

RECIPE: Relaxed Sound Predictive Analysis

Abstract—The abstract goes here.

I. INTRODUCTION

In a multithreaded application, a data race occurs when two concurrent threads access the same memory location, such that (i) at least one of the accesses is write access, and (ii) no explicit mechanism is enforced to prevent the threads from simultaneous access to the location [10]. Race detection is important not only because races often reveal bugs in the implementation, but also as the basis of other techniques and analyses like atomicity checking [8], [9], [12], data-flow analysis [1] and record/replay [2], [7].

In this paper, we focus on *sound* race detection, whereby reported races are guaranteed to be real. This is the key requirement for adoption of the tool by developers [?]. Ensuring soundness is a difficult challenge, which mandates dynamic forms of analysis. Indeed, extensive research has been carried out on dynamic race detection. The general goal has been to derive constraints from a given execution trace on event reordering, and check for the remaining reorderings whether they disclose data races. The key question then becomes about constraint extraction.

Existing Approaches: Numerous techniques have been proposed to date for race detection, which either sacrifice soundness [4], [?], [?], [?] or have significant coverage limitations [?], [?], [?]. Recently *predictive analysis* has emerged as a promising alternative, whereby a single trace is considered, and races are discovered by permuting the execution schedule governing the trace [?], [13].

Predictive analysis guarantees soundness, and has the potential for high coverage, though current predictive analyses suffer from two major limitations:

- 1) All the permutations attempted by the analysis must preserve the flow dependencies exhibited by the original trace. As an example, if in the original trace thread t_1 reads a shared variable that was written by thread t_2 , then the same must hold in the permuted trace.
- 2) The analysis cannot step outside the boundaries of the input trace, e.g. by exploring branches that were not followed in that trace.

Both of these constraints are a conservative means of ensuring soundness. In an empirical study we conducted, which we describe in Section VII, we found that these restrictions often lead to serious loss in coverage.

As an illustration, we refer to the trace in Figure 1, where the statements in red denote an unexplored branch outside the trace. Existing predictive analyses are unable to detect the race between lines 2 and 5 because of the dependence between lines 3 and 4, which is a barrier to the needed reorderings.

$x = 0; y = 0;$	
T_1	T_2
1: $y = 3;$	
2: $x = 1;$	
3: $y = 5;$	
	4: if ($y > 2$)
	5: $\text{print}(1/x);$
	6: else
	7: $\text{print}(2/x);$

Fig. 1. Example illustrating ordering constraints beyond synchronization primitives

A second race that is missed, as it involves the statements in red, is between lines 2 and 7.

Our Approach: We describe a novel approach to predictive race detection, which we have implemented as the RECIPE analysis tool, that is able to relax both of these constraints. Compared to the state of the art [3], RECIPE is able to detect x1.5 more races with only the first restriction relaxed, and x2.5 more races with both restrictions relaxed.

The departure point of our approach is to consider the explicit values of shared memory locations instead of dependencies between trace events. The gain in coverage is twofold:

- 1) Reorderings that violate the original dependence structure but preserve the branching history become possible. The values read by a branching statement may change so long as the branch condition evaluates to its original truth value.
- 2) In addition, in certain cases we can guarantee soundness while directing execution toward branches that diverge from the original trace. This mandates that the effects of an unexplored branch can be modeled precisely, which holds frequently in practice because the original and new branches often access the same set of shared memory locations.

Returning to the example in Figure 1, we demonstrate how value-based reasoning improves coverage. First, value-based reasoning reveals that trace [1, 4, 5, 2, 3] is feasible, since the first assignment, $y = 3$, satisfies the condition $y > 2$. Hence the race is discovered. A second race, between lines 2 and 7, is detected by negating the condition. This is possible because the effects of the `else` branch can be modeled precisely. Negation leads to an execution starting at line 4 (where $y \equiv 0$). The race between lines 2 and 7 then becomes visible.

Concretely, we achieve value-level (rather than dependence-level) granularity computationally via a unique encoding of the input execution trace as a constraint system. The constraints are then processed by a satisfiability checker. This gives us the

flexibility to explore nontrivial constraints, such as forcing a context switch after the first assignment to y or negating the condition in Figure 1. Importantly, these manipulations are beyond what dependence-based reasoning can achieve, since this view of the trace is too conservative.

Contributions: This paper makes the following principal contributions:

II. TECHNICAL OVERVIEW

In this section, we walk the reader through a detailed technical description of our approach based on the example in Figure 1. As input, we assume (i) a program P as well as (ii) a trace of P recorded during a dynamic execution.

A. Preliminaries

To facilitate our presentation, we first introduce some terminologies used throughout this paper. Intuitively, a trace is a sequence of events recorded during the observation run.

Event An event, $e = \langle t, id, inst \rangle$, is a concrete representation of the runtime execution of a static instruction $inst$.

- t refers to the thread that issues the event e , denoted as t^e .
- id refers to the id associated with each event. The key property of id is *uniqueness*, i.e., any two events in the trace own different ids. The other important property of id is *monotonicity*, i.e., the events from the same thread should own the strictly increasing ids. Unless otherwise specified, we use the index of an event in the trace as its id, which satisfies the above properties. Throughout this paper, we denote an event as e_{id} with the id as the subscript.
- $inst$ is the static instruction. The instructions are three-address instructions involving at most three operands, which modern compilers commonly support. Specifically, we are interested in the types of instructions listed in Table I. When the variable does not appear on the left hand of an equation, such as y in $x.f = y$, it may refer to a variable, a constant or event object creation expression $new(\dots)$. The *bop* stands for the binary operator, which may refer to $+$, $-$, $*$, $/$, $\%$, \wedge , \vee in the assignment, or refer to $<$, $>$, $=$, \wedge , \vee in the branch. The target of the branch event is not important in our scope, therefore, we may abbreviate the branch instruction as the boolean expression afterwards. The listed instructions suffice to represent all trace events of interest. This is because a concrete finite execution trace can be reduced to a straight-line loop-free call-free path program (argument passing of the method call is modeled as assignments and the virtual call resolution is modeled as branches). This standard form of simplification preserves all the data-race-related information contained in the original trace.

Trace A trace τ is a sequence of events. It can also be represented more comprehensively as, $\tau = \langle \Gamma, \{\tau_{t_1}, \tau_{t_2}, \dots, \tau_{t_n}\}, O, \theta \rangle$.

