Be Ahead with Sustained Excellence

AIEEE - 2010

Question paper with Solutions

Disclaimer

- The solutions have been created by the teachers of BASE with an intention to help students.
- While care has been taken to ensure accuracy, BASE will not be responsible for any inadvertent error that may exist in the solution. However, BASE will welcome feedback on any errors noticed. The same may be sent via email to info@base-edu.in
- BASE does not stake claim to the fact that the solutions it has provided are the only ways in which questions could be answered.
- The solutions provided here cannot be used as a support for any legal disputes by any individual or organization.

AIEEE – 2010 Question paper (Code – A) with Solutions

Duration: 3 hours
Max. Marks: 432

Marks Pattern

Part A – PHYSICS (144 marks) –Questions No. 1 to 20 and 23 to 26 consist of FOUR (4) marks each and Questions No. 21 to 22 and 27 to 30 consist of EIGHT (8) marks each for each correct response.
Part B – CHEMISTRY (144 marks) – Questions No. 31 to 39 and 43 to 57 consist of FOUR (4) marks each and Questions No. 40 to 42 and 58 to 60 consist of EIGHT (8) marks each for each correct response.
Part C – MATHEMATICS (144 marks) – Questions No. 61 to 66, 70 to 83 and 87 to 90 consist of FOUR (4) marks each and Questions No. 67 to 69 and 84 to 86 consist of EIGHT (8) marks each for each correct response.
Candidates will be awarded marks as stated above for correct response of each question.
¼ (one-fourth) marks will be deducted for indicating incorrect response of each question.
No deduction from the total score will be made if no response is indicated for an item in the answer sheet.

PART – A : PHYSICS

Directions : Questions number 1-3 are based on the following paragraph.
An initially parallel cylindrical beam travels in a medium of refractive index $\mu(I) = \mu_0 + \mu_2 I$, where μ_0 and μ_2 are positive constants and I is the intensity of the light beam. The intensity of the beam is decreasing with increasing radius.

1. The initial shape of the wavefront of the beam is
   (1) Planar (2) Convex
   (3) Concave (4) Convex near the axis and concave near the periphery
   Ans (1)
   Wavefront of a parallel cylindrical beam is planar.

2. The speed of light in the medium is
   (1) Maximum on the axis of the beam (2) Minimum on the axis of the beam
   (3) The same everywhere in the beam (4) Directly proportional to the intensity I
   Ans (2)

3. As the beam enters the medium, it will
   (1) travel as a cylindrical beam
   (2) diverge
   (3) converge
   (4) diverge near the axis and converge near the periphery
   Ans (3)

Directions : Questions number 4-5 are based on the following paragraph.

A nucleus of mass M + Δm is at rest and decays into two daughter nuclei of equal mass $\frac{M}{2}$ each.

Speed of light is c.

4. The speed of daughter nuclei is

(1) $c\sqrt{\frac{\Delta m}{M+\Delta m}}$ (2) $c\frac{\Delta m}{M+\Delta m}$ (3) $c\sqrt{\frac{2\Delta m}{M}}$ (4) $c\sqrt{\frac{\Delta m}{M}}$

Ans (3)

Linear momentum is conserved. $\frac{M}{2}v_1 - \frac{M}{2}v_2 = 0 \Rightarrow v_1 = v_2 = v$ say. Energy released is in the form of kinetic energy of daughter nuclei.

$$\Delta mc^2 = 2\left[\frac{1}{2}\frac{M}{2}v^2\right] \Rightarrow v = c\sqrt{\frac{2\Delta m}{M}}$$

5. The binding energy per nucleon for the parent nucleus is E_1 and that for the daughter nuclei is E_2. Then

(1) $E_1 = 2\,E_2$ (2) $E_2 = 2\,E_1$ (3) $E_1 > E_2$ (4) $E_2 > E_1$

Ans (4)

Stability of the daughter nucleus is greater than stability of the parent nucleus. Hence, the answer.

Directions : Questions number 6-7 contain Statement-I and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.

6. **Statement-1:** When ultraviolet light is incident on a photocell, its stopping potential is V_0 and the maximum kinetic energy of the photoelectrons is K_{max}. When the ultraviolet light is replaced by X-rays, both V_0 and K_{max} increase.

