Modelling of Population Growth

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1 Introduction

Several mathematical models may be used to predict population increase; the most popular ones are limiting and exponential growth models.

Exponential growth models presuppose that population growth is unrestrained and that the rate of rise is proportionate to population size. For example, $\frac{dN}{dt} = r_N$, where N is the population size, **t is the passage of time**, and **r is the per capita growth rate**, may be used to mathematically represent the population growth rate. The growth rate in this model is independent of population size and assumes infinite resources and perfect conditions.

On the other hand, limiting growth models consider the carrying capacity of the environment, or the maximum number of people that the ecosystem can support. Logistic growth is described by the equation $\frac{dN}{dt} = r_N(1 - \frac{N}{K})$, where K is the carrying capacity. This model predicts that when the population size gets closer to the carrying capacity, population growth decreases. Given the limited resources and the competition for those resources, the logistic growth model depicts population expansion more accurately. In this exercise, we tried to model population growth based on these two approaches.

2 Model

2.1 Exponential model of following year

In this exercise, we attempted to model population growth using a simple population growth model. In this model, we assume that the population grows by a factor that corresponds to the rate of reproduction. Mathematically, we express this factor as a product of the current population size, denoted as p, and a constant factor α, which represents the rate of population growth per years. Thus, the rate of population growth can be expressed as $\alpha \cdot p$. Based on this model, we can predict the population size in the following year as a function of the current population size and the value of α. This can be done using a difference equation that relates the population size in the current year (p_{n-1}) to the population size in the following year (p_n). This difference equation is given by:

$$p_n = p_{n-1} + \alpha \cdot p_{n-1}$$

To demonstrate this, let's assume an initial population size of 2 ($p_0 = 2$) and a value of $\alpha = 0.1$. We can then use the difference equation to calculate the population size in the following year, denoted as p_1, as follows:

$$p_1 = p_0 + \alpha \cdot p_0 = 2 + 0.1 \cdot 2 = 2.2$$

2.2 Exponential model for over a long period of time

According to this model, we can expect the population size to increase from 2 to 2.2 individuals in the following year, assuming a constant value of α. When we create a loop out of this model, total population would exponentially increase as growth rate will also increase. This figure shows how the population grows

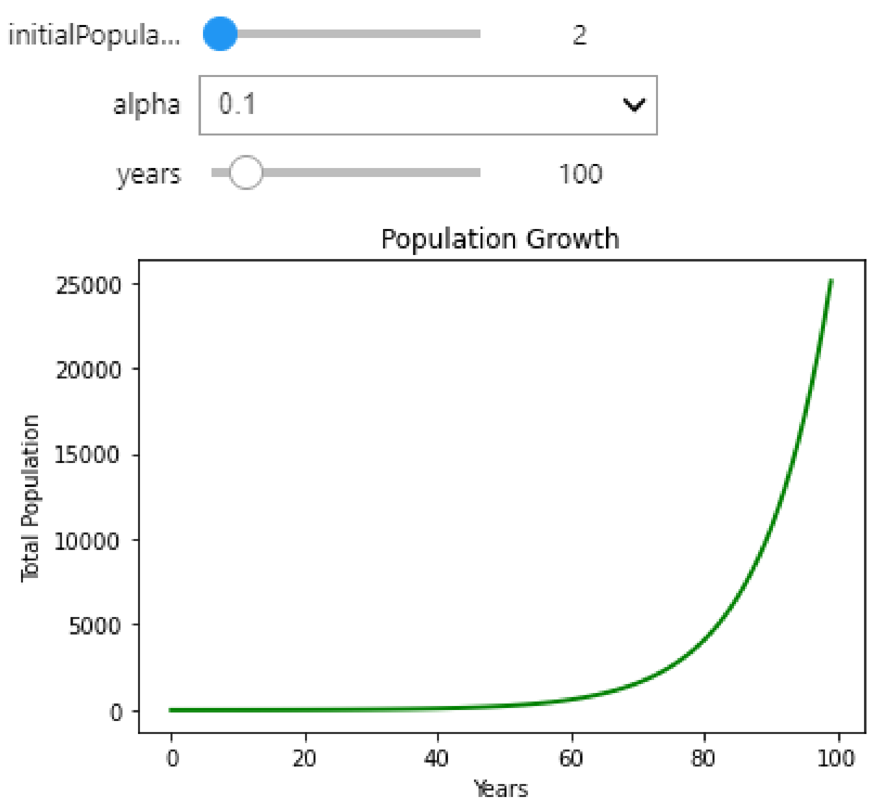

Figure 1: Population growth over 100 year. Using alpha parameter as 0.1 and initial population as 2

over time. We can see that initially, the growth rate is relatively slow, and the population size remains relatively constant for the first 70 years. However, after this point, the population begins to grow exponentially, increasing rapidly from 1.579 to 25.055 people in just 30 years.

