Islamic University Faculty of Computer and Information Systems		الجامعة الإسلامية كلية الحاسب الآلي ونظم المعلومات

Numerical Computing Methods
Assignment (5)

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Grade

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2. Taking $h = .05$, determine the value of y at $x = 0.1$ by Euler's modified method, given that,

$$\frac{dy}{dx} = x^2 + y; \quad y(0) = 1$$

$\Rightarrow y(x_0) = y_0$, $f(x,y) = x^2+y$

$x_0 = 0$, $y_0 = 1$, $h = 0.05$

$x_1 = x_0 + h = 0 + 0.05 = \boxed{0.05}$

$y_1 = y_0 + hf(x_0, y_0) = 1 + 0.05 f(0,1) = 1 + 0.05(1) = \boxed{1.05}$

$\rightarrow x_2 = x_1 + h = 0.05 + 0.05 = \boxed{0.1}$

$y_2 = y_1 + hf(x_1, y_1) = 1.05 + 0.05 f(0.05, 1.05) = 1.05 + 0.05(1.0525)$

$= \boxed{1.102625}$

$\therefore$ the value of y when the $x = 0.1$ will be $\boxed{1.102625}$

3. Given $\frac{dy}{dx} = x^2 + y$, $y(0) = 1$, find $y(.02)$, $y(.04)$ and $y(.06)$ using Euler's modified method.

$f(x,y) = x^2 + y$

$g(x_0) = y_0 \Rightarrow x_0 = 0, y_0 = 1$, let $h = 0.02$

$x_1 = x_0 + h = 0 + 0.02 = 0.02$

$y_1 = y_0 + hf(x_0, y_0) = 1 + 0.02f(0,1) = 1 + 0.02(1) = 1.02$

$x_2 = x_1 + h = 0.02 + 0.02 = 0.04$

$y_2 = y_1 + hf(x_1, y_1) = 1.02 + 0.02 f(0.02, 1.2) = 1.02 + 0.02(1.0204) = 1.040408$

$x_3 = x_2 + h = 0.04 + 0.02 = 0.06$

$y_3 = y_2 + hf(x_2, y_2) = 1.040408 + 0.02 f(0.04, 1.040408) = 1.040408 + 0.02(1.042008)$

$= 1.06124$

x	0	0.02	0.04	0.06
y	1	1.02	1.040408	1.06124

5. Use Euler's method with $h = 0.1$ to solve the differential equation $\frac{dy}{dx} = x^2 + y^2, y(0) = 1$ in the range $x = 0$ to $x = 0.3$.

$f(x,y) = x^2 + y^2$

$x_0 = 0 \quad , y_0 = 1 \quad , h = 0.1$

$x_1 = x_0 + h = 0 + 0.1 = 0.1$

$y_1 = y_0 + hf(x_0, y_0) = 1 + 0.1 f(0,1) = 1 + 0.1(1) = 1.1$

$x_2 = x_1 + h = 0.1 + 0.1 = 0.2$

$y_2 = y_1 + hf(x_1, y_1) = 1.1 + 0.1 f(0.1, 1.1) = 1.1 + 0.1(1.22) = 1.222$

$x_3 = x_2 + h = 0.2 + 0.1 = 0.3$

$y_3 = y_2 + hf(x_2, y_2) = 1.222 + 0.1 f(0.2, 1.222) = 1.222 + 0.1(1.533) = 1.375$

x	0	0.1	0.2	0.3
y	1	1.1	1.222	1.375

6. Solve for y at $x = 1.05$ by Euler's method, the differential equation $\frac{dy}{dx} = 2 - \left(\frac{y}{x}\right)$ where $y = 2$ when $x = 1$. (Take $h = 0.05$).

