GEBZE TEKNİK ÜNİVERSİTESİ

DEPARTMENT OF ELECTRONIC ENGINEERING

MATH214 NUMERICAL METHODS

2020 – 2021 FALL TERM

Final Project

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I. Problem Definition and Formulation

The main purpose of this project is finding current and voltage values of given circuit by numerical methods. Circuit's current can be calculated by Equation(1). Source voltage, inductance and resistor values are given already.

$$\frac{di(t)}{dt} = \frac{1}{L}[V_S - V_D(t) - i(t).R] \tag{1}$$

Only diode's current-voltage relation required to calculate ordinary differential equation(1). Data values can be assumed as xy-coordinates. Best way to find this relation is fitting an exponential line to the data. Least square exponential fitting method is given in equation(2).

$$I_D = A.e^{BV_D} \quad \text{where } B \equiv b \text{ and } A \equiv \exp(a) \tag{2}$$

$$a = \frac{\sum_{i=1}^{n}\left(X_i{}^2Y_i\right).\sum_{i=1}^{n}(Y_i.\ln(Y_i)) - \sum_{i=1}^{n}(X_iY_i).\sum_{i=1}^{n}(X_i.Y_i.\ln(Y_i))}{\sum_{i=1}^{n}(Y_i).\sum_{i=1}^{n}(X_i{}^2Y_i) - (\sum_{i=1}^{n}(X_iY_i))^2}$$

$$b = \frac{\sum_{i=1}^{n}(Y_i).\sum_{i=1}^{n}(X_iY_i.\ln(Y_i)) - \sum_{i=1}^{n}(X_iY_i).\sum_{i=1}^{n}(Y_i.\ln(Y_i))}{\sum_{i=1}^{n}(Y_i).\sum_{i=1}^{n}(X_i{}^2Y_i) - (\sum_{i=1}^{n}(X_iY_i))^2}$$

$$V_D = \frac{\ln\left({}^{I_D}/_{A}\right)}{B} \tag{3}$$

Diode voltage can be found by equation(3) and this form used in matlab code. To find current values numerically by time, Runge-kutta order four method is used and given by equation(4). This method requires multiple steps to calculate differential equation(1)'s solution. All the time variable increasements are not necessary in this case. Because of the needed values are just previous current value and diode voltage at the same current. Order of this method should ensure numerical solutions accurate.

$$K_1 = h.f\left(t_i, y(t_i)\right),$$

$$K_2 = h.f\left(t_i + \frac{h}{2}, y(t_i) + \frac{K_1}{2}\right),$$

$$K_3 = h.f\left(t_i + \frac{h}{2}, y(t_i) + \frac{K_2}{2}\right),$$

$$K_4 = h.f(t_{i+1}, y(t_i) + K_3),$$

$$y(t_{i+1}) = y(t_i) + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4) \tag{4}$$

