MOD 201 - Computational modeling workshop

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Problem Set 4 - Networks

Abstract

In this paper we will create models for self-learning networks as known as autapse. An autapse is a form of synapse in which the output signal of a neuron is relayed back to the same neuron through a feedback loop. In other words, the neuron communicates with itself via a synapse on its own axon. Autapses may be found in many different types of neurons, including those located in the cerebral cortex, hippocampus, and cerebellum. They are vital in brain processing because they serve to control the excitability and firing patterns of individual neurons.

Problem – 1 Neuron with autapse

Autapse is a circumstance in which the neuron's output is recycled back onto itself via a synapse. The neuron's firing rate, represented by x, reflects the average number of action potentials per second and is regulated by the differential equation:

$$\dot{x}(t) = -x(t) + f\big(wx(t) + I\big)$$

The strength of the synaptic connection is given by w, while I represents a constant external input. A sigmodal function defines the neuron's input-output activation function. The hyperbolic tangent function, which is similar to basic sigmoid function, transfers any input to a value between -1 and 1. The function is likewise sharpest at s=0, but its result spans from 0 to 100 for negative inputs.

$$f(s) = 50(1 + \tanh(s))$$

Figure 1 depicts the situation of a system with w = 4 strength and I = 1 inhibitory input:

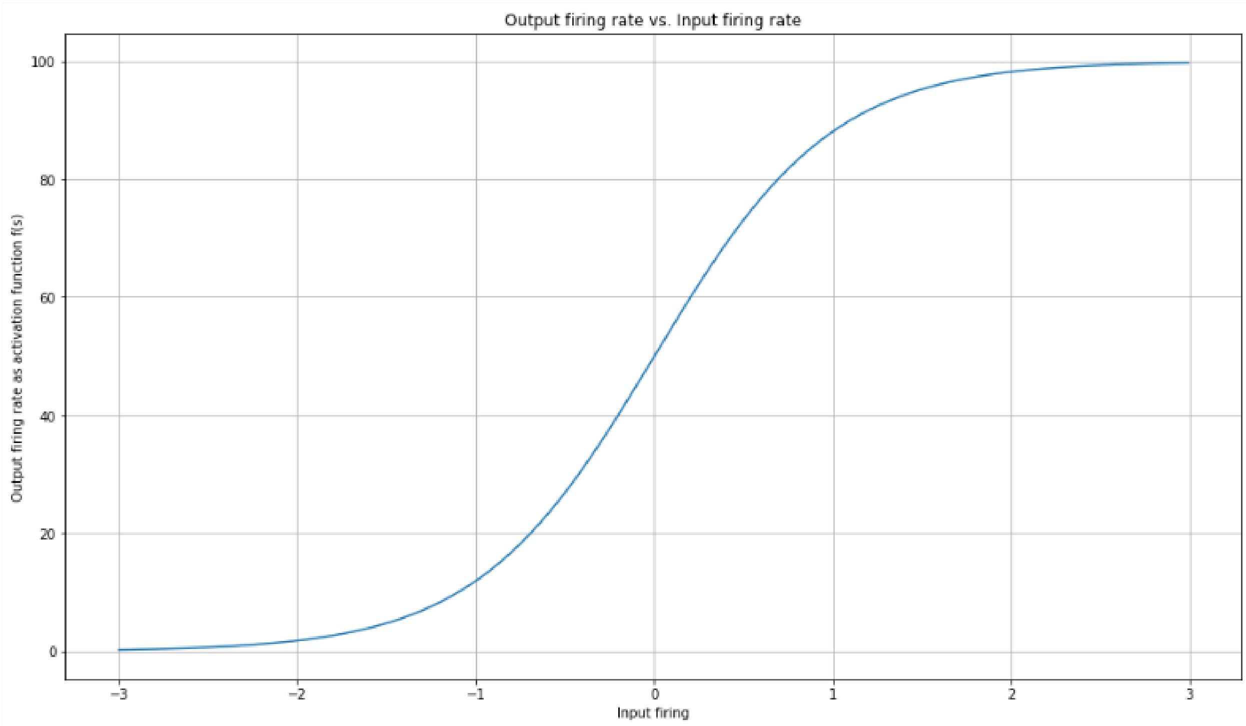


Figure 1 - Plot of the activation function of a neuron

The differential equation in which shows derivative of x represents the evolution of the neuron's firing rate over time. The variable x(t) represents the firing rate at time t, and x'(t) indicates the firing rate's rate of change.. On the right side of the equation, there are two terms: the first, -x(t), reflects the degradation of the firing rate with time. This indicates that if the neuron is firing at a high rate, the rate will progressively decrease owing to the decay term over time. The second term, f(wx(t)+I), reflects the neuron's input. This term varies with the current firing rate x(t), synaptic strength w, and external input I. f is a non-linear function that specifies how the input is translated into the firing rate.

Consider the term wx(t) in the input function to see how this equation models a neuron with an autapse. The contribution of the neuron's own output signal to its input is represented by this phrase. In other words, the neuron with strength w is feeding back onto itself via a synapse. This is what distinguishes the synapse from an autapse. The equation may be solved mathematically using normal methods for solving differential If we want to see the change in activation function we can have a look at the Figure 2