$s ::=$		
	$y = x.f$	(heapr)
	$x.f = y$	(heapw)
	$z = x \text{ bop } y$	(assign)
	$\text{if } (x \text{ bop } y) \text{ goto } \dots$	(branch)
	$\text{lock}(l) \mid \text{unlock}(l)$	(sync)
	$\text{fork}(t) \mid \text{join}(t) \mid \text{begin}(t) \mid \text{end}(t)$	(thread)

TABLE I
INSTRUCTION TYPES

- Γ refers to the set of threads involved in the trace, which include t_1, t_2, \dots, t_n .
- $\{\tau_{t_1}, \tau_{t_2}, \dots, \tau_{t_n}\}$ denotes the event sequences produced by each thread.
- O is a function that assigns to every event an integer value that denotes its scheduling order. For example, e_i is scheduled earlier than e_j if $O(e_i) < O(e_j)$.
- θ is a function that maps every variable to a value. For example, given an event e executing the instruction $x = y + z$, the mapping recorded in the trace may be $\theta(x) = 3$, $\theta(y) = 2$, $\theta(z) = 1$. If the variable x is defined in two events, the mapping for x becomes ambiguous. To avoid the ambiguity, we conduct a pre-processing (Section III) that produces the SSA form of the trace, in which each variable is defined exactly once.

B. Constraint System

The predictive analysis takes the program and a trace as the input. Suppose it starts with the following trace of the program in Figure 1: $e_1 e_2 e_3 e_4 e_5$, where the line number is used as the event id (subscript). A potential race is between the pair, (e_2, e_5) , as they access the shared variable x from different threads and one is a write.

Given the trace, $\tau = \langle \Gamma, \{\tau_{t_1}, \tau_{t_2}, \dots, \tau_{t_n}\}, O, \theta \rangle$, the predictive analysis computes a new trace, $\tau' = \langle \Gamma, \{\tau_{t_1}, \tau_{t_2}, \dots, \tau_{t_n}\}, O', \theta' \rangle$. τ' preserves the same event sequence for each thread, i.e., T_1 still executes $e_1 e_2 e_3$ and T_2 still executes $e_4 e_5$, but it reschedules the events from different threads so that the potential race pair runs concurrently, i.e., e_2 and e_5 run concurrently. When the schedule changes, the variables may get different values. Therefore, we need to compute both the new schedule O' and the new value mapping θ' , in order to witness a race. To guarantee that τ' is a feasible trace of the program P , the computation needs to respect a set of constraints, which include the value constraints, control flow constraints and synchronization constraints¹. We explain them briefly in the following. The full details are explained in Section III and the feasibility guarantee is explained in Section V.

We first preprocess the trace as follows. We introduce symbols to replace the shared accesses in the instructions, e.g., We use W_x^{id}/R_x^{id} to replace the write/read of the shared variable x by the event e_{id} .

¹We omit the synchronization constraints in this paper, which are already well explained in existing techniques [?], [?].

Race Condition The potential race pair involves two accesses of the same shared location from different threads, where at least one access is a write. Given any potential race pair, e.g., (e_2, e_5) , it is a real race if and only if the two accesses can occur at the same time (race condition), which is captured by the race condition constraint

$$O'(e_2) = O'(e_5)$$

. The race condition constraint— together with the other constraints — guarantees the feasibility of the predicted race if a solution is found for the overall constraint system.

Intra-thread Constraints The predictive analysis needs to preserve the same event sequence for each thread, which further requires

- **Control Flow Constraints** Each thread takes the same control flows (or branch decisions) to reproduce the events. For the running example (ignoring the statements in red), this yields:

$$\theta'(R_y^4) > 2 \equiv \text{true}$$

That is, in the predicted trace τ' , the value of variable y read at e_4 should be greater than 2.

- **Intra-thread Order Constraints** The events should follow the same thread-local order as in the original trace. This constraint is imposed by the fact that the two runs share the same instruction sequence and take the same control flows. We yield the following formula:

$$O'(e_1) < O'(e_2) < O'(e_3) \wedge O'(e_4) < O'(e_5)$$

That is, in the predicted trace τ' , the three events from T_1 are ordered in the same way as in τ , and similarly, the branch event e_4 in T_2 still executes before the event e_5 inside the branch body.

- **Intra-thread Value Constraints** The events should respect the value constraints imposed by each instruction. For our running example, we obtain:

$$\begin{aligned} \theta'(W_x^0) = 0 \wedge \theta'(W_y^0) = 0 \wedge \theta'(W_y^1) = 3 \\ \wedge \theta'(W_x^2) = 1 \wedge \theta'(W_y^3) = 5 \end{aligned}$$

As an example, $\theta'(W_y^3) = 5$ denotes that the predicted run still assigns the value 5 to the variable y at event e_3 .

Inter-thread Value Constraints The inter-thread value constraints capture what writes the read events read from and under what scheduling condition the value flow occurs. The resulting formula for our example is

$$\begin{aligned} & (\theta'(R_y^4) = \theta'(W_y^0) \wedge O'(e_4) < O'(e_1)) \\ \vee & (\theta'(R_y^4) = \theta'(W_y^1) \wedge O'(e_1) < O'(e_4) < O'(e_3)) \\ \vee & (\theta'(R_y^4) = \theta'(W_y^3) \wedge O'(e_3) < O'(e_4)) \end{aligned}$$

Notice, importantly, that the formula associates the value constraints with the scheduling order constraints. As an example, $\theta'(R_y^4) = \theta'(W_y^1) \wedge O'(e_1) < O'(e_4) < O'(e_3)$ means that, in the predicted run, the read e_4 reads from the write e_1 , under the condition that e_4 happens after e_1 and no other writes (such as e_3) interleave them.

Unexplored Branches Beyond the encoding steps so far, which focus on the given trace, we can often encode constraints along unexplored branches. In our running example, this is essential to discover the race between lines 2 and 7. We conduct the symbolic execution to exercise the unexplored branches, which returns a set of traces representing possible executions of the unexplored branch. Each trace is combined with the original trace to find races involving the accesses in the unexplored path. For the racy events, e_2 and e_7 , we first specify the race condition constraint $O'(e_2) = O'(e_7)$. In addition, we negate the path condition, thereby obtaining $\theta'(R_y^4) > 2 \equiv \text{false}$ in place of $\theta'(R_y^4) > 2 \equiv \text{true}$. We also model the other necessary constraints such as the intra-thread order constraint $O'(e_4) < O'(e_7)$, similar to the above analysis.

