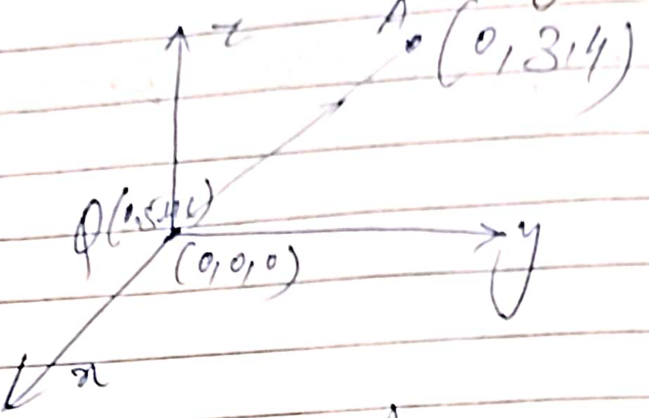
Assignment - 2

Ques-1. Find E - - - - - - origin.



$\vec{r} = 3a\hat{y} + 4a\hat{z}$

$E = \frac{1}{4\pi\epsilon_0} \times \frac{Q}{R^2}\hat{a}_R$

$= \frac{9\times10^9 \times 0.5\times10^{-6}(3a\hat{y} + 4a\hat{z})}{(5)^3}$

$= \frac{9\times10^2}{25}(3a\hat{y} + 4a\hat{z})$ C/m²

$= 36(3a\hat{y} + 4a\hat{z})$

$= 108a\hat{y} + 144a\hat{z}$ C/m²

Q2. a) $F|_{(1,3,7)} = ?$

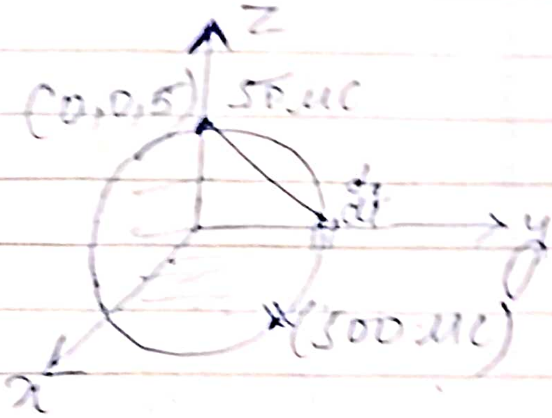
As both the charges are placed at one point, so its distance = 0.

Hence, $F = qE = q\frac{kQ}{r^2} = q\frac{kQ}{0} = \infty$

b) $E|_{(1,-3,7)} = ?$

Similarly, $E = \infty$

Q3. Find the - - - - - - z = 0m.



$$\vec{E} = \iint \frac{\rho_s \, ds \, \hat{a}_r}{4\pi\epsilon_0 r^2}$$

$$= \frac{\rho_s \cdot (a\hat{z})}{2\epsilon_0}\left(1 - \frac{1}{\sqrt{2}}\right)$$

$$= \frac{500 \times 10^{-6} \times 0.293}{\pi \times 25 \times 2\epsilon_0}$$

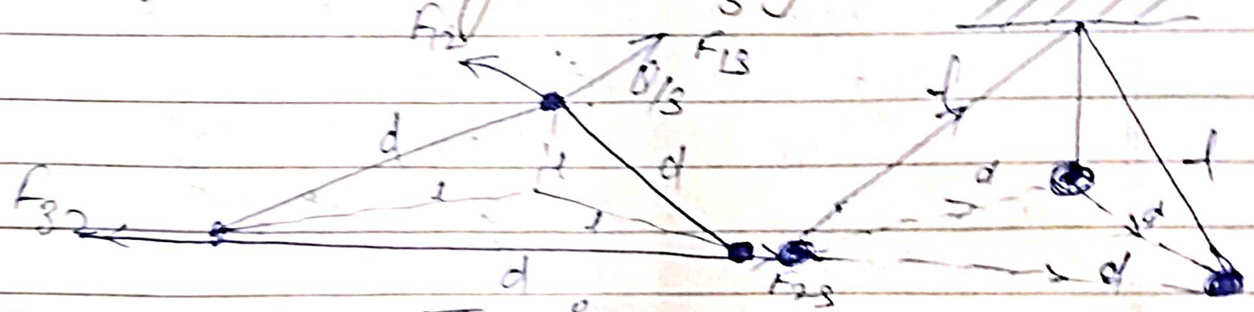
$$\vec{F} = qE = \frac{50 \times 10^{-6} \times 500 \times 10^{-6}}{2\epsilon_0 \pi \times 25} \times \frac{2}{2} \times 0.25$$