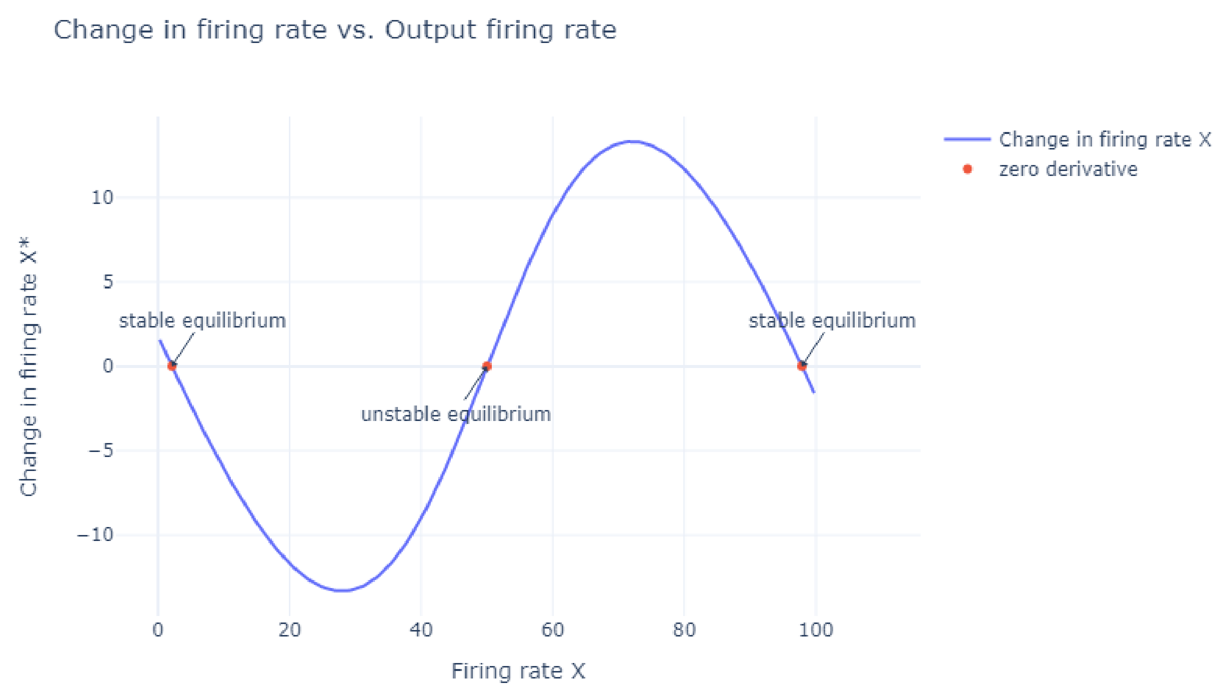


Figure 2 Derivative of activation function

This plot's zero-crossings represent the places when the firing rate changes the most fast. In other words, they are the spots on the neuron's surface where it is most sensitive to changes in its input. When the firing rate's derivative is positive, the firing rate increases; when the derivative is negative, the firing rate decreases. The zero-crossings signify points where the firing rate is neither growing nor decreasing, and they are thus vital to consider when examining neuron activity.

