

DATA 605 - Discussion 7

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Table of Contents

Chapter 5.2 Exercise 16	1
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Chapter 5.2 Exercise 16

Let X be a random variable with density function $f_X(x) = \begin{cases} cx(1-x), & \text{if } 0 < x < 1, \\ 0, & \text{otherwise.} \end{cases}$

- a) What is the value of c ?
- b) What is the cumulative distribution function F_X for X ?
- c) What is the probability that $X < 1/4$?

Part (a)

Probability density function must be positive and integrate to 1.

$$\int_0^1 c x(1-x) dx = \frac{c}{6} = 1, \text{ then } c = 6.$$

So density function is $f_X(x) = \begin{cases} 6x(1-x), & \text{if } 0 < x < 1, \\ 0, & \text{otherwise.} \end{cases}$

Part (b)

$$F_X = \int_{-\infty}^x f(t) dt$$

$$\int f(x) dx = -6\left(\frac{x^3}{3} - \frac{x^2}{2}\right) = -2x^3 + 3x^2 = x^2(3 - 2x)$$

Cumulative distribution function $F_X(x) = \begin{cases} x^2(3 - 2x), & \text{if } 0 < x < 1, \\ 0, & \text{otherwise.} \end{cases}$

Part (c)

$$P(X < 1/4) = 0.25^2 \times (3 - 2 \times 0.25) = 0.15625$$