$f(x,y) = 2 - \left[\frac{y}{x}\right]$

$x_0 = 1 \;,\; y_0 = 2 \;,\; h = 0.05$

$x_1 = x_0 + h = 1 + 0.05 = 1.05$

$y_1 = y_0 + hf(x_0, y_0) = 2 + 0.05\, f(1, 2) = 2 + 0.05(0) = 2$

x	1	1.05
y	2	2

11. Given $\frac{dy}{dx} = x^3 + y, y(0) = 1$ Compute y (0.02) by Euler's method taking h = 0.01.

$f(x,y) = x^3 + y$

$x_0 = 0 \;,\; y_0 = 1 \;,\; h = 0.01$

$x_1 = x_0 + h = 0 + 0.01 = 0.01$

$y_1 = y_0 + hf(x_0, y_0) = 1 + 0.01 f(0,1) = 1 + 0.01(1) = 1.01$

$x_2 = x_1 + h = 0.01 + 0.01 = 0.02$

$y_2 = y_1 + hf(x_1, y_1) = 1.01 + 0.01 f(0.01, 1.01) = 1.01 + 0.01(1.010001)$

$= 1.02010001$

x	0	0.01	0.02
y	1	1.01	1.02010001

12. Find $y(1)$ by Euler's method from the differential equation $\frac{dy}{dx} = \frac{-y}{1+x}$ when $y(0.3) = 2$.
Convert up to four decimal places taking step length $h = 0.1$.

$f(x,y) = \frac{-y}{1+x}$, $x_0 = 0.3$, $y_0 = 2$, $h = 0.1$

$f(x_0, y_0) = \frac{-2}{1+0.3} = -1.538$

$y_1^0 = y_0 + h f(x_0, y_0) = 1.8462$

$y_1^1 = y_0 + h/2 [f(x_0, y_0) + f(x_1, y_1^0)] = 1.6923$

$y_1^2 = y_0 + h/2 [f(x_0, y_0) + f(x_1, y_1^1)] = 1.7115$

$y_1^3 = y_0 + h/2 [f(x_0, y_0) + f(x_1, y_1^2)] = 1.7084$

$y_1^4 = y_0 + h/2 [f(x_0, y_0) + f(x_1, y_1^3)] = 1.7094$

$\therefore y_1^4 = 1.7094$ at $x_1 = 0.4$

$y_2^0 = y_1 + h f(x_1, y_1) = 1.5873$

$y_2^1 = y_1 + h/2 [f(x_1, y_1) + f(x_2, y_2^0)] = 1.5954$

$y_2^2 = y_1 + h/2 [f(x_1, y_1) + f(x_2, y_2^1)] = 1.5954$

$\therefore y_2^2 = 1.5954$, $x_2 = 0.5$

$y_3^0 = y_2 + h f(x_2, y_2) = 1.4889$

$y_3^1 = y_2 + h/2 [f(x_2, y_2) + f(x_3, y_3^0)] = 1.4954$

$y_3^2 = y_2 + h/2 [f(x_2, y_2) + f(x_3, y_3^1)] = 1.4954$

$\therefore y_3^2 = 1.4954$, $x_3 = 0.6$

$y_4^0 = y_3 + h f(x_3, y_3) = 1.4484$

$y_4^1 = y_3 + h/2 [f(x_3, y_3) , f(x_4, y_4^0)] = 1.4058$

$y_4^2 = y_3 + h/2 [f(x_3, y_3) , f(x_4, y_4^1)] = 1.4071$

$y_4^3 = y_3 + h/2 [f(x_3, y_3) , f(x_4, y_4^2)] = 1.4071$

$\therefore y_4^3 = 1.4071$, $x_4 = 0.7$

$y_5^0 = y_4 + h f(x_4, y_4) = 1.3243$

$y_5^1 = y_4 + h/2 [f(x_4, y_4) + f(x_5, y_5^0)] = 1.3289$

$\therefore y_5^2 = 1.3289$, $x_5 = 0.8$

$y_6^0 = y_5 + h f(x_5, y_5) = 1.2549$

$y_6^1 = 1.2588$, $y_6^2 = 1.2588$

$\therefore y_6^2 = 1.2588$, $x_6 = 0.9$

$y_7^0 = y_6 + h f(x_6, y_6) = 1.1925$

$y_7^1 = y_6 + h/2 [f(x_6, y_6) + f(x_7, y_7^0)] = 1.1958$

$y_7^2 = 1.1958$

$\therefore y_7^2 = 1.1958$, $x_7 = 1$