Constraint Solving The formulas from the different encoding steps are conjoined and sent to the off-the-shelf solver such as **z3**. The race under analysis is real if the solver returns a solution, which includes the scheduling order and the values for the variables.

Highlights In this example, our analysis finds two more pairs of races than existing analyses [?], [?], which include (e_2, e_5) and (e_2, e_7) . The key insight for finding the first race is, we do not require e_4 to read from e_3 to retain the original value, as what existing approaches do, rather, we allow e_4 to read from e_1 while still preserving the feasibility of the branch. This relaxation allows the events after e_4 to run concurrently with the events before e_3 , leading to the detection of the first race. The key insight for finding the second race is, we explore the unexplored branches and ask the solver to compute a feasible schedule that exercises them. The neighboring branches are likely to contain racy events if the executed branches contain racy events.

III. RELAXATION OF FLOW DEPENDENCIES

Given the trace, $\tau = \langle \Gamma, \{\tau_{t_1}, \tau_{t_2}, \dots, \tau_{t_n}\}, O, \theta \rangle$, the main goal of our analysis is to derive a new trace, $\tau' = \langle \Gamma, \{\tau_{t_1}, \tau_{t_2}, \dots, \tau_{t_n}\}, O', \theta' \rangle$, which reschedules the events from different threads (the order of events inside each thread should remain unchanged) to witness the race. The key insight underlying our first relaxation is that, the read events in τ' are allowed by our analysis to read different values from different writes (than in trace τ), but they are required by existing approaches to read the same values as in τ . Therefore, our relaxation allows more schedules, which are likely to expose more races. Note that we also need to compute the new value mapping θ' because the rescheduling may alter the value every variable is assigned to. The computation of the new schedule and new value mapping is explained in the following.

A. SSA Form of the Trace

First of all, we rewrite the trace into SSA form, such that every variable is defined exactly once in the trace. This requirement is a prerequisite for the computation of the mapping, in which each variable is mapped to exactly one value. Another

side effect is that the def/use chains become explicit in the SSA form, which simplifies the following analysis steps.

RECIPE handles the local assignment and heap accesses differently.

- **Local Assignment** The SSA form for local assignment resembles the SSA form of static instructions in compiler optimization, except that the loops and recursions are fully resolved in a concrete trace, obviating the need for the Phi node. More concretely, we replace the variable v defined in an event, as well as the following uses of the definition, to a new variable v^{id} , where id is the unique id of the event. The uniqueness of the id guarantees that no two events define the same variable, i.e., each variable is defined exactly once. Consider the example in Figure 2, where the line number is also used as event id, the local variable x defined at line 4 is renamed as x^4 in the trace, while x defined at line 9 is renamed as x^9 , which is used in the following two events. At line 5, where the object $o3$ is created, we define a reference variable w^5 . Internally, the object $o3$ is encoded as a unique integer that indicates its heap address.
- **Heap accesses** The def/use relation between local heap accesses can be locally determined. Therefore, we treat them in the same way as local assignment. The def/use relations between shared heap accesses are more complex. We may link a read with different writes under different schedules, e.g., the read access to y at line 4 in Figure 1 can either obtain the definition before line 1 or at line 1 or at line 3. Therefore, we introduce two symbols to represent the shared read and shared write respectively and use them to establish different links flexibly. The details are as follows.

- **Local Heap Accesses** Given the write $x.f = y$ to a local heap location at event e_{id} , we introduce a fresh local variable $l_{o.f}^{id}$ (o is the runtime object referenced by x) to replace $x.f$. We also replace the uses of the location $o.f$ accordingly. Here the subscript $o.f$ is interpreted as a heap location, i.e., the field f of the object o . Consider the example in Figure 2, at lines 6-7, we replace the local heap accesses as the local variable $l_{o3.f}^6$.
- **Shared Heap Accesses** Given the write $x.f = y$ or the read $y = x.f$ of a shared location at event e_{id} , we introduce two symbols, $W_{o.f}^{id}$ and $R_{o.f}^{id}$ respectively, where o is the runtime object referenced by the x . Similarly, the subscript $o.f$ is interpreted as a heap location, i.e., the field f of the object o .

We use the heap location such as $o.f$ in the the symbolic form of heap accesses. An underlying assumption is that the heap location remains unchanged in our predictive analysis, which we ensure through additional constraints (Section III).

B. Constraint System

Based on the SSA form of the original trace τ , we build the constraints to compute a new trace τ' with the new

	$s1 = \text{new}(); // o1$	
	$s2 = \text{new}(); // o2$	
T_1	T_2	$Trace$
1: $s1.f=3;$		$W_{o1.f}^1=3;$
2: $s2.f=1;$		$W_{o2.f}^2=1;$
3: $s1.f=2;$		$W_{o1.f}^3=2;$
	4: $x=0;$	$x^4=0;$
	5: $w=\text{new}(); // o3$	$w^5=o3;$
	6: $w.f=10;$	$l_{o3.f}^6=10;$
	7: $z=w.f;$	$z^7=l_{o3.f}^6;$
	8: $y=s1.f;$	$y^8=R_{o1.f}^8;$
	9: $x=y+z;$	$x^9=y^8+z^7;$
	10: $\text{if}(x>11)$	$\text{if}(x^9>11)$
	11: $s2.f=x;$	$W_{o2.f}^{11}=x^9;$

Fig. 2. Running Example (shared variables are in bold font, the line number is used as the event id).

schedule O' and new value mapping θ' . Consider the example in Figure 2, in the original trace τ , the event e_8 reads the value of 2 from e_3 , therefore, e_{11} must happen after e_2 . Existing approaches enforce the same value dependence between e_3 and e_8 , leading to the miss of the race between e_2 and e_{11} . Our analysis can derive a new trace τ' to find the race by relaxing the dependence. In τ' , e_8 reads from a different event e_1 , producing the new value mapping, $\theta'(y^8) = 3$. Accordingly, at the event e_9 , the value mapping for x^9 is also updated as 13 ($\theta'(y^8)=3$, $\theta'(z^7) = 10$), enabling the true branch at line 10. Finally, the events e_2 and e_{11} are scheduled to run concurrently. The relaxation needs to respect a set of constraints, which we explain in the following. We omit the synchronization constraints intentionally, as they are well explained in all existing predictive analysis techniques [?], [3].

