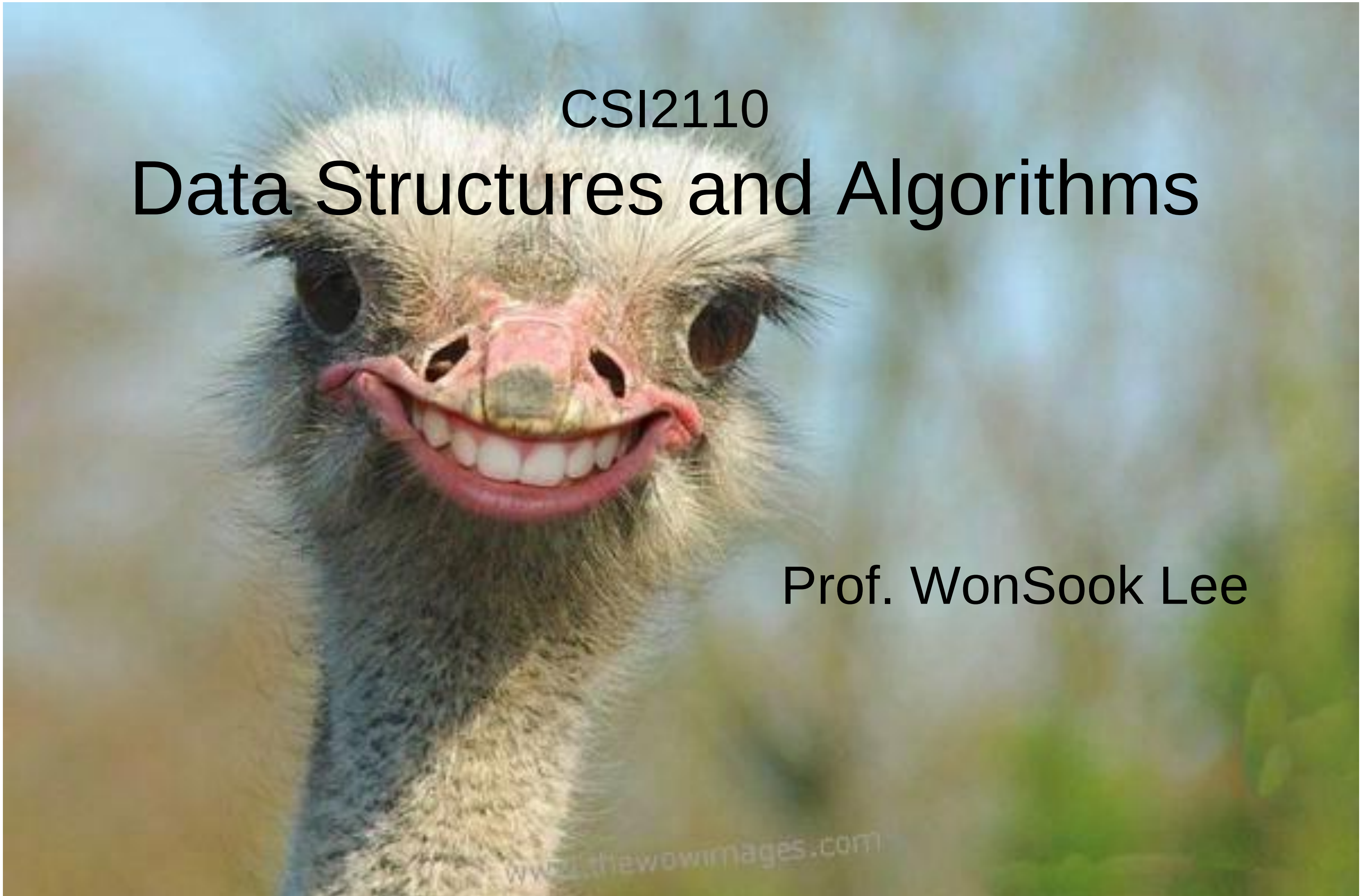
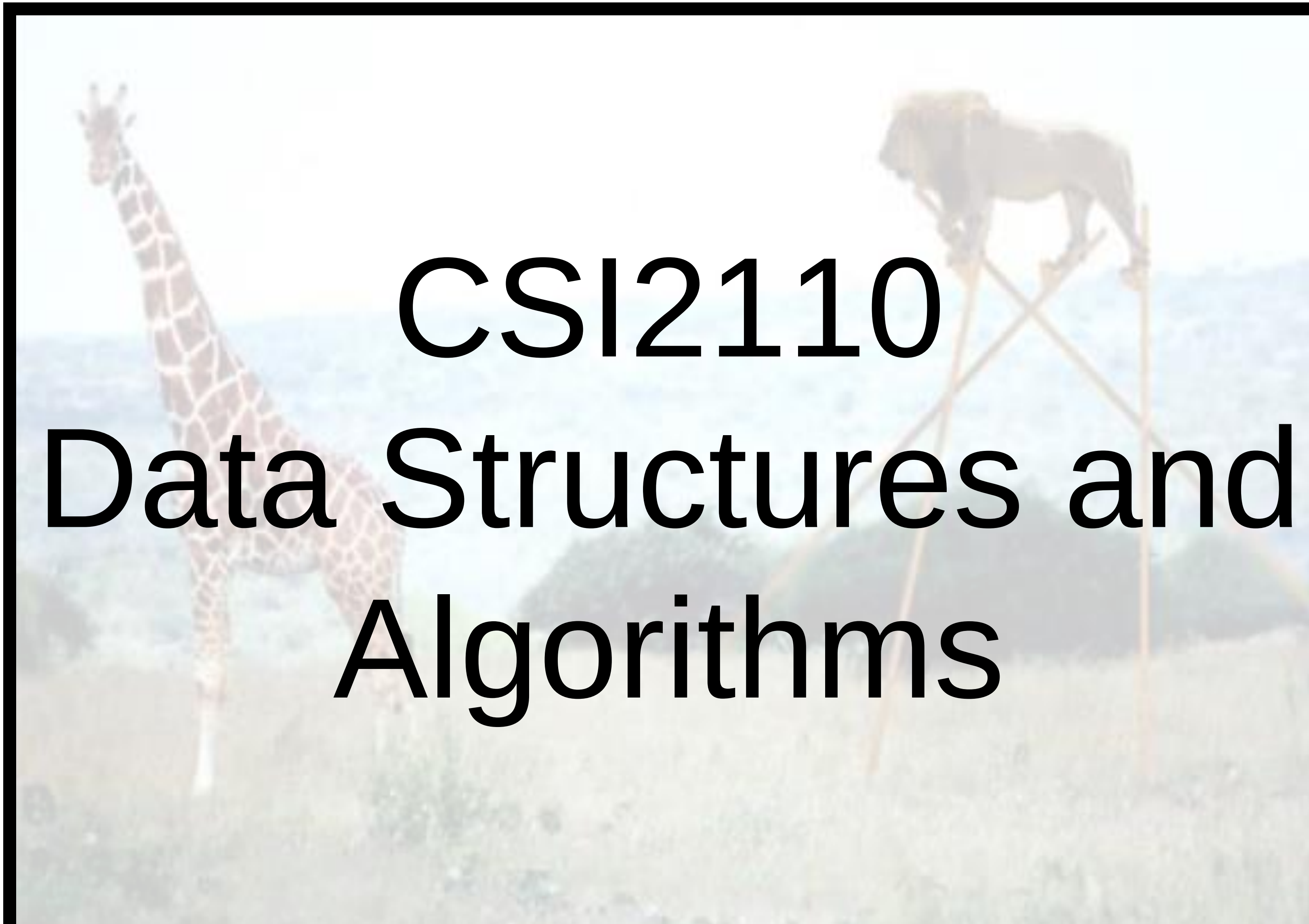


CSI2110

Data Structures and Algorithms

Prof. WonSook Lee



CSI2110
Data Structures and Algorithms

Overview

- Abstract Data Types
- Stack
- Queue
- Deque
  - double ended queue : pronounce “deck”

Abstract Data Types (ADTs)

- An Abstract Data Type is an abstraction of a data structure.

The ADT specifies:
  - what can be stored in the ADT
  - what operations can be done on/by the ADT

- For example, if we are going to model a bag of marbles as an ADT, we could specify that:
  - this ADT stores marbles
  - this ADT supports putting in a marble and getting out a marble.

Abstract Data Types (ADTs)

- Specify precisely the operations that can be performed
- The implementation is HIDDEN and can easily change

EXAMPLES

Objects of type: Phone Book

Operations: find, add, remove …

Abstract Data Types (ADTs)

- There are lots of formalized and standard ADTs.

- In this course we are going to learn a lot of different standard ADTs. (**stacks**, queues, dictionary...)

Stacks, Queues, and Deques

ADT Stack

- Implementation with Arrays
- Implementation with Singly Linked List

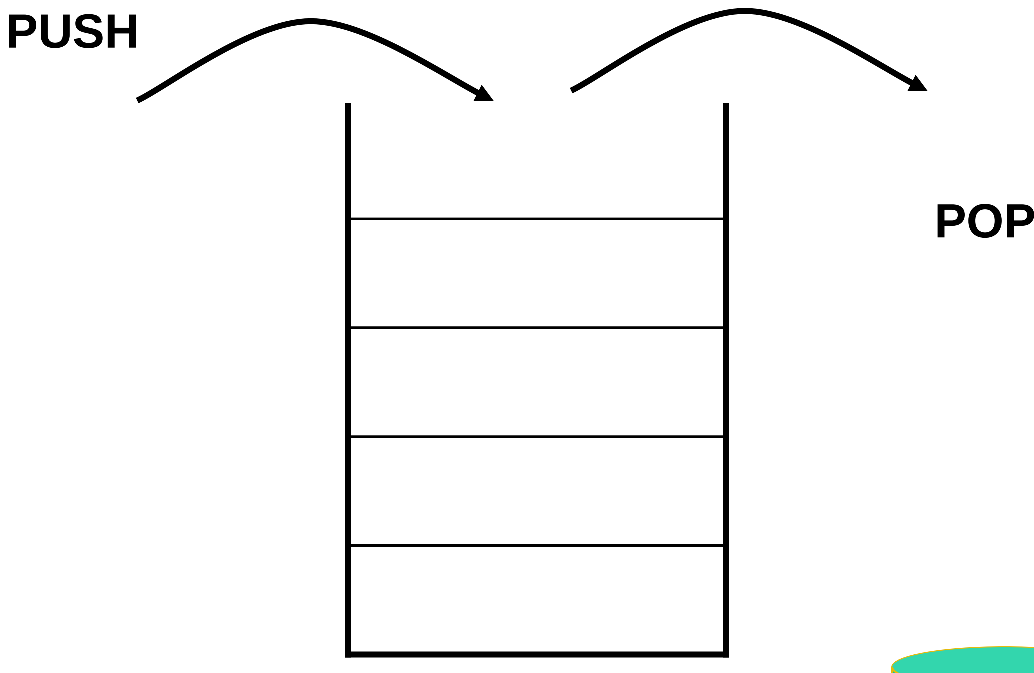
ADT Queue

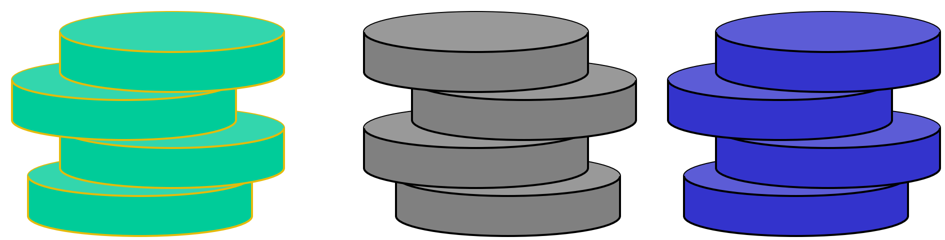
- Implementation with Arrays
- Implementation with Singly Linked List

