Homework Assignment 3 - Coding Part Write-up
Networks and Markets

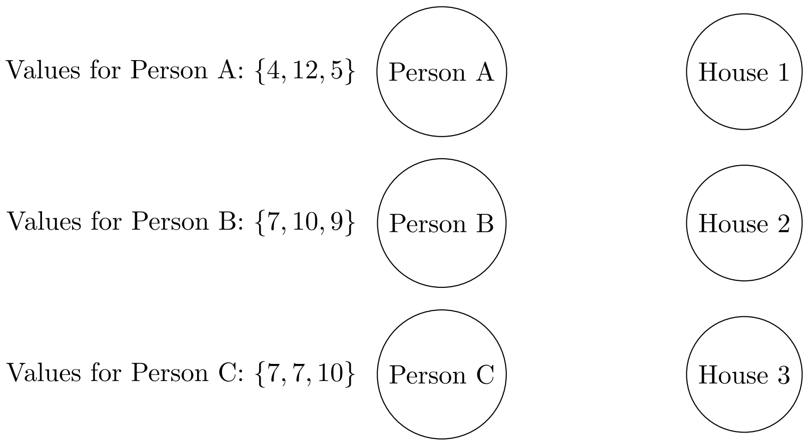
Omer Zohar   Gil Aharoni   Adam Tuby

August 6, 2024

Part 4: Implementing Matching Market Pricing

1 Question 7

(b) Consider the matching market example in Lecture 5 Page 7:



Formally, the matching market context is $\Gamma = (\{A, B, C\}, \{1, 2, 3\}, v)$, where v is the valuation function defined as follows:

$$v_A(1) = 4, v_A(2) = 12, v_A(3) = 5$$
$$v_B(1) = 7, v_B(2) = 10, v_B(3) = 9$$
$$v_C(1) = 7, v_C(2) = 7, v_C(3) = 10$$

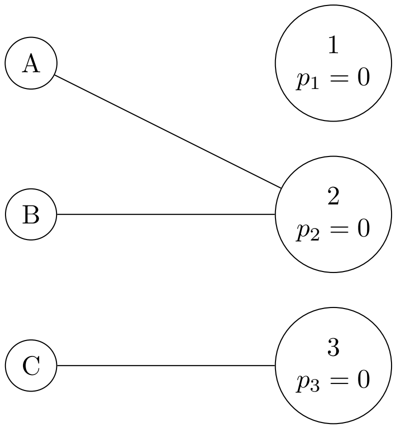
We turn to run the algorithm of Theorem 8.8 to find a market equilibrium (p, M) to find the maximum social value, in order to validate out implementation's output. We begin by initializing the prices vector $\vec{p} \equiv 0$ to be the zero vector. We then proceed to run the algorithm, updating the prices vector until there is a perfect matching M in the induced preferred choice graph for $(\Gamma, \vec{p})$:

1. Observing the following *induced preferred-choice graph* from $(\Gamma, \vec{p})$:

Values for A: $\{1:4, 2:12, 3:5\}$
Utility for A: $\{1:4, 2:12, 3:5\}$

Values for B: $\{1:7, 2:10, 3:9\}$
Utility for B: $\{1:7, 2:10, 3:9\}$

Values for C: $\{1:7, 2:7, 3:10\}$
Utility for C: $\{1:7, 2:7, 3:10\}$



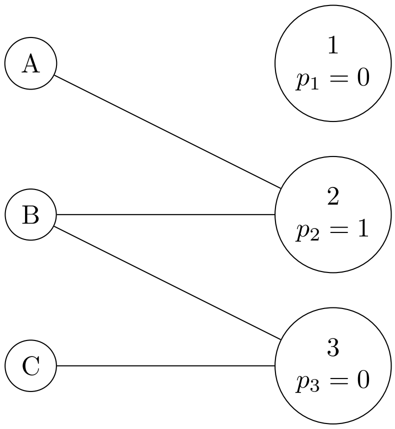
There obviously isn't a perfect matching as $S = \{A, B\}$ is a constricted set with $|N(S)| = |\{2\}| = 1 < 2 = |S|$ (which, by a theorem we've seen in class, implies that there isn't a perfect matching). Thus, we raise the prices for all items in $N(S)$ by 1, and update the prices vector $\vec{p}$ accordingly. The updated prices vector is $\vec{p} = (a : 0, b : 1, c : 0)$. Not all prices are greater than zero, so we don't perform a shift operation, and we proceed to the next iteration.

