HMW 7- Data 605

OMER OZEREN

Table of Contents

## Problem 1

Let be mutually independent random variables, each of which is uniformly distributed on the integers from to . Let denote the minimum of the ’s. Find the distribution of .

### SOLUTION

Number of possible combinations of ’s is (choosing values out of options with replacement).

Consider number of combinations with at least one . It is equal to all combinations () minus all combinations with values between and (). So .

Consider number of combinations with at least one and no . It is euqal to all combinations () minus all combinations with at least one (see above: ) and minus all combinations with values between and (). So .

Similarly considering combinations without or and with at least one ,

.

More generally, we can see that .

## Problem 2.

Your organization owns a copier (future lawyers, etc.) or MRI (future doctors). This machine has a manufacturer’s expected lifetime of 10 years. This means that we expect one failure every ten years. (Include the probability statements and R Code for each part.)

### a.

What is the probability that the machine will fail after 8 years?. Provide also the expected value and standard deviation. Model as a geometric. (Hint: the probability is equivalent to not failing during the first 8 years.)

#### Solution

Let be the probability that the machine fails, and the probability that it doesn’t.

We’re looking for the first failure (success) after 8 years, so this is a geometric distribution. Since we’re only expecting one failure in 10 years, .

0.3874.

10.

90.

Standard deviation = 9.486833.

pdf <- pgeom(8, 0.1, lower.tail = F)  
p <- 0.1  
q <- 1 - p  
ex <- p^-1  
var <- q/p^2  
sd <- sqrt(var)  
cat(sprintf("\n %s = %f \n",   
 c("Probability", "Expected Value", "Variance", "Standard Deviation"),  
 c(pdf, ex, var, sd))  
 )  
##   
## Probability = 0.387420   
##   
## Expected Value = 10.000000   
##   
## Variance = 90.000000   
##   
## Standard Deviation = 9.486833

### b.

What is the probability that the machine will fail after 8 years?. Provide also the expected value and standard deviation. Model as an exponential.

#### Solution

, with .

0.449329.

10.

100.

Standard Deviation .

pdf <- pexp(8, 0.1, lower.tail = F)  
l <- 0.1  
ex <- 1/l  
var <- 1/l^2  
sd <- sqrt(var)  
cat(sprintf("\n %s = %f \n",   
 c("Probability", "Expected Value", "Variance", "Standard Deviation"),  
 c(pdf, ex, var, sd))  
 )  
##   
## Probability = 0.449329   
##   
## Expected Value = 10.000000   
##   
## Variance = 100.000000   
##   
## Standard Deviation = 10.000000

### c.

What is the probability that the machine will fail after 8 years?. Provide also the expected value and standard deviation. Model as a binomial. (Hint: 0 success in 8 years)

#### Solution

We’re looking for 0 successes in 8 years. So, with .

0.4304672.

.

0.72.

Standard Deviation 0.8485281.

pdf <- pbinom(0, 8, 0.1)  
n <- 8  
i <- 0  
p <- 0.1  
q <- 0.9  
ex <- n\*p  
var <- n\*p\*q  
sd <- sqrt(var)  
cat(sprintf("\n %s = %f \n",   
 c("Probability", "Expected Value", "Variance", "Standard Deviation"),  
 c(pdf, ex, var, sd))  
 )  
##   
## Probability = 0.430467   
##   
## Expected Value = 0.800000   
##   
## Variance = 0.720000   
##   
## Standard Deviation = 0.848528

### d.

What is the probability that the machine will fail after 8 years?. Provide also the expected value and standard deviation. Model as a Poisson.

#### Solution

Since Poisson uses averages, and we expect one failure every 10 years, we can say the average yearly failure rate is 0.1. We’re looking for 0 failures in the first 8 years.

0.449329.

.

Standard Deviation 0.8944272.

lambda <- 0.1  
t <- 8  
i <- 0  
ex <- lambda\*t  
var <- lambda\*t  
sd <- sqrt(var)  
pdf <- ppois(i, t\*lambda)  
cat(sprintf("\n %s = %f \n",   
 c("Probability", "Expected Value", "Variance", "Standard Deviation"),  
 c(pdf, ex, var, sd))  
 )  
##   
## Probability = 0.449329   
##   
## Expected Value = 0.800000   
##   
## Variance = 0.800000   
##   
## Standard Deviation = 0.894427