By looking at the equation that describes population growth, we can predict this outcome. For instance, if we assume a constant growth rate of 10%, we can see that even a small increase in the population size would lead to a significant increase in the growth rate. This is because the rate of growth is proportional to the current population size, which means that as the population grows larger, the rate of growth also increases, leading to an exponential increase in the population size over time. Increasing the initial population number does not

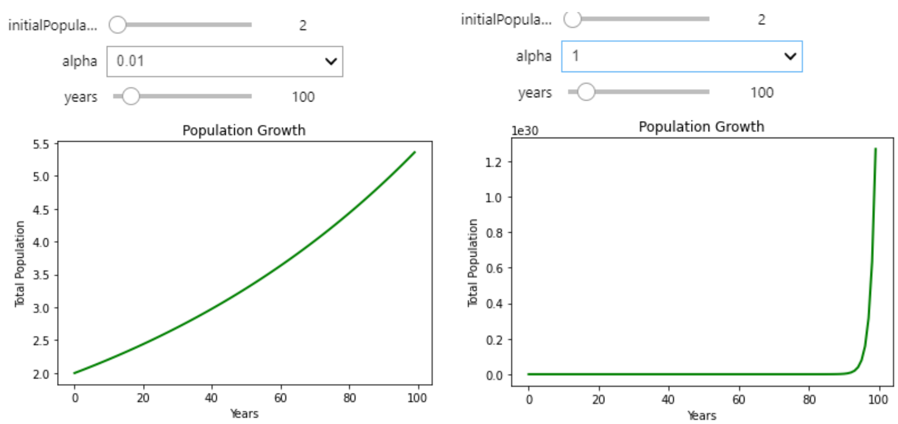

Figure 2: (a) Population growth over 100 year. Using alpha parameter as 0.01 and initial population as 2. (b) Population growth over 100 year. Using alpha parameter as 1 and initial population as 2.

change the slope of the plot, but it can significantly increase the total population after 100 years. On the other hand, decreasing the value of the growth rate coefficient α can cause a notable decrease in the total population. For instance, when α is set to 0.01, the total population after 100 years is only around 5.5. In contrast, if we increase α to 1, the total population grows much faster, and after 100 years, it would reach approximately 1.2×10^{30}.

If we need to have a look at the growth factor to interpret this data. We see from the figure below that the growth factor is linearly increasing between 0 to 500 people. The initial growth rate is increasing by function of total population as a result it becomes 50 when the total population is 500.

2.3 Population growth in limiting enviroment

On the other hand in real world populations growth rate doesnt just continously increase. Since there are other modulator factors in the enviroment which limit growth such as predator/prey interactions, resources, mating limitations and competition, populations are always in tend to limit number of individuals and