Race Condition Following the standard definition, pair (e_i, e_j) of events forms a race iff (1) e_i and e_j are accesses of the same location ℓ by different threads, (2) at least one of them writes to ℓ , and (3) e_i and e_j run concurrently. We refer to the candidate pair throughout this section as (candidate) *racy events*.

We first identify all candidate pairs of events that access the same location from different threads (and at least one is a write). We then check each pair separately. The checking is achieved by encoding all necessary constraints and invoking a constraint solver. The first constraint asserts the feasibility of the concurrent execution of the racy events:

$$O'(e_i) = O'(e_j)$$

In Figure 2, the race condition for the race pair, (e_2, e_{11}) , is $O'(e_2) = O'(e_{11})$

Intra-thread Constraints Given the candidate race pair, e_i and e_j , suppose e_i is after e_j in the original trace τ . The predictive analysis reschedules the events in the prefix of e_i , i.e., prior to e_i , so that e_i and e_j can run concurrently. We refer to the prefix as $\text{prefix}(\tau, e_i)$. Throughout this section, we only consider the events in the prefix.

The predictive analysis by design requires that each thread should follow the same event sequence prior to e as in the original run. The preservation of the event sequence for each thread requires the following intra-thread constraints:

- **Control Flow Constraints** The branches in the predicted run should take the same decisions as in the original trace so that each thread reproduces the same set of events. Specifically, we only need to reason about the branches prior to e_i . Without loss of generality, we assume the branch event e_k is in the form of $if(x < y)$. Then we require that

$$\bigwedge_{e_k \in prefix(\tau, e_i) \wedge inst^{e_k} = if(x < y)} \theta'(x) < \theta'(y) \equiv \theta(x) < \theta(y)$$

The constraint specifies that the branch condition should be evaluated as the same boolean value in the predicted run τ' as in the original trace τ . Importantly, Unlike existing analyses [?], [3], we do not pose the requirement that the values flowing into branching statements remain the same, but adopt the relaxed requirement that the evaluation of branching expressions remains the same. Consider the example in Figure 2, given the branch $if(x^9; 11)$ at e_10 , the original mapping is $\theta(x^9) = 12$. Existing analyses require the same value mapping for x^9 in the predicted run, while we allow a different value mapping, e.g., $\theta'(x^9) = 13$, which retains the same truth value for the branch expression.

- **Intra-thread Order Constraints** The events in the sequence should follow the same order as in the original trace. More formally, we require that

$$\forall e_m, e_n \in prefix(\tau, e_i), s.t., t^{e_m} = t^{e_n}. \\ O(e_m) < O(e_n) \Rightarrow O'(e_m) < O'(e_n)$$

This constraint specifies that two events from the same thread should follow the same order as reflected by the ids of the events.

- **Intra-thread Value Constraints** The value mapping of the variables should not contradict the value constraints imposed by each instruction. Without loss of generality, suppose the instruction is a local assignment in the form of $x = y + z$, then we require that

$$\bigwedge_{e_k \in prefix(\tau, e_i) \wedge inst^{e_k} = x = y + z} \theta'(x) = \theta'(y) + \theta'(z)$$

Remember that every variable may obtain a different value in the new trace τ' because it may read from some shared variable affected by the rescheduling. The constraint specifies that, the variables should be re-assigned to some values that are consistent with the instruction. Consider the example in Figure 2, given the event $x^9 = y^8 + z^7$ at line 9, if the mappings $\theta'(y^8)$ and $\theta'(z^7)$ change to 3 and 10 respectively, the mapping $\theta'(x^9)$ should change to 13 consistently. In the above, we use the local assignment event for demonstration, in general, the intra-thread value constraints should also be specified for the heap/heapw events.

T_1	T_2
1: $x^1 = new(); // \text{ creates } o1$	
2: $x^2 = new(); // \text{ creates } o2$	
3: $W_y^3 = x^1;$	
4: $W_y^4 = x^2;$	
5: $W_{o2.f}^5 = 5;$	
	6: $z^6 = R_y^6;$
	7: $w^7 = R_{o2.f}^7;$

Fig. 3. A trace.

Inter-thread Value Constraints for Relaxation We now move to the first novel feature of RECIPE, which is its ability to explore execution schedules that depart from the value flow exhibited in the original trace. More precisely, RECIPE is able to relax value flow dependencies in the original trace: a read access may read a different value from other write events, as long as the read value enforces feasibility of the following execution. This is strictly beyond the coverage potential of existing predictive analyses, which restrict trace transformations to ones where any read access must read the same value (often from the same write event) as in the original trace.

To ensure feasibility under relaxation of flow dependencies, we need to secure the flow between the read/write with the execution schedule. For example, in Figure 2, for the read at e_8 to read from the write $t e_1$, we need a schedule where e_8 happens after e_1 and other writes such as e_3 do not interleave them.

In general, the constraint formula, given read R_ℓ^m of location ℓ at the event e_m with set \mathcal{W} of matching write events (i.e., events including write access to ℓ), takes the following form:

$$\bigvee_{e_n \in \mathcal{W}} \bigwedge_{e_p \in \mathcal{W} \setminus \{e_n\}} \begin{aligned} & (\theta'(R_\ell^m) = \theta'(W_\ell^n)) \\ & O'(e_n) < O'(e_m) \\ & (O'(e_p) < O'(e_n) \vee O'(e_m) < O'(e_p)) \end{aligned}$$

This disjunctive formula iterates over all matching write events, and demands for each that (i) it occurs prior to the read event $O'(e_n) < O'(e_m)$ and (ii) all other write events either occur before $O'(e_p) < O'(e_n)$ it or after the read event $O'(e_m) < O'(e_p)$.

An important concern that arises due to relaxation of flow dependencies is that heap accesses may change their meaning, i.e., they involve different base objects and no longer match with each other. As an illustration, we refer to Figure 3. While the read at the event e_7 appears to match the write at e_5 , this is conditioned on the read at e_6 being linked to the assignment at e_4 . Suppose the event e_6 reads from e_3 due to the relaxation. Then e_7 and e_5 no longer share the same base object (or the same location). Even worse, we do not know which events e_7 matches, because the base object is no longer known.

