

MOD 201 - Computational modeling workshop

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Final Report: Bayesian Modelling of Population of Neurons for Motion Detection Task

Introduction

This report is based on the groundbreaking study conducted by Mehrdad Jazayeri and J Anthony Movshon titled "Optimal representation of sensory information by neural populations." The study explores the modeling of population neurons and investigates the application of Bayesian inference using log likelihood in the context of motion detection. While motion detection serves as a practical example, the primary objective of the study is to delve into the complex dynamics of neural populations and harness the power of Bayesian modeling.

The research is centered on the encoding model of direction-tuned neurons in area MT, where neurons have self-similar tuning functions and pairwise correlations. The researchers hoped to decipher motion information from brain responses by better understanding the complexities of these tuning functions and correlations.

The use of the log likelihood ratio as a critical parameter for decoding is a significant feature of the work. Meaningful information about the underlying motion signals can be retrieved by evaluating the probability of observable brain responses in response to certain stimuli. Based on the aggregate activity of the neuronal population, this Bayesian framework may determine the most likely motion direction.

Throughout the report, the fundamental concepts of population coding are explored, shedding light on the interplay between individual neuron responses and their collective representation. The effects of tuning width, preferred angles on population dynamics and information decoding are examined.

1.General case: computing log likelihood

Humans have the capacity to optimally integrate likelihoods when assessing a stimulus from several cues rather than relying simply on distinct estimates generated from individual signals. This ideal combination accords with Bayesian theories, which advocate combining sensory input with previous ideas about the stimuli. To do this, the brain must use the likelihood function, which reflects the likelihood of a neuron firing a specific amount of spikes in response to a stimulus.

The researchers demonstrated that their model accurately predicts performance in various perceptual tasks involving motion judgments. Moreover, they utilized the model to make testable

predictions about how individual sensory neurons contribute to perceptual judgments in these tasks.

The log likelihood function for a sensory parameter θ , encoded by the activity of N neurons, is derived by summing the logarithm of the tuning function ($\Phi_i(\theta, S)$) multiplied by the number of spikes fired (n_i) across all neurons. The researchers assumed statistical independence among the neurons.

In situations where the stimulus strength only scales the tuning curves of the neurons without altering their shape, the tuning function $\Phi_i(\theta, S)$ can be expressed as the product of two functions: $f_i(\theta)$ and $g(S)$. Consequently, the log likelihood function (after removing terms independent of θ) simplifies to a summation of n_i multiplied by the logarithm of the tuning function for each neuron.

The technique of computing the log likelihood function in a feedforward network is depicted in Figure 1. The input stimulus is represented at the bottom of the picture by a particular sequence of spikes in the sensory representation of neurons (n_1, n_2, \dots, n_N). Each neuron's contribution to the log likelihood function is calculated by multiplying its response by the logarithm of its own tuning curve, denoted as $\log[f_i]$.

The graph depicts the contributions of individual neurons to the log likelihood function for two different stimulus values, shown in orange and green. The total log likelihood function, $\log L(y)$, is derived by summing the contributions of all neurons. The log likelihood function indicates the likelihood of various stimulus values (y) causing the observed pattern of neural responses.

The orange point at the peak of the log probability function in Figure 1 shows the most likely stimulus based on the given pattern of cell responses. This explains how to utilize the log likelihood function to predict the most likely stimulus value from neuronal population activity.

