



CSI2110

# Data Structures and Algorithms

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We learned...

# Review:

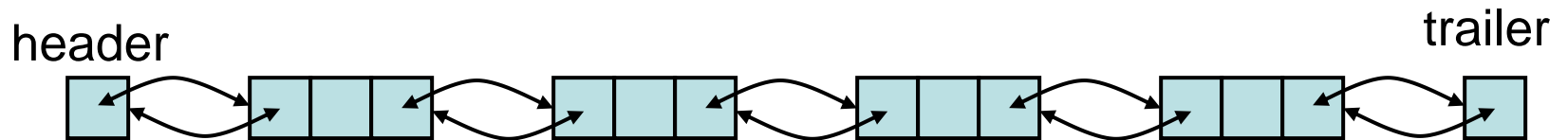
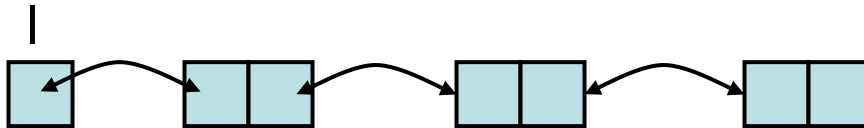
## Basic Data Structures (“concrete” data structures)

Array



Linked Lists

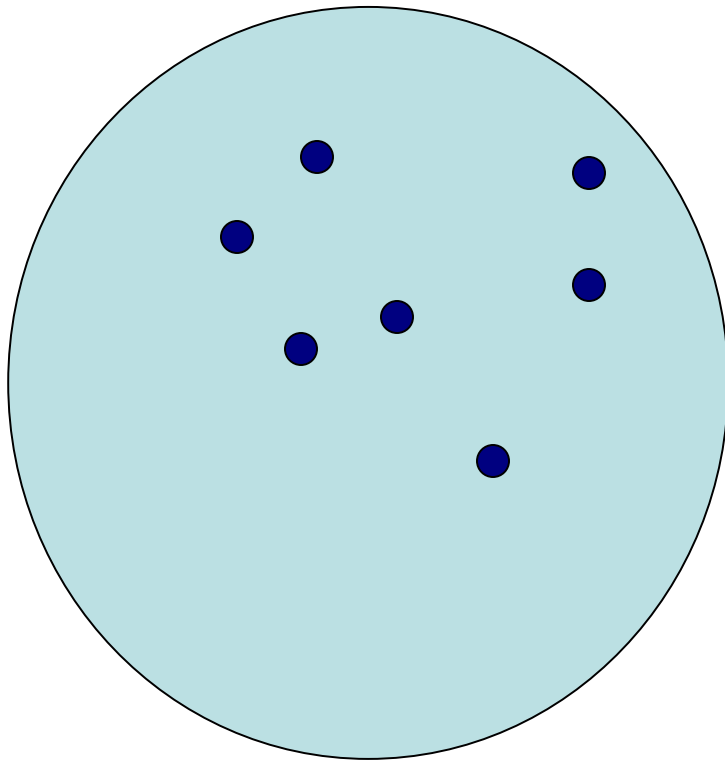
For example:



# Abstract Data Types (ADT)

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**ADT** is an abstraction of a data structure.  
**ADTs** specify **what can be stored** and what **operations** can be performed.



## Containers

Contains objects

I can INSERT

I can REMOVE

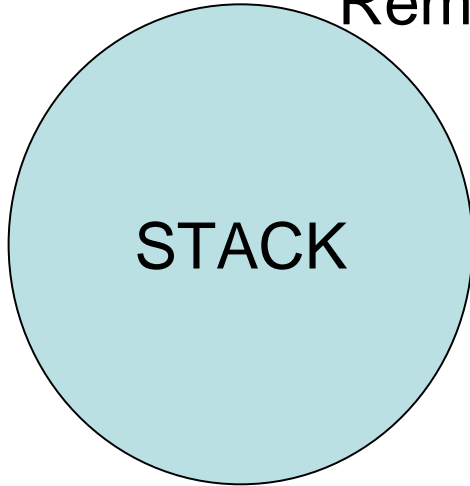
I can .....

# Abstract Data Types seen so far

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Insert = PUSH

Remove = POP

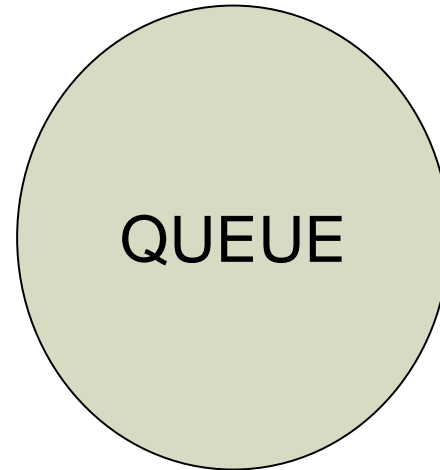


STACK

“last in first out”

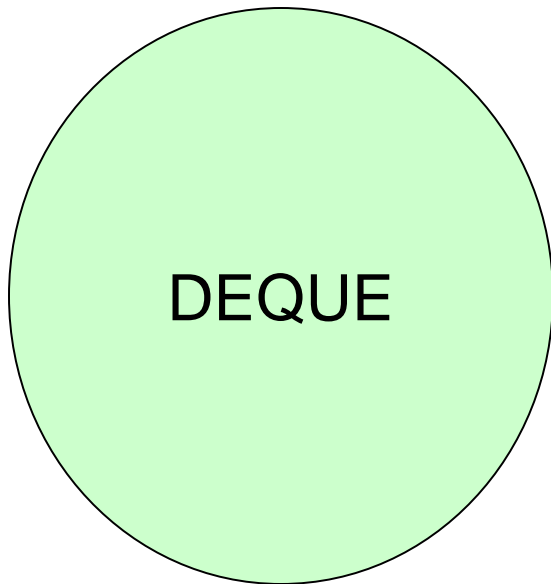
Insert = ENQUEUE

Remove = DEQUEUE



QUEUE

“first in first out”



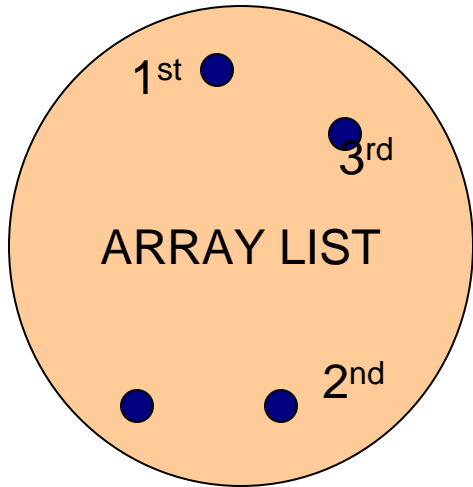
DEQUE

Insert: InsertFirst, InsertLast

Remove: RemoveFirst RemoveLast

What are we going to see next ...