Invariant Base Object Constraints To address this challenge, we enhance the constraint system with the requirement that heap objects that are dereferenced in field access statements before the racy events retain their original address in the predicted trace. This achieves two guarantees: First, matching

heap access events in the original trace are guaranteed to also match in the predicted trace. Second, candidate races in the original trace remain viable in the predicted trace as they still refer to same location. Third, sharing between threads of heap locations remains unchanged because each location is accessed by the same number of threads given that the base object is the same.

Formally, we require that

$$\bigwedge_{e_k \text{ has } x.f \wedge e_i \in \text{prefix}(\tau, e_i)} \theta'(x) \equiv \theta(x)$$

This constraint fixes that all heap dereferences prior to the racy event retain their original base object as in the original trace τ . The array accesses are handled similarly to the field accesses, but we need to additionally require the index variable to be the same as in the original trace. In addition, for local heap accesses, in case that the base variable reads a local object from a shared reference variable, the base variable may change due to the rescheduling. We also fix the base variable as a constant, in order to ensure the validity of the SSA encoding of the local heap accesses and preserve the def/use chains among them. The constraint is specified during the SSA encoding.

By sending the above constraints to a solver, we compute the necessary schedule orders among the events as well as the mapping of the variables. The necessary schedule orders define a partial order among the events, which permit a set of schedules that define the complete order that complies with it.

IV. EXPLORATION OF UNEXECUTED BRANCHES

We now switch to the second feature of RECIPE, which is its ability to reason about unexplored branches. At the high level, we simply need to derive a trace that contains the unexplored branch, after that, we can reduce the analysis to the analysis in Section ?? (starting from the SSA processing). RECIPE derives such traces by symbolically executing the unexplored branches to record the events and including these events in the new traces.

As shown in the following, suppose the symbolic execution $\text{symEngine}(\tau, e)$ returns the single-thread traces starting from the branch e , which now takes a different decision. The resultant trace is computed by first removing the events thread-locally after e (inclusively) and then appending one of the single-thread traces that represents a path in the unexplored branch of e . The negation $\text{neg}(e)$ of e , which is the same as e except the evaluation result of the boolean expression differs, is also appended.

```

for  $e : \tau$  do
  if  $\text{type}(e) = \text{branch}$  then
3:    $S = \text{symEngine}(\tau, e)$ 
    for  $\tau' : S$  do
       $\tau_r = \tau - \tau \downarrow_{t^e \wedge \geq e}$ 
6:    $\tau_r = \tau_r.\text{replace}(e, \text{neg}(e) + \tau')$ 
    end for
  end if
9: end for

```

Symbolic Execution Our symbolic execution is realized on top of the dataflow analysis, which generates events for each instruction within the unexplored branch. The symbolic execution has known limitations in reasoning about the schedules and loops (and recursion). Therefore, our analysis assumes all the events in the unexplored branch happen atomically at the branch event e without interleavings from other threads, as illustrated also in the above algorithm (line 6). In this way, our analysis adopts the sequential reasoning. RECIPE will reschedule the events based on the relaxation. As for loop (or recursion), the analysis terminates the current data flow immediately after one iteration, i.e., we expand the loop for only once. Otherwise, the analysis terminates the data flow normally if the flow goes out of the scope of the unexplored branch.

The analysis starts at the event e (exclusively), with the state σ . Initially, the state includes the runtime heap value for the base objects that are already resolved. During the symbolic execution, the state is updated to maintain the heap value and also the boolean expression values for branches. The values for other variables are not used indeed and therefore treated as symbolic. The update of the state is realized through the standard kill/gen rules for three basic heap instructions. In the object creation, which we separate from the local assignment, we assign a unique integer to represent the newly created object and store it into the state σ . For field access, $\sigma[x.f]$ actually denotes $\sigma[o.f]$, where o is the base object resolved in the state. These kill/gen rules take effect only when $x.f$ is not of the primitive type.

The analysis also generates the events, where the heap values are stored in the map of the event for later use. For the branch, we store its boolean evaluation result. As the boolean result is available only after the branch takes the decision, we delay the generation of the branch event until the first event after the branch.

V. THEORETICAL GUARANTEES

In this section, we first prove the soundness of RECIPE and then discuss the detection capability as compared to existing approaches. Both the proof and discussion are in the context of sequential consistency model [6].

Theorem 1 (Soundness). *The trace τ returned by our solver is a feasible trace for the concurrent program.*

Proof. The proof consists of two parts: (1) the trace is feasible, and (2) the trace can be generated by the program. We sketch the proof as follows.

- τ is feasible. Researchers [?], [11] point out a trace is feasible iff it satisfies the sequential consistency, i.e., $\forall \text{object } o, \tau \downarrow_o$ satisfies the serial specification of the object o . If all the events considered are of the types in Table I, then the sequential consistency precisely means the following: (1) read-write consistency, i.e., each read event of a variable should contain the value written by the most recent write event, and (2) the synchronization consistency, e.g., the lock acquire and release of a lock

Operation	Kill	Gen
$y = \text{new}(\dots)$	$\sigma[y] \mapsto \star$	$\sigma[y] \mapsto \text{newI}, \langle t, \text{cnt}, y = \text{newI}, \perp \rangle$
$y = x.f$	$\sigma[y] \mapsto \star$	$\sigma[y] \mapsto \sigma[x.f], \langle t, \text{cnt}, y = x.f, \llbracket x \rrbracket = \sigma[x] \rangle$
$x.f = y$	$\sigma[x.f] \mapsto \star$	$\sigma[x.f] \mapsto \sigma[y], \langle t, \text{cnt}, x.f = y, \llbracket x \rrbracket = \sigma[x] \rangle$
$z = x \text{ bop } y$		$\langle t, \text{cnt}, z = x \text{ bop } y, \perp \rangle$
$\text{if}(x < y)$		$\langle t, \text{cnt}, \text{if}(x < y), \llbracket x < y \rrbracket = \text{bool} \rangle$

TABLE II
DATAFLOW ANALYSIS

should not be interleaved by the lock operations of the same lock, the begin event of a thread should follow the fork event of the parent thread. Our inter-thread data flow constraints guarantee the read-write consistency among shared locations. The SSA form encodes the read-write consistency among local variables. The synchronization consistency is also captured as constraints in our solver.