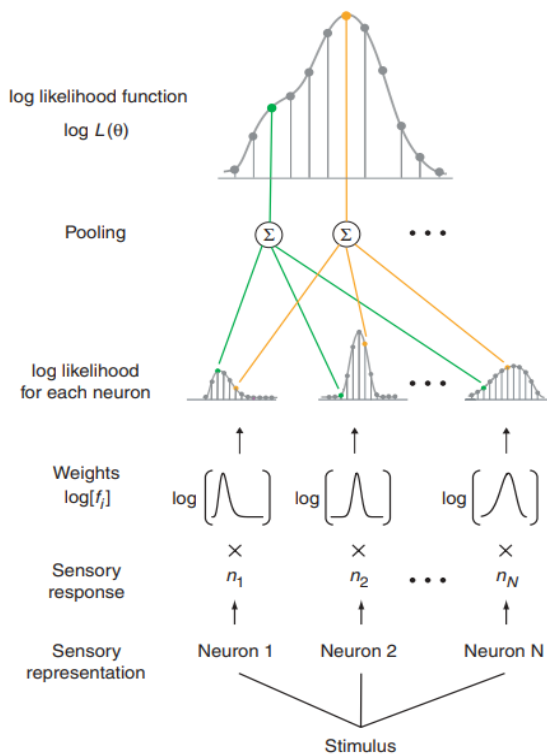


Figure 1: This figure is retrieved from original article of Jazayeri & Movshon(2006). This figure demonstrates using a feedforward network to compute the log likelihood function. A stimulus at its input (bottom) triggers $n_1, n_2 \dots n_N$ spikes in the sensory representation. The contribution of each neuron to the log likelihood function is given by multiplying its response by the logarithm of its own tuning curve, $\log[f_i]$. Adding the contributions of each neurons (shown in orange and green for two sample stimulus values) yields the total log probability function, $\log L(y)$, for all y values that might have evoked this pattern of responses. The orange point at the peak of the log likelihood curve denotes the most likely stimulus in this case.

2. Example case: decoding motion from MT responses

In the given example case, the encoding model involves a population of direction-tuned neurons in area MT. These neurons have uniformly distributed preferred directions, θ_i , and their tuning functions, $f_i(\theta)$, follow a circular Gaussian (von Mises) profile with a concentration parameter κ . The mean firing rate of the neurons represents their response to random-dot stimuli, where the coherence of the motion (C) measures the strength of the signal. The firing rate, represented by n_i , follows Poisson statistics and depends on the coherence, tuning of the cell, $f_i(\theta)$, and stimulation time, t .

Pairwise correlations exist among the neurons in the model, with a correlation coefficient ρ_{ij} . Neurons that favor the same direction have the highest correlation (max), while the strength of pairwise correlations reduces as the neurons' preferred directions diverge. The profile of pairwise correlations takes the shape of a circular Gaussian with a concentration parameter.

The equations provided in the code are used to calculate the log likelihood ratio of the firing rates. The weight of each cell, represented as n_i or $CellWeight$, is determined by the cosine of the difference between the stimulus angle (θ) and the preferred angle of the neuron (θ_i). The tuning function, $f_i(\theta)$, is computed using the preferred angle, stimulus angle, and tuning width (κ). Finally, the log likelihood ratio, $\log(L_\theta)$, is calculated as the sum of the product of the cell weight and the cosine of the difference between the stimulus angle and the preferred angle for each neuron in the population.

Weights

$$n_i = CellWeight = \cos(\theta - \theta_i)$$

Tuning of Cell

$$f_i(\theta) = \exp[K (\cos(\theta - \theta_i) - 1)]$$

Firing Rates Log Likelihood Ratio

$$\log(L_\theta) = \sum_{i=1}^N n_i \cdot \log f_i(\theta) = \sum_{i=1}^N n_i \cdot \cos(\theta - \theta_i)$$

Table 1: Equations for computing log-likelihood of neurons in response to moving stimuli.