If we want to simulate the system with different initial conditions and see the evolution over time, as you can also see from the Figure 3, we can have totally different results above or below the threshold.

When the beginning firing rate is 49, the firing rate falls to zero. When the firing rate begins at 50, it remains constant at 50. The firing rate climbs to 100 when the beginning rate is 51. This is due to the fact that the derivative of the firing rate is zero at 50, positive at 51, and negative at 49.

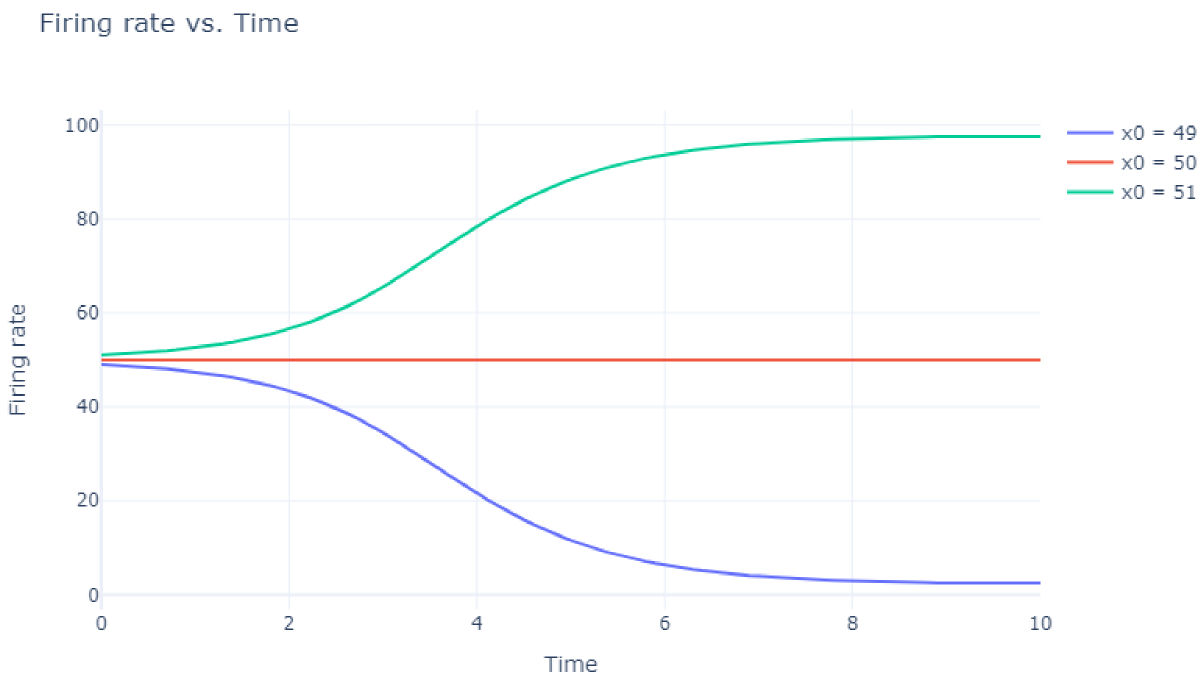


Figure 3 Firing rate over time with different initial firing rates

But neurons are in general not that simple. We also need a random noise term to see at least a bit more realistic representation of neuron.