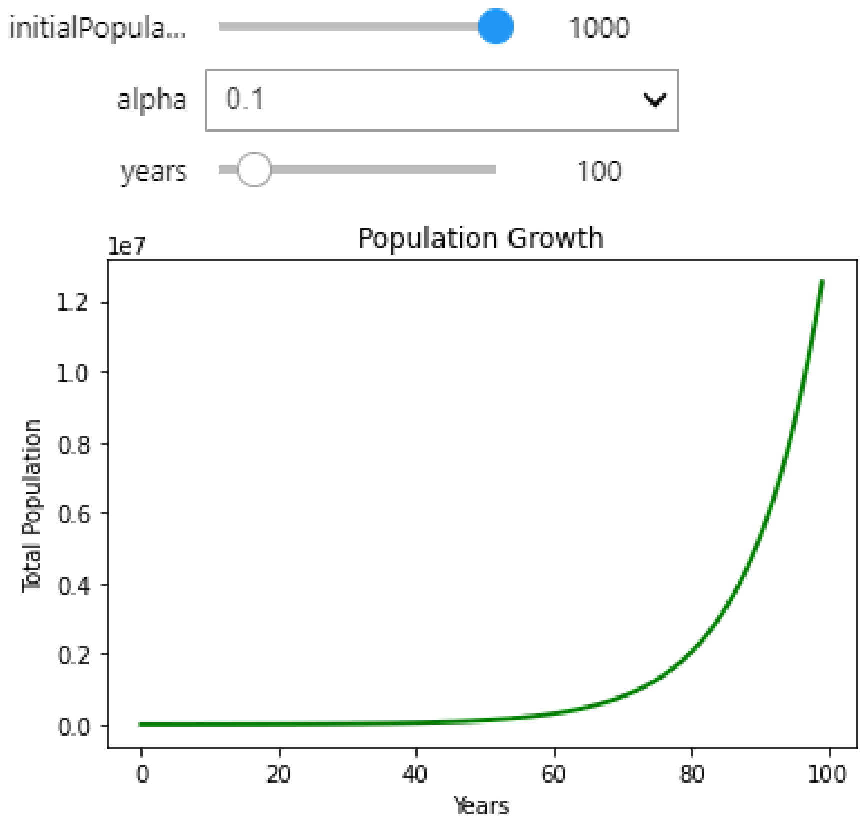

Figure 3: Population growth over 100 year. Using alpha parameter as 0.01 and initial population as 1000.

they in general cycle at some point. So for this we use a growth rate as follows Growth rate:

$$\delta_n = \beta p_{n-1} \left(200 - p_{n-1}\right)$$

If we plot the relation between growth rate and total population, with also considering the limiting factors it would first increase and then linearly decrease as shown in the Figure 5.

The entire population is constrained by this capacity when population growth is modelled with a constant carrying capacity (for example, 200). As a result, whenever this model is simulated, the population is always less than or equal to 200. The population first grows quickly, but with time, the growth rate slows and the population approaches a stable state. The population growth graph shows a high initial surge followed by a level area. A more accurate model for populations with scarce resources is logistic growth, which is this form of growth. The following equation may be used to explain the limiting model. Next year's

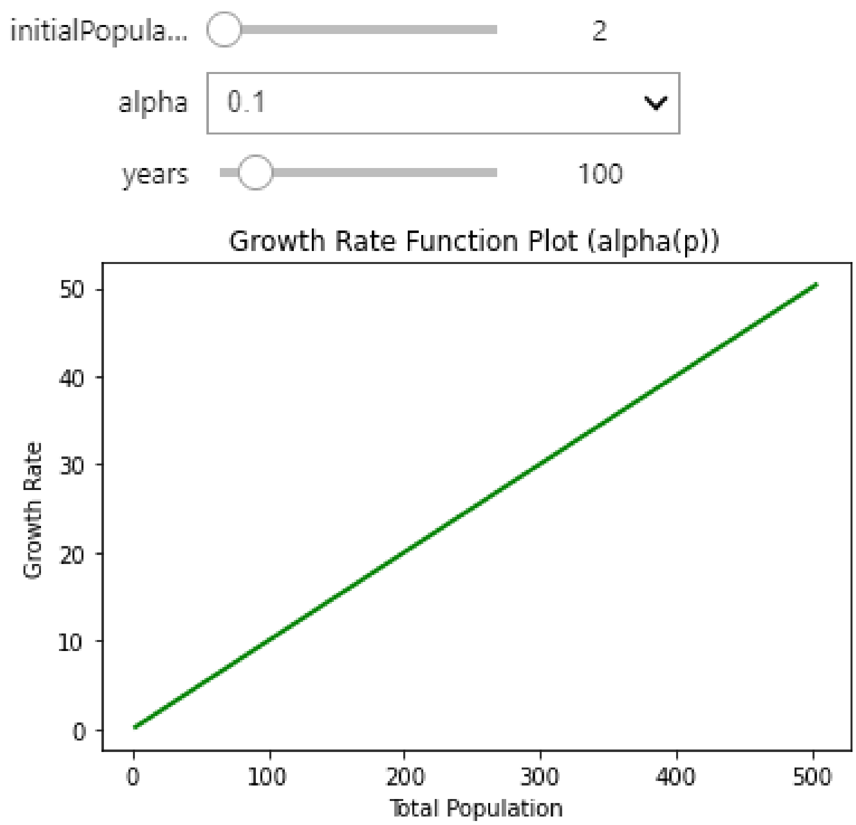

Figure 4: The plot of the function alpha(p) for population rates between 0 and 500 individuals

total population:

$$p_n = p_{n-1} + \beta p_{n-1}\,(200 - p_{n-1})$$

Next year's total population when we take β=0.001:

$$p_n = p_{n-1} + 0.001 p_{n-1}\,(200 - p_{n-1})$$

The population growth model can evolve in intriguing ways if the parameter β is increased. For example, multiplying β by 10 can cause population-wide oscillations. Population cycles are this kind of activity, and they are frequently seen in populations of predators and prey in the real world. A predator-prey system involves a cyclical interaction between the predators and the prey, with the predators eating the prey and the prey population increasing in the absence of predators. Both the predator and prey populations may experience cyclic oscillations as a result of this interaction.

