Polynomial Interpolation: Exploring Stability and Accuracy

The Importance of Chebyshev nodes

Olivia Gette

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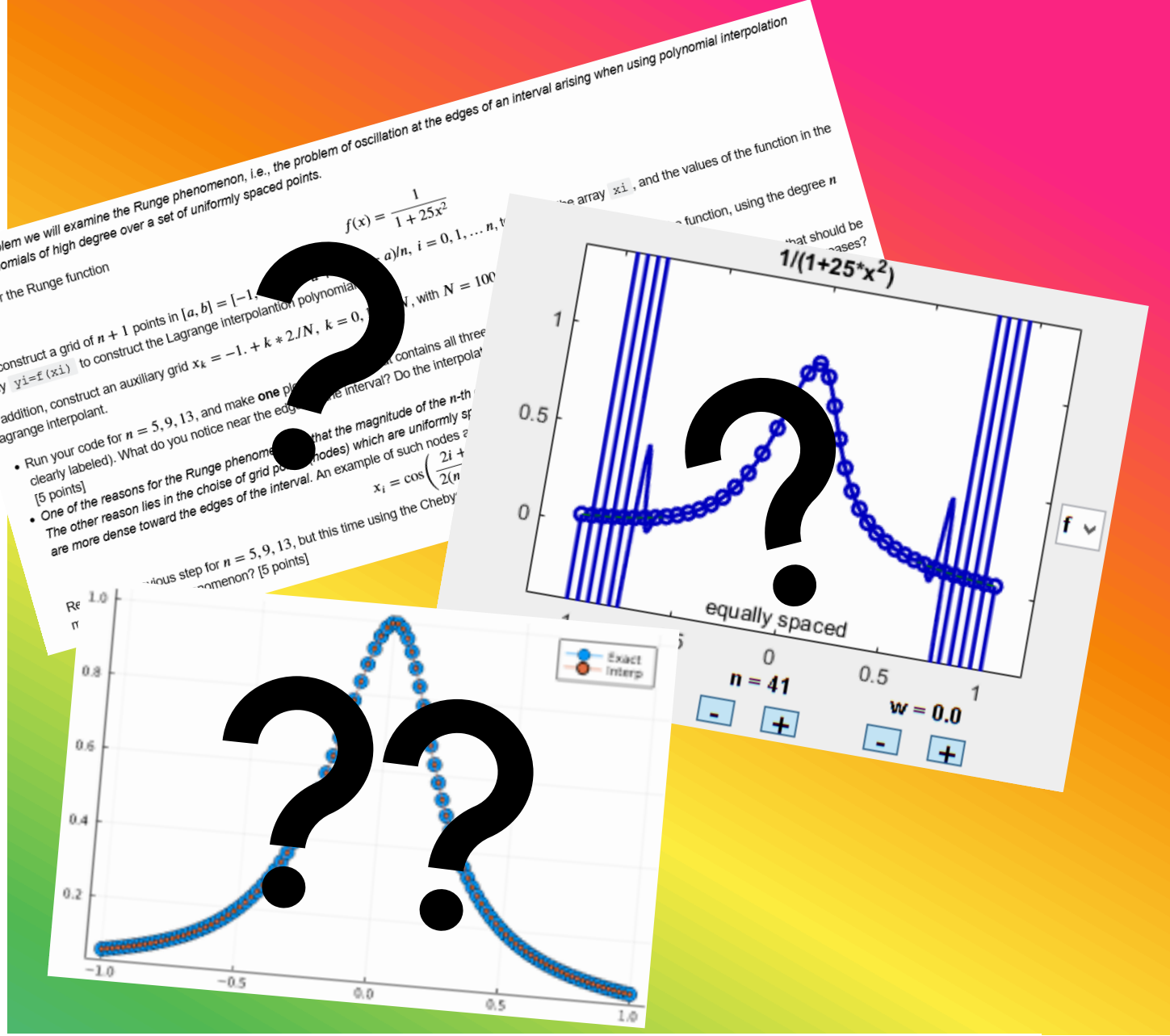
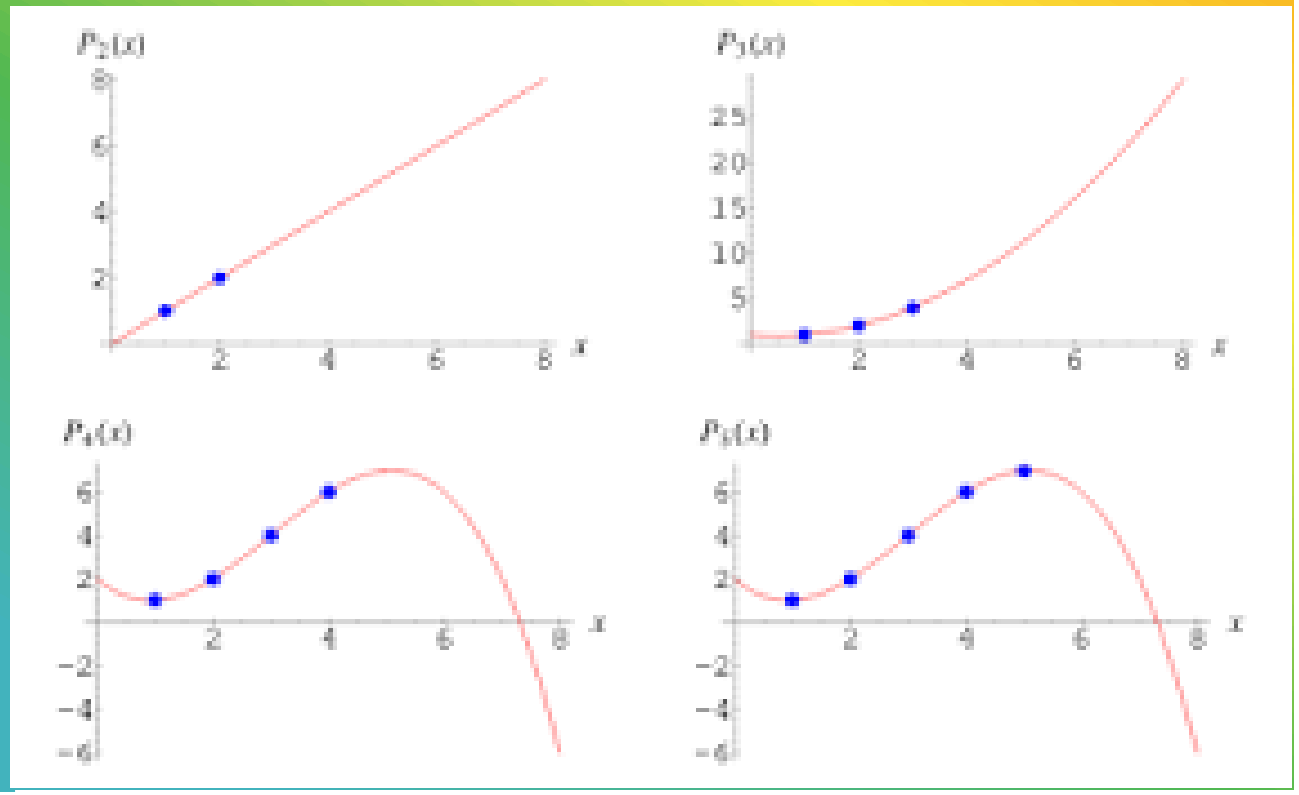


