DATA 605 - Homework 15

Omer Ozeren

Table of Contents

Problem 1

Find the equation of the regression line for the given points. Round any final values to the nearest hundredth, if necessary.

(5.6,8.8), (6.3,12.4), (7,14.8), (7.7,18.2), (8.4,20.8)

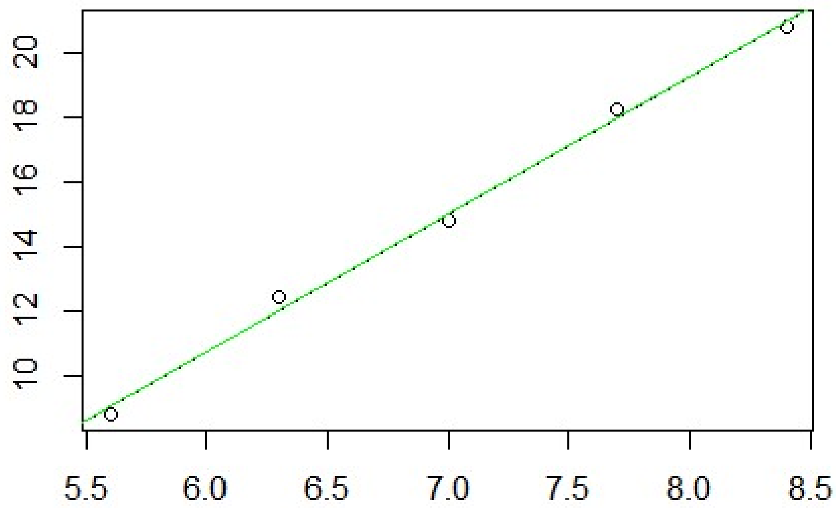
SOLUTION 1

```
x <- c(5.6, 6.3, 7, 7.7, 8.4)
y <- c(8.8, 12.4, 14.8, 18.2, 20.8)
r <- lm(y~x)
r
##
## Call:
## lm(formula = y ~ x)
##
## Coefficients:
## (Intercept)            x
##     -14.800        4.257
```

Based on the linear regression model, the regression line is $y = -14.8 + 4.257x$.

Check with plot.

```
plot(x,y, xlab="", ylab="")
abline(r)
lines(c(5,9), -14.8+4.257*c(5,9), col="green")
```



Problem 2

Find all local maxima, local minima, and saddle points for the function given below. Write your answer(s) in the form (x, y, z). Separate multiple points with a comma.

$f(x, y) = 24x - 6xy^2 - 8y^3$

SOLUTION 2

We first need to find the first and second partial derivatives.

$f_x = 24 - 6y^2 \quad f_y = -12xy - 24y^2$

$24 - 6y^2 = 0 \;\rightarrow\; y^2 = 4 \;\rightarrow\; y = \pm 2$

When $y = 2$: $-12xy - 24y^2 = 0 \;\rightarrow\; -24x = 96 \;\rightarrow\; x = -4.$

When $y = -2$: $-12xy - 24y^2 = 0 \;\rightarrow\; 24x = 96 \;\rightarrow\; x = 4.$

Plugging these values in to get our third coordinate:

$f(-4,2) = 24(-4) - 6(-4)(2^2) - 8(2^3) =$ -64.

$f(4,-2) = 24(4) - 6(4)(-2^2) - 8(-2^3) = 64.$

Our two critical points are $(-4,2,-64)$ and $(4,-2,64)$.

To classify these extrema, we can use the second derivative test.

$D(x, y) = f_{xx} f_{yy} - f_{xy}^2$.

$f_{xx} = 0$.

$f_{yy} = -12x - 48y$.

$f_{xy} = f_{yx} = -12y$.

$D = 0 - (-12y)^2 = -144y^2$.

$D(x, y) < 0 \ \forall (x, y)$, so both critical points are saddle points.

Problem 3

A grocery store sells two brands of a product, the "house" brand and a "name" brand. The manager estimates that if she sells the "house" brand for x dollars and the "name" brand for y dollars, she will be able to sell $81 - 21x + 17y$ units of the "house" brand and $40 + 11x - 23y$ units of the "name" brand.

Step 1. Find the revenue function $R(x, y)$. Step 2. What is the revenue if she sells the "house" brand for \$2.30 and the "name" brand for \$4.10?

SOLUTION 3

$$\begin{aligned} R(x, y) &= (81 - 21x + 17y)x + (40 + 11x - 23y)y \\ &= 81x - 21x^2 + 17xy + 40y + 11xy - 23y^2 \\ &= 81x + 40y + 28xy - 21x^2 - 23y^2 \end{aligned}$$

$R(2.3, 4.1) = 81 \times 2.3 + 40 \times 4.1 + 28 \times 2.3 \times 4.1 - 21 \times (2.3)^2 - 23 \times (4.1)^2 = 116.62$

Problem 4

A company has a plant in Los Angeles and a plant in Denver. The firm is committed to produce a total of 96 units of a product each week. The total weekly cost is given by $C(x, y) = \frac{1}{6}x^2 + \frac{1}{6}y^2 + 7x + 25y + 700$, where x is the number of units produced in Los Angeles and y is the number of units produced in Denver. How many units should be produced in each plant to minimize the total weekly cost?

SOLUTION 4

Consider $x + y = 96$, then $x = 96 - y$.

$$
\begin{aligned}
C(x,y) = C(96-y,y) &= \frac{1}{6}x^2 + \frac{1}{6}y^2 + 7x + 25y + 700 \\
&= \frac{1}{6}(96-y)^2 + \frac{1}{6}y^2 + 7 \times (96-y) + 25y + 700 \\
&= \frac{1}{6}(y^2 - 192y + 9216) + \frac{1}{6}y^2 + 672 - 7y + 25y + 700 \\
&= \frac{1}{6}y^2 - 32y + 1536 + \frac{1}{6}y^2 + 18y + 1372 \\
&= \frac{1}{3}y^2 - 14y + 2908 \\
&= C_1(y)
\end{aligned}
$$

$$C_1{}'(y) = \frac{2}{3}y - 14$$

To find the minimal value consider $C_1{}'(y) = \frac{2}{3}y - 14 = 0$, then $y = 21$. Then $x = 96 - y = 75$.

Problem 5

Evaluate the double integral on the given region.

$$\int\int_R (e^{8x+3y})dA, R: 2 \le x \le 4 \; and \; 2 \le y \le 4$$

SOLUTION 5

$$
\begin{aligned}
\int_2^4 \int_2^4 (e^{8x+3y})\, dy\, dx &= \int_2^4 (\frac{1}{3}e^{8x+3y})|_2^4 \, dx \\
&= \int_2^4 ((\frac{1}{3}e^{8x+12}) - (\frac{1}{3}e^{8x+6}))\, dx \\
&= \int_2^4 \frac{1}{3}e^{8x+6}(e^6 - 1)\, dx \\
&= \frac{1}{24}e^{8x+6}(e^6-1)|_2^4 \\
&= \frac{1}{24}e^{32+6}(e^6-1) - \frac{1}{24}e^{16+6}(e^6-1) \\
&= \frac{1}{24}(e^6-1)(e^{38} - e^{22}) \\
&= \frac{1}{24}(e^{44} - e^{38} - e^{28} + e^{22})
\end{aligned}
$$

There should be 75 units produced in Los Angeles and 21 units produced in Denver.