$$= \frac{9 \times 10^{9} \times 50 \times 10^{-9} \times 0.29}{25}$$

$$= 10 \times 0.29 = 5.22N$$

Q4. Three identical - - - - - show that

$$Q^2 = 12\pi\epsilon_0 mgd^3\left[l^2 - \frac{d^2}{3}\right]^{-1/2}$$



For charge ③

$$F_{32} = \frac{kq^2}{d^2} = \frac{kq^2}{9d^2}$$

$$F_{31} = \frac{kQ^2}{9d^2}$$

F due to gravity $= mg$

Tensions is along AO —
So,

$$F_{net} = F_{31}\cos 30^\circ + F_{32}\cos 30^\circ$$

$$= \frac{KQ^2}{9d^2}\left(\frac{\sqrt{3}}{2} \cancel{\times} \frac{\sqrt{3}}{2}\right)$$

$$= \sqrt{3}\left(\frac{KQ^2}{9d^2}\right)$$

So, $mg\,(AP) = F_{net}\,(OP)$

$$mg\left(\frac{2}{3} \times d\cos 30^\circ\right) = \frac{\sqrt{3}}{9}\frac{KQ^2}{d^2}\left(\sqrt{l^2 - \left(\frac{2}{3}d\cos 30^\circ\right)^2}\right)$$

$$mg \times \frac{\sqrt{3}d}{3} = \frac{4\pi\epsilon_0\sqrt{3}Q^2}{d^2}\left(\sqrt{l^2 - \frac{d^2}{3}}\right)$$

$$\boxed{Q^2 = 12\pi\epsilon_0 mg d^3\left(l^2 - \frac{d^2}{3}\right)^{-1/2}}$$

Q5. An iron - - - - - - - - - - reached.

Balancing all the forces on cesium —

$$\vec{F} = m\vec{a}$$

Electrostatic force is, $\vec{F} = q\vec{E}$

$$q\vec{E} = m\vec{a}$$

$$E = -400\,a_x + 200\,a_y$$

$$\vec{a} = \frac{d^2x}{dt^2}a_x + \frac{d^2y}{dt^2}a_y + \frac{d^2z}{dt^2}a_z$$

Putting all values in eq^n —

$$-1.6\times10^{-19}(-400\,a_x + 200\,a_y)\times10^{3} = 2.22\times10^{-25}\times10^{3}\vec{a}$$

On comparision —

$$\frac{d^2x}{dt^2} = \frac{400\times1.6\times10^{-19}}{2.22\times10^{-25}}$$

Date
Page

Double integrating w.r.t t

$$x = \frac{400 \times 1.6 \times 10^{-19}}{2 \times 2.22 \times 10^{-25}} t^2$$

Similarly,

$$\frac{d^2y}{dt^2} = \frac{-1.6 \times 10^{-19} \times 200}{2.22 \times 10^{-25}} + c \quad - (1)$$

Double integrating w.r.t t

$$y = \frac{-200 \times 1.6 \times 10^{-19} \times t^2}{2 \times 2.22 \times 10^{-25}} + c \quad - (2)$$

from (1) & (2)

$$|x| = 2|y|$$

So, for $y = 40$ cm

max value of x ~~contar~~ obtained is 80 cm.

Q6. Calculate - - - - - - - - $\rho_v = e^{-2r/r^2}$

$$Q = \int \rho_v \, dv$$

$$= \int_0^{0.1} \int_0^{\pi} \int_2^4 \rho^2 z^2 \sin(0.6)\phi \, \rho \, d\rho \, d\phi \, dz$$

$$= \int_0^{0.1} \int_0^{\pi} \rho^2 \sin(0.6)\phi \left(\frac{64}{3} - \frac{8}{3} \right)$$

$$= \frac{-56}{3} \int_0^{0.1} \frac{\rho^3}{0.6} \left[\cos(0.6)\phi \right]_0^{\pi}$$

$$\Rightarrow 1.08 \times 10^{-3} \, C.$$

b) $Q = \int \rho_v \, dv$

$= \int_\infty e^{-2r/r_0} \cdot r^2 \sin\theta \, dr \, d\theta \, d\phi$

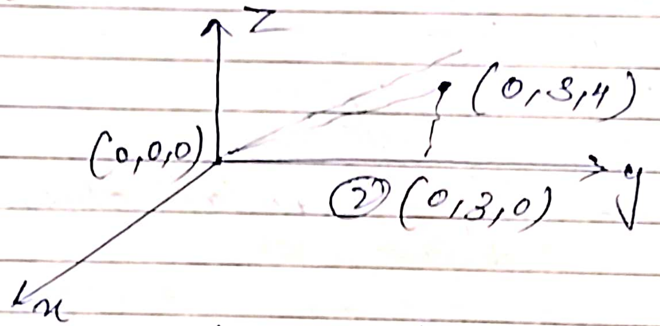
$= \int_0^\infty \int_0^\pi \int_0^{2\pi} \frac{e^{-2r}}{r_0} \cdot r^2 \sin\theta \, dr \, d\theta \, d\phi$