ADT Double Ended Queues

- Implementation with doubly Linked List

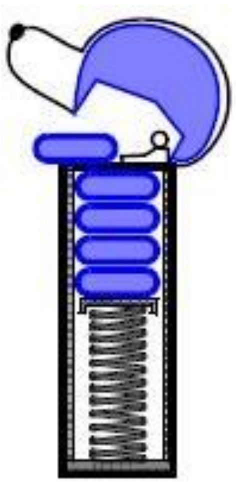
Stacks

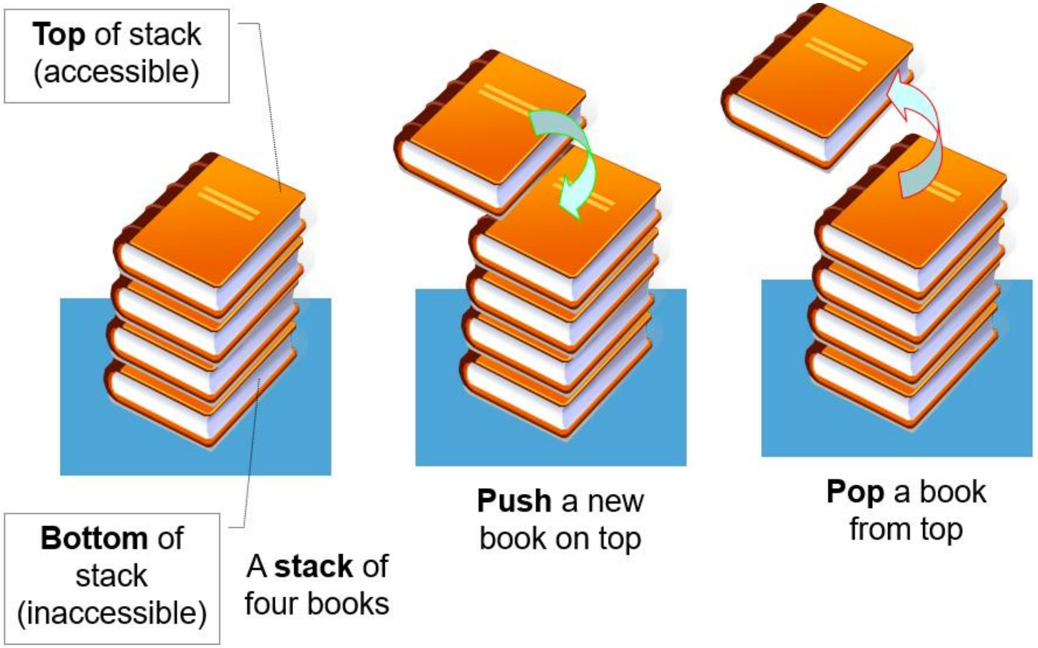




The Stacks

- A stack is a container for inserting and removing objects according to the principle, the last entered element, first go out. (last-in-first-out, or LIFO)
- The objects can be inserted all together, but only the last element (the most recently inserted) can be removed.
- Analogue: PEZ® dispenser





The Stack Abstract Data Type

- Main methods:
  - push(o): Inserts object o onto top of stack
  - pop(): Removes the top object of stack and returns it; if the stack is empty, an error occurs

- Support methods:
  - size(): Returns the number of objects in stack
  - isEmpty(): Return a boolean indicating if stack is empty.
  - top(): Return the top object of the stack, without removing it; if the stack is empty, an error occurs.

Applications of Stacks

- Direct applications
  - Page-visited history in a Web browser
  - Undo sequence in a text editor
  - Chain of method calls in the Java Virtual Machine
- Indirect applications
  - Auxiliary data structure for algorithms
  - Component of other data structures

Examples

Evaluating an expression with two stacks

(((10+5) + 5) / ((2+3) * 2))

How do we solve it ?

One possible sequence of operations

(((10+5) + 5) / ((2+3) * 2))

((15 + 5) / (5 * 2))

(20 / 10)

2

Another one

(((10 + 5) + 5) / ((2 + 3) * 2))

(15 + 5) / ((2 + 3) * 2))

(20 / ((2 + 3) * 2))

(20 / (5 * 2))

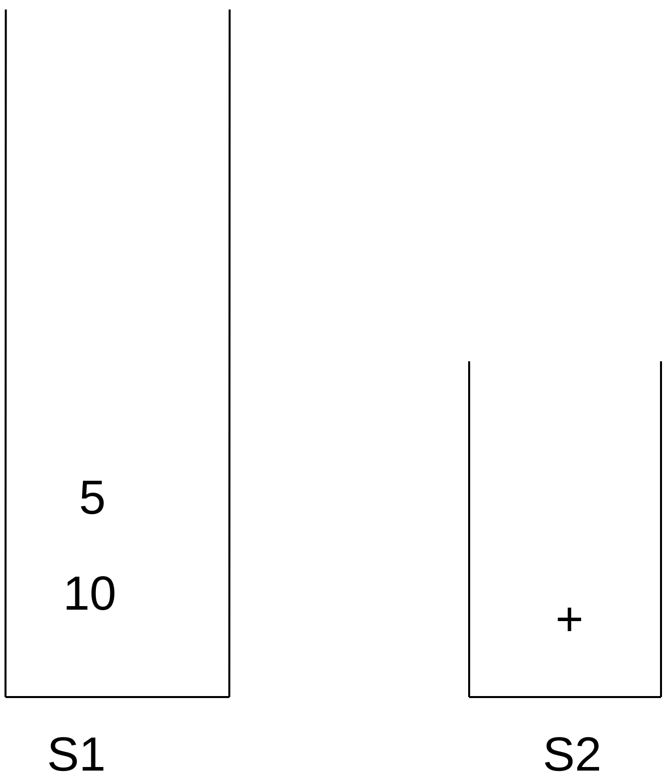
(20 / 10)

2

With two Stacks

One for the operands
One for the operators

(((10+5) + 5) / ((2+3) * 2))



With two Stacks

One for the operands
One for the operators

when find CLOSED parenthesis

((15 + 5) / ((2+3) * 2))

(((10+5) + 5) / ((2+3) * 2))

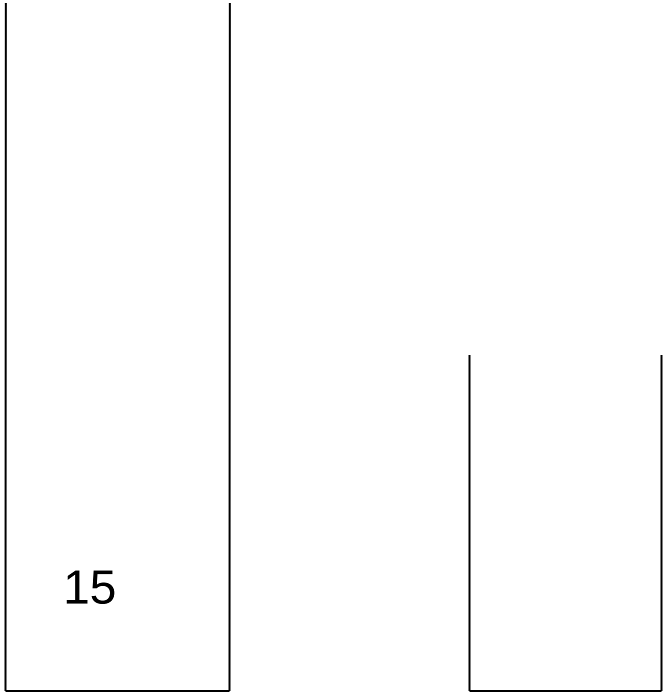
POP S1 5

POP S2 +

POP S1 10

Evaluate: 10 + 5 = 15

PUSH S1 the result



S1 S2

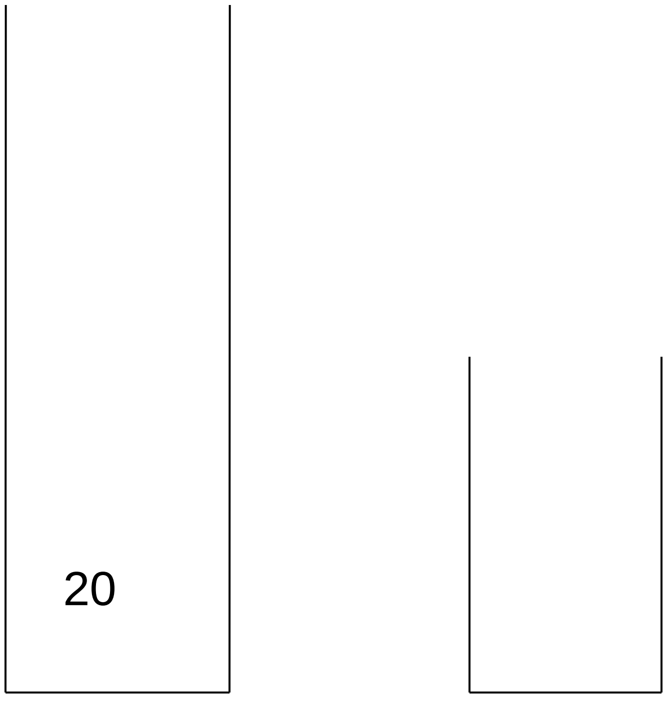
With two Stacks

One for the operands
One for the operators

CLOSED parenthesis

(20 / ((2+3) * 2))

(((10+5) + 5) / ((2+3) * 2))



S1 S2

POP S1 5

POP S2 +

POP S1 15

Evaluate: 15 + 5 = 20

PUSH S1 the result

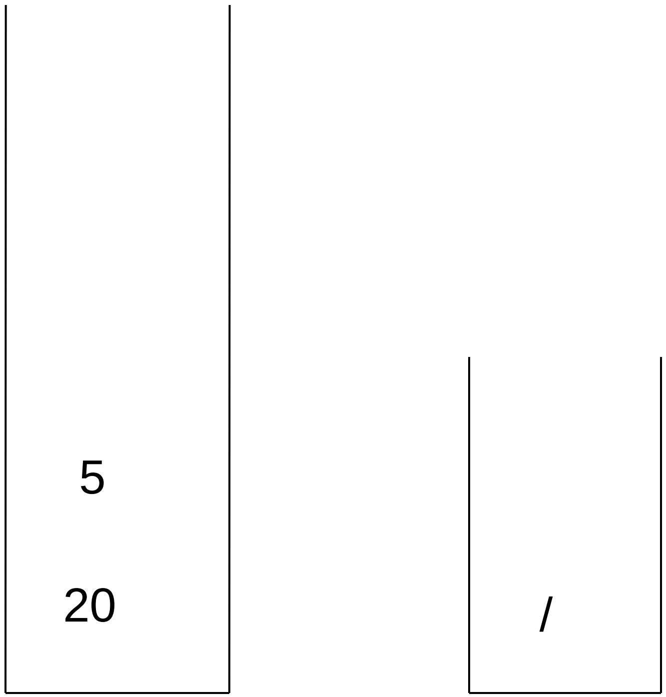
With two Stacks

One for the operands
One for the operators

CLOSED parenthesis

(20 / ((2+3) * 2))