Statement-2: Photoelectrons are emitted with speeds ranging from zero to a maximum value because of the range of frequencies present in the incident light.

(1) Statement-1 is true, Statement-2 is false

(2) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation of Statement-1

(3) Statement-1 is true, Statement-2 is true; Statement-2 is the not the correct explanation of Statement-1

(4) Statement-1 is false, Statement-2 is true

Ans (1)

Frequency and hence the energy of a X-rays is greater than ultraviolet light. Therefore, V_0 and K_{max} both increase.

Though the energy gained by each electron is the same, the energy lost during collision with lattice points is different for different electrons. It thus, results in a range for speed. ∴ Statement II is false.

7. **Statement-1 :** Two particles moving in the same direction do not lose all their energy in a completely inelastic collision.

Statement-2 : Principle of conservation of momentum holds true for all kinds of collisions.

(1) Statement-1 is true, Statement-2 is false

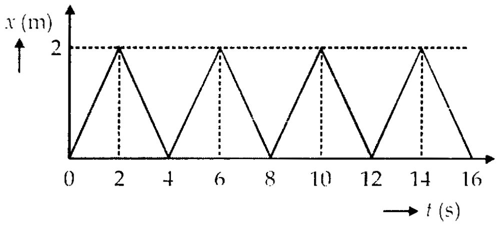
(2) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation of Statement-1

(3) Statement-1 is true, Statement-2 is true; Statement-2 is the not the correct explanation of Statement-1

(4) Statement-1 is false, Statement-2 is true

Ans (2)

8. The figure shows the position-time (x-t) graph of one-dimensional motion of a body of mass 0.4 kg. The magnitude of each impulse is



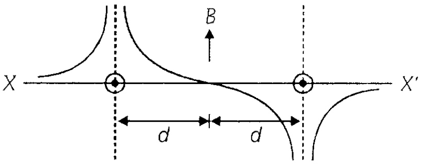
(1) 0.2 Ns (2) 0.4 Ns (3) 0.8 Ns (4) 1.6 Ns

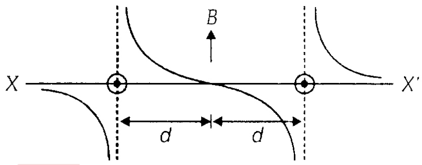
Ans (3)

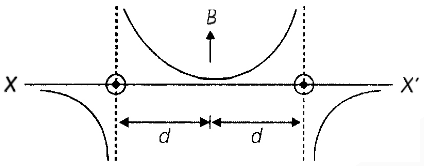
Impulse J = m |(v – u)|

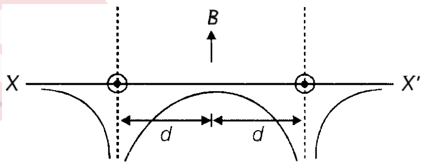
$$= 0.4\left|\frac{0-2}{4-2} - \frac{2-0}{2-0}\right| = 0.4\,|-2| = 0.8 \text{ Ns}$$

9. Two long parallel wires are at a distance 2d apart. They carry steady equal currents flowing out of the plane of the paper as shown. The variation of the magnetic field B along the line XX′ is given by

(1) 

(2) 

(3) 

(4) 

Ans (2)

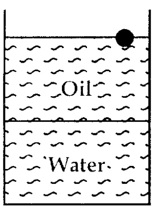
$$B_{bet} = \frac{\mu_0 i}{2\pi d}\hat{j} - \frac{\mu_0 i}{2\pi(2d-x)}(-\hat{j})$$

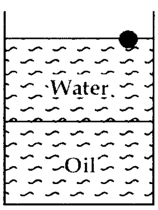
for $x = d$, $B_{bet} = 0$

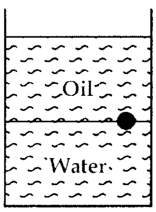
$x < d$, $B_{bet} \Rightarrow \hat{j}$

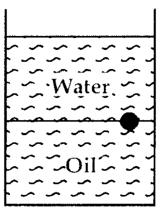
$x > d$, $B_{bet} \Rightarrow (-\hat{j})$

10. A ball is made of a material of density ρ where $\rho_{oil} < \rho < \rho_{water}$ with ρ_{oil} and ρ_{water} representing the densities of oil and water, respectively. The oil and water are immiscible. If the above ball is in equilibrium in a mixture of this oil and water, which of the following pictures represents its equilibrium positions?