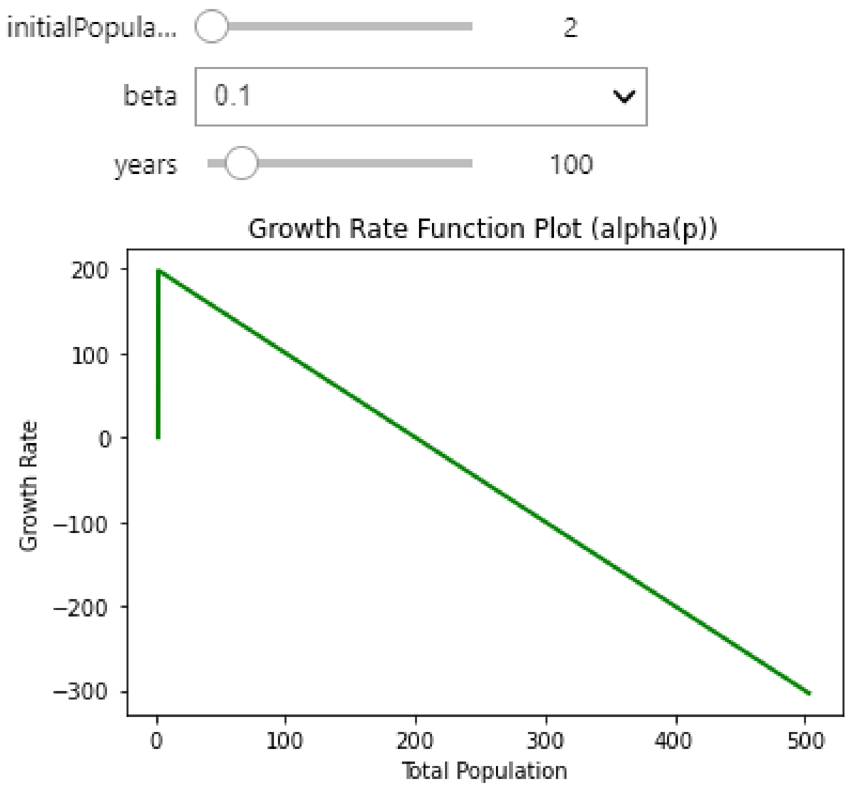

Figure 5: The plot of the function (p) for population rates between 0 and 500 individuals considering a(p)=200-p

3 Conclusion

As a conclusion, population growth models are a crucial tool for comprehending how a population evolves over time. While more sophisticated models, like the logistic growth model, can assist to account for considerations like carrying capacity and resource restrictions, simpler models like the exponential growth model can serve as a good starting point for understanding population dynamics. We may learn more about the potential changes in population that may occur under various situations by taking into account various elements including growth rate and limiting constraints. We may visualize population behavior over time and gain a better understanding of the dynamics of population expansion through simulations and graphical representations. These models may aid in decision-making in a number of situations and have significant applications in disciplines including ecology, epidemiology, and economics.

