

Gene Placement

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1 Introduction

The problem of gene placement in phylogenetic trees is a well known problem in which the goal is to place a new species on a given phylogenetic tree that represents an evolutionary chain of some given species.

2 Gene Rates - Multipliers

Let T be a species phylogenetic tree, consists of M species and their genomes. We denote by g_i the genome of the i -th organism from the tree. Furthermore, let S , be a set of N gene sequences $S = s_1, s_2, \dots, s_N$. Each gene sequence $s_i \in S$ can be aligned to the species' genome using an MSA algorithm. Furthermore, each gene has evolution rate which we denote by r_i the evolution rate of the i -th gene s_i . Note that the topology of T is define by the average over all the genes in S . We define the matrix A as an $M \times N$ binary matrix, such that the j -th entry of the i -th row of the matrix is 1 if the j -th gene, s_j , appears for the i -th specie of the backbone and 0 otherwise.

Given a hamming distance of h , our (asymptotically additive) estimate of the phylogenetic distance under the Jukes Cantor (69) model is $d = \frac{3}{4} \log(1 - \frac{4}{3}h)$.

Given a query sequence, that consist of $k < N$ genes, APPLES minimizes the global (least square) hamming distance between the genes distances. Let $h_{i,j}$ denote the hamming distance for gene i to species j . Let $(t_1, \dots, t_j, \dots, t_M)$ be the distanced on *species* tree if you add query q to a particular location on that tree. Let L_i be the length of the i -th gene and define:

$$d_j = \frac{3}{4} \log(1 - \frac{4}{3} \frac{1}{\sum_i L_i} \sum_i L_i h_{i,j})$$

Then, APPLES finds the position on the tree (thus, t_j values) that minimizes:

$$\sum_j \left(\frac{1}{d_j} \right)^2 (d_j - t_j)^2 .$$

Our goal in this project, is to add sort of normalization/weighted version of the phylogenetic distance calculation, such that the evolution rate of each gene

will be considered as well. That is, to define a weight function, which receives (an estimator of) the evolutionary rate, the length of the gene, and the matrix A , and estimates the weight of the gene. Thus, $w_i = f(r_i, L_i, A)$ represents the weight of the i -th gene, in the phylogenetic distance calculation.

$$d_j = \frac{3}{4} \log(1 - \frac{4}{3} \frac{1}{\sum_i w_i} \sum_i w_i h_{i,j})$$

3 Site Rate Multipliers

In this section, since the results for the gene rate multipliers did not show a significant improvement we decided to extend our approach, and to consider site rates. The site rate were calculated based on the site entropies. Given M species, and Q sites, for the j -th site, we first calculate p_X for $X \in \{A, C, G, T, -\}$, which is the normalized probability to see the base X in the j -th site. Then, the entropy of the j -th site is defined as:

$$H_j = - \sum_{X \in \{A, C, G, T\}} p_X \log(p_X),$$

where the entropy is ∞ if the site is gap in all species, that is if $p_- = 1$.

The site entropies approximate the level of preservation of each site. Hence, the next step is to perform a discretization on the site entropies in order to define k site multipliers on the sites, where k is hyperparameter.

1. The *naive site multipliers* are the k values x_1, x_2, \dots, x_k in the range between 0.5 to 1.5 that partition the segment to k equal parts.
2. Optimized k values using regression. The optimization is done in the following way.
 - (a) Initial values are: x_1, x_2, \dots, x_k .
 - (b) The constraints are: $x_1 < x_2 < \dots < x_k$, and the median **or** the expected multiplier is 1.
 - (c) Given $d_{ti,j}$, the tree distance between the i -th and the j -th species, which their sequences is given by s_i and s_j . The target function is:

$$\arg \min_{x_1, x_2, \dots, x_k} \sum_{s_j, s_i} (JC^{-1}(d_t) - d_H(s_j, s_i, (x_1, x_2, \dots, x_k)))^2,$$

where JC^{-1} denotes the inverse function of the JC transformation, and $d_H(s_j, s_i, (x_1, x_2, \dots, x_k))$ denotes the hamming distance between the species s_i and s_j with site multipliers (x_1, x_2, \dots, x_k) .