(((10+5) + 5) / ((2+3) * 2))



S1 S2

POP S1 3

POP S2 +

POP S1 2

Evaluate 2+ 3 = 5

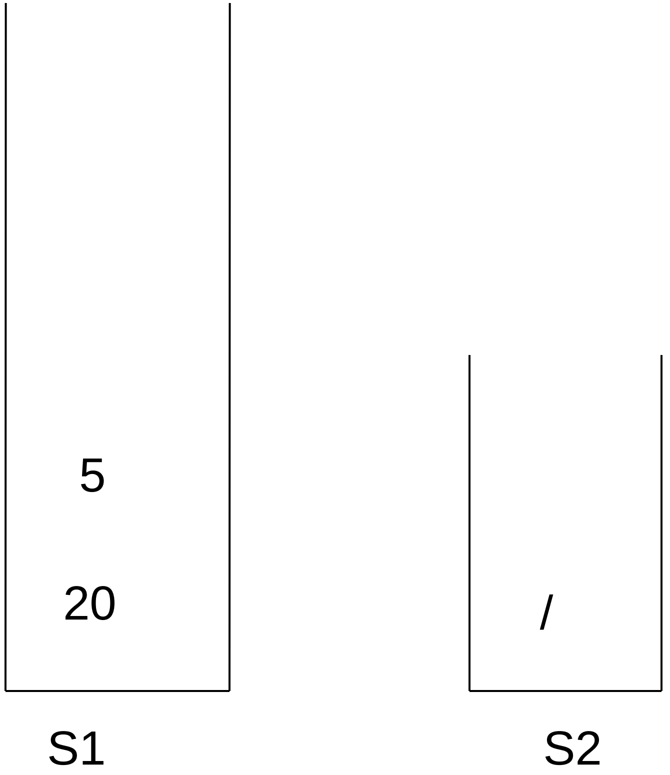
PUSH S1 the result

With two Stacks

One for the operands
One for the operators

(20 / (5 * 2))

(((10+5) + 5) / ((2+3) * 2))



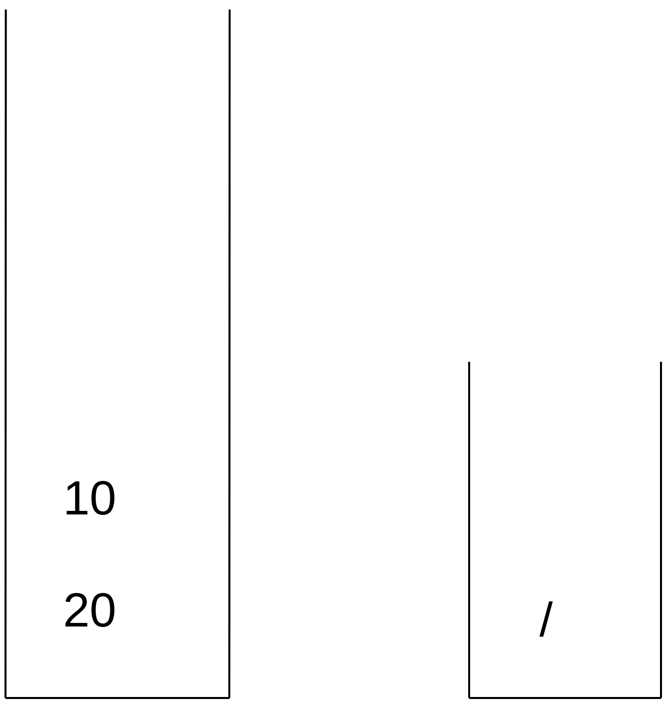
With two Stacks

One for the operands
One for the operators

CLOSED parenthesis

(20 / (5 * 2))

(((10+5) + 5) / ((2+3) * 2))



S1 S2

POP S1 2

POP S2 *

POP S1 5

Evaluate: 5 * 2 = 10

PUSH S1 result

With two Stacks

One for the operands
One for the operators

CLOSED parenthesis

(20 / 10)

(((10+5) + 5) / ((2+3) * 2))

10

/

20

20 / 10= 2

S1

S2

Examples

Checking for balanced parenthesis

{ [(a+b) -c]/d}

{ [a+b) -c]/d}

Example: Evaluation of arithmetic expressions (Postfix notation)

Infix	Postfix
A + B	**A B +**

(x – y)/(u + v)

(x – y) (u+ v) /

x y - u v + /

INFIX	POSTFIX
$a+b$	$a\ b\ +$
$(x - y + z)$	$x\ y\ -\ z\ +$
$(x - y - z)/(u + v)$	$x\ y\ -\ z\ -\ u\ v\ +\ /$

In general:

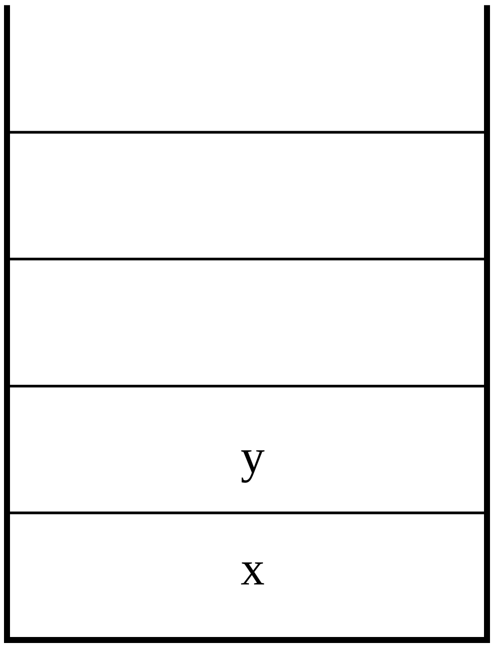
A operator B

A B operator

x y – z – u v + /

x: push(x)

y: push(y)

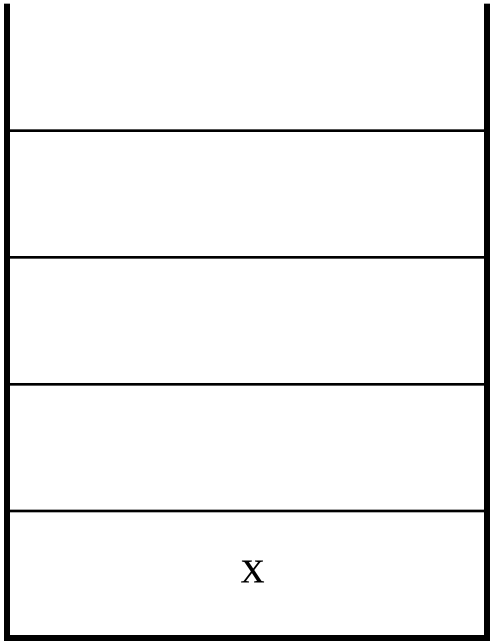


$$x\ y\ -\ z\ -\ u\ v\ +\ /$$

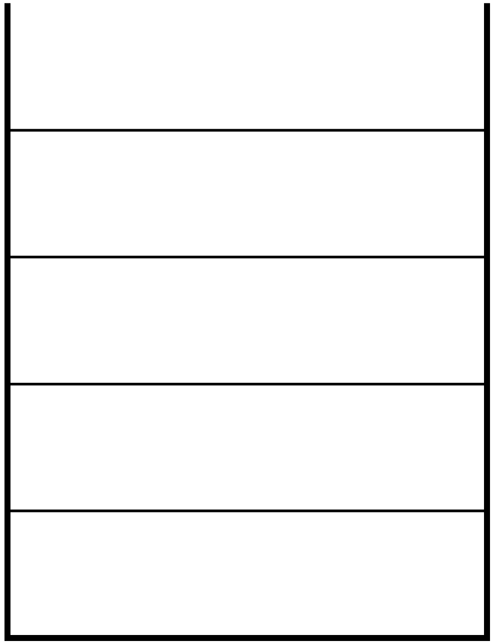
x: push(x)

y: push(y)

– : pop() (we get y)



$$x\ y - z - u\ v + /$$



x: push(x)

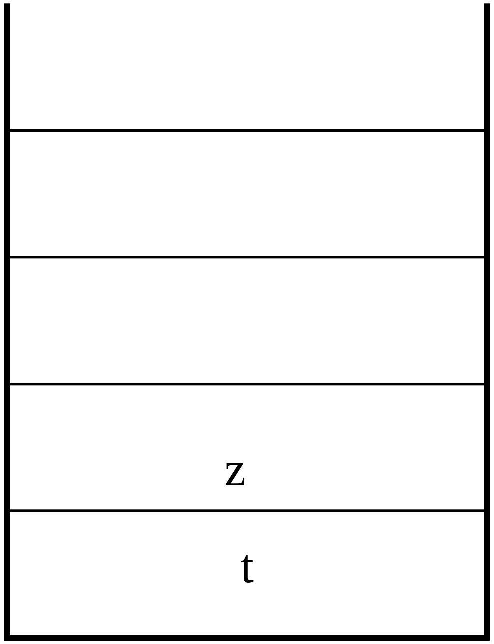
y: push(y)

– : pop() (we get y)

pop() (we get x)

$\boxed{x - y} = t$

x y – z – u v + /



x: push(x)

y: push(y)

– : pop() (we get y)

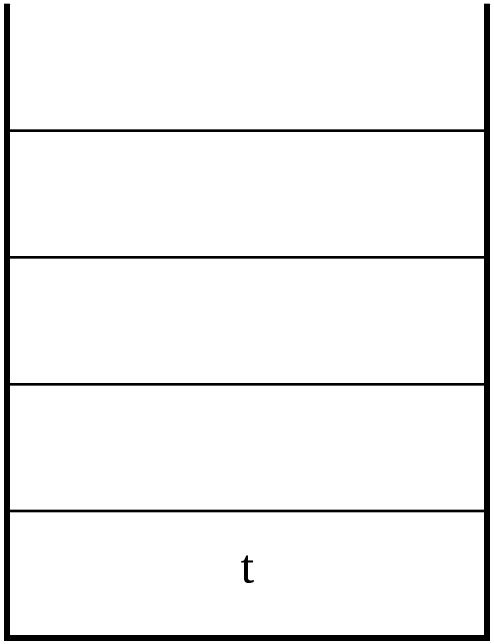
pop() (we get x)

$\boxed{x - y} = t$

push(t)

z: push(z)

x y – z – u v + /



x: push(x)

y: push(y)

– : pop() (we get y)

pop() (we get x)

