

## DATA 605 - Discussion 6

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### Chapter 4 Exercise 26 :

Suppose that A and B are events such that  $P(A|B) = P(B|A)$  and  $P(A \cup B) = 1$  and  $P(A \cap B) > 0$ . Prove that  $P(A) > 1/2$ .

#### Answer :

Using the formula for conditional probability for events A and B:

$$P(A|B) = P(A \cap B)/P(B) \text{ and } P(B|A) = P(B \cap A)/P(A)$$

we can rewrite the equality in the question to be:

a)  $P(A \cap B)P(A) = P(B \cap A)P(B)$  using cross-multiplication.

Per the probability communitive law for two events,  $P(A \cap B) = P(B \cap A)$

thus we can divide the  $P(A \cap B)$  on both sides to get

b)  $P(A) = P(B)$

We have to show  $P(A) > 1/2$  and to do this, recall the formula

of  $P(A \cup B)$  that is the probability of either of the two events A and B happening is:

- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- We can solve for  $P(B)$  to get
- $P(A \cup B) + P(A \cap B) - P(A) = P(B)$  and plug it into b. to get
- $P(A) = P(A \cup B) + P(A \cap B) - P(A)$
- Adding  $P(A)$  on both sides gives
- $2P(A) = P(A \cup B) + P(A \cap B)$

- Dividing on both sides by 2 gives
- $P(A) = P(A \cup B)/2 + P(A \cap B)/2$
- $P(A \cup B) = 1$  as per the information given so
- $P(A) = 1/2 + P(A \cap B)/2$
- Finally note that the second operand  $P(A \cap B)/2$  will always be within  $[0,1]$
- so  $P(A) > 1/2$