- τ can be generated by the program, i.e., for each thread t , $\tau \downarrow_t$ can be generated by the corresponding thread code. This claim requires that, for any two adjacent events e and e' in $\tau \downarrow_t$, (1) if e is not a branch, inst^e and $\text{inst}^{e'}$ should be adjacent in the code; (2) if e is a branch, inst^e and $\text{inst}^{e'}$ should be adjacent and the inst^e should be evaluated as the boolean value indicated by $\text{inst}^{e'}$. We derive the trace through either concrete execution or symbolic execution, which satisfies the requirements and guarantees the correctness of the claim.

□

Discussion 1 (Detection Capability). *Our technique has stronger detection capability than existing techniques [?], [?], [?].*

First, to the best of our knowledge, none of the existing techniques automatically explore the un-executed branches by relaxing the schedules, while ours enables this. Second, assume we disable the exploration of un-executed branches and all techniques start with the same trace. Our technique still explores more traces. Figure 4 illustrates the difference pictorially, where two vertical lines represent the progress of two threads and the circles with numbers denote the events with ids. Existing techniques require the reads to read the same values, therefore, enforce the schedule order $e_5 \rightarrow e_6$ and $e_2 \rightarrow e_3$. We allow the reads to read different values from different writes as long as the feasibility is preserved. Comparatively, we enforce a weaker schedule order $e_4 \rightarrow e_6$ and $e_2 \rightarrow e_3$, which allows more scheduling, e.g., the concurrent execution of e_7 (or some event after e_7) and e_5 .

However, our technique does not identify the maximal set of traces from a single trace. The underlying reason is that, we still enforce the schedule order for preserving the base object, e.g., $e_2 \rightarrow e_3$. It is possible to further relax the schedule so that e_3 reads from e_1 , but this requires challenging reasoning of the field accesses as their base objects become unknown. We plan to explore this idea in future work.

VI. RELATED WORK

In this section, we survey related research on race detection with special emphasis on predictive trace analysis (PTA).

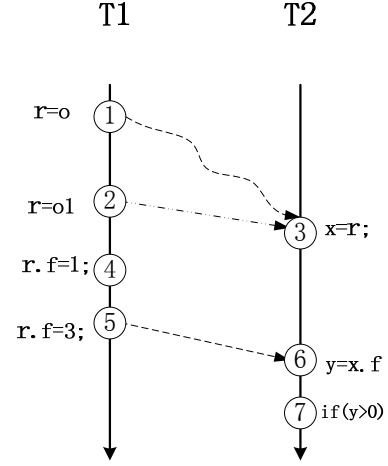


Fig. 4. Illustration of Detection Capability.

A. Race Detection Techniques: Broad Survey

Initial attempts to address the challenge of race detection focused on built-in synchronization primitives [?], [?]. These include locks as well as the wait/notify and start/join scheduling controls. Notable among these efforts is *lockset* analysis, which considers only locks [?]. Because the derived constraints are partial, permitting certain infeasible event reorderings, lockset analysis cannot guarantee soundness [?].

A different tradeoff is struck by the *happens-before* (HB) approach [5]. In this style of analysis, all synchronization primitives are accounted for, though reordering constraints are conservative. As an example, HB inhibits reordering of two synchronized blocks governed by the same lock.

Recently there have been successful attempts to relax HB constraints. Among these are hybrid analysis [?], which permits both orderings of lock-synchronized blocks, as well as the *Universal Causal Graph* (UCG) representation [4], which also enables both orderings but only if these are consistent with wait/notify- and start/join-induced constraints.

B. Predictive Trace Analysis

Given concurrent execution trace t , a *maximal and sound causal model* based on t defines the set of all traces that a program that is able to generate t can generate (maximality), and only those traces (soundness). PTA is founded on the notion of sound causality, as it considers feasible reorderings of the input trace that prove a candidate data race as such.

The first to propose PTA are Smagardakis et al. [?]. In their original work, both synchronization constraints and inter-

thread dependencies are preserved, where inter-thread dependencies are respected by only allowing reorderings that leave the dependence structure exhibited by the original trace intact. This ensures that the values of shared memory locations remain the same, which secures the soundness argument, though maximality is not guaranteed.

Said et al. [?] describe a PTA that is also sound albeit not maximal. Similarly to our analysis, Said et al. perform symbolic analysis of the input trace, and then utilize an SMT solver to search for interleaved schedules that establish the presence of data races. Soundness is guaranteed by their ability to precisely encode the semantics of sequential consistency. ExceptionNULL [?] is another example of a sound PTA without maximality guarantees, where the goal is to detect null dereferences rather than data races.

Serbanuta et al. [?] have recently shown that it is possible to build a model that is not only sound but also maximal: Any extension of the model with a new trace renders the model unsound. This provides a foundation for different forms of PTA, including data races, but also atomicity, serializability and other properties.

Inspired by this result and closer to RECIPE is the PTA technique of Huang et al. [?], which is both sound and maximal. This technique, too, makes use of an SMT solver based on symbolic encoding of the input trace. As part of the encoding, control-flow information is taken into account to enable reorderings that do not violate control dependencies. Still, the two relaxations that RECIPE features, which enable (i) value- rather than dependence-based reasoning about data flow and (ii) consideration of unexplored branches, are strictly outside the scope of Huang et al.’s technique. In Section VII, we demonstrate via direct comparison with Huang et al. the dramatic improvement in coverage thanks to these relaxation methods, which we prove to handle in a sound and maximal manner.

VII. EVALUATION

Our evaluation focuses on the effectiveness and scalability of our approach. To measure the effectiveness, we compare with the predictive analysis, RV [?]. We chose RV for two main reasons: (1) RV is the only open source predictive analysis, (2) RV represents the state-of-the-art approach, which is theoretically proven to have higher detection capability than other approaches.

Evaluation Method We conduct our experiment on a large set of applications, which are also used to evaluate RV. Specifically, the set includes large applications such as Jigsaw, Xalan, Lusearch. To handle large applications that make use of reflection, we applied Tamiflex [?], which replaces reflective calls with the concrete method invocations recorded in the observed run for the purpose of effective static analysis. We omit the benchmark eclipse because the current version of Tamiflex leads to abnormal execution after the instrumentation, which throws an exception when the main service is started. By applying our tool to such abnormal execution, we identify only 3 races, similarly to the report of RV [?].

In addition, the montecarlo benchmark requires an external input that we downloaded from the internet and simplified. For the large applications, we utilized the most lightweight available configuration.