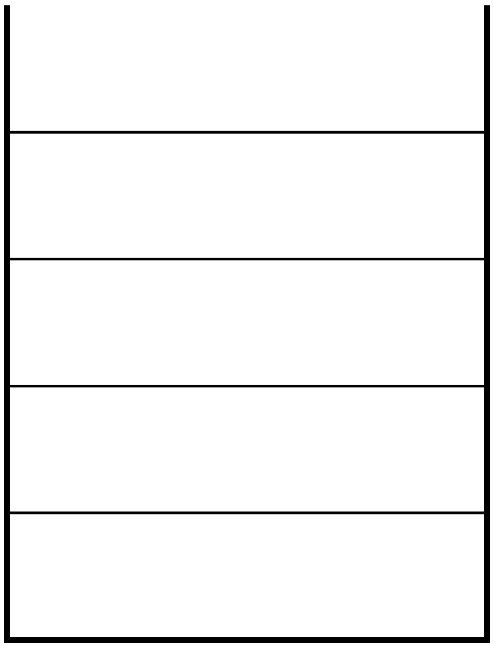
$\boxed{x - y} = t$

push(t)

z: push(z)

– : pop() (we get z)

x y – z – u v + /



x: push(x)

y: push(y)

– : pop() (we get y)

pop() (we get x)

$\boxed{x - y} = t$

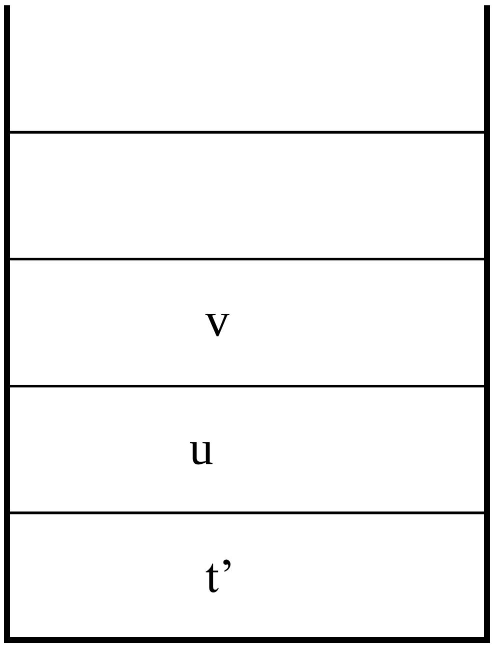
push(t)

z: push(z)

– : pop() (we get z)

pop() (we get t)

x y – z – u v + /



x: push(x)

y: push(y)

– : pop() (we get y)

pop() (we get x)

$\boxed{x - y} = t$

push(t)

z: push(z)

– : pop() (we get z)

pop() (we get t)

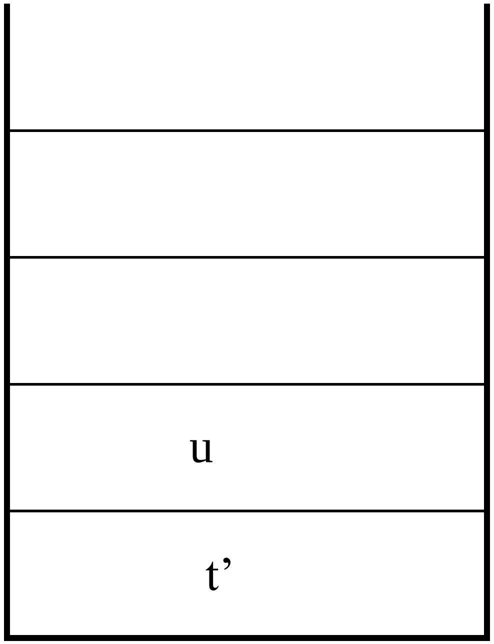
$\boxed{t - z} = t'$

push(t’)

u: push(u)

v: push(v)

x y – z – u v + /



x: push(x)

y: push(y)

– : pop() (we get y)

pop() (we get x)

$\boxed{x - y} = t$

push(t)

z: push(z)

– : pop() (we get z)

pop() (we get t)

$\boxed{t - z} = t'$

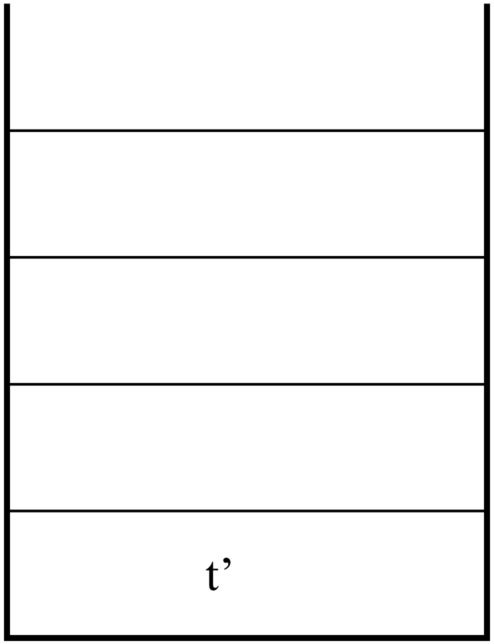
push(t’)

u: push(u)

v: push(v)

\+ : pop() (we get v)

x y – z – u v + /



x: push(x)

y: push(y)

– : pop() (we get y)

pop() (we get x)

$\boxed{x - y} = t$

push(t)

z: push(z)

– : pop() (we get z)

pop() (we get t)

$\boxed{t - z} = t'$

push(t’)

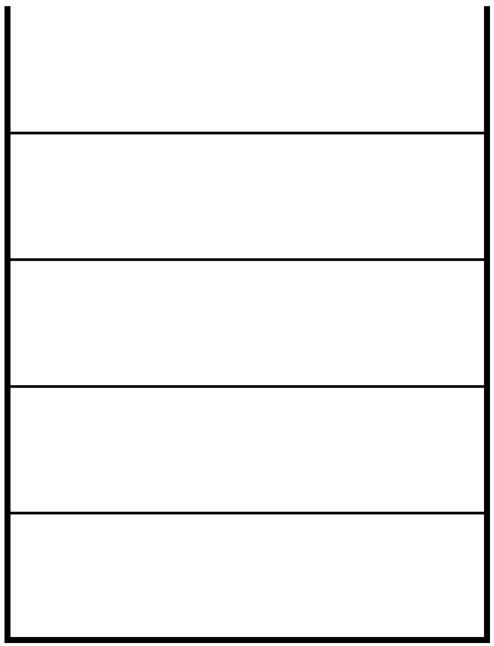
v: push(v)

\+ : pop() (we get v)

pop() (we get u)

$\boxed{u+v} = t''$

x y – z – u v + /



x: push(x)

y: push(y)

– : pop() (we get y)

pop() (we get x)

$\boxed{x - y} = t$

push(t)

z: push(z)

– : pop() (we get z)

pop() (we get t)

$\boxed{t - z} = t'$

push(t')

v: push(v)

+ : pop() (we get v)

pop() (we get u)

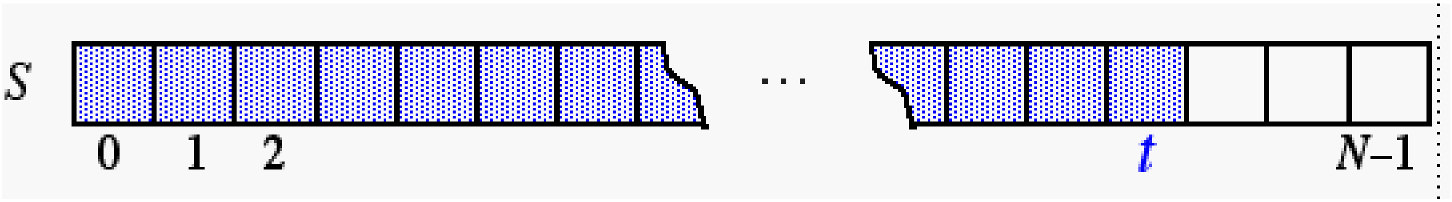
$\boxed{u+v} = t''$

/ : pop() (we get t')

Result = t'/t''

Implementing a Stack with an Array

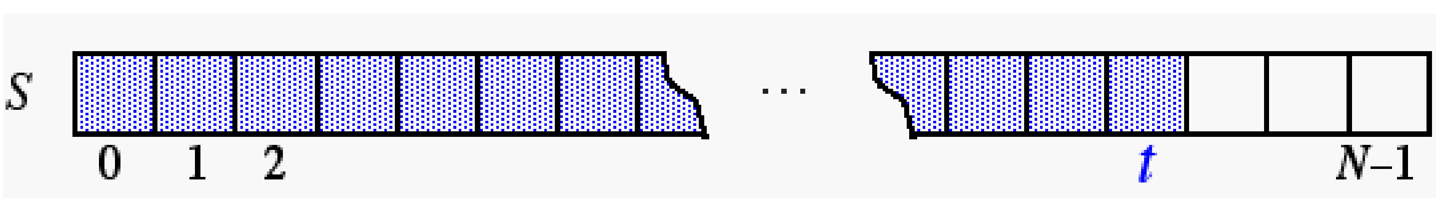
The stack consists of an N-element array S and an integer variable t, the index of the top element in array S.



Algorithm size():
return t +1
Algorithm isEmpty():
return (t < 0)
Algorithm top():
if isEmpty() then
ERROR
return S[t]

```
Algorithm push(obj):
        if size() = N then
                ERROR
        t ← t + 1
        S[t] ← obj
```

```
Algorithm pop():
        if isEmpty() then
                ERROR
        e ← S[t]
        S[t] ← null
        t ← t-1
        return e
```



Performance and Limitations

Time

- Performance

size()	O(1)
isempty()	O(1)
top()	O(1)
push(obj)	O(1)
pop()	O(1)

Space: O(N)

N= size of the Array

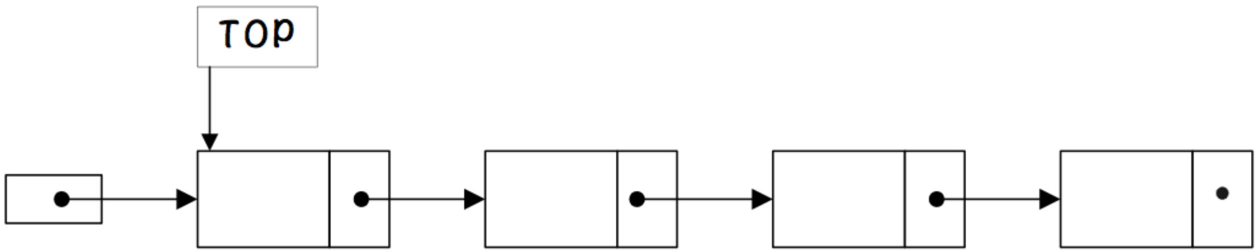
- Limitations

What does it mean O(1) ?

