$$
\begin{aligned}
q_i^j f_j &= \phi f_i \\
q_i^j g_j &= \gamma g_i \\
f^i g_i &= 0
\end{aligned}
$$

$$
Q_{ij}^{kl} = q_j^l \delta_i^k + q_i^k \delta_j^l + q_i^k \delta_i^l \delta_j^k + q_i^l \delta_i^k \delta_j^l - q_j^l \delta_i^k \delta^{kl} - q_i^k \delta_j^l \delta^{kl}
$$

$$
\begin{aligned}
A_{ij} &= f_i g_j - g_i f_j \\
Q_{ij}^{kl} A_{kl} &= \left(q_j^l \delta_i^k + q_i^k \delta_j^l + q_i^k \delta_i^l \delta_j^k + q_i^l \delta_i^k \delta_j^l - q_j^l \delta_i^k \delta^{kl} - q_i^k \delta_j^l \delta^{kl}\right) \cdot (f_k g_l - g_k f_l) \\
&= q_j^l \delta_i^k f_k g_l - q_j^l \delta_i^k g_k f_l + q_i^k \delta_j^l f_k g_l - q_i^k \delta_j^l g_k f_l \\
&\quad + q_i^k \delta_i^l \delta_j^k f_k g_l - q_i^k \delta_i^l \delta_j^k g_k f_l + q_i^l \delta_i^k \delta_j^l f_k g_l - q_i^l \delta_i^k \delta_j^l g_k f_l \\
&\quad - \left(q_j^l \delta_i^k + q_i^k \delta_j^l\right) \delta^{kl} \left(f_k g_l - g_k f_l\right) \\
&= q_j^l f_i g_l - q_j^l g_i f_l + q_i^k f_k g_j - q_i^k g_k f_j \\
&\quad + q_i^j f_j g_i - q_i^j f_i g_j + q_i^j f_i g_j - q_i^j f_j g_i \\
&= \gamma f_i g_j - \phi g_i f_j + \phi f_i g_j - \gamma g_i f_j \\
&= \gamma (f_i g_j - g_i f_j) + \phi (f_i g_j - g_i f_j) \\
&= (\gamma + \phi) A_{ij}
\end{aligned}
$$

$$
\begin{aligned}
S_{ij}^{(1)} &= f_i + f_j \\
Q_{ij}^{kl} S_{kl}^{(1)} &= \left(q_j^l \delta_i^k + q_i^k \delta_j^l + q_i^k \delta_i^l \delta_j^k + q_i^l \delta_i^k \delta_j^l - q_j^l \delta_i^k \delta^{kl} - q_i^k \delta_j^l \delta^{kl}\right) \cdot (f_k + f_l) \\
&= q_j^l \delta_i^k f_k + q_j^l \delta_i^k f_l + q_i^k \delta_j^l f_k + q_i^k \delta_j^l f_l + q_i^k \delta_i^l \delta_j^k f_k + q_i^k \delta_i^l \delta_j^k f_l + q_i^l \delta_i^k \delta_j^l f_k + q_i^l \delta_i^k \delta_j^l f_l \\
&\quad - q_j^l \delta_i^k \delta^{kl} f_k - q_j^l \delta_i^k \delta^{kl} f_l - q_i^k \delta_j^l \delta^{kl} f_k - q_i^k \delta_j^l \delta^{kl} f_l \\
&= \phi f_j + \phi f_i + q_i^j f_j + q_i^j f_i + q_i^j f_i + q_i^j f_j - q_j^i f_i - q_j^i f_i - q_i^j f_j - q_i^j f_j \\
&= \phi (f_i + f_j)
\end{aligned}
$$

$$
\begin{aligned}
S_{ij}^{(2)} &= (n-2)(f_i g_j + g_i f_j) + 2(f_i g_i + f_j g_j) \\
Q_{ij}^{kl} S_{kl}^{(2)} &= \left(q_j^l \delta_i^k + q_i^k \delta_j^l + q_i^k \delta_i^l \delta_j^k + q_i^l \delta_i^k \delta_j^l - q_j^l \delta_i^k \delta^{kl} - q_i^k \delta_j^l \delta^{kl}\right) \cdot ((n-2)(f_k g_l + g_k f_l) + 2(f_k g_k + f_l g_l)) \\
&= (n-2)(q_j^l \delta_i^k f_k g_l + q_i^k \delta_j^l f_k g_l + q_i^k \delta_i^l \delta_j^k f_k g_l + q_i^l \delta_i^k \delta_j^l f_k g_l - q_j^l \delta_i^k \delta^{kl} f_k g_l - q_i^k \delta_j^l \delta^{kl} f_k g_l) + \\
&\quad (n-2)(q_j^l \delta_i^k g_k f_l + q_i^k \delta_j^l g_k f_l + q_i^k \delta_i^l \delta_j^k g_k f_l + q_i^l \delta_i^k \delta_j^l g_k f_l - q_j^l \delta_i^k \delta^{kl} g_k f_l - q_i^k \delta_j^l \delta^{kl} g_k f_l) + \\
&\quad 2(q_j^l \delta_i^k f_k g_k + q_i^k \delta_j^l f_k g_k + q_i^k \delta_i^l \delta_j^k f_k g_k + q_i^l \delta_i^k \delta_j^l f_k g_k - q_j^l \delta_i^k \delta^{kl} f_k g_k - q_i^k \delta_j^l \delta^{kl} f_k g_k) + \\
&\quad 2(q_j^l \delta_i^k g_l f_l + q_i^k \delta_j^l g_l f_l + q_i^k \delta_i^l \delta_j^k g_l f_l + q_i^l \delta_i^k \delta_j^l g_l f_l - q_j^l \delta_i^k \delta^{kl} g_l f_l - q_i^k \delta_j^l \delta^{kl} g_l f_l) \\
&= (n-2)(q_j^l f_i g_l + q_i^k f_k g_j + q_i^j f_j g_i + q_i^j f_i g_j - q_j^i f_i g_i - q_i^j f_j g_j) + \\
&\quad (n-2)(q_j^l g_i f_l + q_i^k g_k f_j + q_i^j g_j f_i + q_i^j g_i f_j - q_j^i g_i f_i - q_i^j g_j f_j) + \\
&\quad 2(q_j^l f_i g_i + q_i^k f_k g_k + q_i^j f_j g_j + q_i^j f_i g_i - q_j^l f_i g_i - q_i^j f_j g_j) +
\end{aligned}
$$

$$
\begin{aligned}
& 2(q^l_j g_l f_l + q^k_i g_j f_j + q^j_i g_i f_i + q^j_i g_j f_j - q^i_j g_i f_i - q^j_i g_j f_j) \\
= \;& (n-2)(\gamma f_i g_j + \phi f_i g_j + q^j_i f_j g_i + q^j_i f_i g_j - q^j_i f_i g_i - q^j_i f_j g_j) + \\
& (n-2)(\phi f_j g_i + \gamma f_j g_i + q^j_i f_i g_j + q^j_i f_j g_i - q^j_i f_i g_i - q^j_i f_j g_j) + \\
& 2\left(q^k_i f_k g_k + q^l_j g_l f_l\right) \\
= \;& (n-2)\left((\gamma+\phi)(f_i g_j + f_j g_i) + 2q^j_i(f_i g_j + f_j g_i - f_i g_i - f_j g_j)\right) + 2(q^k_i + q^k_j) f_k g_k
\end{aligned}
$$