HMW 3- Data 605

OMER OZEREN

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Problem set 1

(1) What is the rank of the matrix A?

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ -1 & 0 & 1 & 3 \\ 0 & 1 & -2 & 1 \\ 5 & 4 & -2 & -3 \end{pmatrix}$$

```
A1 <- matrix(c(1,2,3,4,-1,0,1,3,0,1,-2,1,5,4,-2,-3), nrow = 4, byrow = T)
A1

##      [,1] [,2] [,3] [,4]
## [1,]    1    2    3    4
## [2,]   -1    0    1    3
## [3,]    0    1   -2    1
## [4,]    5    4   -2   -3

#After the row reduced echelon matrix
A<- matrix(c(1,2,3,4,0,2,4,7,0,0,-4,-5/2,0,0,0,9/8), nrow= 4, ncol=4, byrow=
TRUE)
A

##      [,1] [,2] [,3]   [,4]
## [1,]    1    2    3  4.000
## [2,]    0    2    4  7.000
## [3,]    0    0   -4 -2.500
## [4,]    0    0    0  1.125
```

Answer: From the above matrix, its known that its dimension is 4x4 square matrix and lineary independent, therefore it’s rank is 4. We can also check the answer by running the below.

```r
A1 <- matrix(c(1,2,3,4,-1,0,1,3,0,1,-2,1,5,4,-2,-3), nrow = 4, byrow = T)
decompA <- qr(A1)
A1

##      [,1] [,2] [,3] [,4]
## [1,]    1    2    3    4
## [2,]   -1    0    1    3
## [3,]    0    1   -2    1
## [4,]    5    4   -2   -3

decompA$rank

## [1] 4
```

(2) Given an mxn matrix where m > n, what can be the maximum rank? The minimum rank, assuming that the matrix is non-zero?

Answer: If m is greater than n, then the maximum rank of the matrix is n and if m is less than n, then the maximum rank of the matrix is m.

(3) What is the rank of matrix B?

$$B = \begin{pmatrix} 1 & 2 & 1 \\ 3 & 6 & 3 \\ 2 & 4 & 2 \end{pmatrix}$$

```r
B <- matrix(c(1,2,1,3,6,3,2,4,2), nrow = 3, byrow = T)
B

##      [,1] [,2] [,3]
## [1,]    1    2    1
## [2,]    3    6    3
## [3,]    2    4    2

dim(B)

## [1] 3 3

R1 <- B[1, ]
R2 <- B[2, ]
R3 <- B[3, ]
a <- R1-(1/3)%*%R2
b <- R3-(2/3)%*%R2
M <- matrix(c(a,b,R2), nrow = 3, byrow = T)
M

##      [,1] [,2] [,3]
## [1,]    0    0    0
## [2,]    0    0    0
## [3,]    3    6    3
```

Answer: Since the rank is number of non zero row, rank is 1.

Problem Set 2:

Compute the eigenvalues and eigenvectors of the matrix A. You'll need to show your work. You'll need to write out the characteristic polynomial and show your solution.

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{pmatrix}$$

```r
A <- matrix(c(1,2,3,0,4,5,0,0,6), nrow = 3, byrow = T)
```

$$pB(\lambda) = det(B - (\lambda I3))$$

$$= \begin{pmatrix} 1-\lambda & 2 & 3 \\ 0 & 4-\lambda & 5 \\ 0 & 0 & 6-\lambda \end{pmatrix}$$

$$= (1-\lambda)[(4-\lambda)(6-\lambda) - (0)(5)] - 2[0] + 3[0]$$

$$= (1-\lambda)[(4-\lambda)(6-\lambda)]$$

$$\lambda = 1,4,6$$

```r
# Double-check eigenvalues in R
eigen(A)$values

## [1] 6 4 1
```

The **characteristic polynomial** is $p_A(\lambda) = (1-\lambda)(4-\lambda)(6-\lambda)$ **or** $p_A(\lambda) = 24 - 34\lambda + 11\lambda^2 - \lambda^3$.

If $\lambda = 1$, then $A - 1I_3$ is row-reduced to

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Then $v_1 = v_1$ and $v_2 = 0$ and $v_3 = 0$. The **EigenVector** is

$$E_{\lambda=1} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

If $\lambda = 4$, then $A - 4I_3$ is row-reduced to

$$\begin{bmatrix} 1 & -\frac{2}{3} & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Then $v_1 - \frac{2}{3}v_2 = 0$ and $v_3 = 0$.

Or $v_1 = v_1$ and $v_2 = \frac{3}{2}v_1 = 1.5v_1$ and $v_3 = 0$.

The **EigenVector** is

$$E_{\lambda=4} = \begin{bmatrix} 1 \\ 1.5 \\ 0 \end{bmatrix}$$

Finally, if $\lambda = 6$, then $A - 6I_3$ is row-reduced to

$$\begin{bmatrix} 1 & 0 & -1.6 \\ 0 & 1 & -2.5 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Then $v_1 - 1.6v_3 = 0$ and $v_2 - 2.5v_3 = 0$.

Or $v_1 = 1.6v_3$ and $v_2 = 2.5v_3$ and $v_3 = v_3$.

The **EigenVector** is

$$E_{\lambda=6} = \begin{bmatrix} 1.6 \\ 2.5 \\ 1 \end{bmatrix}$$