STATIC STRUCTURE

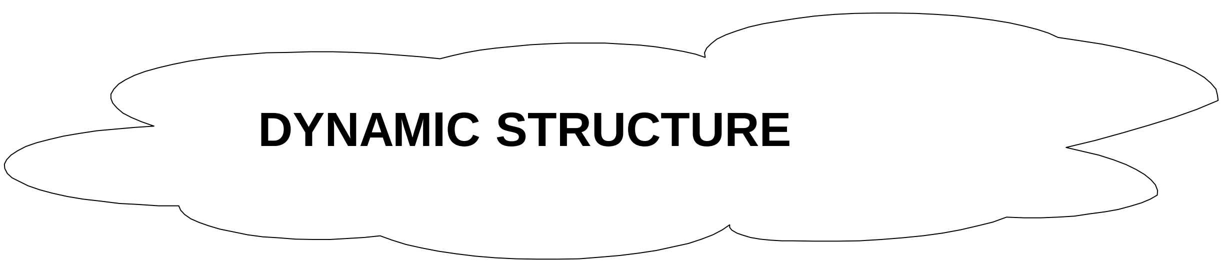
we will see the formal definition next lecture

Implementing a Stack with a Singly Linked List

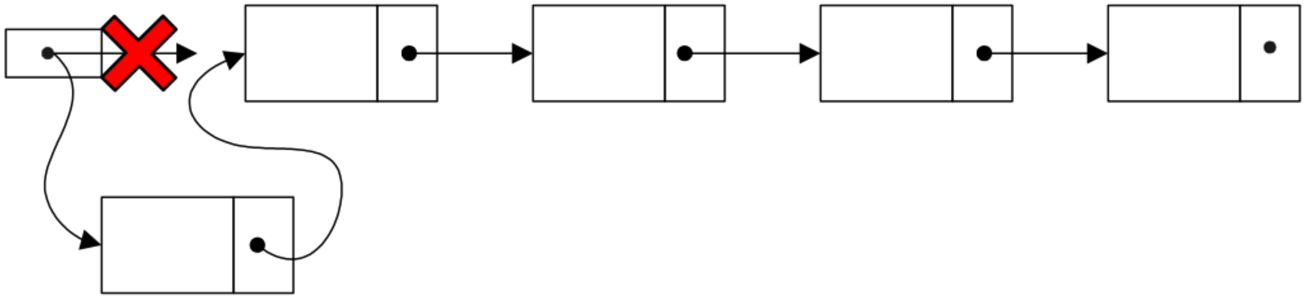


size = 4

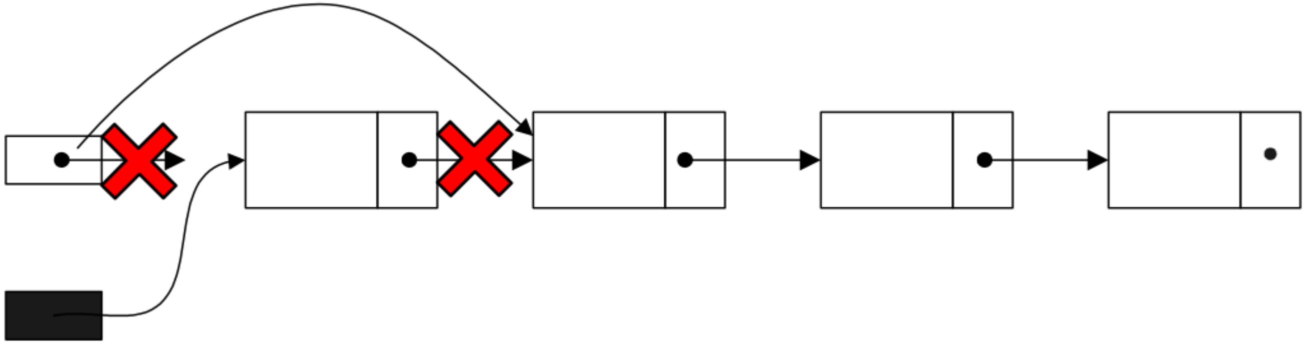
-Singly linked list plus a variable containing the current size of the list



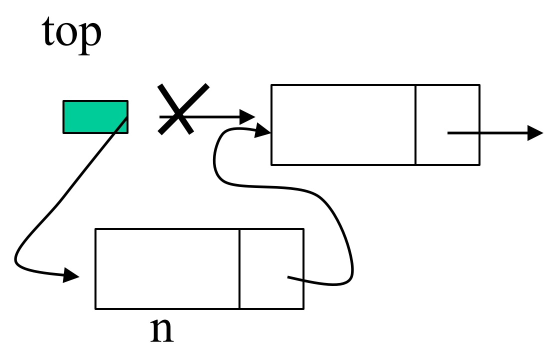
PUSH: Add at the front



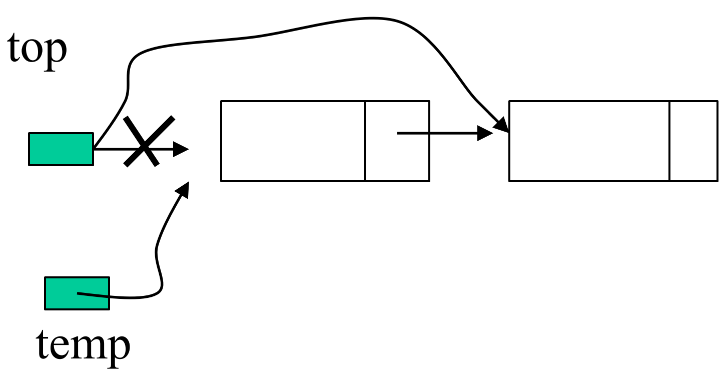
POP: Take the first



node:
node.item
node.next



Algorithm push(obj):
- n □ new Node
- n.item □ obj
- n.next □ top
- top □ n
- size++



Algorithm pop():
- if isEmpty() then
  - ERROR
- temp □ top.item
- top □ top.next
- size- -
- return temp

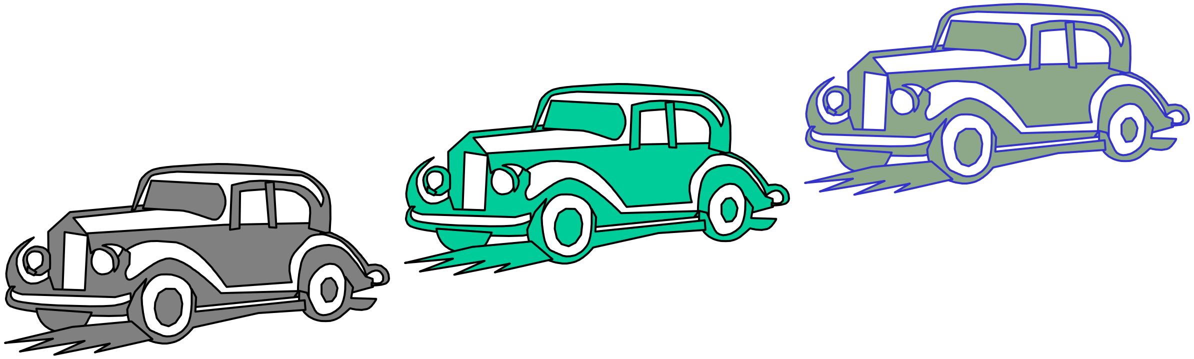
Performance

Time:

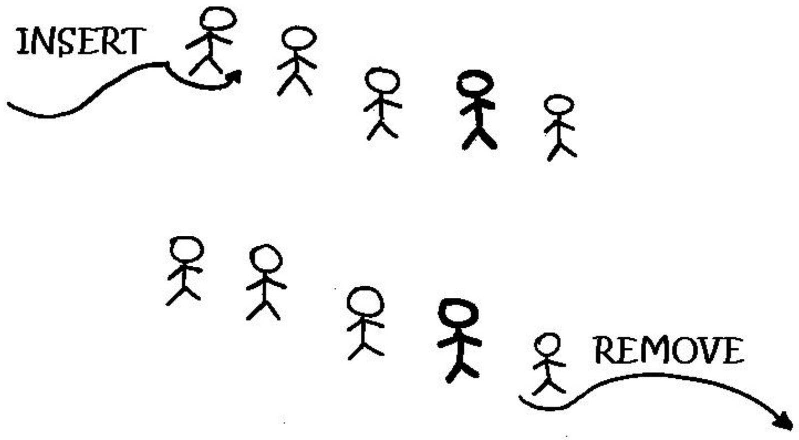
size()	O(1)
isempty()	O(1)
top()	O(1)
push(obj)	O(1)
pop()	O(1)

Space: Variable

The Queue



The Queue



first-in-first-out (FIFO)

1-2-3-4-5-6-7

•A queue is different from a stack in the way how the insertion and removal work.

•The elements can be inserted all together, but only the element which stayed longest time in the queue can be out.

•Elements are inserted at the **rear** (enqueued) and removed from the **front** (dequeued)

Applications of Queues

- Direct applications
  - Waiting lists, bureaucracy
  - Access to shared resources (e.g., printer)
  - Multiprogramming
- Indirect applications
  - Auxiliary data structure for algorithms
  - Component of other data structures

Fun Example: Palindromes

“eye”

“madam”

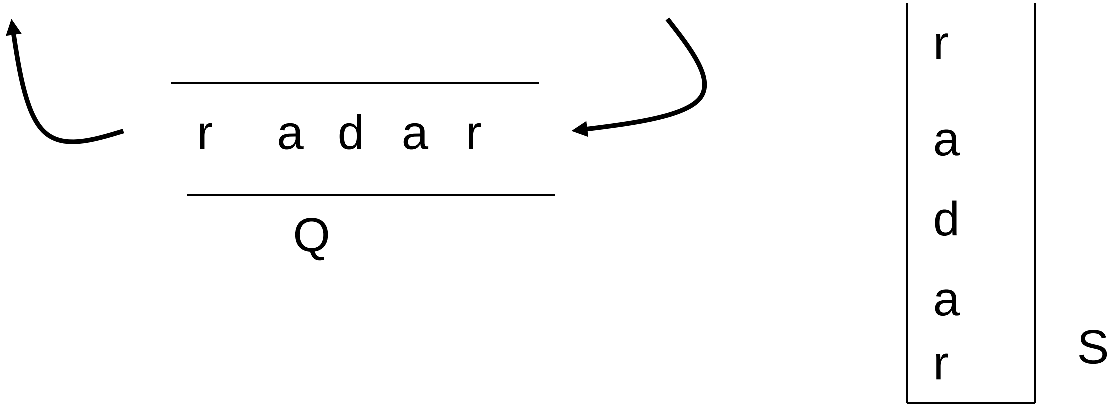
“radar”

“madam i'm adam"

“Able was I ere I saw Elba”

Read the line into a stack and into a queue

Compare the outputs of the queue and the stack

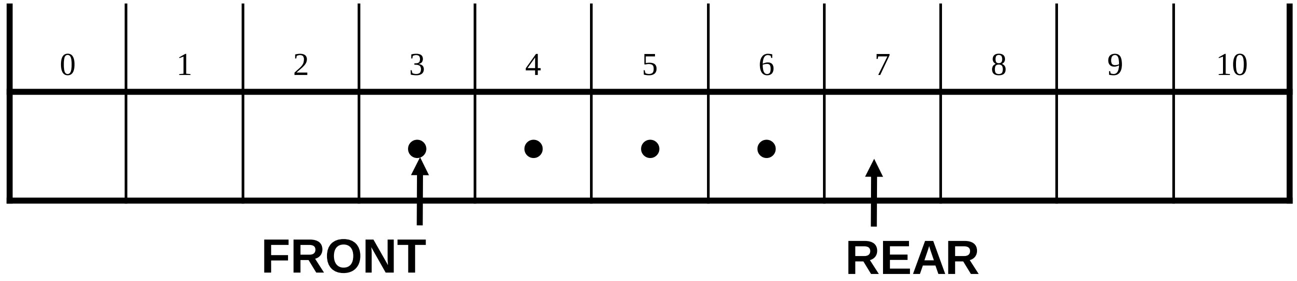


The Queue Abstract Data Type

- Fundamental methods:
  enqueue(o): Insert object o at the rear of the queue
  dequeue(): Remove the object from the front of the queue and return it; an error occurs if the queue is empty