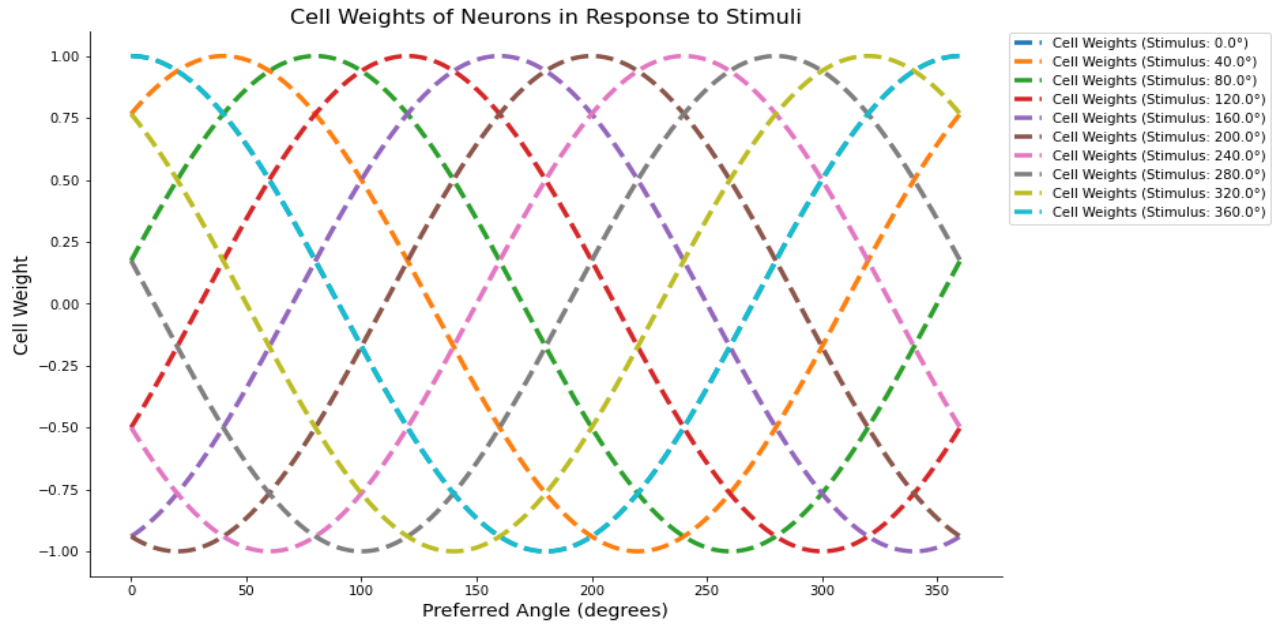


Figure 3: The cell weights of neurons in response to stimuli are plotted for 10 stimuli ranging from 0° to 360° . The cell weights of neurons are calculated using the cosine function with the preferred angle and concentration parameter (κ) as parameters.

So here with Figure 3 you can see the weights of 10 different cell and their response weights for their preferred stimuli (we can think this as their tuning curve). Similarly in figure 4 you can see the firing rates for example neuron and stimuli.

