Exam [maximal number of points: 75 pts.]

Last Name		Mark
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Username		
Student Nr.		
Date		
Workstation Nr.		

1	2	3		Points

- All mobile phones must be switched off and must be stored in your pocket or your bag during the exam.
- Put your student card on the table.
- Fill in this cover sheet first.
- You can start the Spyder editor in Linux by clicking the "Actitivies" icon in the top left corner of the screen and then by searching for "Spyder" and then press "Enter".
- Work in the folder "/questions". Here, a backup of saved files will be created automatically.
- Write your name as a comment line on top of each codefile you submit.
- Start a new page for each of the three problems and write your full name on each one of the sheets.
- Solutions written in pencil, red or green will be considered invalid. Do not use Tipp-Ex.

1. **General questions** [in total 16 pts.]

a) [2 pts.] Let $\{W_t : t \geq 0\}$ denote a one-dimensional Wiener process. Consider the Stochastic Differential Equation (SDE)

$$\mathrm{d}S_t = b(t, S_t)\,\mathrm{d}t + \sigma(t, S_t)\,\mathrm{d}W_t, \qquad S_0 = s_0,$$

with deterministic initial value $s_0 > 0$. State sufficient conditions on $b, \sigma\colon \mathbb{R}_+ \times \mathbb{R} \to \mathbb{R}$ that ensure existence and uniqueness of a solution $\{S_t : t \geq 0\}$ to this SDE (no proof needed). Provide one example which satisfies these conditions and another example which does not.

b) [2 pts.] Let $(\mathcal{V}, \mathcal{H}, \mathcal{V}^*)$ be a Gelfand triple. Let $J = (0, T)$ for some $T > 0$. We consider a solution u to the variational problem:

Find $u \in L^2(J; \mathcal{V}) \cap H^1(J; \mathcal{V}^*)$ such that, for all $\varphi \in \mathcal{V}$,

$$\begin{gathered} \frac{d}{dt}\langle u, \varphi\rangle_{\mathcal{V}^*,\mathcal{V}} + a(u, \varphi) = 0\,, \quad \text{a. e. in } (0,T)\,, \quad T > 0\,, \\ u(0) = u_0 \in \mathcal{H}\,. \end{gathered} \tag{1}$$

Let $v(t) := e^{-\lambda t}u(t)$ for some $\lambda \in \mathbb{R}$. Then v is an element of $L^2(J; \mathcal{V}) \cap H^1(J; \mathcal{V}^*)$ with weak derivative given by

$$v'(t) = -\lambda v(t) + e^{-\lambda t}u'(t)\,.$$

Show that v is the solution of a variational problem of the form (1) for some new bilinear form $\tilde{a} : \mathcal{V} \times \mathcal{V} \to \mathbb{R}$. Show that if a satisfies a Gårding inequality, a suitable choice of λ makes $\tilde{a}$ strongly coercive (i.e., it satisfies a Gårding inequality with $C_2 = 0$).

c) [3 pts.] Let $a^{\mathrm{BS}} : H^1(G) \times H^1(G) \to \mathbb{R}$ be the bilinear form associated to the one-dimensional Black-Scholes (BS) equation in the log-price interval $(-R, R)$ and in time to maturity. Let $g : G \to \mathbb{R}$ be the payoff associated to a put option with strike $K > 0$. Show that for all $\varphi \in C_0^1(G)$, there holds

$$a^{\mathrm{BS}}(g, \varphi) = -\frac{1}{2}K\sigma^2\varphi(\ln K) + rK\int_{-R}^{\ln K} \varphi\, dx\,.$$

d) [2 pts.] State the SDEs satisfied by the short rate processes in the Vasicek and Cox-Ingersoll-Ross (CIR) models. What is the key difference in the qualitative behavior of the short rate between those two models?

e) [2 pts.] Let W be an $\mathbb{R}^n$ valued Brownian motion. Let $X = (X_1, \ldots, X_d)$ be a stochastic process such that, for each $i \in \{1, \ldots d\}$, X_i evolves according to

$$dX_i^t = b_i(X_t)dt + \sum_{j=1}^{n} \Sigma_{ij}(X_t)\, dW_t^j\,, \quad X_0^i = Z^i,$$

for some $b : \mathbb{R}^d \to \mathbb{R}^d$ and $\Sigma : \mathbb{R}^d \to \mathbb{R}^{d\times n}$, and where $Z = (Z^1 \ldots Z^d) \in \mathbb{R}^d$ is a random variable independent of the σ-algebra generated by W. State the expression of the infinitesimal generator $\mathcal{A}$ of X (give the expression of $\mathcal{A}f$ when $f \in C^2(\mathbb{R}^d)$ with bounded derivatives). How to choose b and Σ to obtain the multi-dimensional BS model?