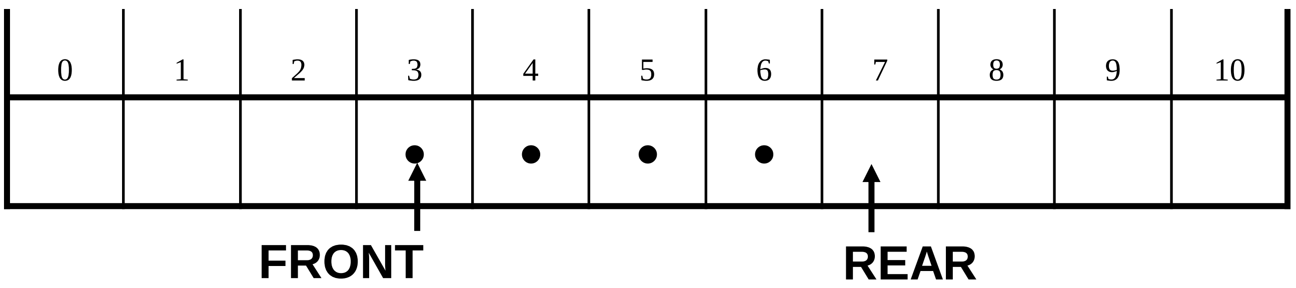
- Support methods:
  size(): Return the number of objects in the queue
  isEmpty(): Return a boolean value that indicates whether the queue is empty
  front(): Return, but do not remove, the front object in the queue; an error occurs if the queue is empty

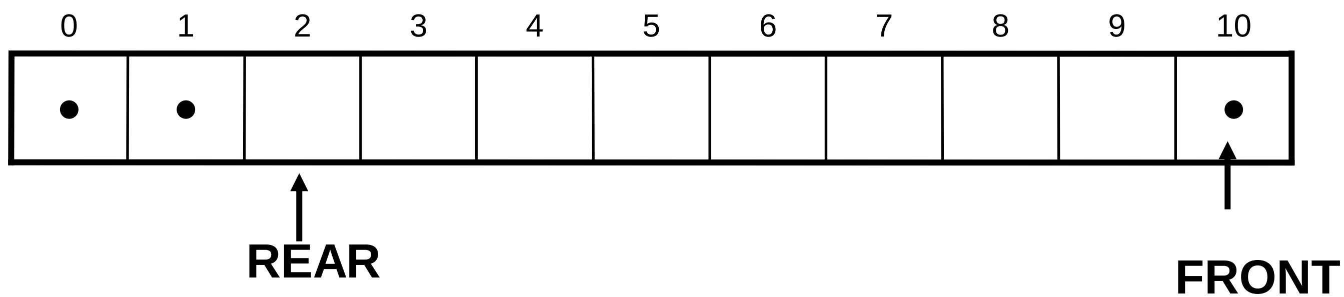
Implementing a Queue with an Array

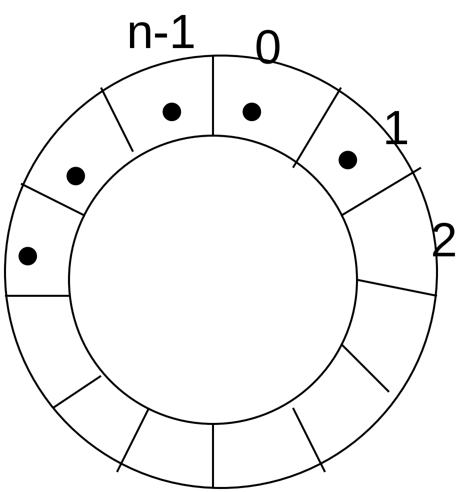


Insert at the **rear** and remove from **front**

Implementing a Queue with an Array







Remove: Front = (Front+1) **mod N**

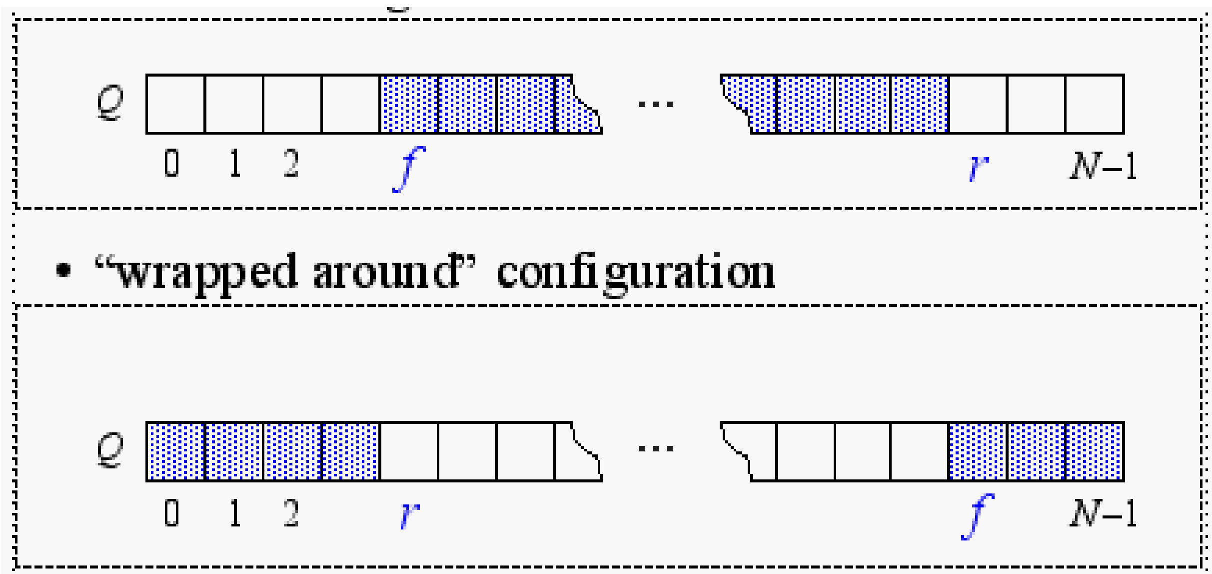
Insert: Rear = (Rear+1) **mod N**

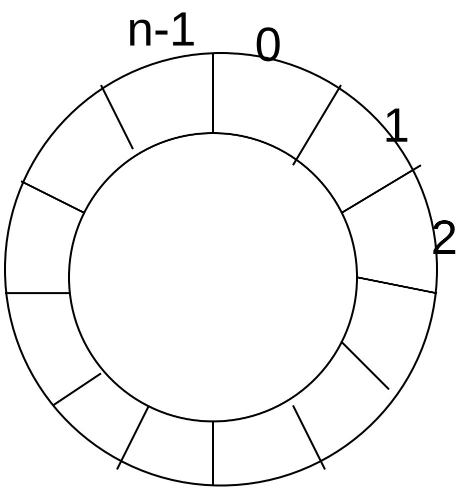
• Array in a circular fashion (“wrapped around”)
• Size   fixed at the beginning

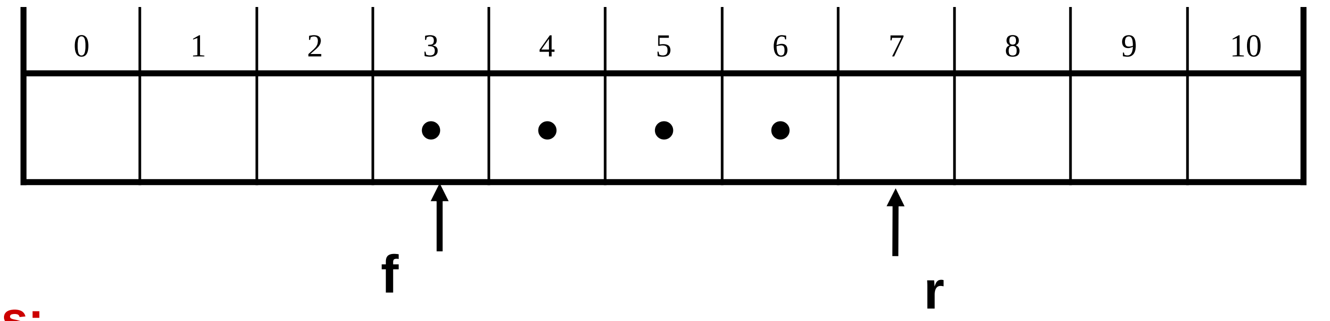
•The queue consists of an N-element array Q and two integer variables:

-f, index of the front element

-r, index of the element after the rear one





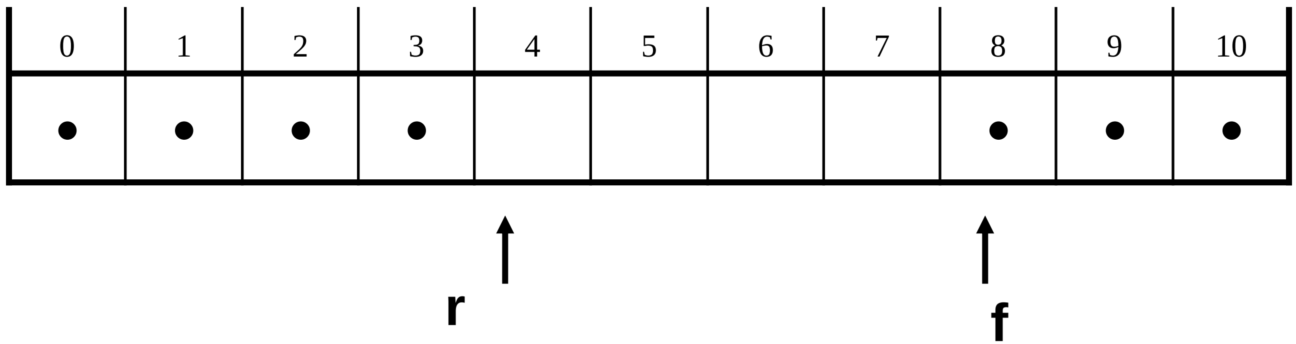


Questions:

What does f = r mean?

The queue is empty

How do we compute the number of elements in the queue from f and r?