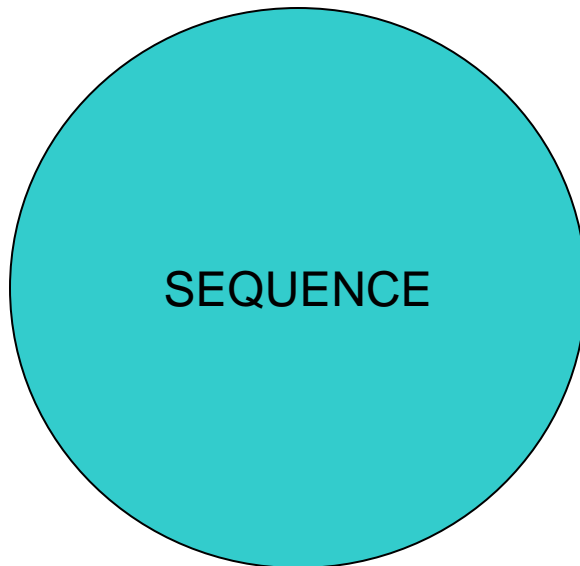
# Generalization...



By “index”



By “position”  
(by address)





# Lists

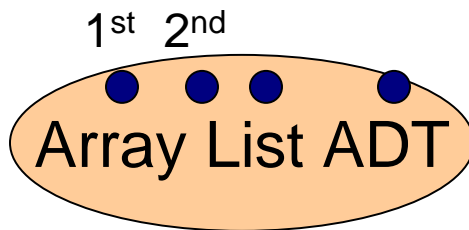
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- Array-List ADT
- Positional-List ADT
- Sequence ADT

# Lists or Sequences

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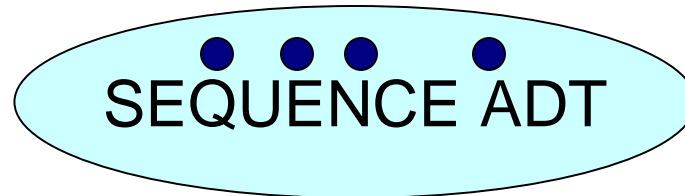
LISTS or SEQUENCES= collection of elements in linear order



To be implemented  
by arrays. Access by  
"index"



To be implemented by linked lists  
Access by "position" (or address)

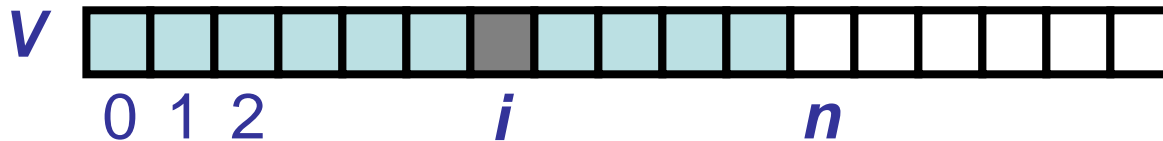


Combination of both

# Array-lists

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- Can access any element directly, not just first or last.
- Elements are accessed by **index** (or **rank**), the number of elements which precede them (if starting from index 0).
- Typically implemented by an array



# The Array-List ADT

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- A sequence  $S$  (with  $n$  elements) that supports the following methods:

- get( $i$ ): Return the element of  $S$  with index  $i$ ;  
an error occurs if  $i < 0$  or  $i > n - 1$
- set( $i, e$ ): Replace the element at index  $i$  with  $e$   
and return the old element; an error  
condition occurs if  $i < 0$  or  $i > n - 1$
- add( $i, e$ ): Insert a new element into  $S$  which  
will have index  $i$ ; an error occurs if  
 $i < 0$  or  $i > n$
- remove( $i$ ): Remove from  $S$  the element at index  $i$ ;  
an error occurs if  $i < 0$  or  $i > n - 1$

# Adapter Pattern

- Two data structures A and B are often similar
- Adapt data structure B to be used as A
- Create a “wrapper class” A holding B

Examples:

- Regular array as an ArrayList, or
- ArrayList can be adapted as a Deque

## Deque

getFirst(), getLast()

addFirst(e), addLast(e)

removeFirst(), removeLast()

## ArrayList

get(0), get(size()-1)

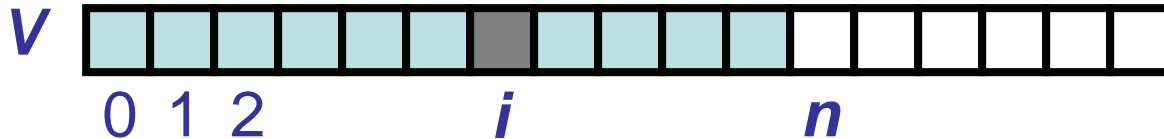
add(0,e), add(size(),e)

remove(0), remove(size()-1)

# Natural Implementation of Array-List: with an Array

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- Array  $V$  of size  $N$
- A variable  $n$  keeps track of the size of the array-list (number of elements stored)
- Operation **get**( $i$ ) is implemented in  $O(1)$  time by returning  $V[i]$



# Insertion

- In operation **add(r, o)**, we need to make room for the new element by shifting forward the  $n - r$  elements  $V[r], \dots, V[n - 1]$
- In the worst case ( $r = 0$ ), this takes  $O(n)$  time

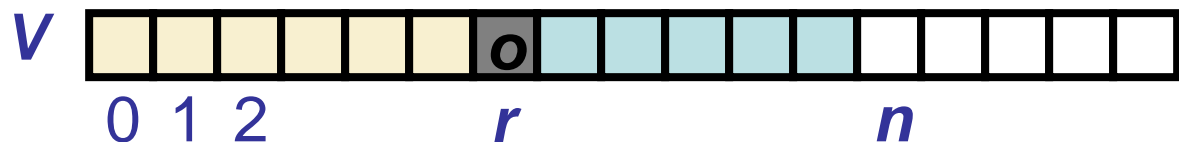
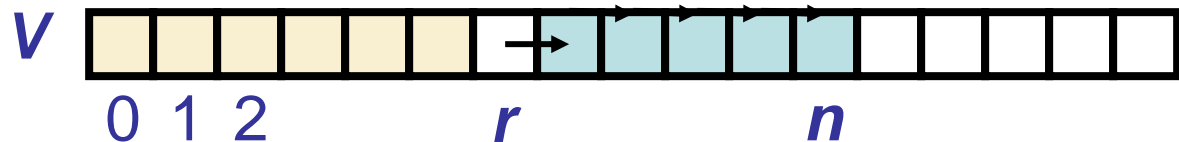
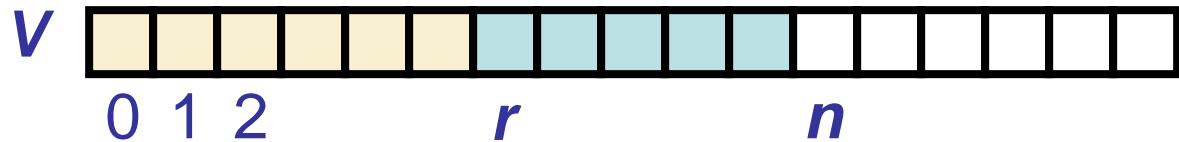
add(r,o):

for  $i = n - 1, n - 2, \dots, r$  do

$S[i+1] \leftarrow s[i]$

$S[r] \leftarrow o$

$n \leftarrow n + 1$



# Deletion

- In operation **remove(r)**, we need to fill the hole left by the removed element by shifting backward the  $n - r - 1$  elements  $V[r + 1], \dots, V[n - 1]$
- In the worst case ( $r = 0$ ), this takes  $O(n)$  time

remove(r):

