DATA 605 - Homework 14

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This week, we'll work out some Taylor Series expansions of popular functions.

- $f(x) = 1/(1-x)$
- $f(x) = e^x$
- $f(x) = \ln(1+x)$

  For each function, only consider its valid ranges as indicated in the notes when you are computing the Taylor Series expansion. Please submit your assignment as a R-Markdown document.

Function 1: $f(x) = 1/(1-x)$

$$f(x) = \frac{1}{(1-x)}$$

$$f'(x) = \frac{1}{(1-x)^2}$$

$$f''(x) = \frac{2}{(1-x)^3}$$

$$f'''(x) = \frac{6}{(1-x)^4}$$

$$= f(x) + \frac{f'(x)}{1!}(x-c)^1 + \frac{f''(x)}{2!}(x-c)^2 + \frac{f'''(x)}{3!}(x-c)^3$$

$$= 1 + x + x^2 + x^3 + x^4 + \ldots$$

$$\sum_{n=0}^{\infty} x^n$$

```
library(pracma)
c = -5
f <- function(x) {1/(1-x)}
```

```
A1 <- taylor(f, x0=c, 5)
A1
## [1]    1.000293   25.007447  250.076532 1250.401437 3126.111100
3126.512977
```

Function 2: $f(x) = e^x$

Find first several derivatives.

$f^0(c) = e^c$

$f'(c) = e^c$

$f''(c) = e^c$

$f'''(c) = e^c$

$f''''(c) = e^c$

Per definition,

$$\begin{aligned} f(x) &= \frac{e^c}{0!}(x-c)^0 + \frac{e^c}{1!}(x-c)^1 + \frac{e^c}{2!}(x-c)^2 + \frac{e^c}{3!}(x-c)^3 + \ldots \\ &= e^c + e^c(x-c) + e^c\frac{(x-c)^2}{2!} + e^c\frac{(x-c)^3}{3!} + \ldots \\ &= e^c \sum_{n=0}^{\infty} \frac{(x-c)^n}{n!} \end{aligned}$$

The Maclaurin Series of $f(x)$, $c = 0$, $f(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \ldots$

```
c = -5
f <- function(x) {exp(x)}
A2 <- taylor(f, x0=c, 5)
A2
## [1]  0.008334245  0.208636869  2.090299109 10.480131597 26.309539362
## [6] 26.485006242
```

Function 3: $f(x) = ln(1 + x)$

$$f(x) = ln(1+x)$$

$$f'(x) = \frac{1}{1+x}$$

$$f''(x) = \frac{-1}{(1+x)^2}$$

$$f'''(x) = \frac{2}{(1+x)^3}$$

$$f(x)+\frac{f'(x)}{1!}x^1+\frac{f''(x)}{2!}x^2$$

$$f(x)=f(0)+f'(0)(x-c)+\frac{f''(c)}{2!}(x-c)^2+\frac{f'''(c)}{3!}(x-c)^3+$$

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$$f(x)=x-\frac{1}{2}x^2+\frac{1}{3}x^3-\frac{1}{4}x^4+\ldots$$

$$\sum_{n=1}^{\infty}(-1)^{n+1}*\frac{x^n}{n}$$

```
c = 0
f <- function(x) {log(1+x)}
A3 <- taylor(f, x0=c, 5)
A3
## [1]  0.2000413 -0.2500044  0.3333339 -0.5000000  1.0000000  0.0000000
```