f) [2 pt.] For $d \in \mathbb{N}$ consider a d-dimensional BS market model and let

$$a^{\mathrm{BS}} \colon H^1(\mathbb{R}^d) \times H^1(\mathbb{R}^d) \to \mathbb{R}$$

be the corresponding bilinear form in log-price. Give the expression of a^{BS} and state a sufficient condition so that a^{BS} satisfies a Gårding inequality (no proof needed).

g) [2 pts.] Describe the smooth pasting condition for the value of an American option. Give a sufficient condition on the Lévy triplet (σ^2, ν, γ) such that smooth pasting holds in an exponential Lévy model.

h) [1 pts.] State one advantage and one limitation of the FEM on tensor product grids (compared to FEM on a triangular mesh).

2. Pricing of a compound option [in total 37 pts.]

Let $V_1(t,s)$ be the value of a European option with payoff given by a Lipschitz function $g_1 : \mathbb{R}^+ \to \mathbb{R}$, and maturity T_1. We assume that the price process S of the underlying asset is given by the Black-Scholes (BS) model with interest rate $r \geq 0$ and volatility $\sigma > 0$.

We further consider a compound option, which is an option on the European option, whose value satisfies

$$V_c(t,s) = \mathbb{E}\left[e^{-r(T_c-t)} g_c(V_1(T_c, S_{T_c})) \;\middle|\; S_t = s\right],$$

where the payoff $g_c : \mathbb{R}^+ \to \mathbb{R}$ is a Lipschitz function and $T_c < T_1$ is the maturity of the compound option.

a) [3 pts.] Express $V_1(t,s)$ in terms of a conditional expectation. Assuming that V_c and V_1 are sufficiently regular with bounded derivatives in s, state with a succinct justification the PDEs that V_c and V_1 satisfy.

b) [2 pts.] Let $J_1 = (0,T_1)$, $J_c = (0,T_c)$, and let $v_1(t,x)$ and $v_c(t,x)$ be the option values V_1 and V_c at **time-to-maturity** t and in **log-price** x. Show that there holds

$$\begin{cases} \partial_t v_1 - \mathcal{A}^{\mathrm{BS}} v_1 + r v_1 &= 0 & \text{in } J_1 \times \mathbb{R}\,, \\ v_1(0,x) &= g_1(e^x) & \text{for } x \in \mathbb{R}\,, \end{cases}$$

$$\begin{cases} \partial_t v_c - \mathcal{A}^{\mathrm{BS}} v_c + r v_c &= 0 & \text{in } J_c \times \mathbb{R}\,, \\ v_c(0,x) &= g_c(v_1(T_1 - T_c, x)) & \text{for } x \in \mathbb{R}\,, \end{cases}$$

and derive the operator $\mathcal{A}^{\mathrm{BS}}$.

c) [1 pts.] We consider the solutions $v_{c,R} : J_c \times G \to \mathbb{R}$ and $v_{1,R}(t,x) : J_1 \times G \to \mathbb{R}$ to the PDEs

$$\begin{cases} \partial_t v_{1,R} - \mathcal{A}^{\mathrm{BS}} v_{1,R} + r v_{1,R} &= 0 & \text{in } J_1 \times G\,, \\ v_{1,R}(0,x) &= g_1(e^x) & \text{for } x \in G\,, \\ v_{1,R}(t,\pm R) &= 0\,, & \text{for } t \in J\,, \end{cases} \tag{2}$$

$$\begin{cases} \partial_t v_{c,R} - \mathcal{A}^{\mathrm{BS}} v_{c,R} + r v_{c,R} &= 0 & \text{in } J_c \times G\,, \\ v_{c,R}(0,x) &= g_c(v_{1,R}(T_1 - T_c, x)) & \text{for } x \in G\,, \\ v_{c,R}(t,\pm R) &= 0\,. & \text{for } t \in J\,, \end{cases} \tag{3}$$

where $G = (-R,R)$. Give a financial interpretation of $v_{1,R}$ and $v_{c,R}$, and motivate the introduction of these two functions.