$= \left[\frac{e^{-2r}}{2}\right]_0^\infty \left[-\cos\theta\right]_0^\pi \left[\phi\right]_0^{2\pi}$

$= -\frac{1}{2}\left[1+1\right] \times 2\pi$

$= 2\pi \Rightarrow 6.28C.$

Q7. Infinite ———— E/(0,3,4)



Due to ∞ line of charge (1) -

$\vec{E} = \frac{\rho_l}{2\pi\epsilon_0 \rho}\left(\frac{3\hat{a_y} + 4\hat{a_z}}{5}\right)$ {As ∴ distance = 5}

$= \frac{54\hat{a_y} + 72\hat{a_y}}{5}$

Due to ∞ line charge (2) -

$E_{(2)} = \frac{\rho_l \hat{a_\rho}}{2\pi\epsilon_0 \rho} = \frac{5\times10^{-9}\hat{a_z}}{2\pi\epsilon_0\times4}$

$= \frac{5\times10^{-9}}{8\pi\epsilon_0}\hat{a_z} \Rightarrow \frac{5\times10^{-9}}{8\times3.14\times8.85\times10^{-12}}$

$= 0.0225\times10^{3}\hat{a_z}$

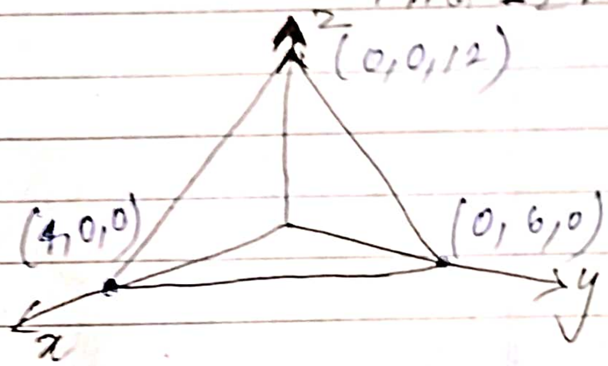
$= 22.5\hat{a_z}$

$$\vec{E}_{net} = \vec{E_1} + \vec{E_2}$$
$$= 10.6 a_y + 86.2 a_z$$
$$=$$

Q8 A dielectric — — — — — — find E_2.

$\epsilon_{r1} = 3$

$E_1 = 2a_x + 5a_y$



Given, $3x + 2y + z = 12$

$\phi \rightarrow 3x + 2y + z = 12$

$$\hat{A} = \frac{\vec{A}}{|A|} = \frac{3a_x + 2a_y + a_z}{\sqrt{14}}$$

where, $\hat{A}$ = Unit normal vector on freespace side

Projection of $\vec{E_1}$ on $\hat{A}$ is normal component of $\vec{E_1}$ at interface —

$$\vec{E_1} \cdot \hat{A} = \frac{2 \times 3}{\sqrt{14}} + 0 + \frac{5 \times 1}{\sqrt{14}} = \frac{11}{\sqrt{14}}$$

$$\text{So, } \vec{E}_{n1} = \frac{11}{\sqrt{14}} \hat{A} = \frac{11}{\sqrt{14}} \left(\frac{3a_x + 2a_y + a_z}{\sqrt{14}} \right)$$

$$= 2.36 a_x + 1.57 a_y + 4.21 a_z$$

$$\vec{E_1} = \vec{E_{t1}} + \vec{E_{n1}}$$

$$\vec{E_{t1}} = \vec{E_{n1}} - E_{n1} = -0.36 a_x + 1.57 a_y + 4.21 a_z$$

Using boundary condn

$$\vec{E_{t1}} = \vec{E_{t2}}$$

$$\vec{E_{n2}} = \frac{\vec{D_{n2}}}{\epsilon_0} = \frac{\epsilon_0 \vec{E_{n2}}}{\epsilon_0}$$