2. Observing the following *induced preferred-choice graph* from $(\Gamma, \vec{p})$:

Values for A: $\{1:4, 2:12, 3:5\}$
Utility for A: $\{1:4, 2:11, 3:5\}$

Values for B: $\{1:7, 2:10, 3:9\}$
Utility for B: $\{1:7, 2:9, 3:9\}$

Values for C: $\{1:7, 2:7, 3:10\}$
Utility for C: $\{1:7, 2:6, 3:10\}$



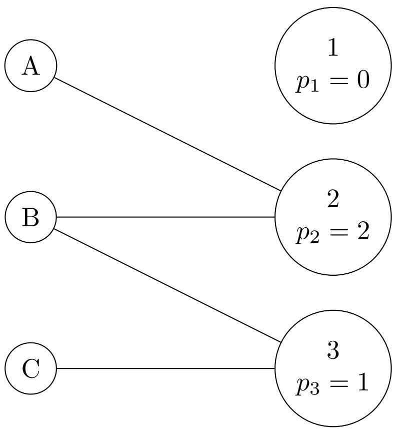
There obviously isn't a perfect matching as $S = \{A, B, C\}$ is a constricted set with $|N(S)| = |\{2, 3\}| = 2 < 3 = |S|$ (which, by a theorem we've seen in class, implies that there isn't a perfect matching). Thus, we raise the prices for all items in $N(S)$ by 1, and update the prices vector $\vec{p}$ accordingly. The updated prices vector is $\vec{p} = (a : 0, b : 2, c : 1)$. Not all prices are greater than zero, so we don't perform a shift operation, and we proceed to the next iteration.

3. Observing the following *induced preferred-choice graph* from $(\Gamma, \vec{p})$:

Values for A: $\{1:4, 2:12, 3:5\}$
Utility for A: $\{1:4, 2:10, 3:4\}$

Values for B: $\{1:7, 2:10, 3:9\}$
Utility for B: $\{1:7, 2:8, 3:8\}$

Values for C: $\{1:7, 2:7, 3:10\}$
Utility for C: $\{1:7, 2:5, 3:9\}$



Similar to the previous iteration, we raise the prices for $\{2, 3\}$, and update the prices vector

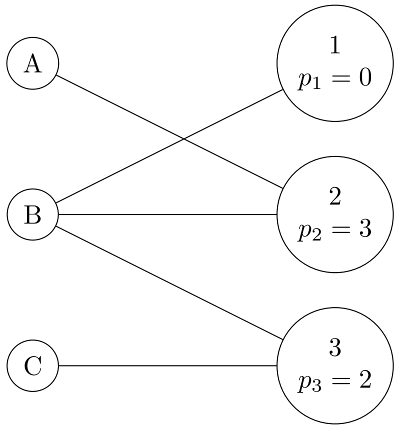
$\vec{p}$ accordingly. The updated prices vector is $\vec{p} = (a : 0, b : 3, c : 2)$. Not all prices are greater than zero, so we don't perform a shift operation, and we proceed to the next iteration.

4. Observing the following *induced preferred-choice graph* from $(\Gamma, \vec{p})$:

Values for A: $\{1 : 4, 2 : 12, 3 : 5\}$
Utility for A: $\{1 : 4, 2 : 9, 3 : 3\}$

Values for B: $\{1 : 7, 2 : 10, 3 : 9\}$
Utility for B: $\{1 : 7, 2 : 7, 3 : 7\}$

Values for C: $\{1 : 7, 2 : 7, 3 : 10\}$
Utility for C: $\{1 : 7, 2 : 4, 3 : 8\}$



And there is a perfect matching in the induced preferred choice graph, which is $M = \{\{A, 2\}, \{B, 1\}, \{C, 3\}\}$. Thus, the market equilibrium is $(\vec{p}, M) = ((1 : 0, 2 : 3, 3 : 2), \{\{A, 2\}, \{B, 1\}, \{C, 3\}\})$, and we are done.

We found the market equilibrium to be $(\vec{p}, M) = ((1 : 0, 2 : 3, 3 : 2), \{\{A, 2\}, \{B, 1\}, \{C, 3\}\})$. The maximum social value is therefore $v(A, 2) + v(B, 1) + v(C, 3) = 12 + 7 + 10 = 29$.

Our algorithm found exactly this market equilibrium.

2 Question 8

(a)

(b)

3 Bonus Question 2

(a)

(b)

(c)

(d)

Part 5: Exchange Networks for Uber

1 Question 9

2 Question 10

(a)

(b)

3 Question 11

4 Bonus Question 3

(a)

(b)

References