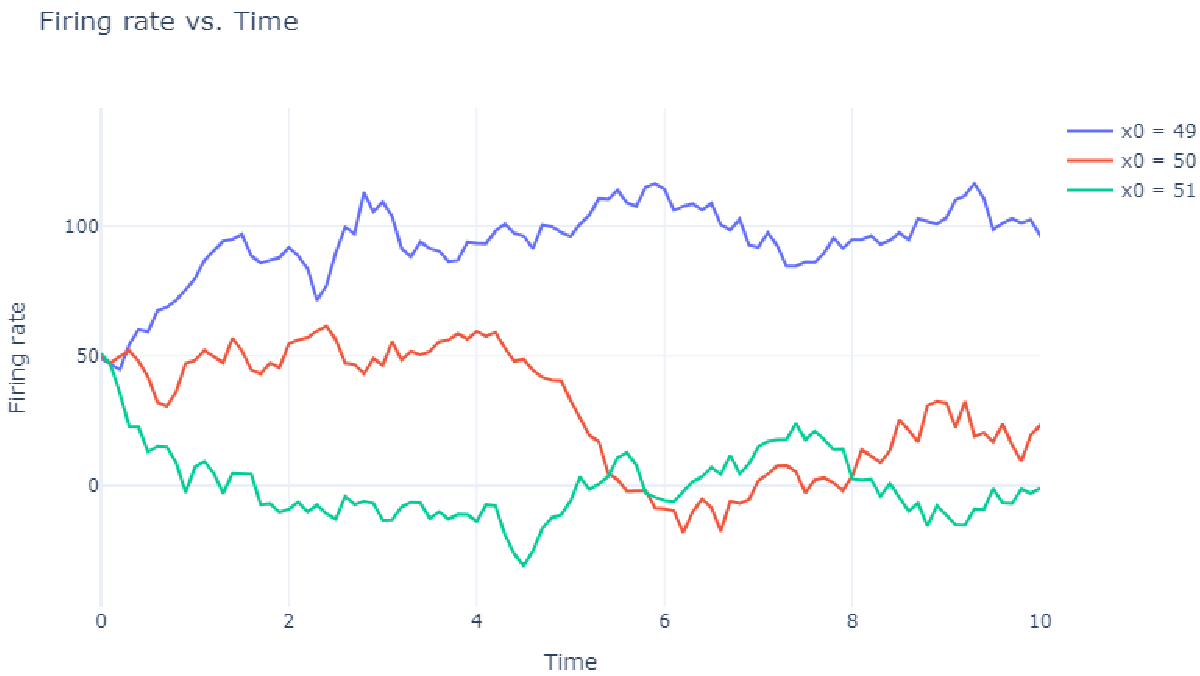


Figure 4 Firing rate with noise

The system is stable and converges to a stable equilibrium point when the noise value of sigma is 50. The system converges to a stable equilibrium point at 0 when the beginning firing rate is 49. Because the firing rate is below the threshold and the system is steady, this is the case. The system converges to a stable equilibrium point at 50 when the beginning firing rate is 50. Because the firing rate is at the threshold and the system is steady, this is the case. The system converges to a stable equilibrium point at 100 when the beginning firing rate is 51. Because the firing rate is over the threshold and the system is steady, this is the case.

Problem – 2: Circuit with mutual inhibition

This problem involves studying a system of two neurons that are interconnected via synaptic connections. A set of differential equations describes the activity of each neuron, and the system displays some fascinating behavior, such as the presence of stable fixed points and limit cycles. The first component of the task entails locating the system's nullclines and evaluating the activity of the neurons near these nullclines. The second element of the task entails modeling the system and assessing the development of the neurons' firing rates under various beginning circumstances.