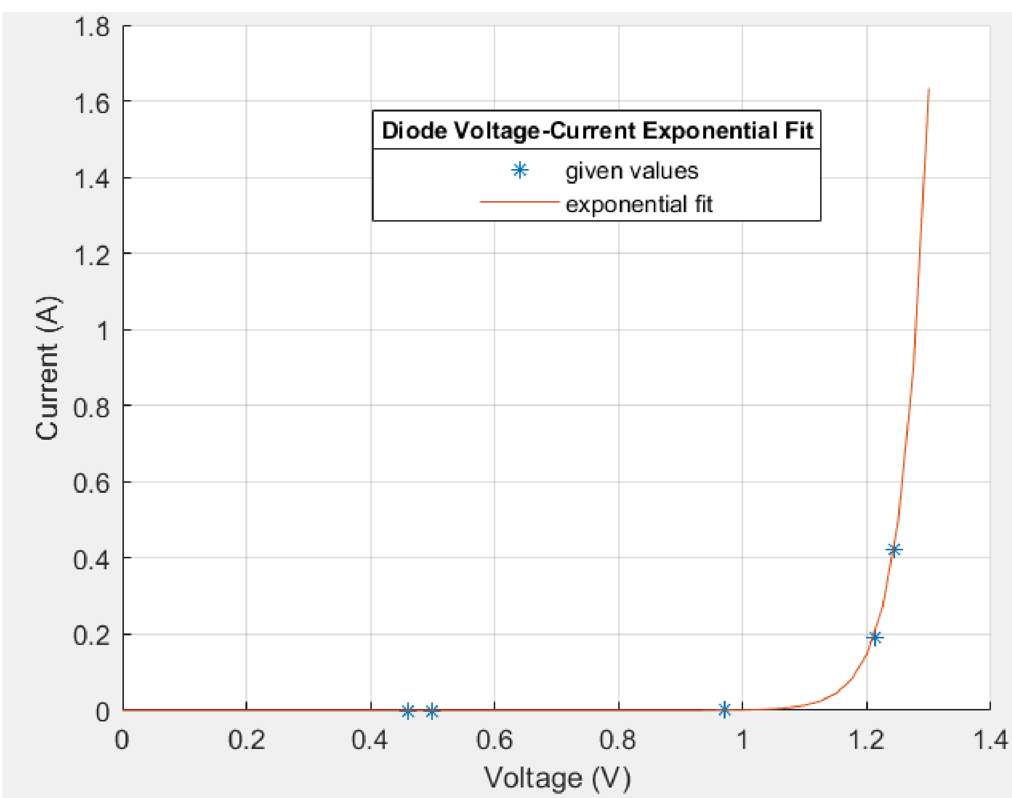
In this project, unknown values are given as follows: resistor $14.2\ \Omega$, inductance $0.98\ H$, source voltage $2\ V$. Two different time step sizes are $25\ ms$, $2.5ms$.

II. Explanation of Matlab Code

Code gives plots of $i(t), V_1(t), V_2(t), V_D(t)$ values. Other values like coefficients and sum of series are shown at workspace section in matlab. Input variables are data file, time step sizes and other constant values like source voltage. fprdata.dat file is imported and first column assigned to x array, second column assigned to y array. After that, program finds a and b constants to be able to fit exponential line. In this case, a value is founded as 4.501e-14 and b value founded as 24. Then, this line plotting with the measured data to make it more comparable. Runge-kutta method is using initial current and diode voltage values as zero. Other required values are calculated in the same loop as ODE solution method. Same process applied to different time step size at further code lines. In the end of the code, there is a part for comparing, numerical solutions of diode voltage and diode voltage values by given data. Also an error analysis for the fitted function.

III. Exponential Fit

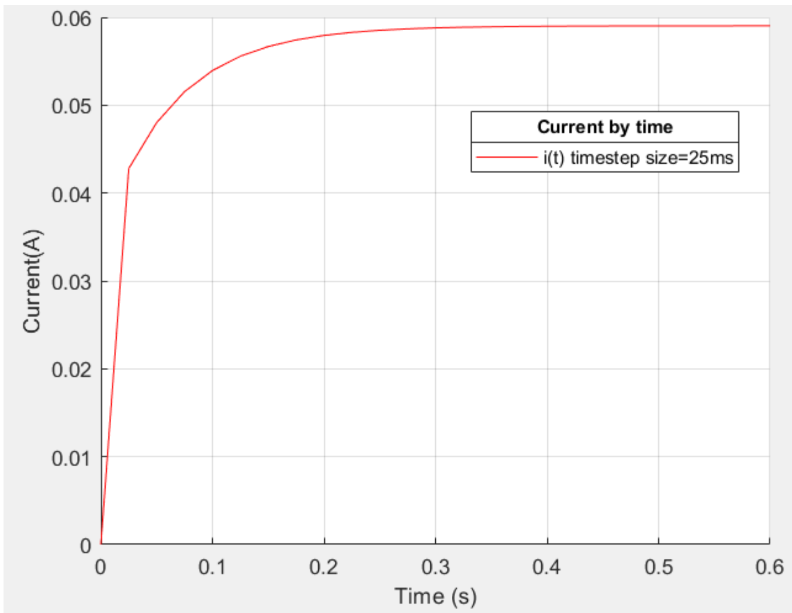
Founded a and b values are respectively 4.501e-14 and 24 by using equation(2). Exponential fit is given by graph-1 as seen in below and fitted function is quite ok. Polynomial fit would be inaccurate because of the small data set. In any least square method, lines should not diverge into the points, they should go between the points. This exponential fit done that perfectly. When five data points are tested in this function by taking difference between current points at same voltage value, maximum error of the fit is around 0.006 ampere.