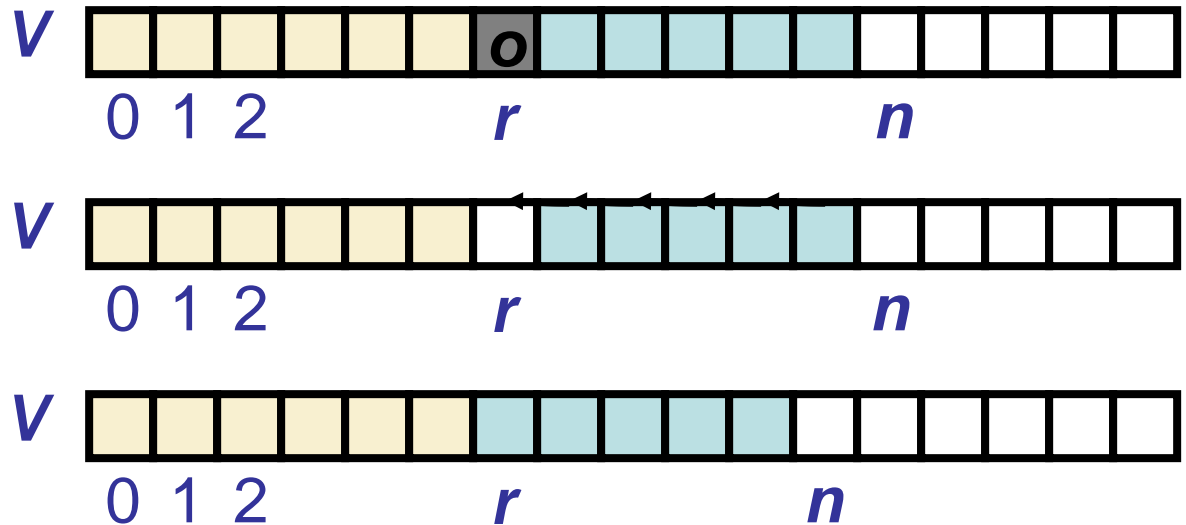
$e \leftarrow S[r]$

for  $i = r, r + 1, \dots, n - 2$  do

$S[i] \leftarrow S[i + 1]$

$n \leftarrow n - 1$

return





# Performance

---

- In the array based implementation of an array-list
  - The space used by the data structure is  $O(n)$
  - **size**, **isEmpty**, **get** and **replace** run in  $O(1)$  time
  - **insert** and **remove** run in  $O(n)$  time
- In an **insert** operation, when the array is full, instead of having an ERROR, we can replace the array with a larger one: extendable arrays seen earlier

## Performance (contd.)

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- Time time complexity of the various methods:

**size**  $O(1)$

**isEmpty**  $O(1)$

**get**  $O(1)$

**replace**  $O(1)$

**insert**  $O(n)$

**remove**  $O(n)$

# Class java.util.ArrayList<E>

- Inherits from

- java.util.AbstractCollection<E>
- java.util.AbstractList<E>

– Implements

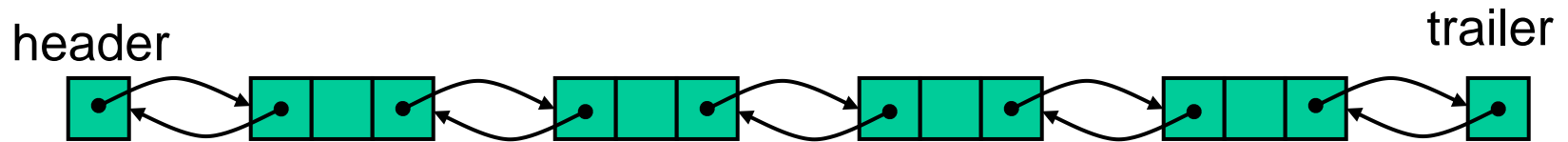
- Iterable<E>
- Collection<E>
- List<E>
- RandomAccess

**Implementation with  
extendable arrays**

• The methods

- size(), isEmpty(), get(int) and set(int,E) in time  $O(1)$
- add(int,E) and remove(int) in time  $O(n)$

If we were to implement an array-list with a  
**doubly linked list** it would be **quite inefficient** !



get(rank) ?

# Finding an element at a certain rank

---

Algorithm get(rank)

```
    if (rank <= size()/2) { //scan forward from head
        node □ header.next
        for (int i=0; i < rank; i++)
            node □ node.next
    } else { // scan backward from the tail
        node □ trailer.prev
        for (int i=0; i < size()-rank-1 ; i++)
            node □ node.prev
    }
    return node;
```

## Performance with linked list ...

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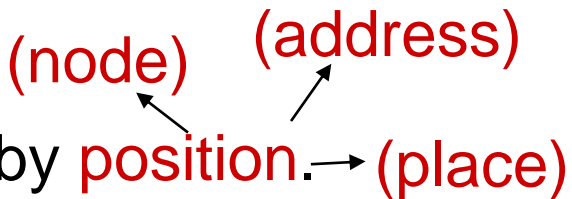
<b>size</b>	<b><math>O(1)</math></b>
<b>isEmpty</b>	<b><math>O(1)</math></b>
<b>get</b>	<b><math>O(n)</math></b>
<b>replace</b>	<b><math>O(n)</math></b>
<b>insert</b>	<b><math>O(n)</math></b>
<b>remove</b>	<b><math>O(n)</math></b>

# Positional Lists

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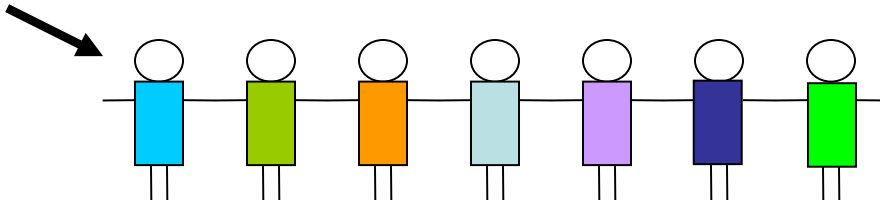
Container of elements that store each element at a **position** and that keeps these positions arranged in a linear order

- Cannot access any element directly, can access just first or last.

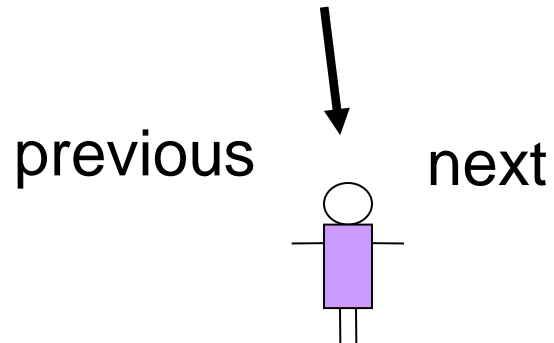
- Elements are accessed by **position**.