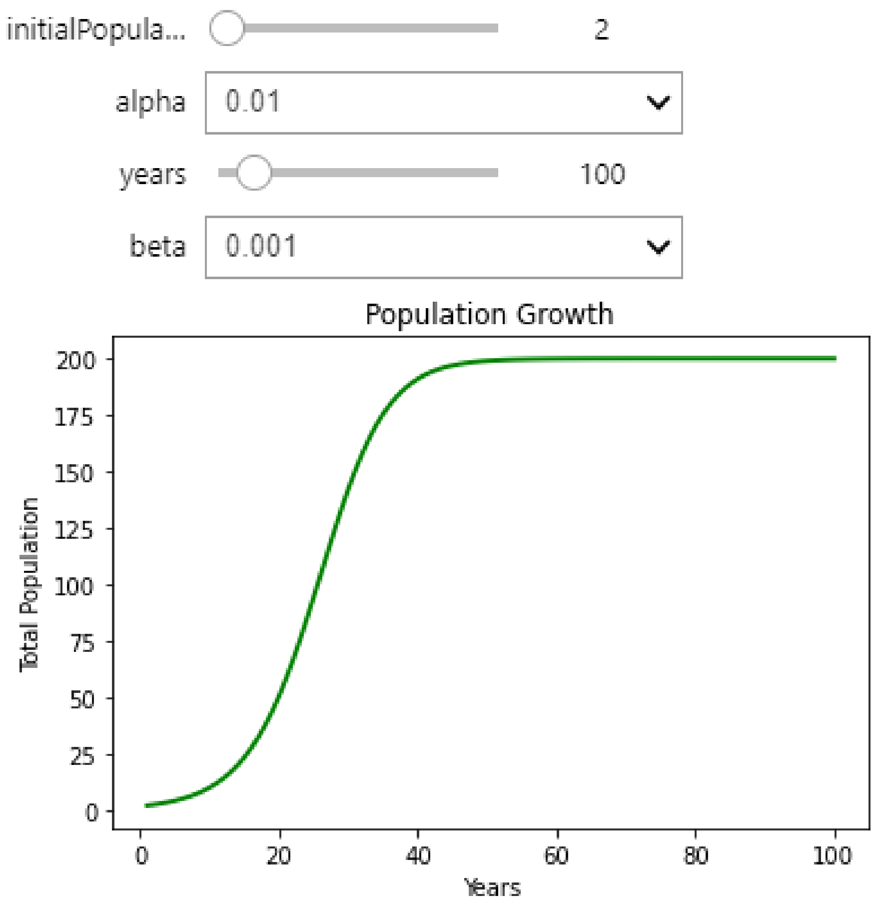

Figure 6: Plot of the variation of the population values over time when considering the beta factor as a limiting population. Beta=0.001

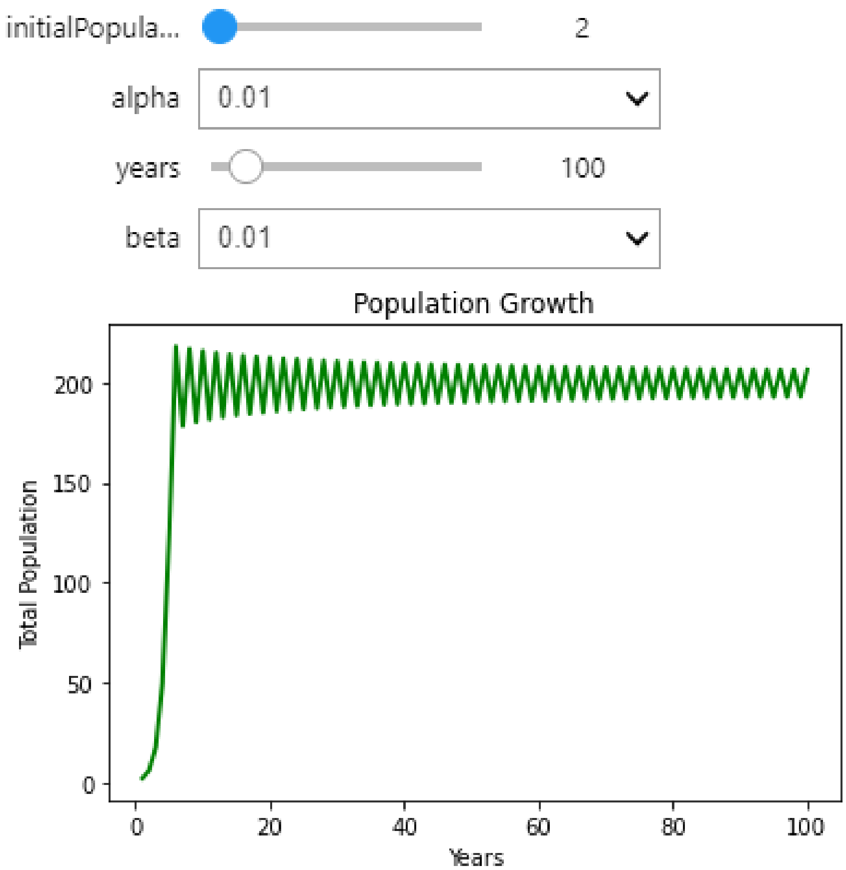

Figure 7: Plot of the variation of the population values over time when considering the beta factor as a limiting population. Beta=0.01