HMW 5- Data 605

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Problem

Choose independently two numbers B and C at random from the interval [0,1] with uniform density. Prove that B and C are proper probability distributions.

Find the probability that

(a) $B + C < 1/2$
(b) $BC < 1/2$
(c) $|B - C| < 1/2$
(d) $max\{B, C\} < 1/2$
(e) $min\{B, C\} < 1/2$

Solution

(a) B + C < 1/2.

$$P(B + C < 1/2)$$

$$= P(X + Y < 1/2)$$

$$= P(0 < X < 1/2, 0 < Y < 1/2 - x)$$

$$= \int_0^{1/2} \int_0^{1/2-x} f\,(x, y) dxdy$$

$$= \int_0^{1/2} \int_0^{1/2-x} 1\,.\,dydx$$

$$= \int_0^{1/2} [1/2 - x]dx$$

$$= [1/2x - x^2/2]_0^{1/2}$$

$$= 1/4 - 1/8$$

$$= 1/8$$

(b) BC < 1/2.

$$P(B.C < 1/2) = \int_0^{1/2} f(z)dz$$

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f <- function(x) {x}
res = integrate(f, lower = 0, upper = 0.5)
res

## 0.125 with absolute error < 1.4e-15
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(c) |B -C| < 1/2

P (|B - C| < 1/2 given that 0 < x +y < 1)

$$P(|X - Y| < 1/2 given that 0 < x + y < 1)$$

$$= P(-1/2 < X - Y < 1/2; 0 < X < 1 - Y)$$

$$= P(0 < X < 1/2 - Y; 0 < Y < 1/2)$$

$$= \int_0^{1/2} \int_0^{1/2-y} f(x, y)dxdy$$

$$= \int_0^{1/2} \int_0^{1/2-y} 1.dxdy$$

$$= \int_0^{1/2} [x]_0^{1/2-y} dy$$

$$= [Y/2 - Y^2/2]_0^{1/2}$$

$$= 1/4 - 1/8$$

$$= 0.125$$

(d) max{B,C} < 1/2.

$$= P(max(B, C) < 1/2)$$

$$= P(B <= 1/2, C <= 1/2)$$

$$= P(B <= 1/2)P(C <= 1/2)$$

$$= 1/2 * 1/2$$

$$= 1/4$$

(e) min{B,C} < 1/2.

$$= P(min(B, C) <= 1/2)$$

$$= 1 - P(min(B, C) > 1/2)$$

$$= 1 - P(B > 1/2, C > 1/2)$$

$$= 1 - P(B > 1/2)P(C > 1/2)$$

$$= 1 - [1 - P(B > 1/2)][1 - P(C > 1/2)]$$

$$= 1 - [1 - 1/2][1 - 1/2]$$

$$= 1 - [1/2][1/2]$$

$$= 1 - 1/4$$

$$= 3/4$$