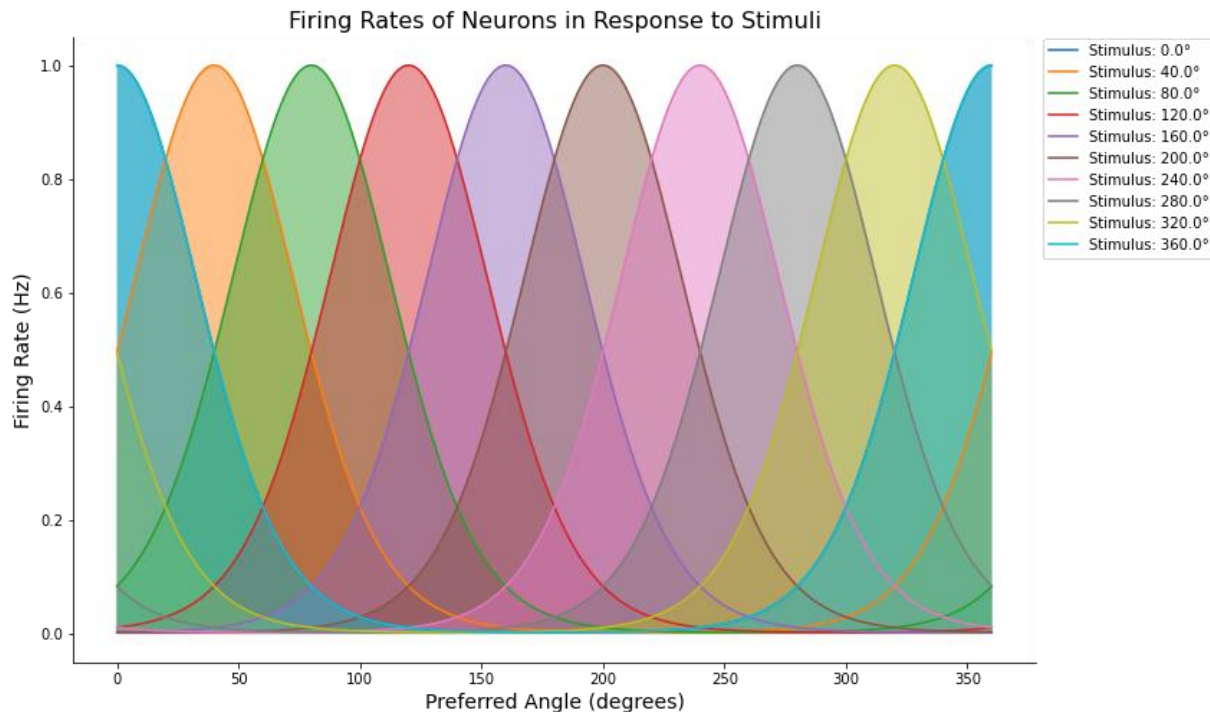


Figure 4: The firing rates of neurons in response to 10 different stimuli.

On the other hand, Figure 5 depicts the log likelihood of neurons in response to 110° stimulus is plotted. The peak of the average likelihood function—the expected maximum-likelihood estimate of the stimulus direction—is shown as orange dashed line.

The contribution of neurons to the likelihood function is dependent on their preferred direction. Neurons that are tuned to a specific direction contribute more effectively, while those with preferences further away make smaller contributions. Interestingly, neurons that prefer opposite directions are pooled with negative weights, resulting in a decrease in likelihood. This behavior resembles a generalized motion-opponent mechanism proposed previously.

Due to the unequal weighting, neurons that are optimally tuned to a particular direction and have a higher signal-to-noise ratio (SNR) exert greater influence compared to those with a lower SNR located farther away from the center of the neural pool. The overall likelihood function is represented by an array of output neurons. Each neuron computes the likelihood of a specific direction by pooling the responses from MT neurons using a cosine-weighting profile centered at that direction.