(1) 

(2) 

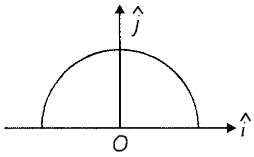
(3) 

(4) 

Ans (3)

High density liquid (water) will be at the bottom. As density of ball is in between oil and water, it floats in between.

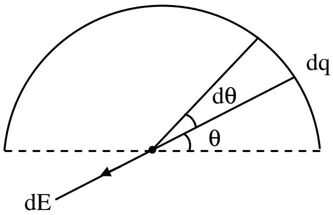
11. A thin semi-circular ring of radius r has a positive charge q distributed uniformly over it. The net field E at the centre O is



(1) $\frac{q}{2\pi^2 \varepsilon_0 r^2}\hat{j}$ (2) $\frac{q}{4\pi^2\varepsilon_0 r^2}\hat{j}$ (3) $-\frac{q}{4\pi^2\varepsilon_0 r^2}\hat{j}$ (4) $-\frac{q}{2\pi^2\varepsilon_0 r^2}\hat{j}$

Ans (4)

$\lambda = \frac{\varepsilon}{\pi r}$



$E = \int dE \sin\theta(-\hat{j})$

$= \int \frac{K}{r^2}\frac{qr}{\pi r^2}\, d\theta\, \sin\theta(-\hat{j})$

$= -\frac{q}{2\pi^2\varepsilon_0 r^2}\hat{j}$

12. A diatomic ideal gas is used in a Carnot engine as the working substance. If during the adiabatic expansion part of the cycle the volume of the gas increases from V to 32 V, the efficiency of the engine is

(1) 0.25 (2) 0.5 (3) 0.75 (4) 0.99

Ans (3)

For adiabatic process

$TV^{\gamma-1}$ = constant

$TV^{\frac{7}{5}-1} = T^1(32V)^{\frac{7}{5}-1}$

$T = 4T' \Rightarrow T' = \frac{T}{4}$

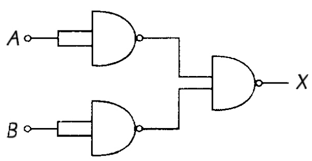
$\Rightarrow \eta = 1 - \frac{T'}{T} = 0.75$

13. The respective number of significant figures for the numbers 23.023, 0.0003 and 2.1×10^{-3} are

(1) 4, 4, 2 (2) 5, 1, 2 (3) 5, 1, 5 (4) 5, 5, 2

Ans (2)

14. The combination of gates shown below yields



(1) NAND gate (2) OR gate (3) NOT gate (4) XOR gate

Ans (2)

$\overline{\overline{A}\cdot\overline{B}} = \overline{\overline{A}} + \overline{\overline{B}} = A + B$

15. If a source of power 4 kW produces 10^{20} photons/second, the radiation belongs to a part of the spectrum called

(1) γ-rays (2) X-rays (3) Ultraviolet rays (4) Microwaves

Ans (2)

$p = nh\gamma$

$$\gamma = \frac{4\times10^{3}}{10^{20}\times6.023\times10^{-34}}$$

$= 6 \times 10^{16}$ Hz

This frequency belongs to X-rays.

16. A radioactive nucleus (initial mass number A and atomic number Z) emits 3 α-particles and 2 positrons. The ratio of number of neutrons to that of protons in the final nucleus will be

(1) $\dfrac{A-Z-4}{Z-2}$ (2) $\dfrac{A-Z-8}{Z-4}$

(3) $\dfrac{A-Z-4}{Z-8}$ (4) $\dfrac{A-Z-12}{Z-4}$

Ans (3)

Emission of α-particle results in decrease of mass number by 4 and decrease of atomic number by 2. Emission of position results in decrease of atomic (proton) number by 1 and increase neutron number by 1.

