DATA 605 - Discussion 13

Omer Ozeren

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Chapter 7 Sec 4 Exercise 5

Find the arc length of the function on the given interval.

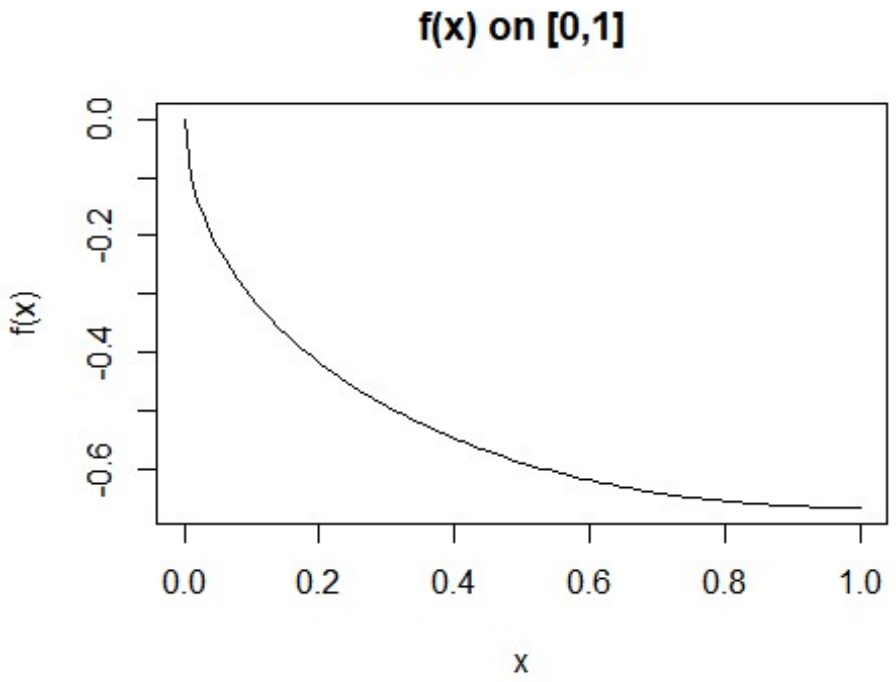
$f(x) = \frac{1}{3}x^{\frac{3}{2}} - x^{\frac{1}{2}}$ on $[0,1]$

Define Function

```
func <- function(x) 1/3 * x^(3/2) - x^(1/2)
```

Plot

```
x <- seq(0,1,0.01)
plot(x,func(x), type="l",
     xlab="x", ylab="f(x)", main="f(x) on [0,1]")
```



Solution

$$f'(x) = \frac{1}{3} \times \frac{3}{2} x^{\frac{1}{2}} - \frac{1}{2} x^{-\frac{1}{2}} = \frac{\sqrt{x}}{2} - \frac{1}{2\sqrt{x}} = \frac{x-1}{2\sqrt{x}}$$

$$\begin{aligned}
L_{arc} &= \int_0^1 \sqrt{1 + f'(x)^2}\, dx = \\
&= \int_0^1 \sqrt{1 + \frac{(x-1)^2}{4x}}\, dx = \\
&= \int_0^1 \sqrt{\frac{4x + x^2 - 2x + 1}{4x}}\, dx = \\
&= \int_0^1 \sqrt{\frac{x^2 + 2x + 1}{4x}}\, dx = \\
&= \int_0^1 \sqrt{\frac{(x+1)^2}{4x}}\, dx = \\
&= \int_0^1 \frac{x+1}{2\sqrt{x}}\, dx = \\
&= \frac{4}{3} \approx 1.333
\end{aligned}$$