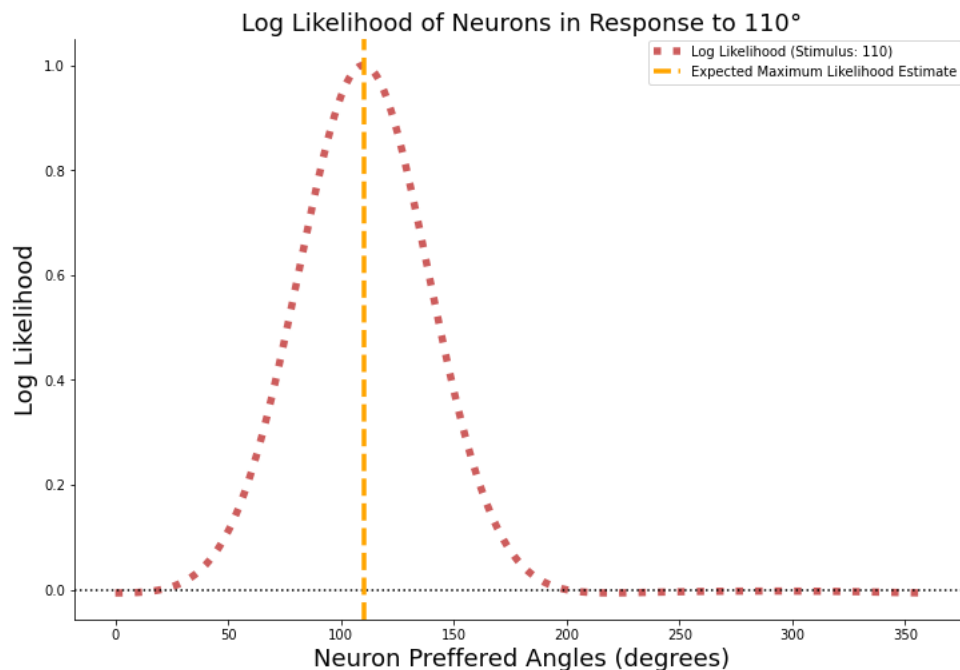


Figure 5: Overall log-likelihood of neurons in response to 110 degree motion stimuli. Orange line represents expected maximum likelihood estimation.

3. Detecting, identifying and discriminating motion

The responses of MT neurons are pooled in this model to produce the likelihood function, which has a direct impact on the model's performance in perceptual tasks. Let's look at how information flows through the model, concentrating on motion detection with a moving dot field as an example.

When the stimulus is presented, the firing rate of each MT neuron is determined by its tuning function and is subject to random Poisson variability. In a motion detection task, the observer is required to determine whether a known direction of motion is present in a field of randomly moving dots. The optimal strategy is to compare the likelihood of that specific motion to a predefined criterion. Signal detection theory, often measured by the receiver operating characteristic (ROC), is commonly used to differentiate the influence of the criterion choice from sensitivity (d' -prime).

We used this model to project the hit and false alarm rates across varied motion strengths to evaluate the model's performance in a yes-no motion detection test. We also looked at the equivalent d' -prime values, which show the sensitivity to motion detection.

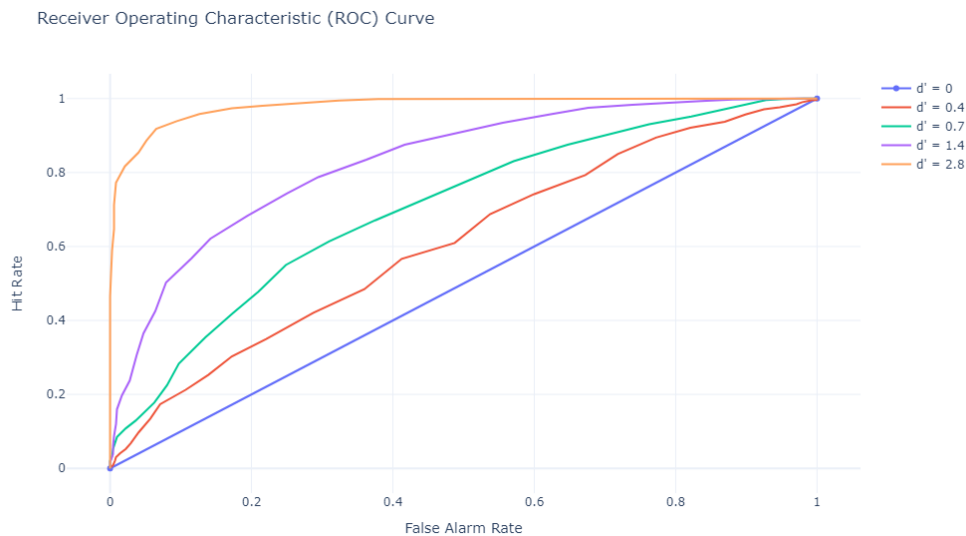


Figure 6: *Model predictions for behavioral performance in psychophysical experiments. A motion-detection task's receiver operating characteristic (ROC). The detectability (d' -prime) of the stimulus and the area under the ROC curve (that is, the % right) rise with the intensity of the motion signal (coherence). High (low) criteria values correspond to the lower-left (upper-right) portion of each curve, where the hit and false alarm rates are both low (high).*

These findings (Fig. 6) provide concrete and verifiable predictions regarding the performance of monkeys in various standard psychophysical tasks, leveraging our understanding of MT cells' characteristics and the implementation of our likelihood model. It is reasonable to extend these predictions to human observers, and we are currently conducting meticulous psychophysical experiments to investigate these predictions further.

