DATA 605 - Homework 13

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```r
library(ggplot2)
```

Question 1.

Use integration by substitution to solve the integral $\int 4e^{-7x}dx$

Answer

Let $u = -7x$. Then $du = -7dx \ \rightarrow \ dx = \frac{du}{-7}$.

Our integral is now $\int \frac{4e^{u}du}{-7}$. Taking out the constants: $\frac{4}{-7}\int e^{u}du$.

Replacing u with our original substitution: $\frac{-4}{7}e^{-7x} + C$.

Question 2.

Biologists are treating a pond contaminated with bacteria. The level of contamination is changing at a rate of $\frac{\mathrm{d}N}{\mathrm{d}t} = -\frac{3150}{t^4} - 220$ bacteria per cubic centimeter per day, where t is the number of days since treatment began. Find a function $N(t)$ to estimate the level of contamination if the level after day 1 was 6530 per cubic centimeter.

Answer

$$\frac{\mathrm{d}N}{\mathrm{d}t} = -\frac{3150}{t^4} - 220 \;\rightarrow\; \mathrm{d}N = (-\frac{3150}{t^4} - 220)\mathrm{d}t$$

To find N, we can take the antiderivative, i.e. the integral.

$N = \int (-\frac{3150}{t^4} - 220)\mathrm{d}t \quad = \int -3150(t^{-4})\mathrm{d}t - \int 220\mathrm{d}t$

Using the power rule for integration: $N = \frac{-31}{-3}(t^{-3}) - 220t + C$.

Solving for $N(1) = 6530$:

$N(1)\frac{-3150}{-3}(1^{-3}) - 220(1) + C = 6530$

$C = 6530 - 1050 + 220 = 5700.$

$N(t) = -1050(t^{-3}) - 200(t) + 5700.$

Question 3.

Find the total area of the red rectangles in the figure below, where the equation of the lines is $f(x) = 2x - 9$.

Answer

The equation is given as $2x - 9$, and the ends of the rectangles look to be 4.5 and 8.5. Since we're looking for the area, we can integrate this function over these boundaries.

$$\int_{4.5}^{8.5} (2x - 9)dx$$

Using the power rule for integration:

$$ (x^2 - 9x)\Big|_{4.5}^{8.5} = \Big[(8.5)^2 - 9(8.5)\Big] - \Big[(4.5)^2 - 9(4.5)\Big] \\ = [72.25 - 76.5] - [20.25 - 40.5] = 16 $$

Question 4.

Find the area of the region bounded by the graphs of the given equations.

$y = x^2 - 2x - 2, y = x + 2$

Answer:

- Solving for x gives

$$\begin{aligned} x + 2 &= x^2 - 2x - 2 \\ x^2 - 3x - 4 &= 0 \\ (x-4)(x+1) &= 0 \\ x = 4, x &= -1 \end{aligned}$$

- Now that I have the endpoints, I can compute and plot or graph the two functions and see that $x + 2 \geq x^2 - 2x - 2$ for all x $\epsilon[-1,4]$.

- Both functions are continous everywhere in the region and we can find the area between curves as

- $\int_{-1}^{4}(\, x + 2 - (x^2 - 2x - 2))\, dx$ Solving the integral gives

$$\begin{aligned} &\int_{-1}^{4}(\, x + 2 - (x^2 - 2x - 2))\, dx \\ &= \int_{-1}^{4} -x^2 + 3x + 4\, dx \\ &= [-x^3/3 + 3x^2/2 + 4x]_{-1}^{4} \\ &= -64/3 + 24 + 16 - (1/3 + 3/2 - 4) \approx 20.833 \end{aligned}$$

Question 5.

A beauty supply store expects to sell 110 flat irons during the next year. It costs \$3.75 to store one flat iron for one year. There is a fixed cost of \$8.25 for each order. Find the lot size and number of orders per year that will minimize inventory costs.

Answer:

lot size = x units/order Annual Cost = Annual storage cost * Average no. of items carried out a year Annual Cost = 3.75.x/2 =1.875x

$$AnnualOrderCost = CostofOrder * numberoforders/year = (8.25 * 110/x)$$

$$Totalinventorycost = 1.875x + (8.25 * 110/x)$$

$$y = 1.875x + (907.5/x)$$

$$y` = 1.875x + (907.5/x) = 0$$

$$1.875 - (907.5/x^2) = 0$$

$$x^2 = 907.5/1.875$$

$$x^2 = 206.25$$

```
A <- sqrt(206.25)
A
## [1] 14.36141
```

lot size =14.36/order

```


ordperyear <- 110/14.36
ordperyear
## [1] 7.660167
```

Orders per year = 7.6

Question 6.

Use integration by parts to solve the integral below.

$$\int ln(9x).x^6 dx$$

Answer:

$$U = ln(9x)$$

$$dU = \frac{1}{x}dx$$

$$dV = x^6 dx$$

$$V = \frac{1}{7}x^7$$

$$UdV = UV - \int VdU$$

$$ln(9x)\frac{1}{7}x^7 - \int \frac{1}{7}x^7\frac{1}{x}dx$$

$$\frac{1}{7}ln(9x)x^7 - \frac{1}{7}\int x^6 dx$$

$$\frac{1}{7}x^7[ln(9x)] - \frac{1}{7}]dx$$

Question 7.

Determine whether f(x) is a probability density function on the interval $[1, e^6]$. If not, determine the value of the definite integral.

$f(x) = 1/6x$

Answer :

- Compute $\int_1^{e^6} 1/6x\,dx$

$$\int_1^{e^6} 1/6x\,dx = [\ln(6x)]_1^{e^6}$$
$$= \ln(6e^6) - \ln(6)$$
$$\ln(6) + \ln(e^6) - \ln(6) = 6 \neq 1$$