$\therefore\ n_p = Z - 3\times2 - 2\times1 = Z - 8$

$n_\eta = A - Z - 3\times2 + 2\times1 = A - A - 4$

$\therefore$ Ratio $= \dfrac{A-Z-4}{Z-8}$

17. Let there be a spherically symmetric charge distribution with charge density varying as

$\rho\ (r) = \rho_0\left(\dfrac{5}{4} - \dfrac{r}{R}\right)$ upto r = R, and $\rho(r) = 0$ for r > R, where r is the distance from the origin. The electric field at a distance r(r < R) from the origin is given by

(1) $\dfrac{\rho_0 r}{3\varepsilon_0}\left(\dfrac{5}{4} - \dfrac{r}{R}\right)$ (2) $\dfrac{4\pi\rho_0 r}{3\varepsilon_0}\left(\dfrac{3}{5} - \dfrac{r}{R}\right)$

(3) $\dfrac{\rho_0 r}{4\varepsilon_0}\left(\dfrac{5}{3} - \dfrac{r}{R}\right)$ (4) $\dfrac{4\rho_0 r}{3\varepsilon_0}\left(\dfrac{5}{4} - \dfrac{r}{R}\right)$

Ans (3)

$$dq = 4\pi r^2\ dr\rho_0\left(\frac{5}{4} - \frac{r}{R}\right)$$

$$\therefore\ q = 4\pi\rho_0\int_0^r\left(\frac{5}{4}r^2dr - \frac{r^3}{R}dr\right)$$

Using $\displaystyle\int EdA = \frac{q}{\varepsilon_0}$

$$E = \frac{\rho_0 r}{4\varepsilon_0}\left(\frac{5}{3} - \frac{r}{R}\right)$$

18. In a series LCR circuit R = 200 Ω and the voltage and the frequency of the main supply is 220 V and 50 Hz respectively. On taking out the capacitance from the circuit the current lags behind the voltage by 30º. On taking out the inductor from the circuit the current leads the voltage by 30º. The power dissipated in the LCR circuit is

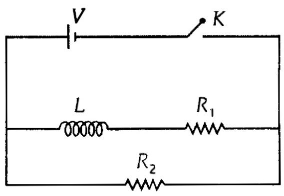
(1) 242 W (2) 305 W (3) 210 W (4) 0 W

Ans (1)

$X_L = X_C$

$\therefore\ P = \frac{V^2}{R} = 242\ W$

19. In the circuit shown below, the key K is closed at t = 0. The current through the battery is



(1) $\frac{V(R_1+R_2)}{R_1R_2}$ at $t=0$ and $\frac{V}{R_2}$ at $t=\infty$

(2) $\frac{VR_1R_2}{\sqrt{R_1^2+R_2^2}}$ at $t=0$ and $\frac{V}{R_2}$ at $t=\infty$

(3) $\frac{V}{R_2}$ at $t=0$ and $\frac{V(R_1+R_2)}{R_1R_2}$ at $t=\infty$

(4) $\frac{V}{R_2}$ at $t=0$ and $\frac{VR_1R_2}{\sqrt{R_1^2+R_2^2}}$ at $t=\infty$

Ans (3)

At $t=0,\ i=\frac{V}{R_2}$

At $t=\infty,\ i=\frac{V}{R_{eq}}=\frac{V(R_1+R_2)}{R_1R_2}$

20. A particles is moving with velocity $\vec{v} = K(y\hat{i} + x\hat{j})$, where K is a constant. The general equation for its path is

(1) $y^2 = x^2$ + constant (2) $y = x^2$ + constant

(3) $y^2 = x$ + constant (4) xy = constant

Ans (1)

$\vec{v} = K(y\hat{i} + x\hat{j})$

$\Rightarrow v_x = \frac{dx}{dt} = ky$

$v_y = \frac{dy}{dt} = Kx$

or $\frac{dy}{dx} = \frac{x}{y}$

or ydy = x dx

or $\frac{y^2}{2} = \frac{x^2}{2} + C$

or $y^2 = x^2$ + constant.