d) [5 pts.] The weak formulation of Problem (2) reads

Find $u \in L^2(J_1; H_0^1(G)) \times H^1(J_1; H^{-1}(G))$ such that, for all $v \in H_0^1(G)$,

$$\begin{cases} (\partial_t u, v)_G + a^{\mathrm{BS}}(u,v) &= 0 & \text{a.e. in } J_1\,, \\ u(0,\cdot) &= g_1 \circ \exp & \text{a.e. in } G\,, \end{cases} \tag{4}$$

where

$$(u,v)_G := \int_G u\,v\,dx\,, \quad a^{\mathrm{BS}}(u,v) := \frac{1}{2}\sigma^2(\partial_x u, \partial_x v)_G + \left(\sigma^2/2 - r\right)(\partial_x u, v)_G + r(u,v)_G\,.$$

Prove that this variational problem admits a unique solution. Without proof, state an analogous weak formulation for Problem (3).

Let $N \in \mathbb{N}$, and let

$$x_i = -R + hi\,, \quad 0 \le i \le N+1\,, \quad h := \frac{2R}{N+1}\,.$$

Let $V_N = \{v \in H_0^1(G) \mid v_{|[x_i,x_{i+1}]} \text{ is affine}\}$ and let $\{b_i\}_{1\le i\le N}$ be the basis of V_N defined by

$$b_i(x_j) = \delta_{ij}\,, \quad 1 \le i,j \le N\,.$$

The semi-discrete formulation of Problem (4) reads

$$\text{Find } u_N \in C^1(J_1; V_N) \text{ such that, for all } v_N \in V_N\,,$$
$$\begin{cases} (\partial_t u_N, v_N)_G + a^{\mathrm{BS}}(u_N, v_N) &= 0 \qquad \text{a.e. in } J_1 \\ u_N(0,x) &= g_{N,1}(x) \qquad \text{a.e. in } G\,, \end{cases} \tag{5}$$

where $g_{N,1} \in V_N$ is the nodal interpolant of $g_1 \circ \exp$, i.e.

$$\forall i \in \{1,\dots,N\}\,, \quad g_{N,1}(x_i) = g_1(e^{x_i})\,.$$

e) [2 pts.] Reformulate Problem (5) as a $\mathbb{R}^N$-valued differential equation of the form

$$\begin{cases} \mathbf{M}\dfrac{d\underline{u}_N}{dt} + \mathbf{A}^{\mathrm{BS}}\underline{u}_N &= 0\,, \\ \underline{u}_N(0,x) &= \underline{g}_{N,1}\,. \end{cases} \tag{6}$$

where $\underline{u}_N(t) := (u_N(t,x_1),\dots,u_N(t,x_N))^T$ and $\underline{g}_{N,1} = (g_1(e^{x_1}),\dots,g_1(e^{x_N}))^T$. Derive the $N \times N$ matrices $\mathbf{M}$ and $\mathbf{A}^{\mathrm{BS}}$ in terms of the nodal basis functions $\{b_i\}_{1\le i\le N}$.

f) [5 pts.] Let $\mathbf{M}$ and $\mathbf{A}^{\mathrm{BS}}$ be as in part **e**). Let $\mathbf{S}$ and $\mathbf{W}$ be the $N \times N$ matrices defined by

$$\mathbf{S}_{ij} := (\partial_x b_j, \partial_x b_i)\,, \quad \mathbf{W}_{ij} := (\partial_x b_j, b_i)\,, \quad 1 \le i,j \le N\,.$$

Give an expression of $\mathbf{A}^{\mathrm{BS}}$ in terms of the three matrices $\mathbf{M}$, $\mathbf{S}$ and $\mathbf{W}$. Show that there holds

$$\mathbf{W}_{ij} = \begin{cases} -\frac{1}{2} & \text{if } j = i+1\,, \\ \frac{1}{2} & \text{if } j = i-1\,, \\ 0 & \text{otherwise.} \end{cases}$$

Without proof, we admit the following

$$\mathbf{M}_{ij} = \begin{cases} \frac{2}{3}h & \text{if } i = j\,, \\ \frac{1}{6}h & \text{if } |i-j| = 1\,, \\ 0 & \text{otherwise}\,, \end{cases} \qquad \mathbf{S}_{ij} = \begin{cases} \frac{2}{h} & \text{if } i = j\,, \\ -\frac{1}{h} & \text{if } |i-j| = 1\,, \\ 0 & \text{otherwise}\,. \end{cases}$$