4. Contribution of different stimuli to overall MT discrimination task

When examining cases requiring discriminating between more than two choices, the importance of having access to the entire probability function becomes clear. The brain must calculate and evaluate many sensory likelihoods in order to perform optimal discrimination of multiple choices. Consider the following scenario: a subject is entrusted with judging between N known alternate motion directions, the number and directions of which fluctuate from trial to trial. For best discrimination, the subject must calculate and compare a separate set of N sensory likelihoods in each trial. Having access to the complete probability function allows for greater flexibility. According to our model, the coherence threshold will rise as the number of possibilities increases.

In the study they used a simplified population of MT neurons to show how this model may explain the involvement of individual neurons with different direction preferences in motion detection and classification tasks. In particular, the log probability associated with the existence of a certain motion direction is compared to a predetermined criterion in a detection job.

Consequently, the activity of MT neurons that are tuned to the anticipated direction of motion is expected to exhibit a positive correlation with the subject's responses. On the other hand, the influence of neurons tuned to different directions diminishes in a cosinusoidal manner, reaching zero for neurons tuned to directions perpendicular to the expected direction, and reversing sign for neurons with preferences for the opposite direction.

The problem of decoding brain responses to distinguish between opposing directions of motion has received a lot of attention. Initially, it was thought of as an opponent process, in which discrimination between two directions might be accomplished by subtracting the activity of a neuron tuned to one way from the activity of its matching "anti-neuron" tuned to the opposite direction.

To illustrate this concept, they utilize following equation to calculate the log likelihood ratio for two alternatives, denoted as θ_1 and θ_2 . The log likelihood ratio is obtained by taking the difference between the log likelihoods of the two alternatives and can be expressed as:

$$\begin{aligned} \log LR &= \log L(\theta_1) - \log L(\theta_2) \\ &= \kappa \sum_{i=1}^N n_i [\cos(\theta_1 - \theta_i) - \cos(\theta_2 - \theta_i)] \end{aligned}$$

This formulation shows that the contribution of each neuron to the log likelihood ratio is determined by its activity n_i and its preferred direction θ_i relative to the two alternatives.