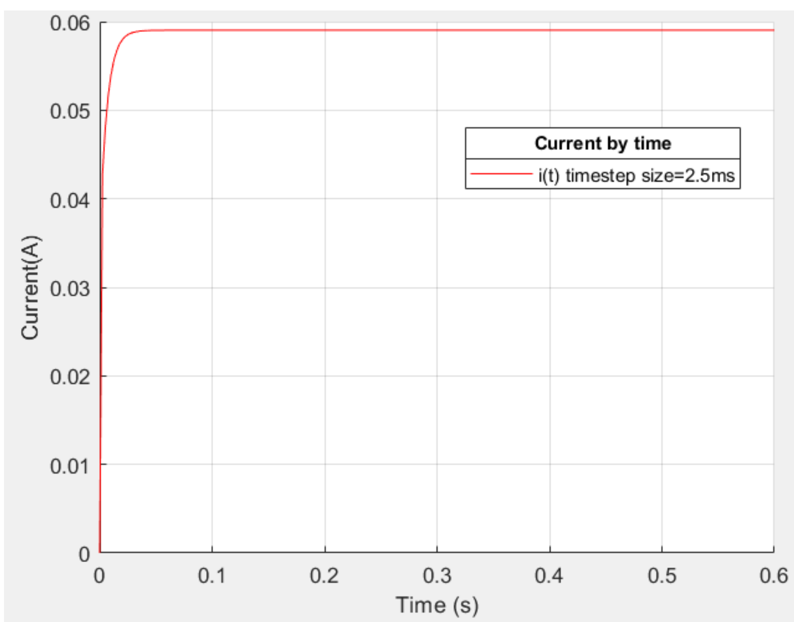
Graph-1: Exponential fit

IV. Numerical Solution of ODE

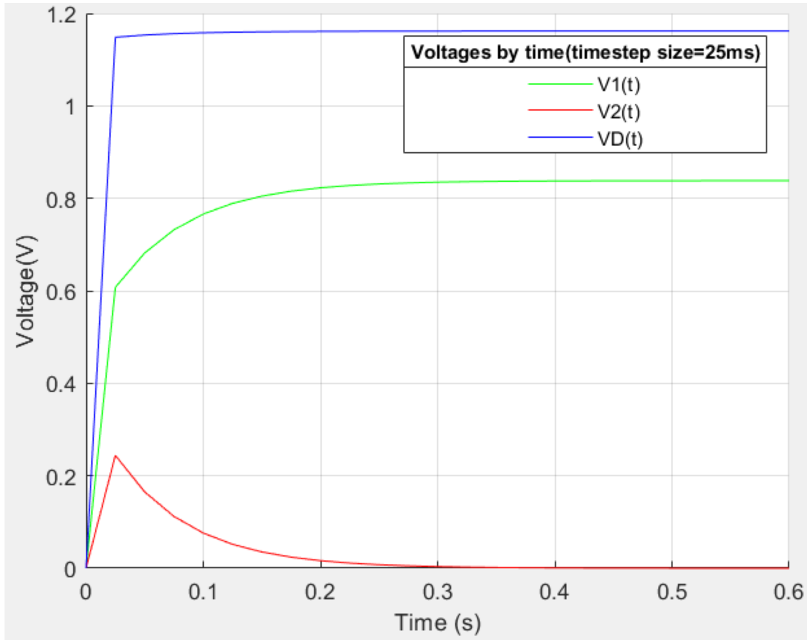
Runge-kutta order four method is chosen because it is considered as order five method and should be more accurate than other low order differential solution methods such as euler, midpoint and modified euler method. Although I did not; If I used these methods, results still would be similar. Equation(1) gives derivative of current which is the current value at exact known time. This current flow through the circuit without changing. So it is possible to determine all the current values by time steps based on numerical method. Solution of runge-kutta method is given graph-2 and graph-3 with different time step sizes.