21. Let C be the capacitance of a capacitor discharging through a resistor R. Suppose t_1 is the time taken for the energy stored in the capacitor to reduce to half its initial value and t_2 is the time taken for the charge to reduce to one-fourth its initial value. Then the ratio $\frac{t_1}{t_2}$ will be

(1) 2 (2) 1 (3) $\frac{1}{2}$ (4) $\frac{1}{4}$

Ans (4)

$$U = \frac{q^2}{2C} = \frac{1}{2C}\left(q_0\, e^{-\frac{t}{\tau}}\right)^2 = \frac{q_0^2}{2C}\, e^{-\frac{2t}{\tau}}$$

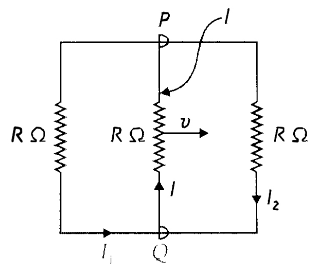
$$U = U_i\, e^{-\frac{2t}{\tau}}$$

$$\frac{U_1}{2} = U_i\, e^{-\frac{2t_1}{\tau}} \Rightarrow t_1 = \frac{T}{2} ln 2$$

$$q = q_0\; e^{-\frac{t}{\tau}}$$

$$\frac{q_0}{4} = q_0 e^{-\frac{t}{2T}} \Rightarrow \frac{t_1}{t_2} = \frac{1}{4}$$

22. A rectangular loop has a sliding connector PQ of length l and resistance R Ω and it is moving with a speed v as shown. The set-up is placed in a uniform magnetic field going into the plane of the paper. The three currents I_1, I_2 and I are



(1) $I_1 = -I_2 = \frac{Blv}{R},\quad I = \frac{Blv}{R}$

(2) $I_1 = -I_2 = \frac{Blv}{3R},\ I = \frac{2Blv}{3R}$

(3) $I_1 = I_2, I = \frac{2Blv}{3R}$

(4) $I_1 = I_2 = I = \frac{Blv}{6R}, I = \frac{Blv}{3R}$

Ans (2)

Induced emf, ε = Blv.

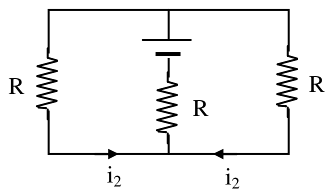
$i = i_1 + i_2$

KVL: $i_1R + iR - Blv = 0$

and $i_2R + iR - Blv = 0$

$$\Rightarrow i = \frac{2Blv}{3R}$$

$$i_1 = i_2 = \frac{Blv}{3R}.$$



23. The equation of a wave on a string of linear mass density 0.04 kg m^{-1} is given by

$$y = 0.02\ (m)\ \sin\left[2\pi\left(\frac{t}{0.04(s)} - \frac{x}{0.50\,(m)}\right)\right]$$. The tension in the string is

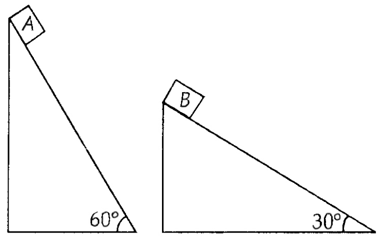
(1) 6.25 N (2) 4.0 N (3) 12.5 N (4) 0.5 N

Ans (1)

$$v = \frac{\omega}{k} = 12.5\ \ ms^{-1}$$

$$v = \sqrt{\frac{T}{\mu}} \Rightarrow T = \mu v^2 \ = \mu \frac{\omega^2}{k^2} \ = 6.25 \text{ N}$$

24. Two fixed frictionless inclined planes making an angle 30° and 60° with the vertical are shown in the figure. Two blocks A and B are placed on the two planes. What is the relative vertical acceleration of A with respect to B?



(1) 4.9 m s^{-2} in vertical direction
(2) 4.9 m s^{-2} in horizontal direction
(3) 9.8 m s^{-2} in vertical direction
(4) Zero

Ans (1)

$a_{Ay} = g \sin^2 60°$

$$\Rightarrow \ a_{Ay} = \frac{9.8 \times 3}{4}$$

$a_{By} = g \sin^2 30°$

$$\Rightarrow a_{By} = \frac{9.8}{4}$$

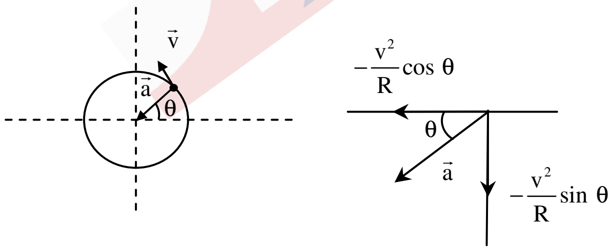
$$\Rightarrow a_{Ay} - a_{By} = \frac{9.8}{4}(3-1) = 4.9 \ \text{ms}^{-2}$$