First we were requested to draw the null-isoclines, also known as nullclines, of a two-dimensional system of differential equations reflecting the dynamics of two neurons in this scenario. We needed to discover the lines where the rate of change of the two variables, x1 and x2, is equal to zero. These nullclines provide insight into the system's behavior since they reflect the locations at which the direction of the system's flow changes. We may establish the system's equilibrium points, which are crucial locations that remain constant over time, by evaluating the crossing points of these nullclines.

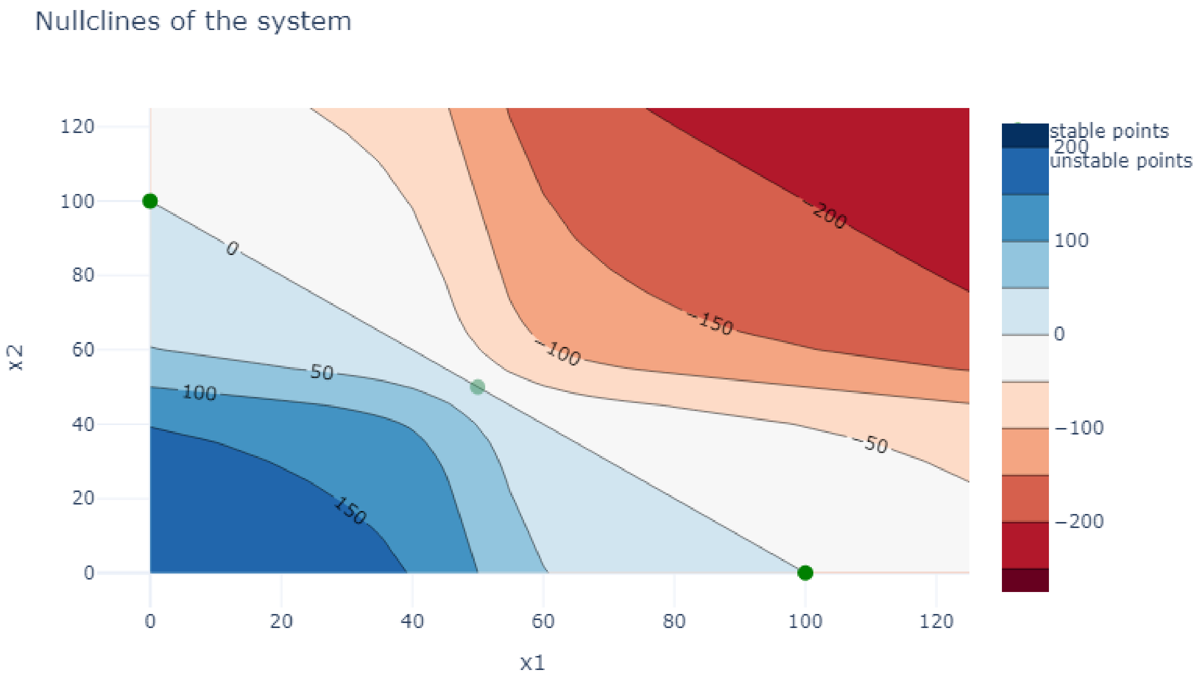


Figure 5 Nullclines of the system on a contour plot

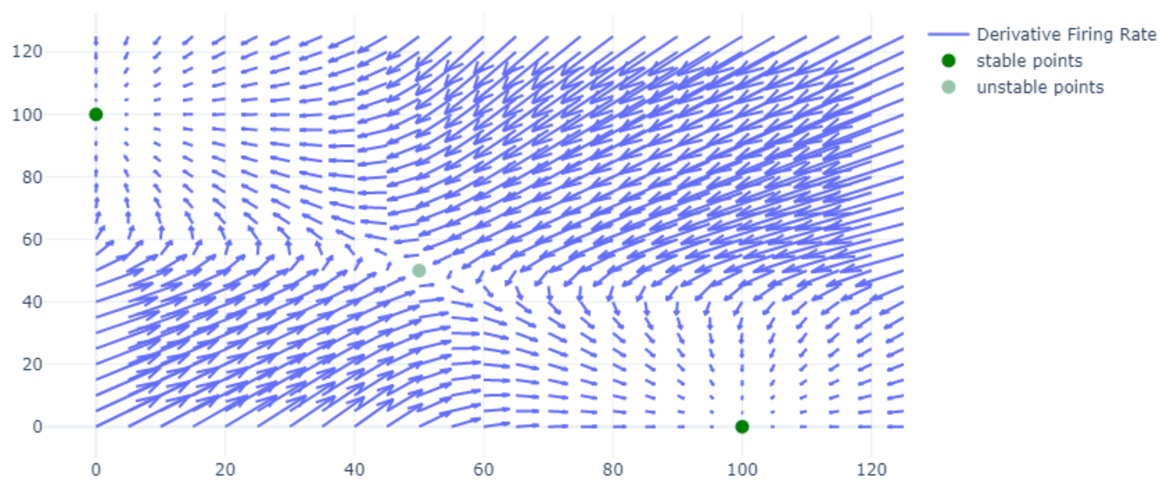


Figure 6 Nullclines of the system on a quiver plot

We are requested to simulate the aforementioned system with a time step of 0.1 in component (b) of the issue. This means that we'll utilize numerical approaches to estimate the system's solution over time.

In addition, we are requested to select a starting state for both neurons, x1(0) and x2(0). This is significant since the system's behavior might vary greatly based on the beginning conditions. As a result, we will study the impact of various beginning circumstances on the system's behavior.