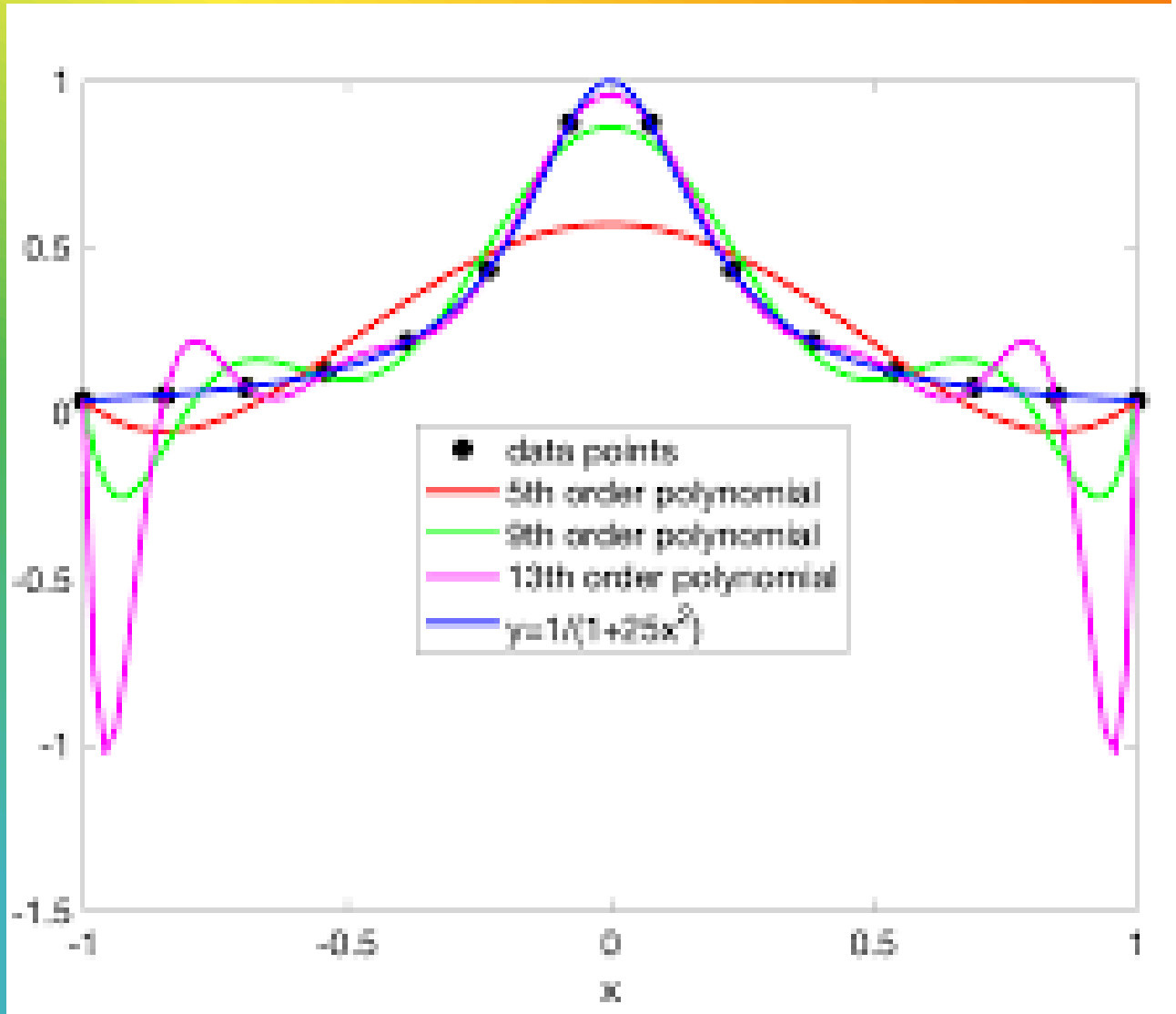
The Problem



What is Polynomial Interpolation?