Based on those formulas, complete the functions `assemble_M(N)`, `assemble_S(N)`, `assemble_W(N)` and `assemble_A(N,sigma,r)` in the template `CompoundBS.py`.

g) [2 pts.] Using the θ-scheme, write a fully discrete scheme for the resolution of Problem (6) of the form

$$\begin{cases} \underline{u}_N^0 &= \underline{g}_{N,1}\,, \\ \mathbf{B}_\theta \underline{u}_N^{m+1} &= \mathbf{C}_\theta \underline{u}_N^m\,, \qquad 0 \le m \le M-1\,, \end{cases} \tag{7}$$

where $M \in \mathbb{N}$. State the expression of the $N \times N$ matrices $\mathbf{B}_\theta$ and $\mathbf{C}_\theta$. Accordingly, complete the functions `assemble_B` and `assemble_C` in the template `CompoundBS.py`.

h) [7 pts.] Complete the function `computeBSEuropeanOption` which implements the finite element - θ scheme to compute an approximation of the value $V(t,\cdot)$ of a European option in a BS market, with payoff g and maturity T on G, for some $t \in [0,T]$. Choose the number M of time steps as

$$M := \left\lceil \frac{T-t}{h} \right\rceil .$$

i) [2 pts.] Use `computeBSEuropeanOption` coded in part **h)** to compute the function $V_1(0,\cdot)$. Take the payoff g_1 as

$$g_1(s) = (s - K_1)_+\,,$$

with $K_1 = 20$ and the maturity $T_1 = 1$. Use the following parameters for the BS model:

$$r = 0.01, \quad \sigma = 0.3\,.$$

Choose the discretization parameters as

$$R = 5.5\,, \quad N = 1000\,.$$

Plot the function on the interval $(10, 30)$ and compare your solution to the exact value that can be computed with the BS formula implemented in `bs_formula_C` in the template.

j) [3 pts.] For $\theta \in \{0, 0.5, 1\}$ and $k \approx h$ as above, what is the theoretical behavior of the scheme (7) in terms of stability and order of convergence for the error $\|\underline{u}_N^M - u(T)\|_{L^2(G)}$? Justify your answer. Using the function `orderOfConvergence`, which plots the numerical error of the fully discrete scheme in log-log scale (keeping otherwise the same parameters as in part **i)** above, compare the numerical results to the expected behavior.

k) [5 pts.] Complete the remainder of the code in the template `CompoundBS.py` to compute an approximation of $V_c(0,\cdot)$. For the underlying call option, keep the same parameters as in item **i)** above. For the compound call option, use $T_c = 0.5$ and $g_c = (s - K_c)_+$ where $K_c = 4$. In the same graph, plot the functions $s \mapsto V_c(0,s)$ (the compound call) $s \mapsto V_1(0,\cdot)$ (the underlying call) and $s \mapsto g(V_1(T_1 - T,\cdot))$ (the compound payoff) for $s \in (10, 30)$. Interpret qualitatively the relative positions of the two graphs.

3. Pricing of an American option in a CEV model [in total 22 pts.]

In this problem, we consider a constant elasticity of variance (CEV) market model and wish to price an American put option with payoff

$$g(s) = (K - s)_+ , \quad K > 0 .$$

Let $T > 0$ denote the option maturity, $J = (0, T]$ and let $v_{\mathrm{am}}(t, s)$ denote the value of the option at **time-to-maturity** $t \in J$ and **spot price** $s > 0$.