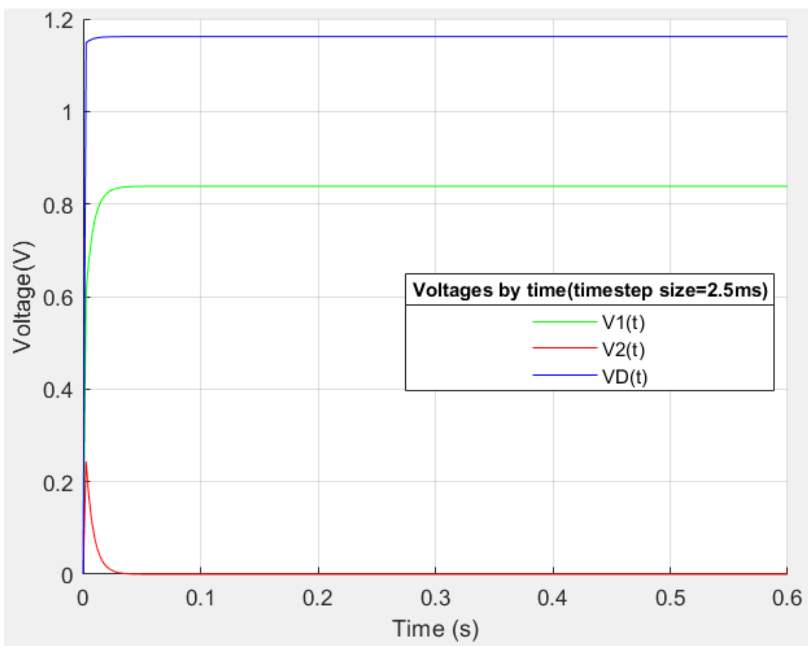
Graph-2: Id(t) $\Delta t = 25ms$



Graph-3: Id(t) $\Delta t = 2.5ms$



Graph-4: V(t) plots $\Delta t = 25ms$



Graph-5: V(t) plots $\Delta t = 2.5ms$

Graph-4 and graph-5 are show $V_1(t), V_2(t), V_D(t)$ voltage changes by time. These voltages are calculated with using numerical current values. All lines are jumping and then continue horizontal. Reason of that jump is exponentially voltage increasement at low current values. It is the only thing that effect result besides that is a RL- circuit.

V. Project Results and Discussions

In the first part of project, the coefficients of exponential fit are found. 4.5e-14 is a value and 24 is b value. After solving ordinary differential equations, derivative of current converges zero to 0.06 ampere decreasingly. At First few iteration, current is increasing excessively because of the diode voltage increase exponentially. After the diode reaching knee voltage, current could not effect that much to voltage of the diode. Also less time step size makes line much more angled and reached maximum current earlier.

Voltage lines seen as they should be except $V_2(t)$ which is at graph-4. Probably larger time step size caused that. Because inductance charging slowly and then discharge slowly as well. In graph-5, inductance having same process but converges much faster. All plots are quite similar to RL circuit plots, only difference is instantaneous jump because of the diode characteristic.

Code works without a problem and gives correct values. Least square fitting method and runge-kutta method are accurate and work fine. Even the comparison between the diode voltage founded by numerical methods and given current values are too close at the same current levels. For instance, at 0.057 ampere, exponential fitted function gives 1.16 Volt and calculation which is assisted by numerical solution gives 1.13 Volt. This comparison also proves the accuracy of the used methods.