Positions are defined relatively to other positions  
(before/after relation)

first



me



There is no notion of rank - I don't know my rank.  
I only know who is next and who is before



# The Positional-List ADT

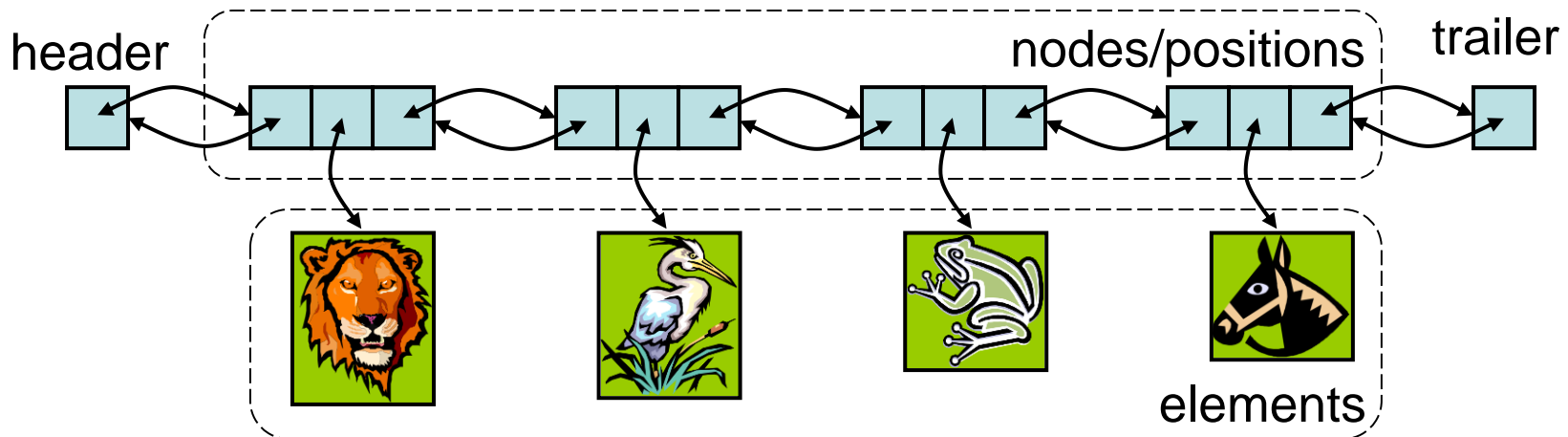
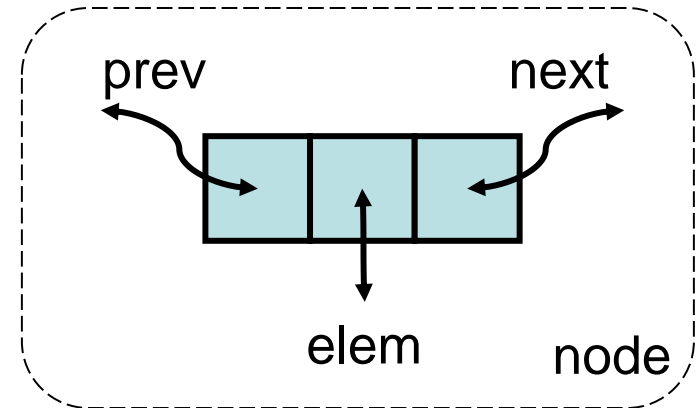
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ADT with position-based methods

- generic methods            `size(), isEmpty()`
- accessor methods            `first(), last()`  
                                 `before(p), after(p)`
- update methods  
                         `addFirst(e), addLast(e)`  
                         `addBefore(p,e), addAfter(p,e)`  
                         `set(p,e), remove(p)`

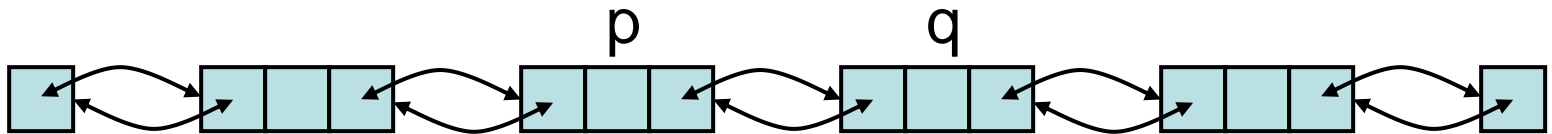
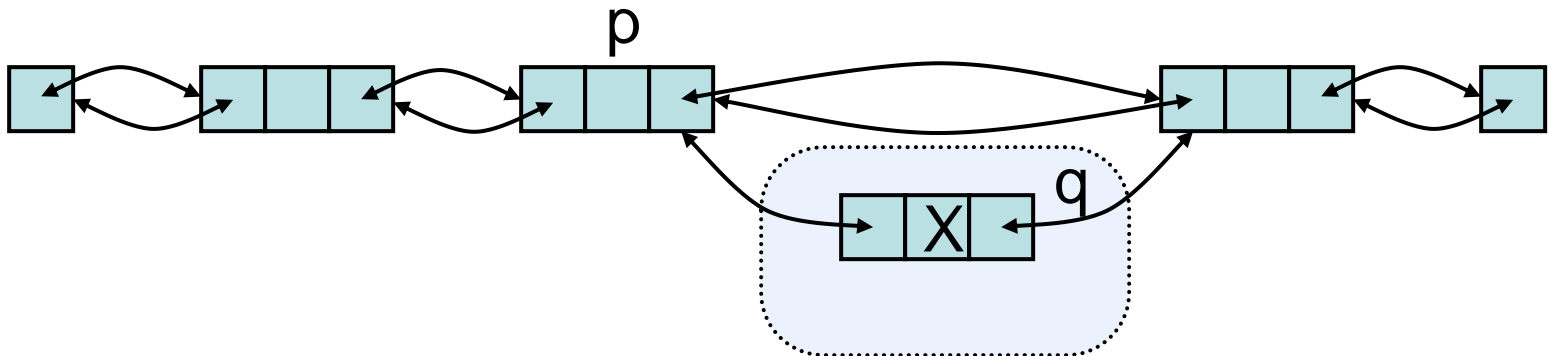
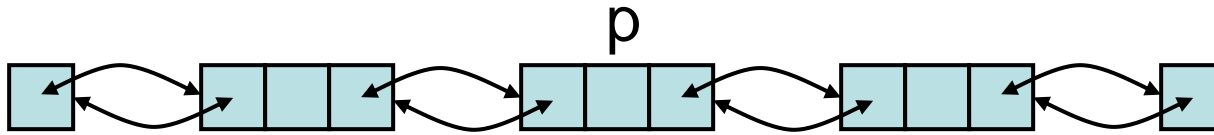
# Natural Implementation: with a Linked List

- A doubly linked list provides a natural implementation of the Positional-List ADT
- Nodes implement Position and store:
  - element
  - link to the previous node
  - link to the next node
- Special trailer and header nodes



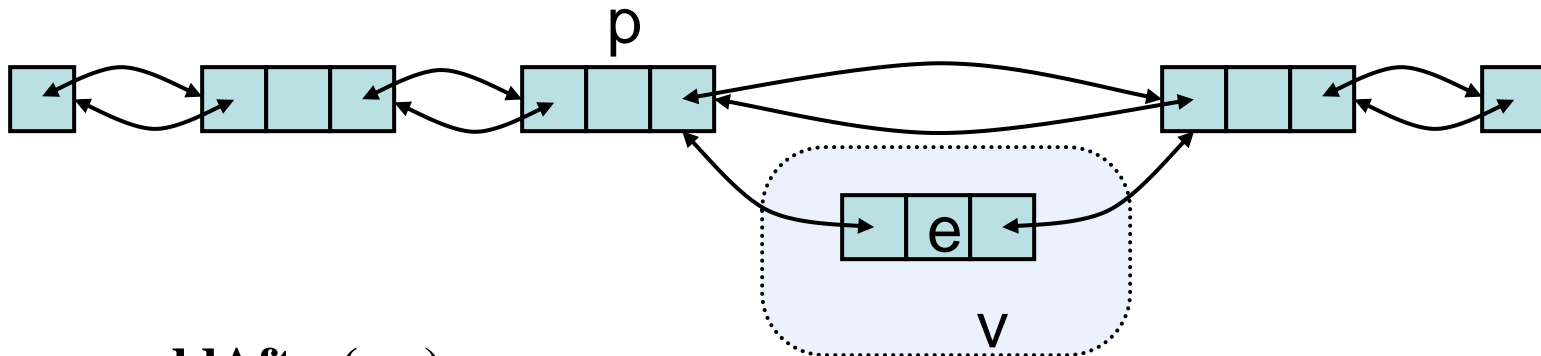
# Insertion

- We visualize operation `addAfter(p, X)`, which returns position `q`



# Insertion

- We visualize operation `addAfter(p, e)`, which returns position `v`



**`addAfter(p,e)`**

Create a new node `v`

Correct order??

