**.

"Poisson Distribution gives us the probability of a given number of events happening in a fixed interval of time."

Formula

The Poisson Distribution pmf is

P (x; μ) = ($\mu^x e^{-\mu}$) / x!,   x=0,1,2,….,∞

The symbol "!" is a factorial.
μ (the expected number of occurrences) is sometimes written as λ. Sometimes called the event rate or rate parameter.

Example

A textbook store rents an average of 200 books every Saturday night. Using this data, you can predict the probability that more books will sell (perhaps 300 or 400) on the following Saturday nights.

Binomial Distribution

"The binomial is a type of distribution that has two possible values."

- The outcome of each trial may be classified into one of two categories, conventionally called Success(S) and Failure(F).
- The outcome of interest is called Success and the other, a failure.

Formula

$$P(X) = \frac{n!}{(n-X)!\,X!} \cdot (p)^X \cdot (q)^{n-X}$$

Example

- A coin toss has only two possible outcomes: heads or tails
- Taking a test could have two possible outcomes: pass or fail.

Multinomial Distribution

"Multinomial distribution has more than one possible value and have fixed probabilities for each independently generated value."

Formula

$$\mathrm{P} = \frac{n!}{(n_1!)(n_2!)\ldots(n_x!)} P_1{}^{n1} P_2{}^{n2} \ldots P_x{}^{nx}$$

Example

You roll a die ten times to see what number you roll. There are 6 possibilities (1, 2, 3, 4, 5, 6), so this is a multinomial experiment.

Bernoulli Distribution

"A Bernoulli distribution is a discrete probability distribution for a Bernoulli trial, a random experiment that has only two outcomes (usually called a "Success" or a "Failure")."

Formula

The possible outcomes labelled by n=0 and n=1 in which n=1 ("success") occurs with probability P and n=0 ("failure") occurs with probability $q \equiv 1 - p$, where $0 < p < 1$. It therefore has probability density function

$$P(n) = \begin{cases} 1 - p & \text{for } n = 0 \\ p & \text{for } n = 1, 0 < p < 1 \end{cases}$$

which can also be written

$$P(n) = p^n (1 - p)^{1-n}.$$

The corresponding distribution function is

$$D(n) = \begin{cases} 1 - p & \text{for } n = 0 \\ 1 & \text{for } n = 1. \end{cases}$$

Example

A student will pass or fail an exam, and a rolled dice will either show a 6 or any other number.

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