(N - f + r) mod N

In the example:
(11 - 8 + 4) mod 11 = 7

```
Algorithm size():
        return (N - f + r) mod N

Algorithm isEmpty():
        return (f = r)

Algorithm front():
        if isEmpty() then
                ERROR
        return Q[f]
```

```
Algorithm dequeue():
        if isEmpty() then
                ERROR
        temp □ Q[f]
        Q[f] □ null
        f □ (f + 1) mod N
        return temp

Algorithm enqueue(o):
        if size = N - 1 then
                ERROR
         Q[r] □ o
```

Performance

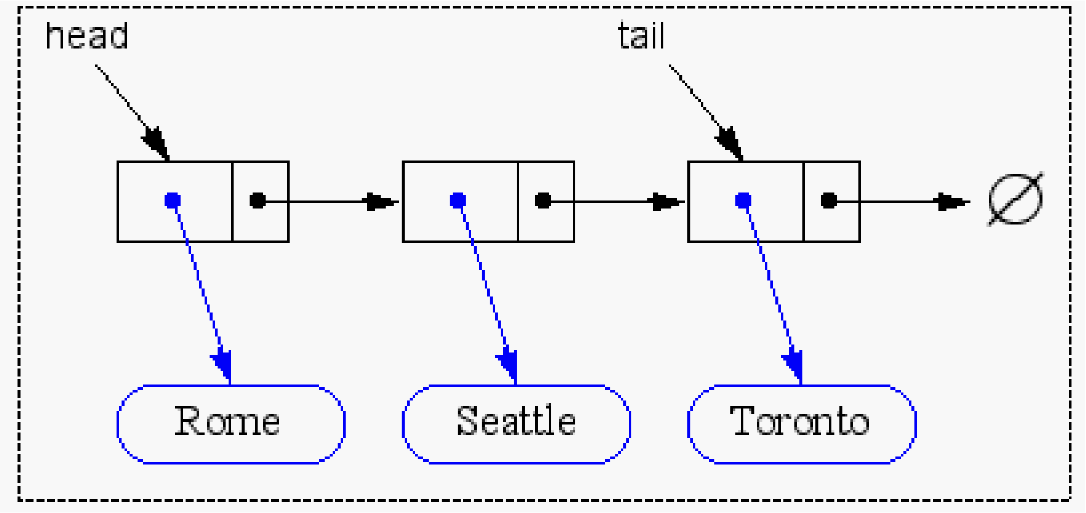
Time:

size()	O(1)
isempty()	O(1)
front()	O(1)
enqueue(o)	O(1)
dequeue()	O(1)

Space: O(N)

Implementing a Queue with a Singly Linked List

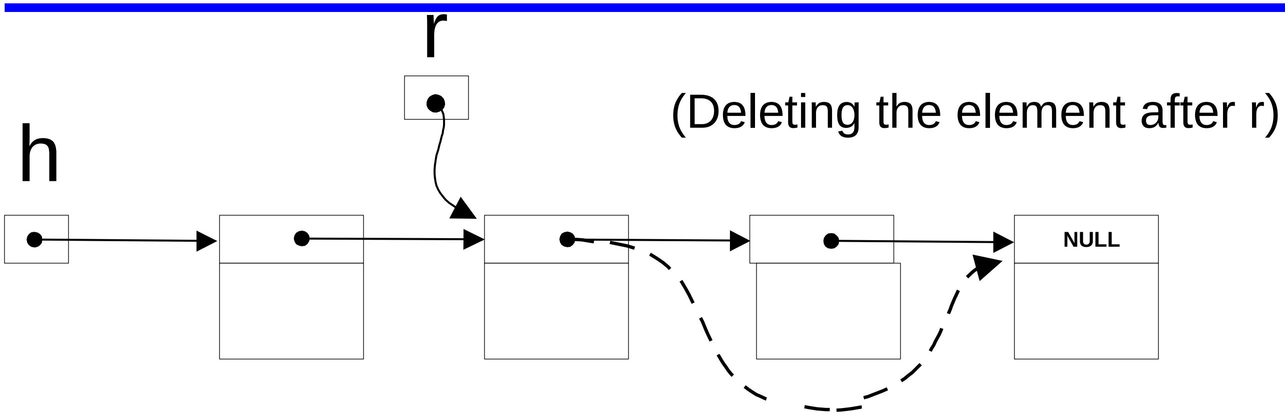
Nodes connected in singly linked list
We keep a pointer to the head and one to the tail



The head of the list is the front of the queue, the tail of the list is the rear of the queue.

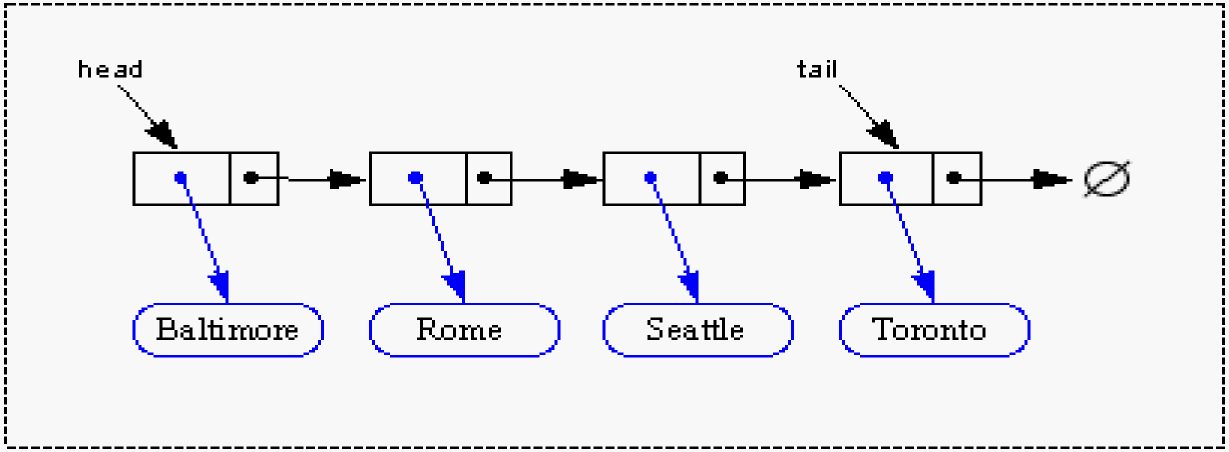
Why not the opposite?

From the last lesson : Deletion

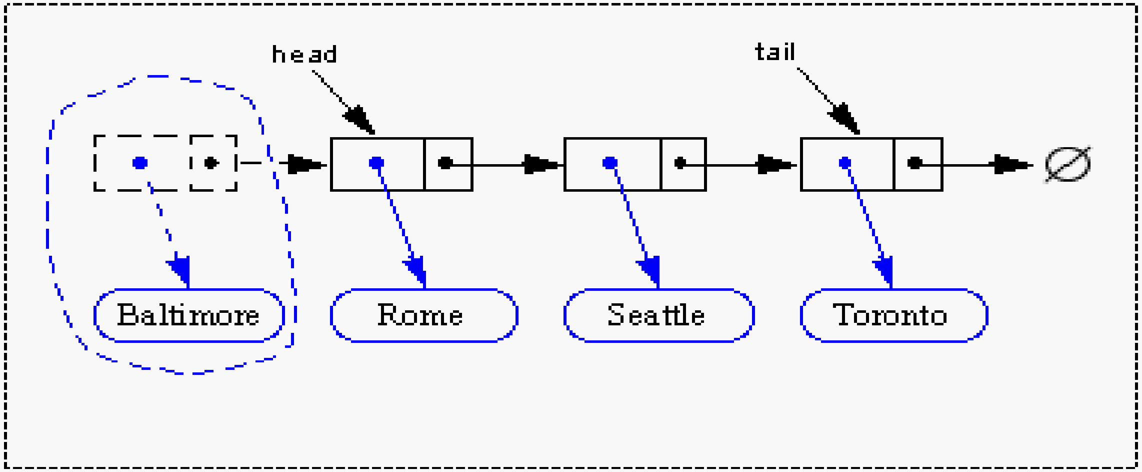


First element (easy)	h ← h.Next
Element after r (easy)	r.Next ← r.Next.Next
Element at r (difficult)	• Use a pointer to the preceding element, or • Exchange the contents of the element at r with the contents of the element following r, and delete The element after r. **Very difficult if r points to the last element!

Removing at the Head

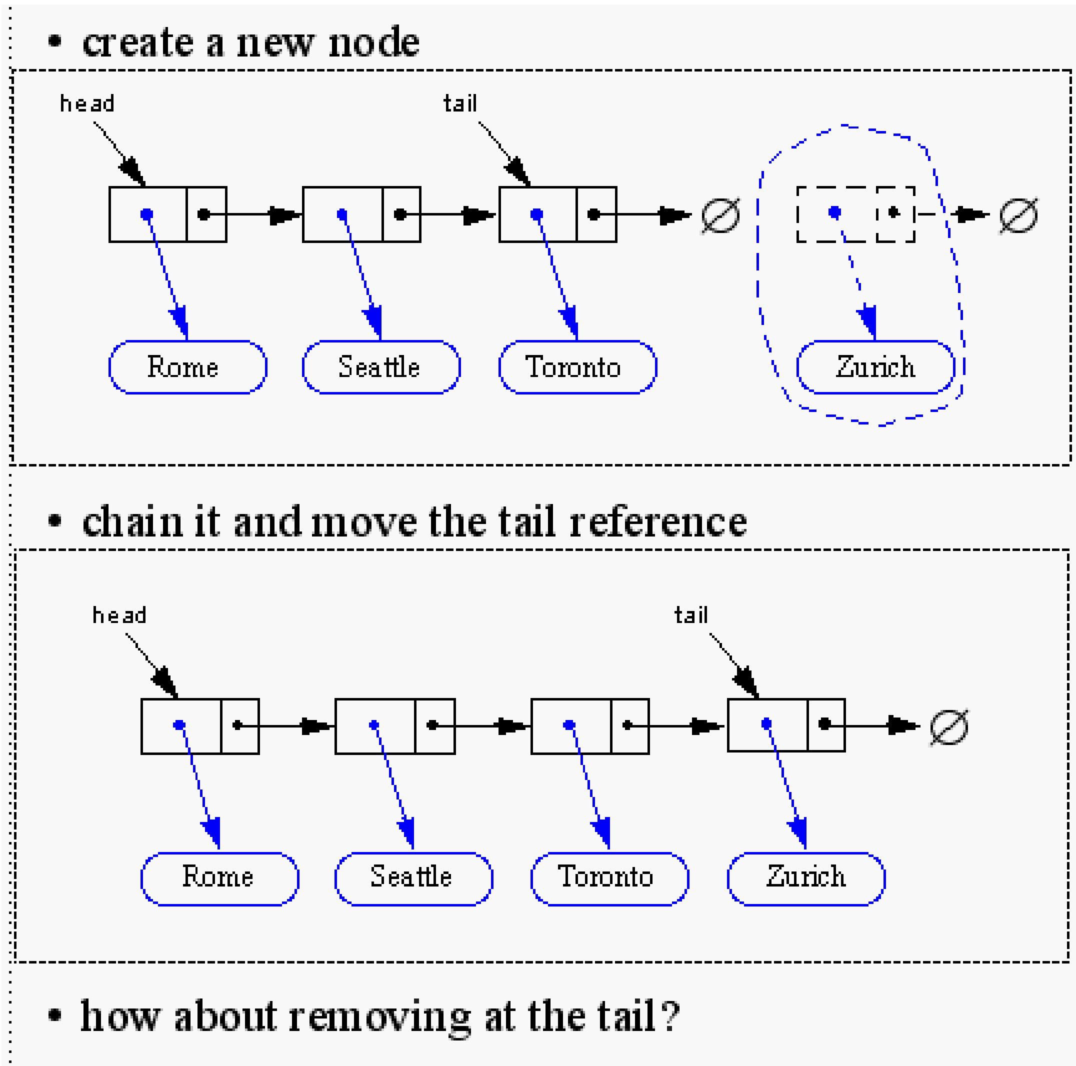


- advance head reference



- inserting at the head is just as easy

Inserting at the Tail



Performance

Time:

size()	O(1)
isempty()	O(1)
front()	O(1)
enqueue(o)	O(1)
dequeue()	O(1)

Space: Variable

If we know in advance a reasonable upper bound for the number of elements in the queue, then

⇨ ARRAYS

Otherwise

⇨ LISTS

A more general ADT: Double-Ended Queues (Deque)

A double-ended queue, or deque, supports insertion and deletion from the front and back.