`v.setNext(p.getNext())`

`v.setElement(e)`

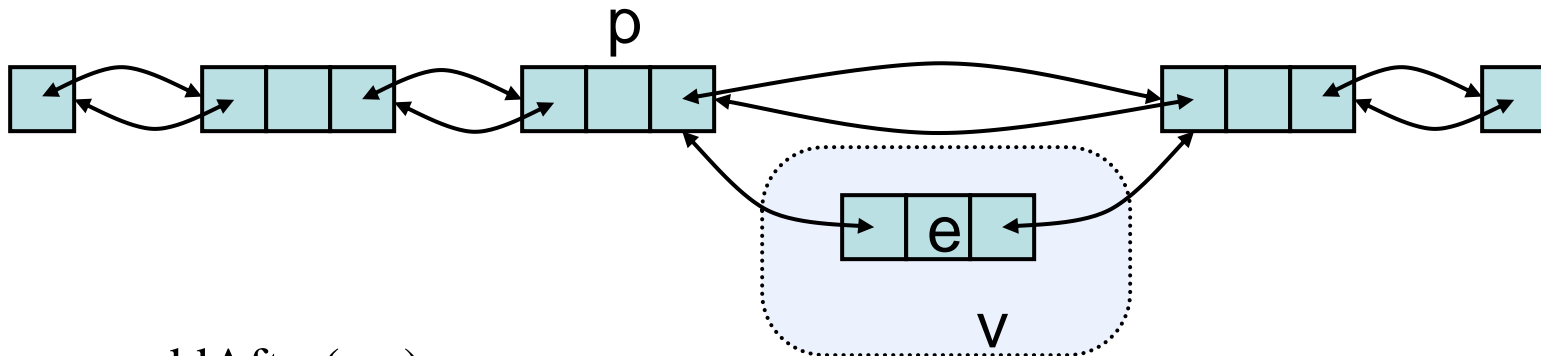
`(p.getNext()).setPrev(v)`

`v.setPrev(p)`

`p.setNext(v)`

# Insertion

- We visualize operation `addAfter(p, e)`, which returns position `v`



`addAfter(p,e)`

Create a new node `v`

`v.setElement(e)`

`v.setPrev(p)`

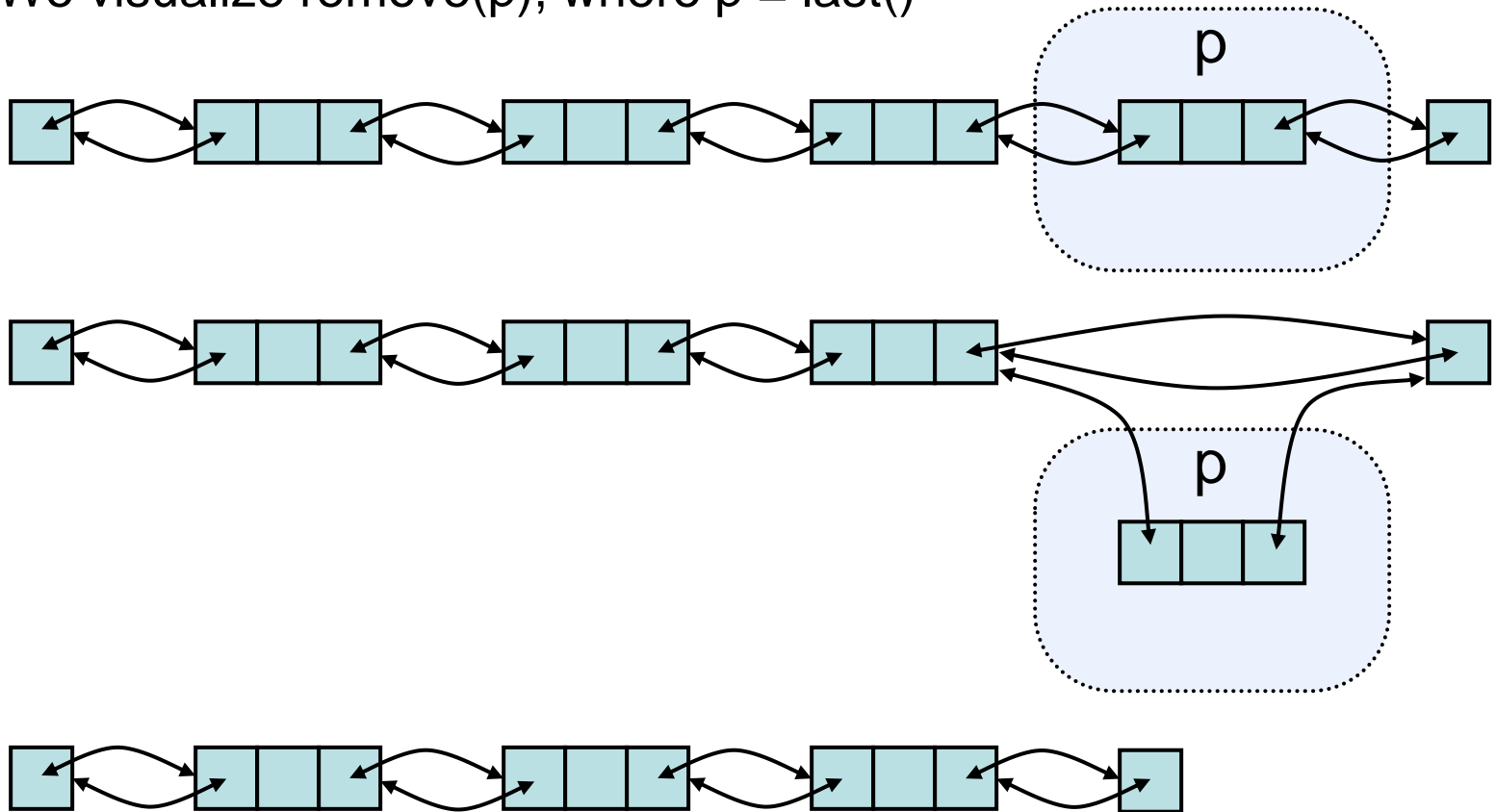
`v.setNext(p.getNext())`

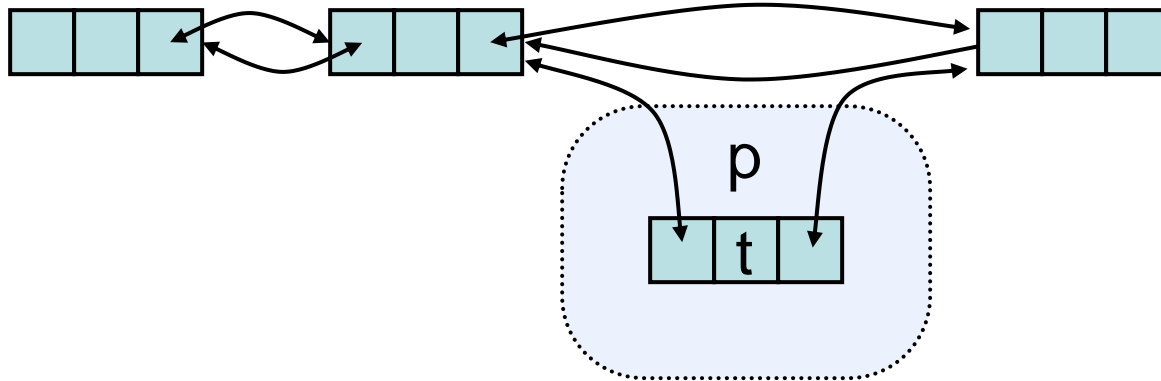
`(p.getNext()).setPrev(v)`

`p.setNext(v)`

# Deletion

- We visualize `remove(p)`, where `p = last()`





```
remove(p)
    t ← p.element
    (p.getPrev()).setNext(p.getNext())
    (p.getNext()).setPrev(p.getPrev())
    p.setPrev(null)
    p.setNext(null)
    return t
```