Let $\mathcal{A}_\varrho^{\mathrm{CEV}}$ be the infinitesimal generator given by

$$\mathcal{A}_\varrho^{\mathrm{CEV}} := \frac{1}{2}\sigma^2 s^{2\varrho}\, \partial_{ss} + rs\, \partial_s ,$$

for $r, \sigma > 0$ and $0 < \varrho \leq 1$. We truncate the price interval to $G = (0, R)$, $R > 0$, and let $\mathcal{H}_\mu := L^2(G; s^{2\mu}\, ds)$ with inner product

$$(\varphi, \psi)_\mu := \int_G \varphi(s)\, \psi(s)\, s^{2\mu}\, ds , \quad \text{where} \quad \begin{cases} \mu \in (-1/2, 1/2 - \varrho) & \text{for } \varrho \in [1/2, 1) , \\ \mu = 0 & \text{for } \varrho \in (0, 1/2) \cup \{1\} . \end{cases}$$

Furthermore, let $W_{\varrho,\mu}$ be the completion of the set $C_0^\infty(G)$ under the norm

$$\|\varphi\|_{\varrho,\mu}^2 := \int_G s^{2\varrho+2\mu} |\partial_s \varphi|^2 + s^{2\mu} |\varphi|^2\, ds .$$

The approximation u_R of the **excess-to-payoff** value $u(t, s) := v_{\mathrm{am}}(t, s) - g(s)$ in this truncated price interval is defined by the following weak formulation:

Find $u_R \in L^2(J; W_{\varrho,\mu}) \times H^1(J; W_{\varrho,\mu}^*)$ such that $u_R(t, \cdot) \in \mathcal{K}_{0,R}$ and for all $\varphi \in \mathcal{K}_{0,R}$,

$$\begin{cases} (\partial_t u_R, \varphi - u_R)_\mu + a_{\varrho,\mu}^{\mathrm{CEV}}(u_R, \varphi - u_R) \;\geq\; -a_{\varrho,\mu}^{\mathrm{CEV}}(g, \varphi - u_R) & \text{a.e. in } J , \\ u_R(0, x) \;=\; 0 & \text{a.e. in } G , \end{cases} \tag{8}$$

where $\mathcal{K}_{0,R}$ is defined by

$$\mathcal{K}_{0,R} := \{\varphi \in W_{\varrho,\mu} : \varphi \geq 0 \;\; \text{a.e. in } G\},$$

and $a_{\varrho,\mu}^{\mathrm{CEV}}$ is given by

$$a_{\varrho,\mu}^{\mathrm{CEV}}(\varphi, \psi) := \frac{1}{2}\sigma^2 (s^{2\varrho}\partial_s\varphi, \partial_s\psi)_\mu + \sigma^2(\varrho + \mu)(s^{2\varrho-1}\partial_s\varphi, \psi)_\mu - r(s\partial_s\varphi, \psi)_\mu + r(\varphi, \psi)_\mu .$$

a) [2 pts.] Give the stock price dynamics for the CEV model as an SDE under the risk-neutral measure. What particular feature of the volatility function makes this problem more difficult than a BS model?

b) [4 pts.] Show that the bilinear form $a^{\mathrm{CEV}}_{\varrho,\mu}$ is continuous on $W_{\varrho,\mu}$, i.e. there exists $C > 0$ such that, for all $\varphi, \psi \in W_{\varrho,\mu}$,

$$a^{\mathrm{CEV}}_{\varrho,\mu}(\varphi,\psi) \le C \|\varphi\|_{\varrho,\mu} \|\psi\|_{\varrho,\mu} \,.$$

You may use the following theorem:

Theorem 1 (Hardy's inequality)**.** *For all $\varphi \in C_0^\infty(G)$ and for all $\epsilon \ge 0$, $\epsilon \ne 1$, there holds*

$$\int_0^R s^{\epsilon-2} |\varphi(s)|^2 \, ds \le \left(\frac{2}{\epsilon-1}\right)^2 \int_0^R s^{\epsilon} |\partial_s \varphi(s)|^2 \, ds \,.$$

Let $N \in \mathbb{N}$, and let

$$x_i = hi \,, \quad 0 \le i \le N+1 \,, \quad h := \frac{R}{N+1} \,.$$

Let

$$V_N = \{ v \in C^0(G) \mid v_{|[x_i, x_{i+1}]} \text{ is affine and } v(0) = v(R) = 0 \}$$

and let $\{b_i\}_{1 \le i \le N}$ be the basis of V_N defined by

$$b_i(x_j) = \delta_{ij} \,, \quad 1 \le i, j \le N \,.$$

Discretizing Problem (8) with finite elements in V_N and the fully implicit Euler scheme (i.e. with $\theta = 1$) with M time steps leads to the approximations

$$\underline{u}_N^m \approx (u_R(km, x_1), \dots, u_R(km, x_N))^T \,, \quad m \in \{0, \dots, M\} \,, \quad k = \frac{T}{M} \,,$$

defined by the fully discrete scheme:

Let $\underline{u}_N^0 = \underline{0}$, and for $m = 0, \dots, M-1$, find $\underline{u}_N^{m+1} \in \mathbb{R}^N$ such that for and for all $\underline{\varphi} \in \mathbb{R}^N_{\ge 0}$,

$$(\underline{\varphi} - \underline{u}_N^{m+1})^T \mathbf{B} \underline{u}_N^{m+1} \ge (\underline{\varphi} - \underline{u}_N^{m+1})^T \underline{F}^m \,. \tag{9}$$

c) [2 pts.] Express the matrix $\mathbf{B} \in \mathbb{R}^{N \times N}$ and the vector $\underline{F}^m \in \mathbb{R}^N$ in terms of the basis functions $\{b_i\}_{1 \le i \le N}$ (don't try to compute the entries explicitly).

In what follows, we will use the fact that for each $m = 0, \dots, M-1$, the vector $\underline{u}_N^{m+1}$ is equal to the solution $\underline{x}$ of the following problem: Find $\underline{x}, \underline{\lambda} \in \mathbb{R}^N$ such that

$$\begin{cases} \mathbf{B}\underline{x} + \underline{\lambda} = \underline{F}^m \,, \\ \underline{x} \ge \underline{0} \,, \\ \underline{\lambda} \ge \underline{0} \,, \\ \underline{x}^T \underline{\lambda} = 0 \,. \end{cases} \tag{10}$$

A problem of the form (10) can be solved numerically by calling `PDAS(`$\mathbf{B}$`,`$\underline{F}^m$`,`$\underline{x}_0$`,`$\underline{\lambda}_0$`)` where $\underline{x}_0$ and $\underline{\lambda}_0$ are some initial guesses in $\mathbb{R}^N$. The function `PDAS` is already implemented for you.

Furthermore, we admit that the vector $\underline{G}$ defined by $\underline{G}_i := a^{\mathrm{CEV}}_{\varrho,\mu}(g, b_i)$ for $i \in \{1, \ldots, N\}$ can be approximated by

$$\underline{G} \approx \mathbf{A}^{\mathrm{CEV}} \underline{g}\,, \quad \underline{g} := (g(x_1), \ldots, g(x_N))^T\,,$$

where $\mathbf{A}^{\mathrm{CEV}}$ is the $N \times N$ matrix defined by

$$\mathbf{A}^{\mathrm{CEV}}_{ij} := a^{\mathrm{CEV}}_{\varrho,\mu}(b_j, b_i)\,.$$

d) [3 pts.] In the template `AmericanCEV.py`, complete the functions `assemble_M`, `assemble_A` and `assemble_B` which return the $N \times N$ matrices $\mathbf{M}^{\mathrm{CEV}}$ and $\mathbf{A}^{\mathrm{CEV}}$ and $\mathbf{B}$ from part c). Here, $\mathbf{M}^{\mathrm{CEV}}$ is defined by

$$\mathbf{M}^{\mathrm{CEV}}_{ij} := (b_i, b_j)_\mu\,, \quad 0 \le i, j \le N\,.$$

Use the function `assembleMatrix(`N`,`a`,`b`,`A`,`B`,`C`)` which, given the functions $A, B, C : (0, R) \to \mathbb{R}$, returns an approximation of the $N \times N$ matrix $\mathbf{K}$ defined by

$$\mathbf{K}_{ij} := \int_a^b A(x) b_i'(x)\, b_j'(x) + B(x) b_i(x) b_j'(x) + C(x) b_i(x) b_j(x)\, dx\,.$$

e) [9 pts.] Complete the function `computeCEVAmericanOption` which computes an approximation of the American put option value $v_{\mathrm{am}}(T, \cdot)$ on G. Plot the graph of the option value in the spot price interval $s \in (4, 6)$ for the parameters

$$N = 1000, \quad T = 1, \quad R = 10, \quad M = \left\lceil \frac{T}{h} \right\rceil, \quad \varrho = 0.7, \quad \mu = -0.4, \quad r = 0.05, \quad \sigma = 0.3.$$

f) [2 pts.] Assume that v_{eu} is the value of a European put option with the same maturity T and strike price K (also in time-to-maturity). What can you say about the relative values of v_{am} and v_{eu}? Justify your answer. What would happen in the case of call options?