As the detection capability depends on the observed run, for fair comparison we monitor the execution once by recording all necessary information required by both approaches, and then apply both techniques to the monitored run. Besides, our reported data for RV may be different from the original report for two reasons: (1) the different observed runs lead to different sets of races, (2) the original implementation of RV contains a bug in identification of the branches, which leads to misses of many branches and incorrect reduction of dependences during the analysis. We confirmed the bug with the author and fixed the bug in our experiments.²

Our experiments and measurements were all conducted on an x86 64 Thinkpad W530 workstation with eight 2.30GHz Intel Core i7-3610QM processors, 16GB of RAM and 6M caches. The workstation runs version 12.04 of the Ubuntu Linux distribution, and has the Sun 64-Bit 1.6.0_26 Java virtual machine (JVM) installed³.

A. Effectiveness

Table III shows the main results of our analysis, which includes four sections: Trace (details about the trace), Races (detected races), Difference (comparison with RV) and Running time (the time taken by the analysis).

The Trace section includes the number of threads (*Th*), the number of shared reads (*Reads*) and shared writes (*Writes*), the subset of shared reads that read the base object references (*Base*), where the base object references are references used as the base/target in the following field reference or the method invocation, the number of branches (*Br*), the synchronization events (*Sync*) and the total number of events (*Total*), which includes local accesses in addition to the aforementioned events. Specifically, for the branch, we report it in the form of A/B , where A refers to the number of branches used by our analysis and B refers to the number of branches used by RV. To ensure that the predicted run sees the same base object at each shared read/write, RV inserts the artificial branch immediately in front of each shared access (and array accesses). We do not use such artificial branches.

We make interesting observations about the Trace section. The non-local events, i.e., all the events listed in Table III, occupy around 30% of the total trace in the first 7 benchmarks, but occupy less than 1% of the trace in the rest benchmarks, which have relatively more complex logic. The reads of the shared base objects occupy a small portion (1/3-1/10) of the total shared reads. The rest shared reads read only the primitive values or the references that are not the base references, e.g., the references involved only in the nullness check. Besides, our analysis involves significantly less branch events as compared

²We have created an in-depth report on the bug, along with a witness test case. (See: <https://sites.google.com/site/recipe3141/>.)

³Note that Sun JDK 1.7 does not support the transformation of dacapo applications with tamiflex.

to the RV approach. The difference plays an important role in the detection, which we will explain shortly.

The *Races* section shows the number of races detected by Recipe-s, i.e., Recipe without exploring un-executed paths, Recipe, i.e., the fully-fledged version, and RV. By comparing the Recipe-s and Recipe, we find Recipe finds 100+ more races, which demonstrates the strength of exploring un-executed paths. Intuitively, Recipe-s predicts based on a single trace, while Recipe predicts based on multiple traces containing different execution paths. We also compare the Recipe version with RV version, as illustrated in the *Difference* section, where *Diff* shows the races found by Recipe but missed by RV, *Diff'* shows the races found by RV but missed by Recipe.

First of all, Recipe finds many 150+ more races than RV. The reasons are multifold: (1) Recipe can reason about the accesses in the un-executed paths, while RV and Recipe-s can only reason about the accesses in the executed paths. (2) Recipe or Recipe-s allows the relaxation of the scheduling even if it breaks the inter-thread read/write dependence in the observed run. Recipe allows the majority of the shared reads, i.e., the reads of primitive values, to freely read from a different value from a different write as long as the value leads to the same branch decisions. RV, however, requires them to read the same values as in the observed run. (3) An critical optimization proposed by RV is to preserve dependences only for the reads before the preceding branches of the racy events, rather than all the reads. However, this optimization is underplayed by the fact that RV introduces huge amount of artificial branches, i.e., one before each shared field access, which ensures the use of the same base objects. We get rid of such artificial branches and instead, rely on the small amount of base read events to ensure the use of the same base objects (Section ??). Our strategy reduces the number of branches greatly and amplifies the effectiveness of the optimization. We also conduct case studies (Section ??) to better illustrate the scenarios.

Another interesting observation is, although Recipe should produce all races found by RV in theory, Recipe may miss races found by RV in practice (Column *Diff'*), i.e., Recipe is not strictly more effective than RV in practice. The underlying reason is due to the limits of the constraint solver: (1) the solver cannot compute constraints with very complex arithmetic operations (2) the solver does not support some program constants such as the scientific notation, $3E - 10$.

The last section, Running time, compares the analysis time for both approaches. We find our approach is significantly slower than RV, e.g., RV often finishes within 200 seconds, while our approach may take more than 1 hour. This is because our approach needs to reason about the computation among variables inside the local access events, while RV needs to only reason about the order relations among the events.

Summary of advantage and weakness In general, our approach is flexible, which allows the relaxation of schedules and paths based on the fine-grained reasoning of value flows. As a result, it detects many more races compared to existing

approaches. On the other hand, it is heavy weighted. The suggestion is to combine it with the lightweight approach with soundness guarantee such as RV, by treating RV as the preprocessing and instructing Recipe to skip those confirmed by RV. In this way, we can complement Recipe by finding those missed by it due to the limit of the constraint solver.

B. Case studies

We manually inspect the reported races in small applications to gain better understanding about our approach.

Relaxing the Inter-thread dependence Figure 5 demonstrates the relaxation of inter-thread dependence enabled by our approach. The code is from the benchmark *bbuffer*, where the line number is marked. Huang et al. [] detects the race between line 291 and line 400, but fails to detect the race between line 294 and line 400. The reason is as follows. In the observation run, the execution follows the order, lines 400, 291 and 294. For the event at line 294, its preceding branch at line 291 reads from line 400. Therefore, Huang et al. [] requires the predicted run to preserve the dependence between line 291 and line 400 so that line 291 reads exactly the same value and the branch takes the same branch decision. The dependence enforces the order constraint, $400 \rightarrow 291$, which further enforces the order $400 \rightarrow 291 \rightarrow 294$. Our approach does not require the existence of such dependence. Specifically, we allow line 291 to happen before line 400 in the predicted run as long as the value read by it leads to the same branch decision, which is true in this case. As a result, there is no order constraint between line 294 and line 400, and the two forms a race. We re-replay such race easily using the eclipse IDE breakpoints.