# Performance

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- In the implementation of the Positional-List ADT by means of a doubly linked list
  - The space used by a list with  $n$  elements is  $O(n)$
  - The space used by each position of the list is  $O(1)$
  - All the operations of the Positional-List ADT run in  $O(1)$  time



# A more general ADT: Sequence ADT

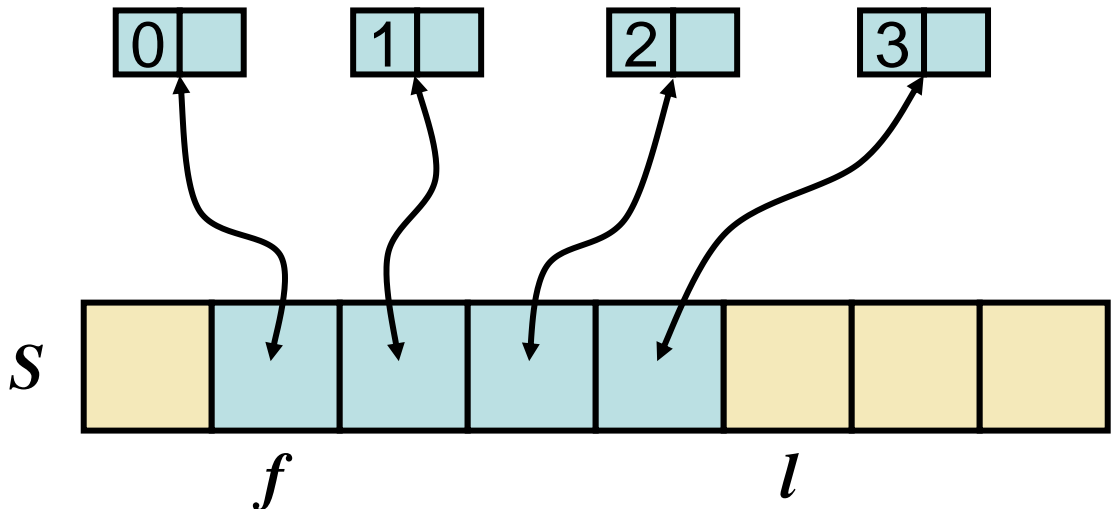
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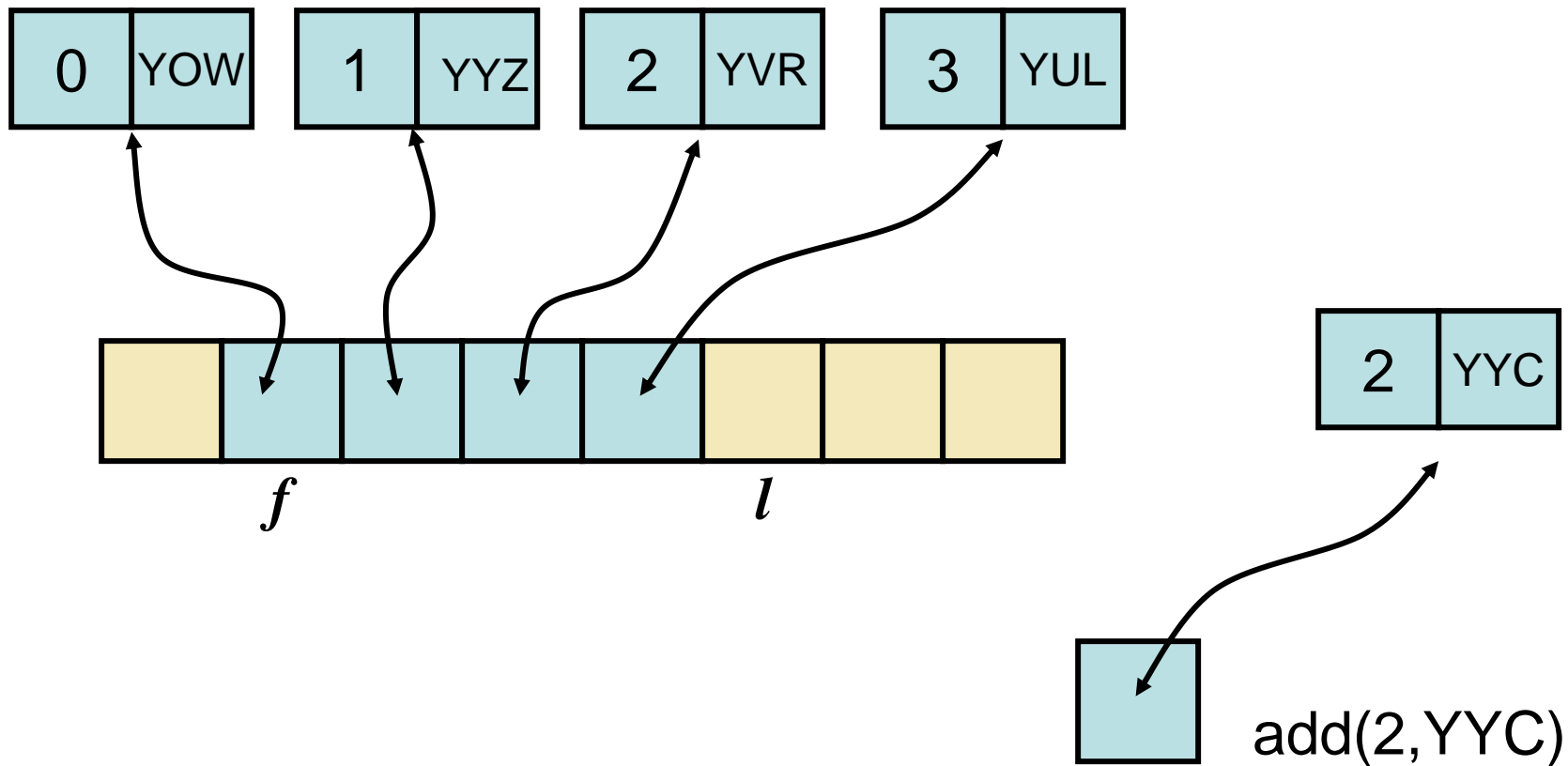
- Combines the Array-List and Positional-List ADT
- Adds methods that **bridge between index and positions**
  - atIndex(i)            returns a position
  - indexOf(p)            returns an integer index