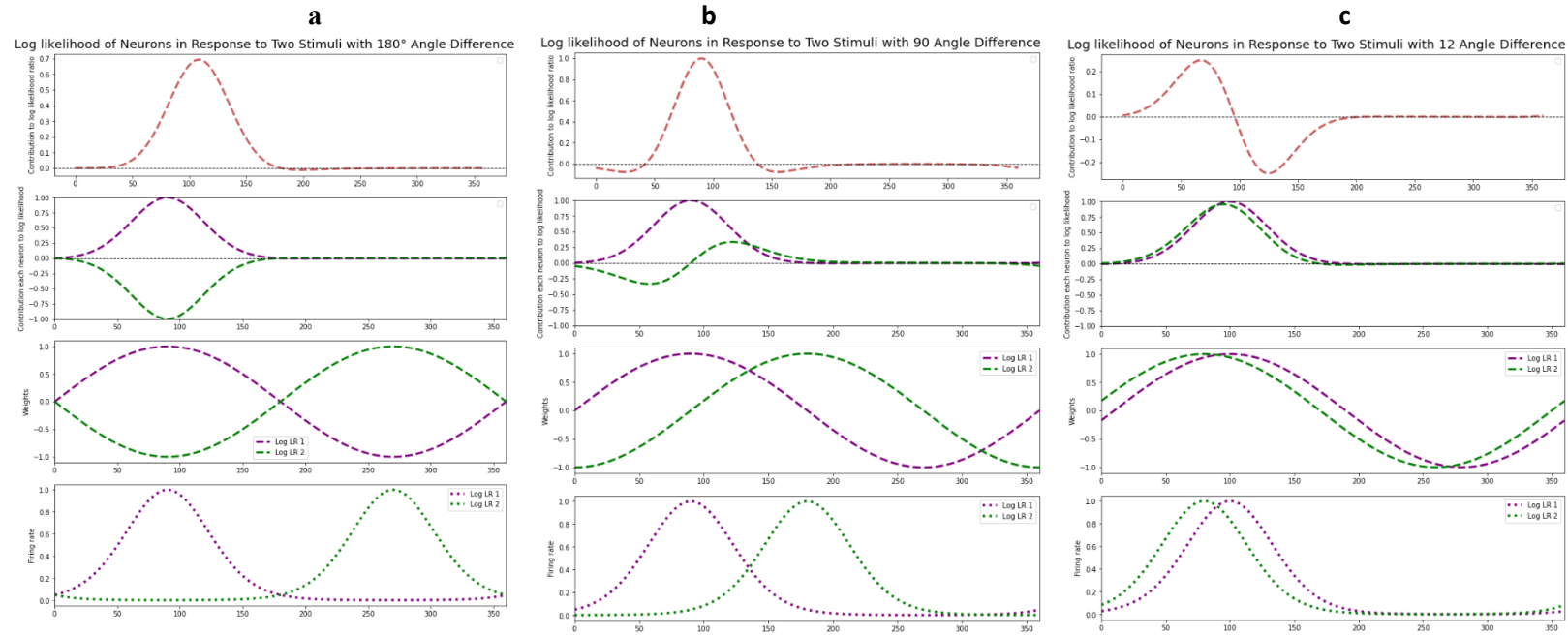


Figure 1 The examples presented demonstrate the role of MT signals in discriminating between two choices in motion. In the lower panels, the activity profiles of area MT neurons in response to a robust motion stimulus are displayed, focusing on one of the two directions being discriminated. The neuron most responsive to this stimulus is indicated by the dashed black line. The angular differences between the alternatives are 180°, 90°, and 12° in panels (a), (b), and (c), respectively.

The model's cosinusoidal weighting profiles are shown in the second panel from the bottom. The contribution of each neuron to the log likelihoods is calculated by multiplying its activity by its associated weight.

The average contribution of each neuron to the two log likelihoods is shown in the third row from the bottom, which is calculated by multiplying the neuron's average firing rate by its weight. The top panels show each neuron's average contribution to the log likelihood ratio. The contribution of each neuron to the two log likelihoods is subtracted in this computation, which is shown by the difference between the blue and red curves in the third row from the bottom.

For all three conditions, neurons with preferences located halfway between the two alternatives exhibit similar weights and, consequently, do not significantly contribute to the log likelihood ratio. In cases where finer discriminations are required, resulting in a smaller overall log likelihood ratio, the contribution becomes more dependent on neurons with preferences shifted away from the two alternatives. The overlapping weighting profiles play a crucial role in determining the log likelihood ratio's dependence on these specific neurons.

Discussion

This study's model provides vital insights into the brain processes underpinning motion detection and discrimination. The model explains how individual neurons with distinct direction preferences contribute to various perceptual tasks by concentrating on the activity of MT neurons.

The capacity of the model to compute the log likelihood function enables a thorough examination of sensory information and its influence on behavioral performance. This is especially significant when several options must be differentiated, as the model emphasizes the flexibility necessary to compute and compare sensory likelihoods for various inputs.

The model also includes a realistic neural computation for computing the probability function, bridging the gap between sensory neuron activity and behavioral responses. The model captures the covariation between neuronal activity and subject responses by taking into account the contributions of MT neurons with varied direction preferences.

The model's predictions match known features of MT cells, implying that it can be used by both monkeys and humans. These predictions apply to psychophysical activities and may be tested further by thorough measurements. Overall, this model adds to a better understanding of how neuronal activity in the MT contributes to motion detection and discrimination. It gives vital

Running head: Bayesian modelling of motion detection

insights into the brain computations behind perceptual tasks linked to motion by explaining the involvement of individual neurons and their contributions to probability functions.