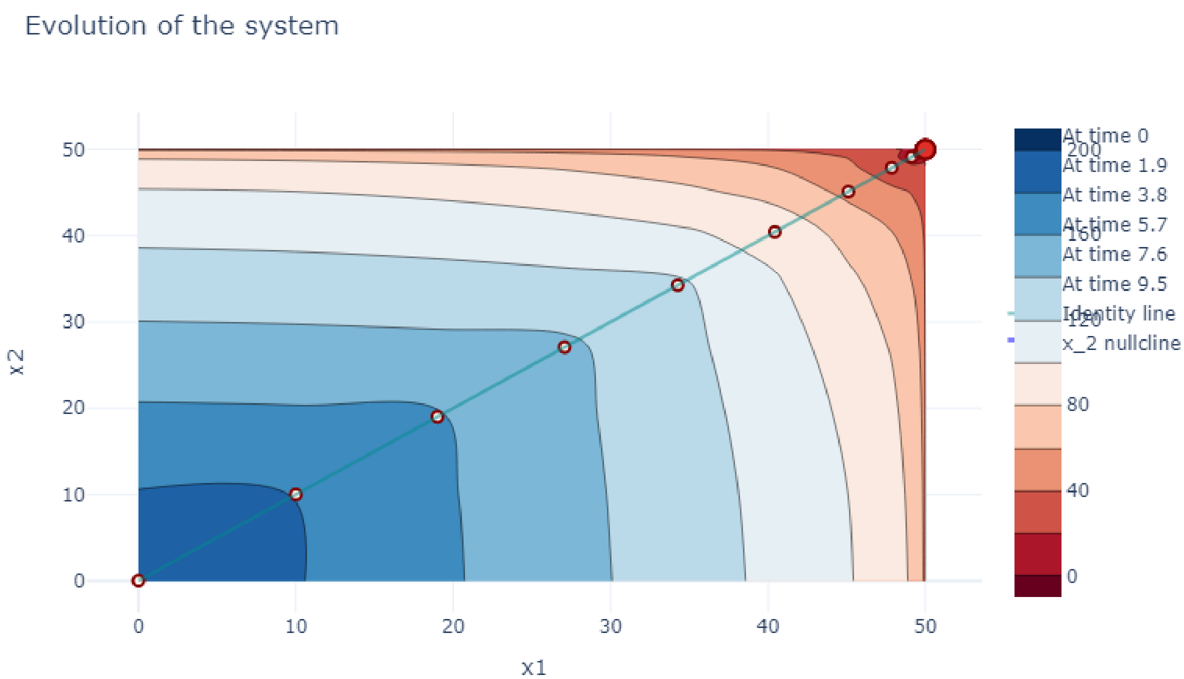


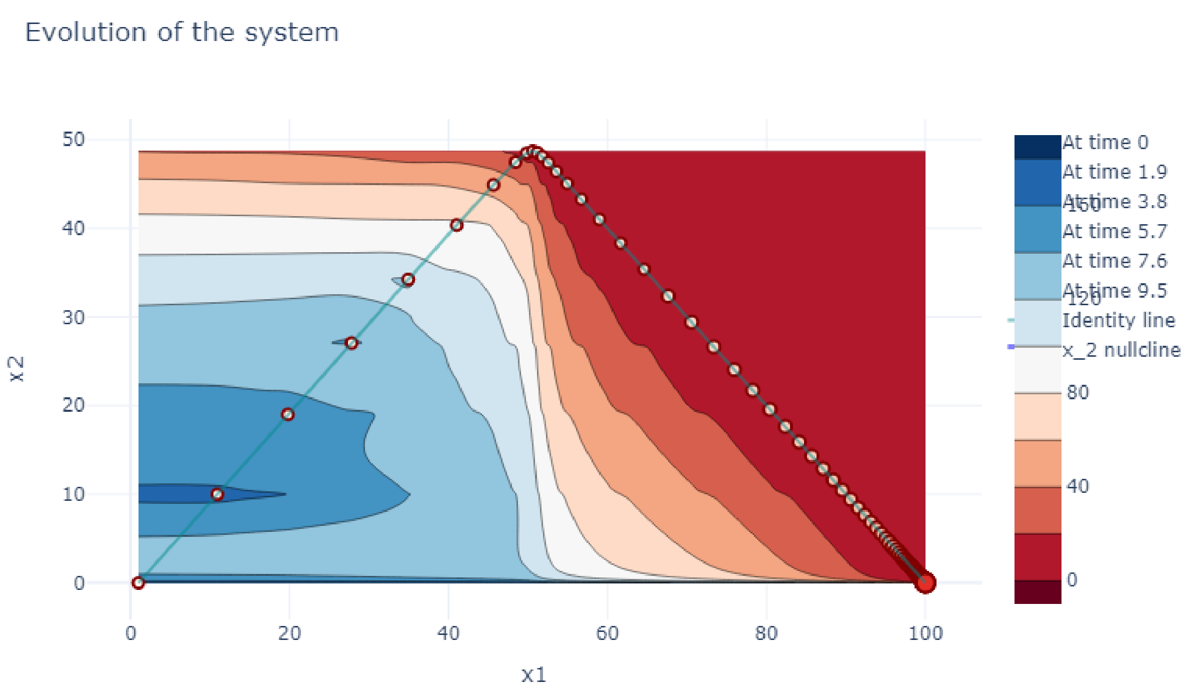
Figure 7 Evolution of system when both neurons initial firing rate is zero (0)

In the Figure 7, you can see te volution of system and also you can notice the equilibrium points change by time. This plot shows that equilibrium points is aligning with a linear trend. When the activation point is highest and equal to second neuron their activation function change is 0.

