Islamic University Faculty of Computer and Information Systems		الجامعة الإسلامية كلية الحاسب الآلي ونظم المعلومات

Numerical Computing Methods
Assignment (1)

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Grade

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1. **The following table gives the distance in nautical miles of the visible horizon for the given heights in feet above the earth's surface.**

x:	100	150	200	250	300	350	400
y:	10.63	13.03	15.04	16.81	18.42	19.9	21.27

Use Newton's forward formula to find y when $x = 218$ ft.

$a=100$, $h=50$, $x=218$, $u=\frac{x-a}{h}=\frac{218-100}{50}=2.36$

Difference table :

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$
100	10.63						
		2.4					
150	13.03		-0.39				
		2.01		0.15			
200	15.04		-0.24		-0.07		
		1.77		0.08		0.02	
250	16.81		-0.16		-0.05		0.02
		1.61		0.03		0.04	
300	18.28		-0.13		-0.01		
		1.48		0.02			
350	19.9		-0.11				
		1.37					
400	21.27						

By difference formula in forward :

$$f(a+hu)=f(a)+u\,\Delta f(a)+\frac{u(u-1)}{2!}\Delta^2 f(a)+\cdots+\frac{u(u-1)(u-2)\cdots(u-n+1)}{n!}\Delta^n f(a)$$

$$\Rightarrow f(218)=f(100)+(2.36)(2.4)+\frac{(2.36)(2.36-1)}{2!}(-0.39)+\frac{(2.36)(2.36-1)(2.36-2)}{3!}(0.15)+$$

$$\frac{(2.36)(2.36-1)(2.36-2)(2.36-3)(2.36-4)}{4!}(-0.07)+\frac{(2.36)(2.36-1)(2.36-2)(2.36-3)(2.36-4)}{5!}$$

$$(0.02)+\frac{(2.36)(2.36-1)(2.36-2)(2.36-3)(2.36-4)(2.36-5)}{6!}(0.02)=16.29734$$

$\therefore f(218)=16.29734$

3. The values of $f(x)$ for $x = 0, 1, 2, \ldots\ldots, 6$ are given by

x:	0	1	2	3	4	5	6
$f(x)$:	1	3	11	31	69	131	223

Estimate the value of $f(3.4)$, using only four of the given values.

$a = 0$, $h = 1$, $x = 3.4$, $u = \frac{x-a}{h} = \frac{3.4-0}{1} = 3.4$

Difference Table:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$
0	1						
		2					
1	3		6				
		8		6			
2	11		12		0		
		20		6		0	
3	31		18		0		0
		38		6		0	
4	69		24		0		
		62		6			
5	131		30				
		92					
6	223						

By forward difference formula:

$$f(a+hu) = f(a) + u\Delta f(a) + \frac{u(u-1)}{2!}\Delta^2 f(a) \ldots + \frac{u(u-1)(u-2)\cdots(u-n+1)}{n!}\Delta^n f(a)$$

$$\Rightarrow f(3.4) = 1 + (3.4)(2) + \frac{(3.4)(3.4-1)}{2!}(6) + \frac{(3.4)(3.4-1)(3.4-2)}{3!}(6) + \frac{(3.4)(3.4-1)(3.4-2)(3.4-3)}{4!}(0)$$

$$f(3.4) = 1 + 6.8 + 24.48 + 11.424 + 0 = 43.704$$

$\therefore f(3.4) = 43.704$

4. Given that:

x:	1	2	3	4	5	6
$y(x)$:	0	1	8	27	64	125

Find the value of $f(2.5)$.

$x_0 = 1$, $h = 1$, $x = 2.5$, $u = \frac{x - x_0}{h} = \frac{2.5 - 1}{1} = 1.5$

Difference Table:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
1	0					
		1				
2	1		6			
		7		6		
3	8		12		0	
		19		6		0
4	27		18		0	
		37		6		
5	64		24			
		61				
6	125					

By forward difference formula:

$$f(x_0 + hu) = f(x_0) + u\Delta f(x_0) + \frac{u(u-1)}{2!}\Delta^2 f(x_0) + \cdots + \frac{u(u-1)(u-2)\cdots(u-n+1)}{n!}\Delta^n f(x_0)$$

$$\Rightarrow f(2.5) = 0 + (1.5)(1) + \frac{(1.5)(1.5-1)}{2!}(6) + \frac{(1.5)(1.5-1)(1.5-2)}{3!}(6) + 0 + 0$$

$$= 1.5 + 2.25 - 0.375 + 0 + 0 = 3.375$$

$\therefore f(2.5) = 3.375$

6. Using Newton's formula for interpolation, estimate the population for the year 1905 from the table:

Year	Population
1891	98,752
1901	132,285
1911	168,076
1921	195,690
1931	246,050

$a = 1891, \ h = 10, \ x = 1905, \ u = \frac{x-a}{h} = \frac{1905-1891}{10} = 1.4$

Difference Table:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1891	98,752				
		33533			
1901	132,285		2258		
		35791		-10435	
1911	168,076		-8177		41358
		27614		30923	
1921	195,690		22746		
		50360			
1931	246,050				

By forward difference formula:

$$f(a+hu) = f(a) + u\Delta f(a) + \frac{u(u-1)}{2!}\Delta^2 f(a) + \dots + \frac{u(u-1)(u-2)\dots(u-n+1)}{n!}\Delta^n f(a)$$

$$\Rightarrow f(1905) = 98752 + (1.4)(33533) + \frac{(1.4)(1.4-1)}{2!}(2258) + \frac{(1.4)(1.4-1)(1.4-2)}{3!}(-10435) +$$

$$\frac{(1.4)(1.4-1)(1.4-2)(1.4-3)}{4!}(41358)$$

$$= 98752 + 46946.2 + 632.24 + 584.36 + 926.4192 = 147841.2192$$

$$\therefore f(1905) = 147841.2192$$

9. Following are the scores obtained by 492 candidates in a certain examination

Scores	*Number of candidates*
0—40	210
40—45	43
45—50	54
50—55	74
55—60	32
60—65	79

Find out the number of candidates
(*a*) who secured scores more than 48 but not more than 50;
(*b*) who secured scores less than 48 but not less than 45.

Scores	Candidates
less than 40	210
less than 45	253
less than 50	307
less than 55	381
less than 60	413
less than 65	492

(a) $a = 40$, $h = 5$, $x = 48$, $u = \frac{x-a}{h} = \frac{48-40}{5} = 1.6$

Difference Table:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
40	210					
		43				
45	253		11			
		54		9		
50	307		20		-71	
		74		-62		222
55	381		-42		151	
		32		89		
60	413		47			
		79				
65	492					

$$f(48) = 210 + (1.6)(43) + \frac{(1.6)(1.6-1)}{2!}(11) + \frac{(1.6)(1.6-1)(1.6-2)}{3!}(9) + \frac{(1.6)(1.6-1)(1.6-2)(1.6-3)}{4!}(-71) + \frac{(1.6)(1.6-1)(1.6-2)(1.6-3)(1.6-4)}{5!}(222)$$

$$= 210 + 68.8 + 5.28 + (-0.576) + (-1.59) + (-2.386)$$

$f(48) = 279.62 \simeq 280$.

(a) $307 - 280 = 27$

(b) $280 - 253 = 27$