- Finds a smooth polynomial that passes through a set of points
- Commonly used to estimate unknown values or create smooth curves from data
- Using evenly spaced points can make interpolation unstable and lead to oscillations (Runge phenomenon)

Mathematical Background

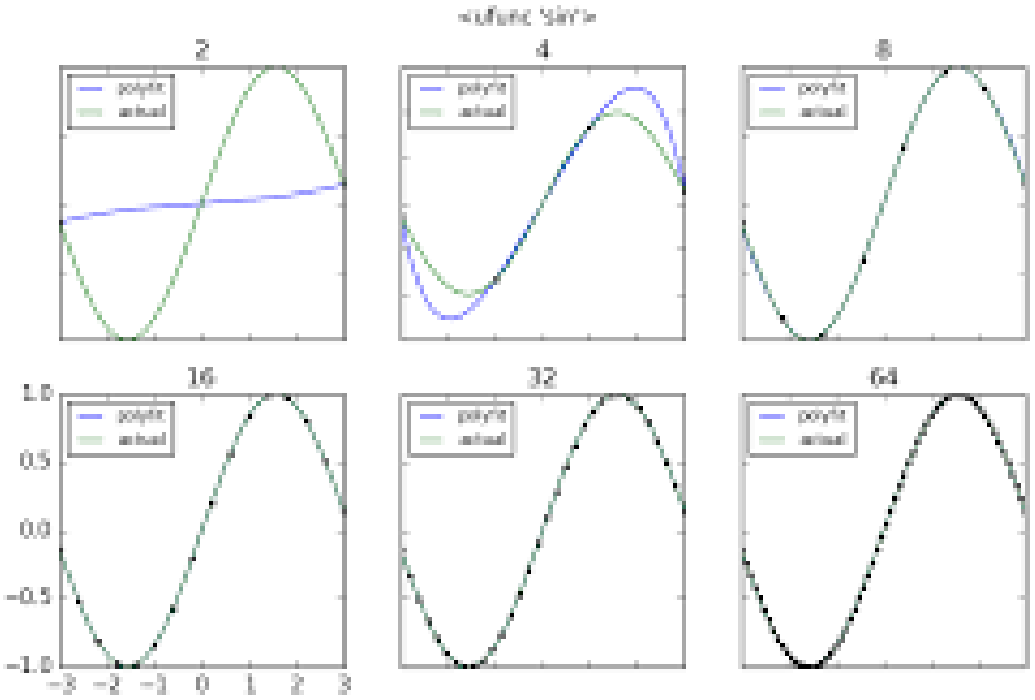


Different ways to Interpolate

- Vandermonde Method
  - Constructs a system of linear equations to find polynomial coefficients
  - Can become unstable with too many points
- Barycentric Method
  - Builds the interpolating polynomial more efficiently and avoids stability issues
  - Uses precomputed weights