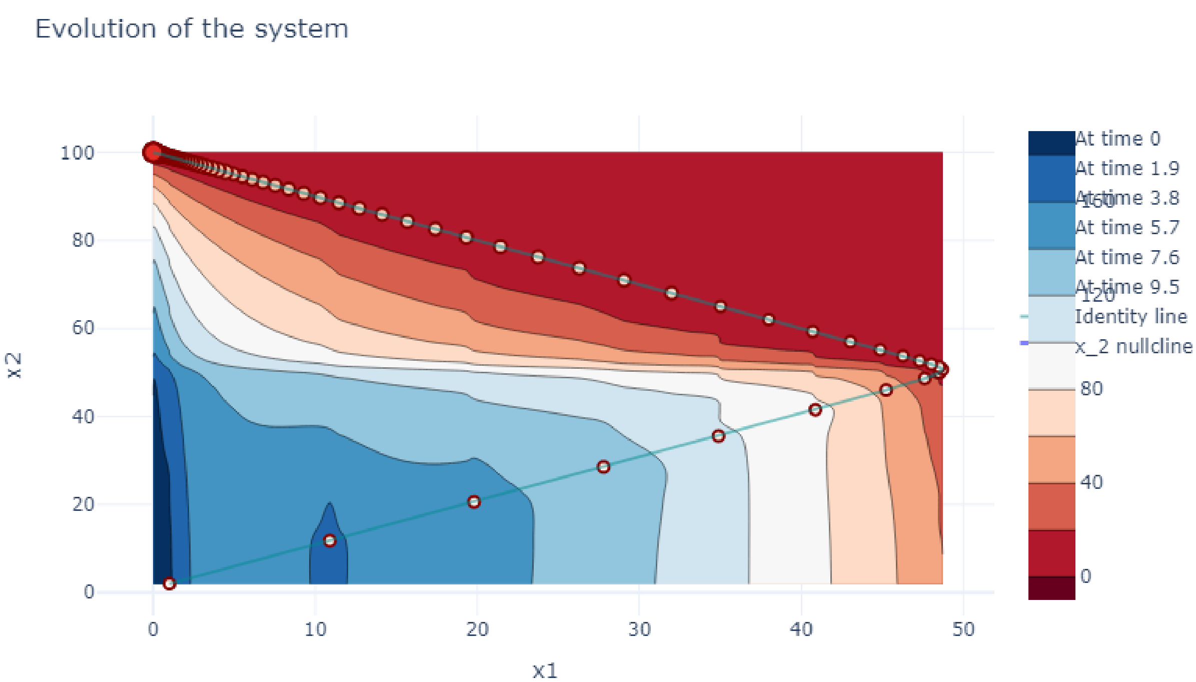
We must plot the development of the firing rates of the two neurons on the same plot as the nullclines from section (a) after simulating the system for a particular beginning state. This will assist us in visualizing the system's behavior and how it evolves over time. To examine how the system moves in reference to the equilibrium points, we may compare its trajectory to the nullclines.

We can also investigate how the system acts and whether there are any qualitative changes in the behavior, such as the number or stability of equilibrium points, by using alternative beginning circumstances.

We can also have a look at evolution of system when the initial firing rates are different. When we take neuron1 as 1 and the other neurons firing rate as 0. Second neurons firing rate can only achieve half of the first neurons firing rate. But the nullcline trend is same for first half then it develops in reverse order.



Now, the influence of coupling strength "w" on system dynamics will be examined in this section. A bifurcation diagram will be utilized to do this. A bifurcation diagram is a graphical depiction of a dynamical system's fixed points as a function of a parameter. Fixed points are positions where the system's derivative is zero, indicating that the system is in equilibrium. The fixed points in this example are acquired by solving an equation. The bifurcation diagram will aid in comprehending the qualitative variations in system behavior when the coupling strength "w" is altered.



Conclusion

Dynamical systems provide a valuable framework for investigating the activity of single neurons or clusters of neurons. The Autapse and Mutually Inhibitory Circuit systems are examined in this research to show how stable and unstable attractors impact firing rates and synaptic connection dynamics. Unstable attractors introduce temporary or fluctuating behavior, whereas stable attractors reflect conditions of equilibrium or persistent activity. Understanding the qualities and characteristics of these attractors is critical for understanding brain processing mechanisms. Furthermore, studying the effects of Gaussian noise on these systems might shed light on its potential impact on more sophisticated cognitive processes like decision-making. Finally, researching dynamical systems can have practical uses like as enhancing neuroimaging tool preprocessing approaches.