**My Robotic Arm: 3-Link Robotic Manipulator With 6 Degrees of Freedom**

1. **Kinematics**

The system will be broken up into two 2 dimensional uncoupled models. The coupling will be modeled as noise which the controller will handle on its own. The x-y plane will have a gravitational term (side view) and the x-z plane will not (top view).

x-y plane:

The position for each link can be written as

The time derivative of the position for each link then be given as

x-z plane:

The position for each link can be written as

The time derivative of the position for each link then be given as

1. **Lagrangian Dynamics**

The Lagrangian is defined as

x-y plane:

Combining all these terms to create the Lagrangian results in

The Lagrangian dynamics can be then be written through the equation

Where is the control torque and torque applied from non-conservative forces/moments, and , and are the states.

The time derivative of the second series of equations can then be written as

The link dynamic equations of motion can now be written as

The dynamics can be written in matrix form

x-z plane:

There is no gravitational effect, therefore, no potential energy acting on this system.

Combining all these terms to create the Lagrangian results in

The Lagrangian dynamics can be then be written through the equation

Where is the control torque and torque applied from non-conservative forces/moments, and , and are the states.

The time derivative of the second series of equations can then be written as

The link dynamic equations of motion can now be written as

The dynamics can be written in matrix form

1. **Control Law (Active-Based Control)**

The Dynamics written in matrix form can be written as

The left-hand side can be re-written as a series of parameters (to be estimated) linearly multiplied by a regressor matrix.

Using the control law as

Where (hat) sign represents the estimated parameters. The dynamics can then be written as

It can then be re-written as

The regressor function can be slightly modified to create the following

Where the represents the parameter errors. is the desired angular acceleration which can be represented in terms of the angular error dynamics, and is noise from the unmodeled dynamics and errors in the system. Let us say

Where and are the linear control gains guiding the angular/angular rate errors to zero, and is the additional control term to help overcome the potentially destabilizing effect of the uncertainty. Assume the noise magnitude is bounded by

Again, where is the magnitude of the unmodeled noise. The noise magnitude can be determined by either a fixed value or through the trajectory error equation given as:

Where , , and are constants determined by the control designer, and is a potentially time varying term (though often considered constant as well). It is also common to bound the term by a max that way the noise is not considered to be extremely large (though this will make the system unstable for when the unmodeled noise is actually larger than the bounded noise limit). This results in a defined as

Where is a convergence criteria determined by the designer and P comes from The Lyapunov equation

Where Q is a tunable parameter and is a symmetric positive definite matrix (often only the diagonal terms are filled out). The linear error control gains can be found using the Linear Quadratic Regulator (LQR). This linearizes the error dynamics. The state-space representation can be written as

The Parameter estimate equations are also given as

Where

is tunable parameter that is a symmetric positive definite matrix (often only the diagonal terms are filled out). defines the rate of adaptivity, and represent the maximum and minimum eigenvalues, respectively, and (probably would be wise to let from the robust control term for consistence in the control design). can simply be determined to be a value of 1, though this would potentially not guarantee parameter convergence. Therefore, the control can be written as

or

Therefore, all the tunable parameters can be given as

1. **Control Law (Passivity-Based Control)**

Consider the control design given as

The left-hand side can be re-written as a series of parameters (to be estimated) linearly multiplied by a regressor matrix.

Where

Where K and are diagonal matrices of constant, positive gains. If is constant the term can be expressed as

Where is a fixed `nominal` parameter vector and is an additional control term. Assume that the parameter uncertainty is bounded by a non-negative constant such that the additional control term can be written as

We can define an equation of motion for the estimated parameter vector if is changing over time, which is given as

This, however, does not guarantee parameter convergence, so the dead-zone rule can potentially also be applied here with the following equation

Where is defined similarly to the active-based control (would like to confirm this is true/possible). This time-varying/adaptive system, however, is not robust to disturbances and unmodeled dynamics as the constant time parameter system. The two can potentially be combined in order to account for time varying parameters as well as disturbances/unmodeled dynamics (would also like to confirm this is possible via simulation).

1. **Parameters and Regressor Matrices**

Let us write the parameters as

The dynamics matrices can be re-written in terms of the parameters as

The equation form of the dynamics can then be written in terms of the parameters as

With the equations written out the regressor matrix can then be written as

If no gravity is being used, then the last three parameter terms can be removed and the last three regressor terms can be removed, this results in the following

If using passivity-based control then the regressor function can be modified to the following

If no gravity is being used, then the last three parameter terms can be removed and the last three regressor terms can be removed, this results in the following

It can be seen through the regressor function that if the control designer is using non-passivity-based control then the control designer MUST have feedback information on the angular acceleration for each link.