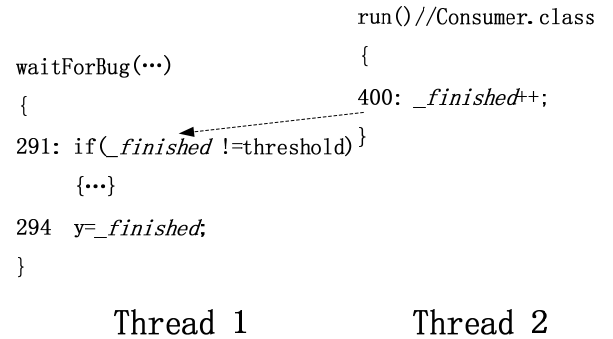


Fig. 5. Relaxation of Inter-thread dependence

Relaxing the Paths Figure 6 demonstrates how our approach relaxes the schedulings to account for the unexecuted paths. Each thread invokes the method *Sorting*, which recursively starts two children threads if there are two more available entries in the thread pool (lines 8-11), or starts one child thread if there is only one available entry (lines 4-5). The availability is computed through the static method *available* with the constant *total*, which is equal to 5. The shared variable *alive* denotes the used entries.

Initially there is one sorting thread. After it starts Thread 1 and Thread 2, there are three threads alive and only two

TABLE III
RELAXED ANALYSIS

Benchmarks	Trace							Races			Difference		Running time (sec)	
	Th	Reads	Writes	Base	Br	sync	Total	Recipe-s	Recipe	RV	Diff	Diff'	Recipe	RV
critical	3	13	7	5	2/13	6	78	8	8	8	0	0	8	2
airline	11	45	15	4	32/82	30	317	9	9	9	0	0	490	4
account	3	46	21	10	3/47	6	227	2	5	5	3	3	41	4
pingpong	4	7	7	3	0/15	6	111	1	1	1	0	0	19	1
bbuffer	4	640	118	10	634/1.1K	217	3.3K	13	25	9	21	5	62	5
bubblesort	26	1.3k	966	121	155/2.8K	322	8.4K	7	7	7	0	0	3295	3
bufwriter	5	165	52	75	16/130	44	525	4	10	2	8	0	63	9
mergesort	5	38	33	5	15/472	28	1.7K	3	10	3	10	0	37	5
raytracer	2	31	5	12	314/8.2K	676	94.5K	4	6	4	2	0	47	2
montecarlo	2	5	86	2	1.9K/38.2K	21.1K	1.9M	1	4	1	3	0	1	17
moldyn	2	605	61	104	19.6K/52.6K	62	203.4K	6	14	2	12	0	2842	1
ftpsrvr	28	684	299	71	4.4K/233.3K	78.2K	3.9M	99	152	57	108	13	811	153
jigsaw	12	525	702	211	63.2K/467.9K	86.7K	5.5M	17	23	8	15	0	33	7
sunflow	9	2.1K	1.3K	473	201.3K/827.0K	50K	7.1M	38	78	20	69	11	4520	22
xalan	9	1.4K	0.9K	209	15.7K/103.2K	190.1K	6.6M	2	6	2	4	0	5317	10
lusearch	10	2.3K	0.5K	715	22.2K/164K	93.2K	9.1M	27	49	14	38	5	5430	8

more entries are available. Suppose in the observation run, Thread 2 consumes both thread entries and starts the children Thread 3 and Thread 4 (not shown), updating `alive` to 5, then Thread 1 cannot execute the branch at lines 4-5. Huang et al. [] require the predicted run to preserve the dependence denoted by the dotted line since the dependence affects the branch condition at line 2. As a result, the predicted run follows the same branch decision and cannot reason about the code at lines 4-5. Our analysis does not have such limitation. Instead, it allows the predicted run to reason about the unexplored code. Specifically, it does not need to preserve the dependence from line 11 to line 1 and it allows line 1 (Thread 1) to read from line 9 (Thread 2). As a result, the branch condition guarding the unexplored branch is evaluated to be true, enabling the unexplored path in the constraint solver. Finally, the solver identifies the race between line 5 (Thread 1) and line 1 (Thread 3). Note that the two lines are synchronized on different locks ⁴.

VIII. VALIDITY THREAT

Java method can have 65535 bytecode instructions maximally. Therefore, we count the number of bytecode instructions inside each method, if the number exceeds 65525, we avoid instrumenting the method. The consequence is that, we will miss the races inside the method. In this case, we specify the variables read from the method to be equal to their concrete values in the constraints. The constraint solver can proceed safely without being affected by such methods.

We do not support the boolean operations such as `&`, bit operators `<<`, which contributes to most of our misses.

IX. CONCLUSION

The conclusion goes here.

ACKNOWLEDGMENT

The authors would like to thank...

⁴We abbreviate the `synchronized` keyword as `sync`.

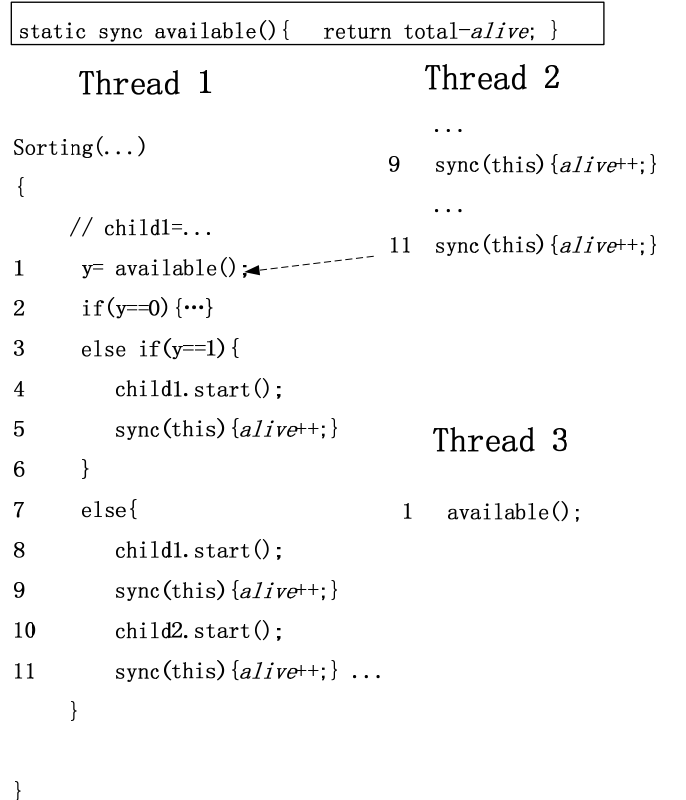


Fig. 6. Relaxation of Paths

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