Matlab Code

```
clear all;
close all;
format long;
Vs=2;
R=14.2;
L=0.98;
f=importdata('fprdata.dat');
x=f(:,1); % Voltage
y=f(:,2); % Current
%------------least square approximation (Exponential)------------
-
xi2yi=0;yilnyi=0;xiyi=0;xiyilnyi=0;yi=0;
for i=1:1:length(f)
    xi2yi=xi2yi+((x(i)^2)*y(i));
    yilnyi=yilnyi+(y(i)*log(y(i)));
    xiyi=xiyi+(x(i)*y(i));
    xiyilnyi=xiyilnyi+(x(i)*y(i)*log(y(i)));
    yi=yi+y(i);
end
a=exp(((xi2yi*yilnyi)-(xiyi*xiyilnyi))/((yi*xi2yi)-(xiyi^2)));
```

```
b=((yi*xiyilnyi)-(xiyi*yilnyi))/((yi*xi2yi)-(xiyi^2));
%------------plot-fit---------------
tx=0:0.025:1.3;
p=a*exp(b*tx);
figure
grid on;
hold on;
plot(x,y,'*');
plot(tx,p);
xlabel('Voltage (V)');
ylabel('Current (A)');
g1=legend('given values','exponential fit');
title(g1,'Diode Voltage-Current Exponential Fit');
%-----------runge kutta-Order4--timestep=0.025---------
T1=0:0.025:0.6;
idt=zeros(0,length(T1));
idt(1)=0;
Vd(1)=0;
for i=2:1:length(T1)
    k1=(0.025)*((1/L)*(Vs-Vd(i-1)-(idt(i-1)*R)));
    k2=(0.025)*((1/L)*(Vs-Vd(i-1)-((idt(i-1)+k1/2)*R)));
    k3=(0.025)*((1/L)*(Vs-Vd(i-1)-((idt(i-1)+k2/2)*R)));
    k4=(0.025)*((1/L)*(Vs-Vd(i-1)-((idt(i-1)+k3)*R)));
    idt(i)=idt(i-1)+(1/6)*(k1+2*k2+2*k3+k4);
    Vd(i)=(log(idt(i)/a))/b;
    VR(i)=R*idt(i);
    VL(i)=Vs-Vd(i)-VR(i);
end
%-----------plot-results--timestep=0.025------------
tx2=0:0.025:0.6;
figure
grid on;
hold on;
plot(tx2,idt,'r');
ylabel('Current(A)');
xlabel('Time (s)');
g2=legend('i(t) timestep size=25ms');
title(g2,'Current by time');
figure
grid on;
hold on;
plot(tx2,VR,'g');
plot(tx2,VL,'r');
plot(tx2,Vd,'b');
ylabel('Voltage(V)');
xlabel('Time (s)');
g3=legend('V1(t)','V2(t)','VD(t)');
```

```
title(g3,'Voltages by time(timestep size=25ms)');
%-------------runge kutta-Order4--timestep=0.0025------
T1=0:0.0025:0.6;
idt=zeros(0,length(T1));
idt(1)=0;
Vd(1)=0;
for i=2:1:length(T1)
    k1=(0.025)*((1/L)*(Vs-Vd(i-1)-(idt(i-1)*R)));
    k2=(0.025)*((1/L)*(Vs-Vd(i-1)-((idt(i-1)+k1/2)*R)));
    k3=(0.025)*((1/L)*(Vs-Vd(i-1)-((idt(i-1)+k2/2)*R)));
    k4=(0.025)*((1/L)*(Vs-Vd(i-1)-((idt(i-1)+k3)*R)));
    idt(i)=idt(i-1)+(1/6)*(k1+2*k2+2*k3+k4);
    Vd(i)=(log(idt(i)/a))/b;
    VR(i)=R*idt(i);
    VL(i)=Vs-Vd(i)-VR(i);
end
%-----------plot-results--timestep=0.0025-----------
tx2=0:0.0025:0.6;
figure
grid on;
hold on;
plot(tx2,idt,'r');
ylabel('Current(A)');
xlabel('Time (s)');
g2=legend('i(t) timestep size=2.5ms');
title(g2,'Current by time');
figure
grid on;
hold on;
plot(tx2,VR,'g');
plot(tx2,VL,'r');
plot(tx2,Vd,'b');
ylabel('Voltage(V)');
xlabel('Time (s)');
g3=legend('V1(t)','V2(t)','VD(t)');
title(g3,'Voltages by time(timestep size=2.5ms)');
%proof ODE numerical solution - given data
%test=0.057;
%test1=(log(test/a))/b
%test2=Vs-test*R-test*L
%error of fitted function
%for i=1:1:5
   % display(abs(y(i)-a*exp(b*x(i))));
%end
```