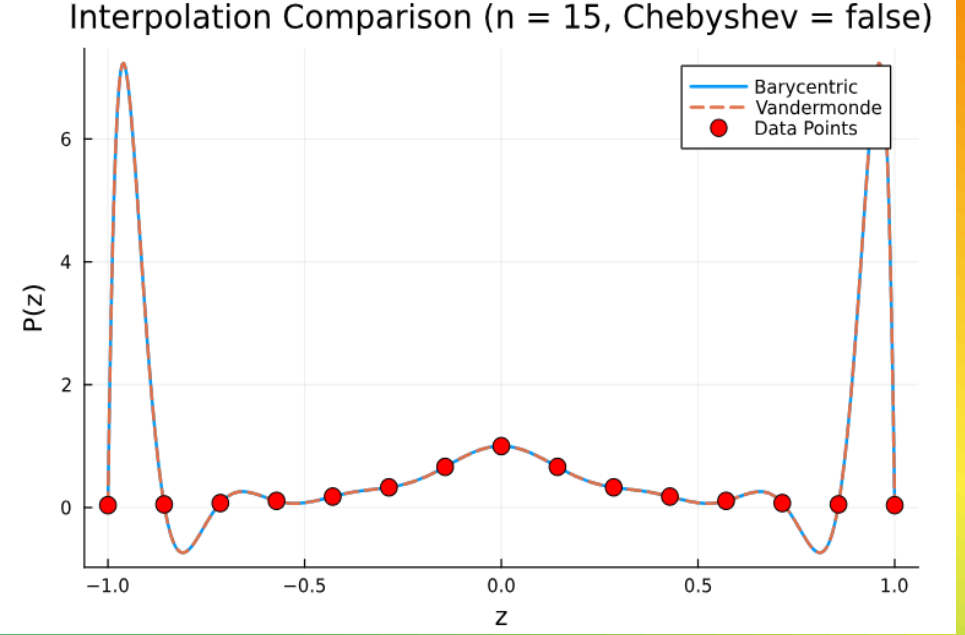
The Role of Chebyshev Nodes

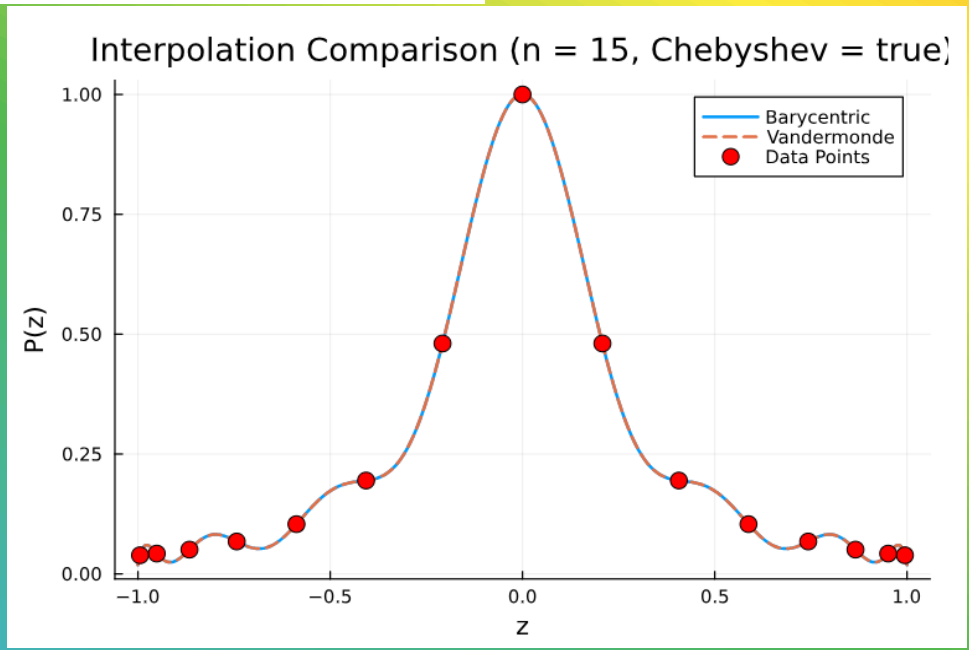
Why They Work



- Chebyshev nodes are clustered near the edges of the interval, reducing oscillations
- They make interpolation more accurate and stable by minimizing the Runge phenomenon

