

Numerical Computing Methods Assignment (3)

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Grade

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Section: 1105

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1. Apply Gauss's forward formula to find the value of $f(x)$ at $x = 3.75$ from the table:

x :	2.5	3.0	3.5	4.0	4.5	5.0
$f(x)$:	24.145	22.043	20.225	18.644	17.262	16.047

$$a = 2.5, \quad h = 0.5, \quad u = 0.5, \quad x = 3.75$$

p	x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$	$\Delta^5 f(x)$
-2	2.5	24.145	-2.102				
-1	3.0	22.043	-1.818	0.284			
0	3.5	20.225	-1.581	0.237	-0.047	9×10^{-3}	-3×10^{-3}
1	4.0	18.644	-1.382	0.199	-0.038	6×10^{-3}	
2	4.5	17.262	-1.215	0.167	-0.032		
3	5.0	16.047					

$$f(u) = f(x_0) + u \Delta f(x_0) + \frac{u(u-1)}{2!} \Delta^2 f(x_0) + \frac{(u+1)u(u-1)}{3!} \Delta^3 f(x_0) + \dots + \frac{(u+1)u(u-1)(u-2)}{4!} \Delta^4 f(x_0)$$

$$f(3.75) = (20.225) + (0.5)(-1.581) + \frac{(0.5)(0.5-1)}{2!} (0.237) + \frac{(0.5+1)(0.5)(0.5-1)}{3!} (-0.038) +$$

$$\frac{(0.5+1)(0.5)(0.5-1)(0.5-2)}{4!} (9 \times 10^{-3}) + \frac{(0.5+1)(0.5+2)(0.5)(0.5-1)(0.5-2)}{5!} (-3 \times 10^{-3})$$

$$= 20.225 + (-0.7905) + (-0.0296) + (2.385 \times 10^{-3}) + (2.1093 \times 10^{-4})$$

$$= \boxed{19.407}$$

3. Find the value of $f(41)$ by applying Gauss's forward formula from the following data:

x :	30	35	40 ↓	45	50
$f(x)$:	3678.2	2995.1	2400.1	1876.2	1416.3

$$a = 40, h = 5, u = 0.2, x = 41$$

p	x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
-2	30	3678.2	-683.1			
-1	35	2995.1	-595	88.1		
0	40	2400.1	-523.9	71.1	-17	9.9
1	45	1876.2	-459.9	64	-7.1	
2	50	1416.3				

$$f(u) = f(x) + u \Delta f(x) + \frac{u(u-1)}{2!} \Delta^2 f(x) + \frac{(u+1)u(u-1)}{3!} \Delta^3 f(x) + \frac{(u+1)u(u-1)(u-2)}{4!} \Delta^4 f(x)$$

$$f = (2400.1) + (0.2)(-523.9) + \frac{(0.2)(0.2-1)}{2!} (71.1) + \frac{(0.2+1)(0.2)(0.2-1)}{3!} (-7.1) + \frac{(0.2+1)(0.2)(0.2-1)(0.2-2)}{4!} (9.9)$$

$$= 2400.1 + (-104.78) + (-5.688) + 0.2272 + 0.14256$$

$$= \boxed{2290.001}$$

5. From the following table find y when $x = 1.45$

x :	1.0	1.2	1.4	1.6	1.8	2.0
y :	0.0	-0.112	-0.016	0.336	0.992	2.0

$$a = 1.4, \quad h = 0.2, \quad u = 0.25, \quad x = 1.45$$

p	x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$	$\Delta^5 f(x)$
-2	1.0	0.0	-0.112				
-1	1.2	-0.112	-0.048	0.064			
0	1.4	-0.16	0.496	0.544	0.48	-0.864	
1	1.6	0.336	0.656	0.16	-0.384	0.576	1.44
2	1.8	0.992	1.008	0.352	0.192		
3	2.0	2.0					

$$f_p(u) = (-0.16) + (0.25)(0.496) + \frac{(0.25)(0.25-1)}{2!}(0.544) + \frac{(0.25+1)(0.25)(0.25-1)}{3!}(-0.864) + \frac{(0.25+1)(0.25)(0.25-1)(0.25-2)}{4!}(-0.384) + \frac{(0.25+2)(0.25+1)(0.25)(0.25-1)(0.25-2)}{5!}(1.44)$$

$$= (-0.16) + 0.124 + (-0.051) + 0.015 + (-0.01476) + 0.011074$$

$$= -0.075$$