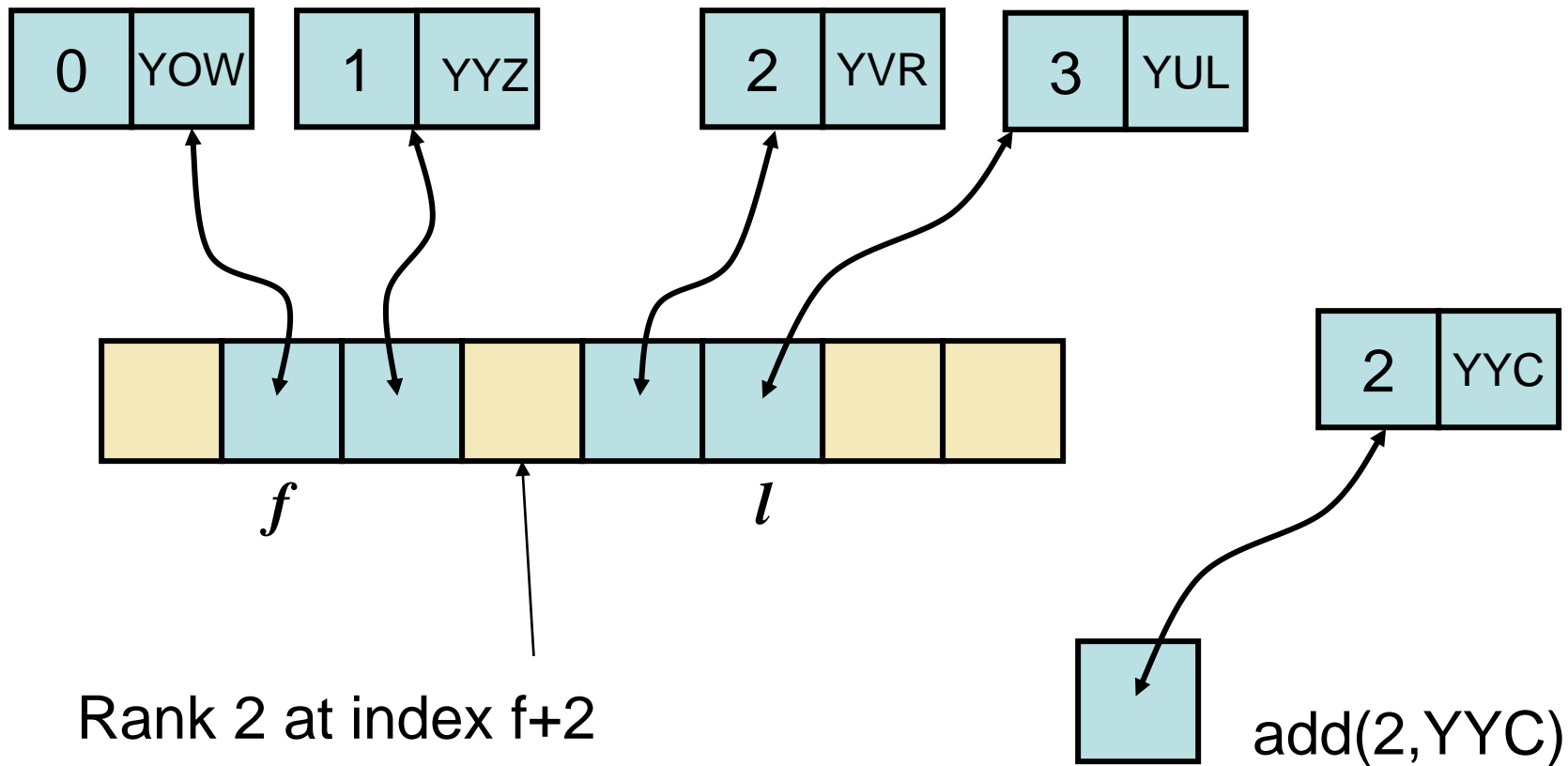
# An array-based Implementation

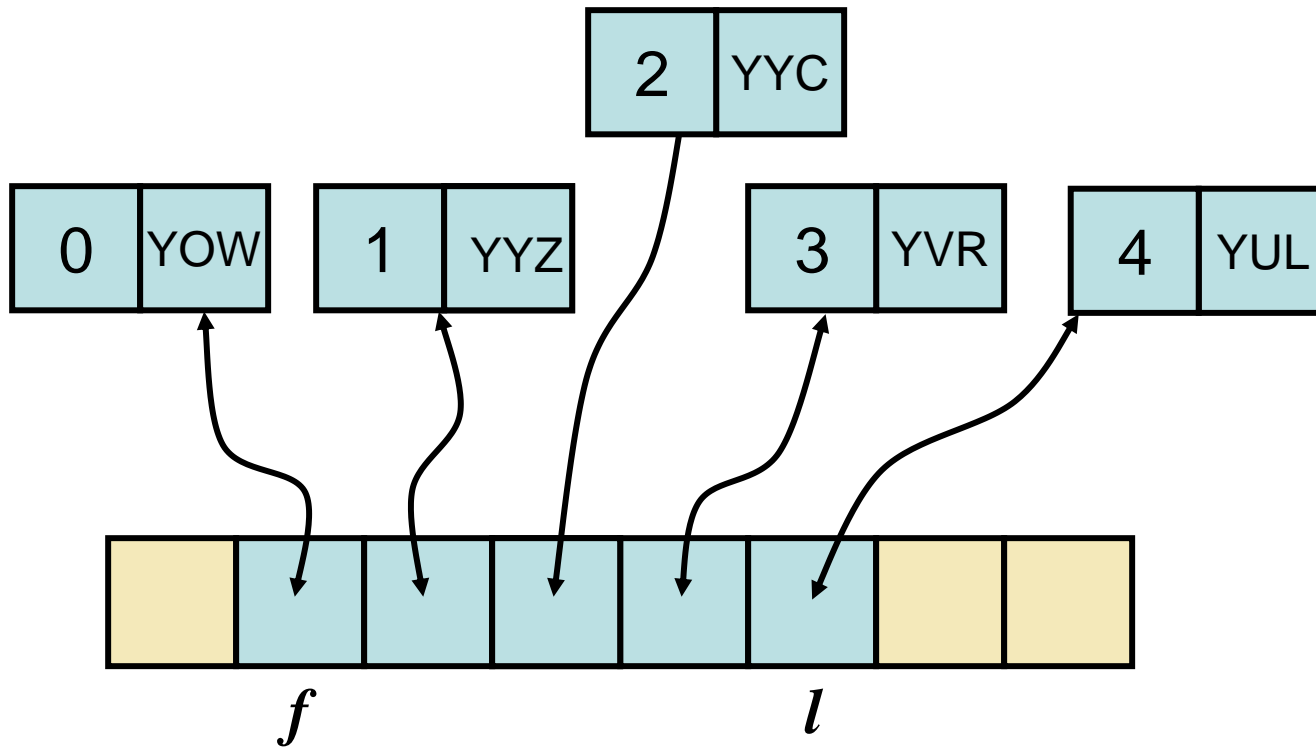
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- Circular array storing positions
- A position object stores:
  - Element
  - index
- $f$  and  $l$  keep track of first and last positions

