HMW 2-Problem Set 1

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1. PROBLEM SET 1

(1) Show that $A^T A \neq A A^T$ in general. (Proof and demonstration.)

(2) For a special type of square matrix A, we get $A^T A = A A^T$. Under what conditions could this be true? (Hint: The Identity matrix I is an example of such a matrix).

Question 1 :

```r
A = matrix(seq(1, 9), nrow=3, byrow = T)
AT = matrix(seq(1, 9), nrow=3, byrow = F)
```

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} * \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix} =$$

```r
A %*% AT
```

```
##      [,1] [,2] [,3]
## [1,]   14   32   50
## [2,]   32   77  122
## [3,]   50  122  194
```

$$\begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix} * \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} =$$

```r
AT %*% A
```

```
##      [,1] [,2] [,3]
## [1,]   66   78   90
## [2,]   78   93  108
## [3,]   90  108  126
```

```r
(AT %*% A) == (A %*% AT)
```

```
##       [,1]  [,2]  [,3]
## [1,] FALSE FALSE FALSE
## [2,] FALSE FALSE FALSE
## [3,] FALSE FALSE FALSE
```

Question 2 :

$A^T A = AA^T$ is true if A is a diagonal matrix. So An identity matrix when transposed and multiplied to itself, are equal to each other.

Let $A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$ then $A^T = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$ therefore:

$$A^T A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} = \begin{bmatrix} a^2 & 0 & 0 \\ 0 & b^2 & 0 \\ 0 & 0 & c^2 \end{bmatrix}$$

$$AA^T = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} = \begin{bmatrix} a^2 & 0 & 0 \\ 0 & b^2 & 0 \\ 0 & 0 & c^2 \end{bmatrix}$$

This gives a case when $A^T A = AA^T$ is true.

```
A = matrix(c(1, 0, 0, 1), nrow=2, byrow = T)
AT = t(A)
(A %*% AT) == (A %*% AT)

##      [,1] [,2]
## [1,] TRUE TRUE
## [2,] TRUE TRUE
```