Main methods:

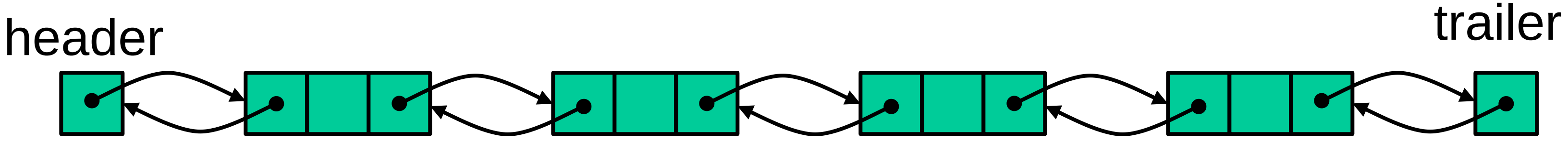
- insertFirst(e): Insert e at the beginning of deque.
- insertLast(e): Insert e at end of deque
- removeFirst(): Removes and returns first element
- removeLast(): Removes and returns last element

Support methods:

- first()
- last()
- size()
- isEmpty()

Implementing Deques with Doubly Linked Lists

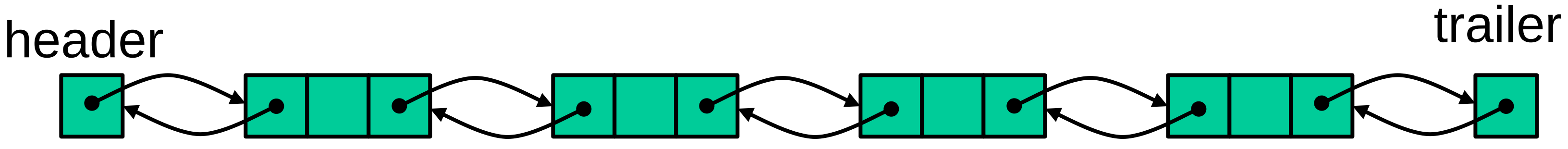
Deletions at the tail of a singly linked list cannot be done efficiently



To implement a deque, we use a doubly linked list
with special header and trailer nodes

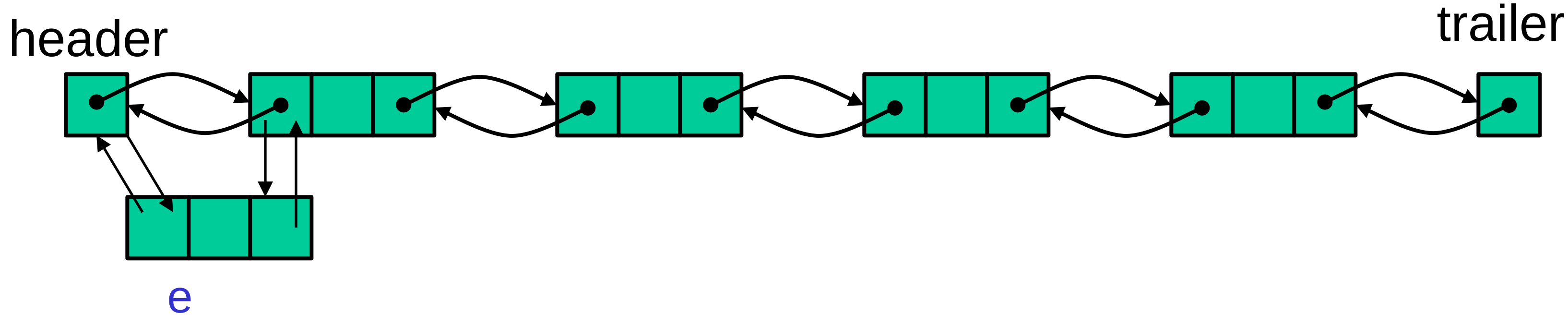
•The **header** node goes before the first list element. It has a valid next link but a null prev link.

•The **trailer** node goes after the last element. It has a valid prev reference but a null next reference.



<u>NOTE</u>: the header and trailer nodes are sentinel or “dummy” nodes because they do not store elements.

insertFirst(e):



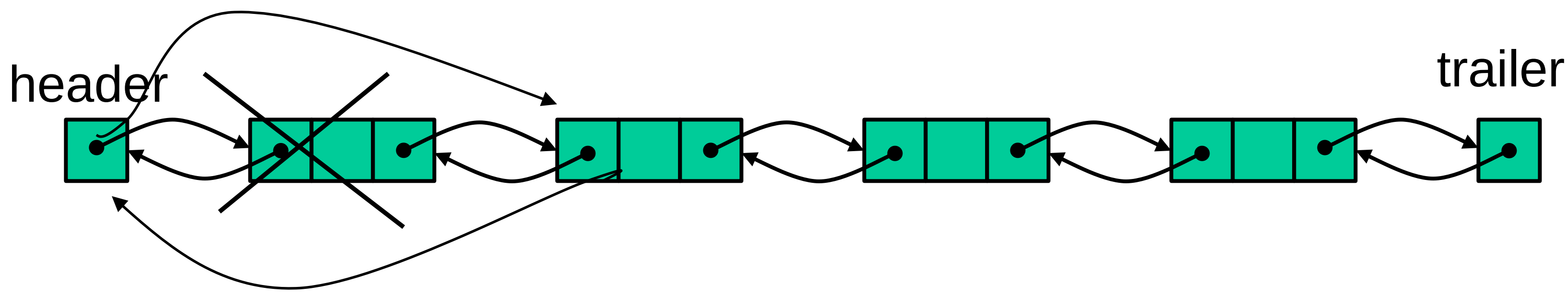
e.next ← header.next

header.next ← e

e.prev ← header

e.next.prev ← e

removeFirst():



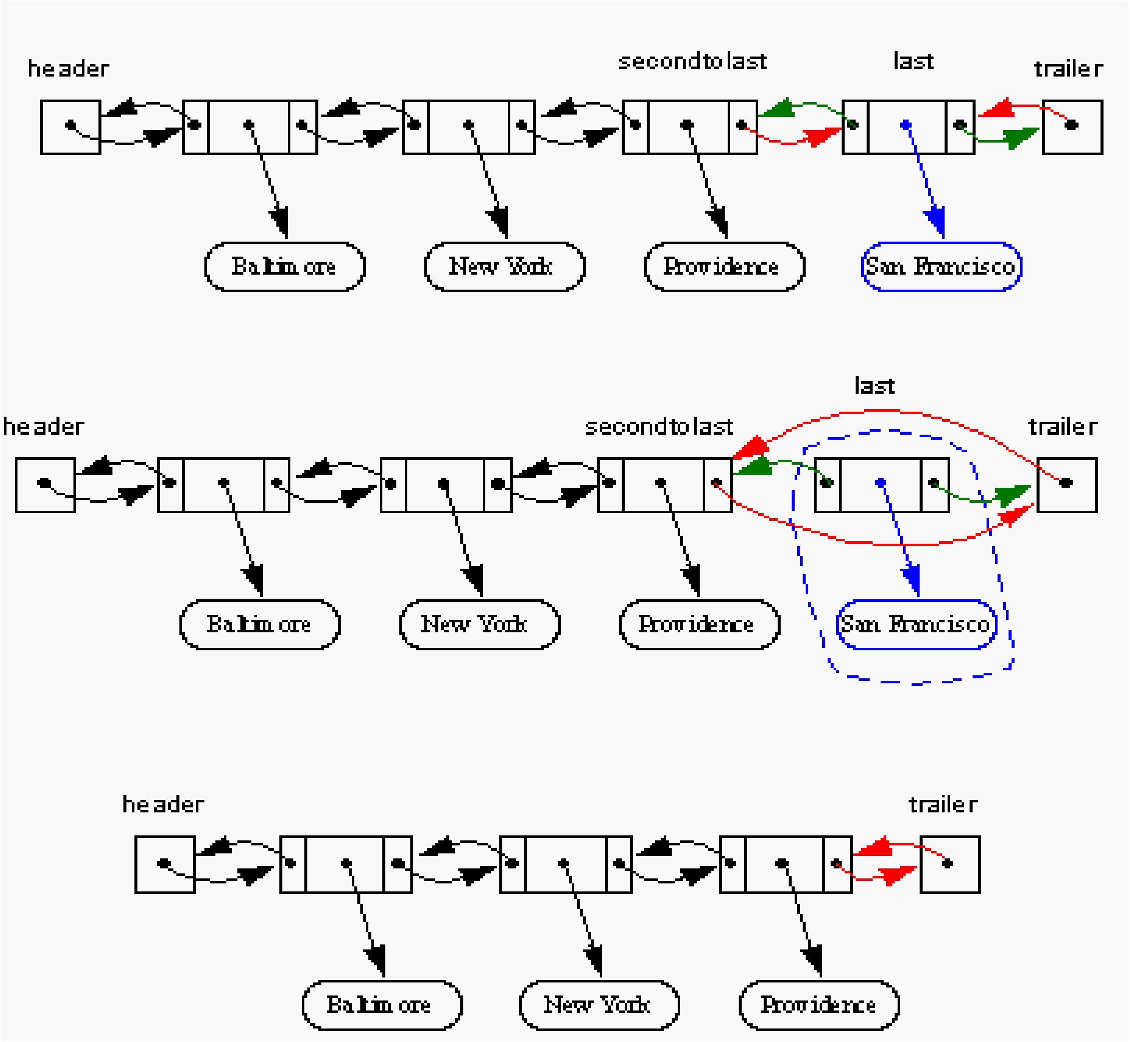
header.next.next.prev ← header

header.next ← header.next.next

OR:

header.next ← header.next.next
header.next.prev ← header

Here's a visualization of the code for removeLast().



With this implementation, all methods have constant execution time (complexity O(1))

Implementing Stacks and Queues with Deques

Stacks with Deques:

Stack Method	Deque Implementation
size()	size()
isEmpty()	isEmpty()
top()	last()
push(e)	insertLast(e)
pop()	removeLast()

Queues with Deques:

Queue Method	Deque Implementation
size()	size()
isEmpty()	isEmpty()
front()	first()
enqueue()	insertLast(e)
dequeue()	removeFirst()

Java implementation of Doubly Linked List

A node of a doubly linked list has a next and a prev link.

The doubly linked list supports the following methods:

•setElement(Object e)
•setNext(Object newNext)
•setPrev(Object newPrev)
•getElement()
•getNext()
•getPrev()