$= 7.08\hat{a_x} + 4.71\hat{a_y} + 2.37\hat{a_z}$

So,

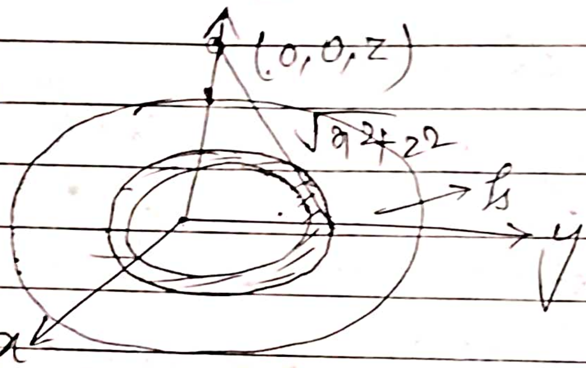
$\overrightarrow{E_2} = \overrightarrow{E_{L2}} + \overrightarrow{E_{H2}}$

$= 6.72\hat{a_x} + 3.14\hat{a_y} + 6.50\hat{a_z}$

Q9. A circular - - - - - - - - ρ_s.

$A = 2\pi x\, dx$

ρ_s = Charge density of disc



$dq = \rho_s \cdot 2\pi x\, dx$

$dv = \frac{dq}{4\pi\epsilon_0\sqrt{x^2+z^2}}$

On integrating —

$V = \int dv = \int_0^R \frac{\rho_s \cdot 2\pi x\, dx}{4\pi\epsilon_0\sqrt{x^2+z^2}}$

$= \int_0^R \frac{\rho_s\, x\, dx}{2\epsilon_0\sqrt{x^2+z^2}}$

let,

$x^2+z^2 = t^2$

$2x\,dx = 2t\,dt$

$x \to 0,\ t = z$

$x \to R\ \ t = \sqrt{R^2+y^2}$

$= \int_z^{\sqrt{R^2+z^2}} \frac{\rho_s \times t \cdot dt}{2\epsilon_0 \cdot t}$

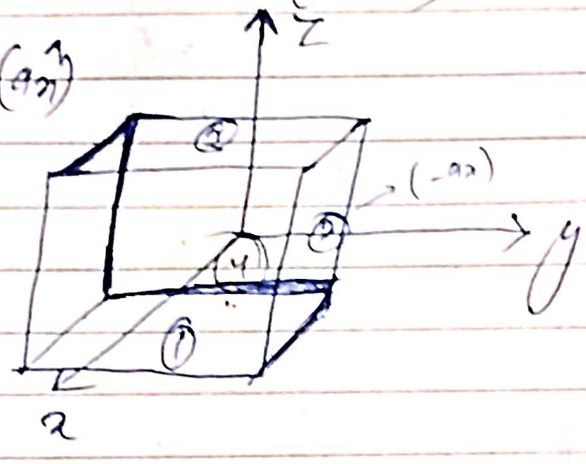
$= \frac{\rho_s}{2\epsilon_0}[t]_z^{\sqrt{R^2+z^2}}$

$V_P = \frac{\rho_s}{2\epsilon_0}\left[\sqrt{R^2+z^2} - z\right]$ cm

Date
Page

Q 10. Given $\vec{D}$ - - - - - to the axis.

$$\vec{D} = 5x^2\hat{a_x} + 10z\hat{a_z} \ (C/m^2)$$



For S_1, $ds = dy\,dz(\hat{a_x})$

$$\phi_{S_1} = \int D \cdot ds$$

$$= \int 5x^2 \cdot dy\,dz$$

$$\Rightarrow 5x^2 \left[y\right]_{-1}^{1} \left[z\right]_{-1}^{1}$$

$$\Rightarrow 5x^2 [2][2]$$

$$\Rightarrow 20x^2 \Rightarrow 20x^2\Big|_{x=1} = 20C$$

For S_2, $ds = -dx\,dy\,\hat{a_x} = -4\hat{a_x}$

$$\phi_{S_2} = \int D \cdot ds = 5x^2 \cdot (-4\hat{a_x})$$

$$\Rightarrow -20x^2\Big|_{x=1} \Rightarrow -20C.$$

For S_3, $ds = 4\hat{a_z}$,

$$\phi_{S_3} = \int (10z\,\hat{a_z}) \cdot 4\hat{a_z} \Rightarrow$$

$$\Rightarrow 40z\Big|_{z=1} \Rightarrow 40$$

For S_4, $ds = -4\hat{a_z}$

$$\phi_{S_4} = \int (10z\,\hat{a_z})(-4\hat{a_z})$$

$$\Rightarrow -40z\Big|_{z=-1} \Rightarrow 40C$$

There is no component in y dir^n

hence, $\phi_{S_5} = \phi_{S_6} = 0$

Total flux $= 26 - 26 + 40 + 40$

$= 80\ C.$