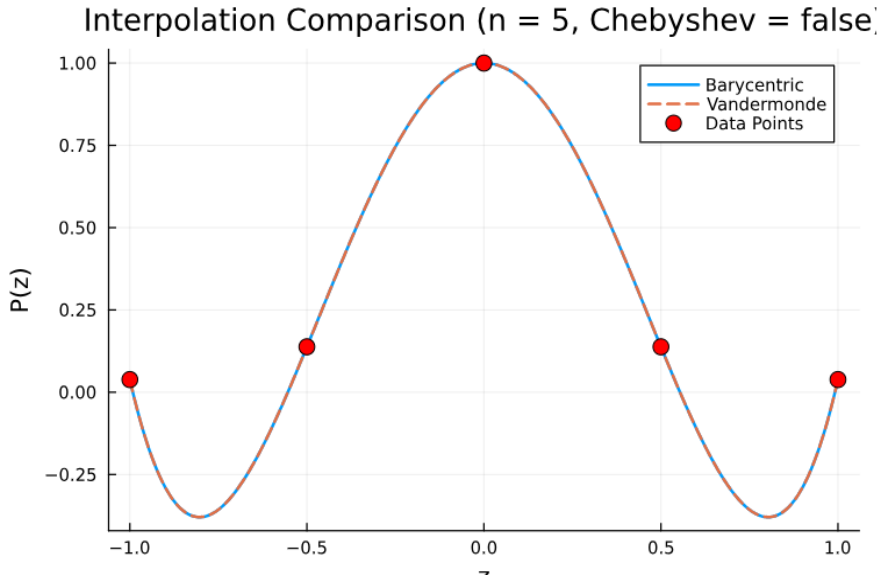
Results – Interpolation Comparisons

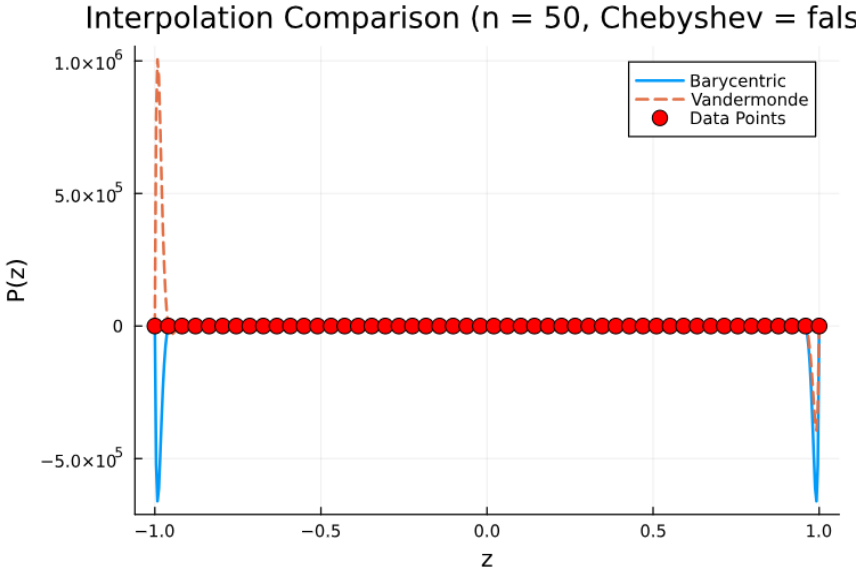


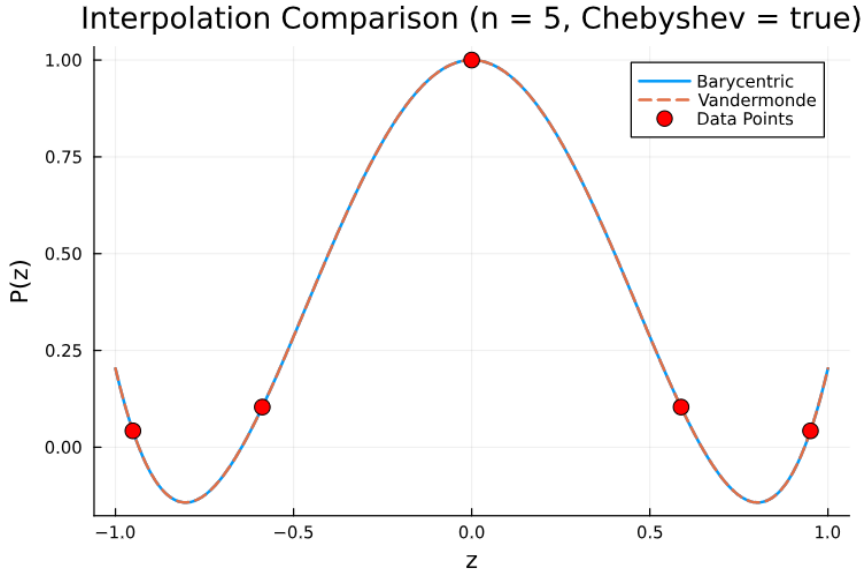


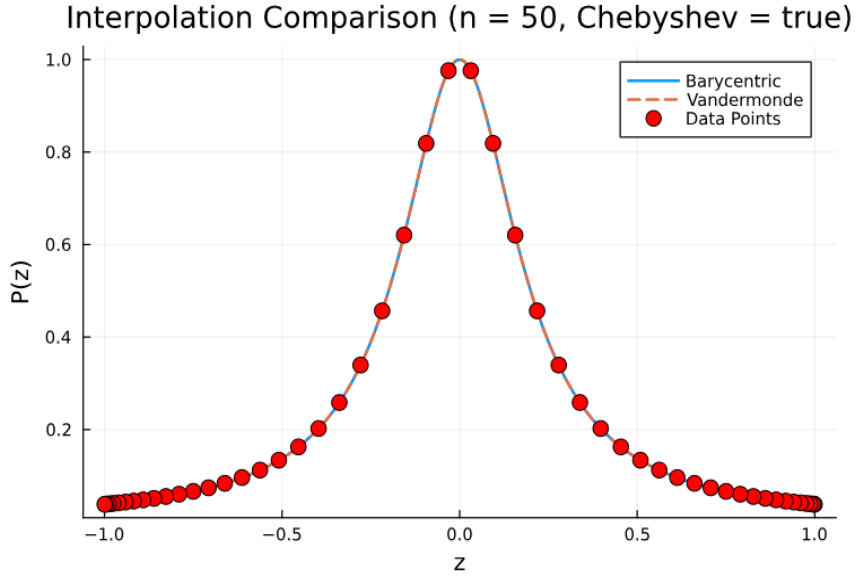
- Figure 1
  - Interpolation with **Equispaced Nodes** (Runge phenomenon appears at higher n)
- Figure 2
  - Interpolation with **Chebyshev Nodes** (reduced oscillations and better accuracy)

Different n Values



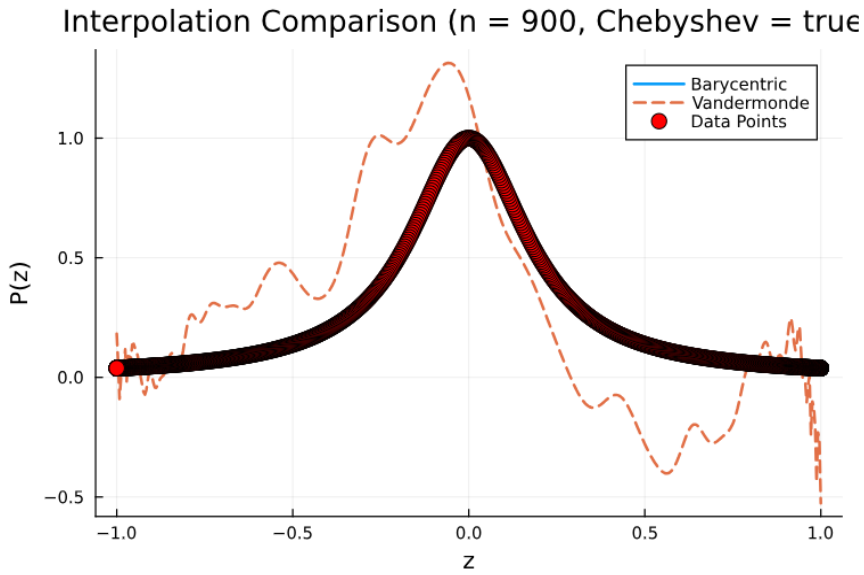






Takeaway and Extensions

What I Learned and What's Next



- What I Learned:
  - Barycentric and Vandermonde methods can both accurately interpolate data when nodes are chosen well
  - Chebyshev nodes significantly improve stability and reduce errors

- Ideas for Extensions:
  - Interpolation in higher dimensions
  - Try with noisy data
  - Look into other methods to interpolate

References

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