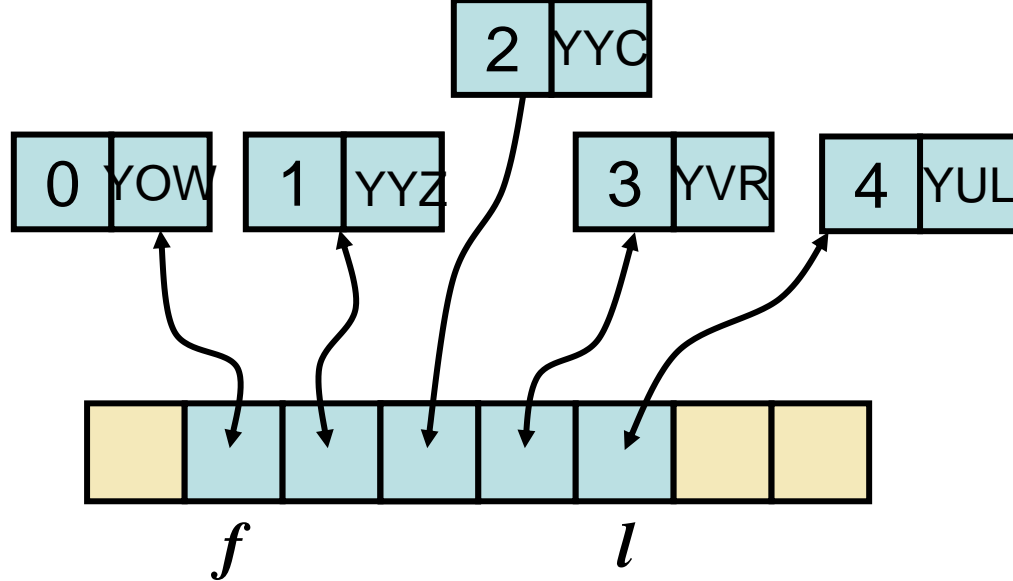




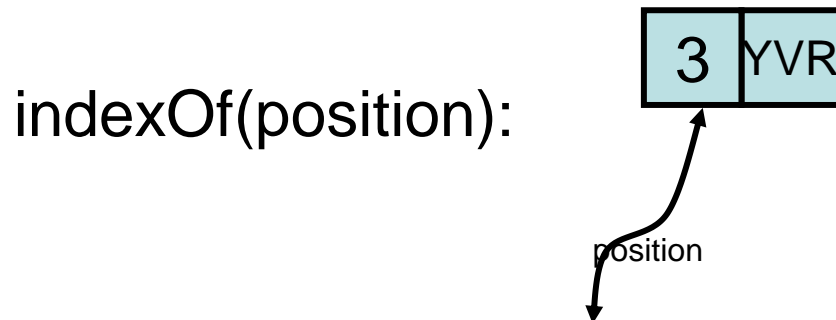




Change all other ranks



`atIndex(i)`      Direct access to the position at index  $f+i$



Immediate access to the corresponding index

# Array-based Implementation

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addFirst, addBefore, addAfter, remove

$O(n)$

Also: add, remove based on the index

$O(n)$

Other methods

$O(1)$

# Implementation with Doubly Linked List

---

All methods are inherited ....

a b c d e f g H i j k l

Bridges:

atIndex(i), indexOf(p):  $O(n)$



Must traverse the list



# Summary: Array-based implementation of Sequences

---

Need to move elements

addFirst, addBefore, addAfter, add(i,e) ----  $O(n)$

remove(position) remove(index) ----  $O(n)$

Bridges: atIndex(i), indexOf(p): ----  $O(1)$

$r = f - i$

$p = r, i = f + r$

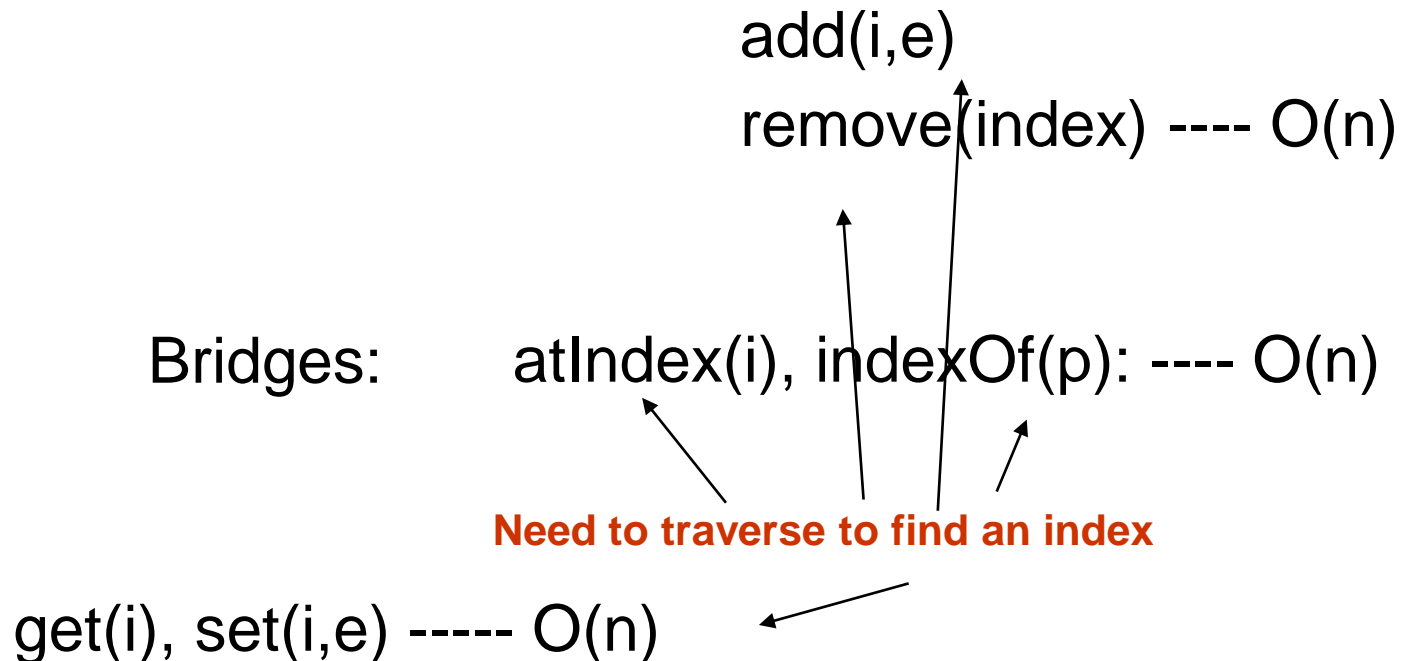
get(i), set(i,e) -----  $O(1)$

Because the position contains  
also the index

# Summary: Implementation of Sequences by Doubly-linked lists

---

addFirst, addBefore, addAfter, remove(position) ----  $O(1)$



# facebook in real life

Hi Friend!

Poke!