25. For a particle in uniform circular motion, the acceleration **a** at a point P(R, θ) on the circle of radius R is (Here θ is measured from the x-axis)

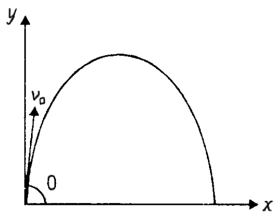
(1) $\frac{v^2}{R}\hat{i} + \frac{v^2}{R}\hat{j}$
(2) $-\frac{v^2}{R}\cos\theta\hat{i} + \frac{v^2}{R}\sin\theta\hat{j}$
(3) $-\frac{v^2}{R}\sin\theta\hat{i} + \frac{v^2}{R}\cos\theta\hat{j}$
(4) $-\frac{v^2}{R}\cos\theta\hat{i} - \frac{v^2}{R}\sin\theta\hat{j}$

Ans (4)

Refer to the diagram,



26. A small particle of mass m is projected at an angle θ with the x-axis with an initial velocity v_0 in the x-y plane as shown in the figure. At a time $t < \frac{v_0 \sin\theta}{g}$, the angular momentum of the particle is



(1) $\frac{1}{2}mgv_0 t^2 \cos\theta\hat{i}$
(2) $-mgv_0 t^2 \cos\theta\hat{j}$
(3) $mgv_0 t \cos\theta\hat{k}$
(4) $-\frac{1}{2} mgv_0 t^2 \cos\theta\hat{k}$

where $\hat{i}$, $\hat{j}$ and $\hat{k}$ are unit vectors along x, y and z-axis respectively.

Ans (4)

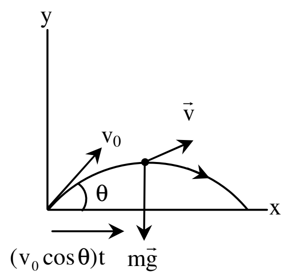
$$\frac{d\vec{L}}{dt} = \vec{\tau}$$

or $d\vec{L} = \vec{\tau}\ dt$

or $d\vec{L} = \left\{-(mg\ v_0\ \cos\ \theta t\ \hat{k}\right\} dt$

or $\vec{L} = \int d\vec{L} = \left\{-\ mg\ v_0\ \cos\ \theta\ \hat{k}\right\} \int t\, dt$

or $\vec{L} = -\frac{1}{2} mg\ v_0\ t^2 \cos\theta\ \hat{k}$



27. Two identical charged spheres are suspended by strings of equal lengths. The strings make an angle of 30° with each other. When suspended in a liquid of density 0.8 g cm^{-3}, the angle remains the same. If density of the material of the sphere is 1.6 g cm^{-3}, the dielectric constant of the liquid is

(1) 1 (2) 4 (3) 3 (4) 2

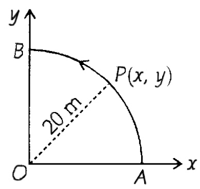
Ans (4)

$$\frac{F}{mg} = \tan\theta$$

and $$\frac{(F/K)}{mg\left(1 - \frac{\rho}{d}\right)} = \tan\theta$$

$$\Rightarrow K = \frac{1}{1 - \frac{\rho}{d}} = 2$$

28. A point P moves in counter-clockwise direction on a circular path as shown in the figure. The movement of P is such that it sweeps out a length s = t^3 + 5, where s is in metres and t is in seconds. The radius of the path is 20 m. The acceleration of P when t = 2 s is nearly



(1) 14 m / s^2 (2) 13 m / s^2

(3) 12 m / s^2 (4) 7.2 m / s^2

Ans (1)

$s = t^3 = 5$

$$v = \frac{ds}{dt} = 3t^2$$

at t = 2s, v = 12 ms^{-1}.

So $$a_r = \frac{v^2}{r} = \frac{144}{20} ms^{-2}$$

$$a_t = \frac{dv}{dt} = 6t$$

at t = 2s, a_t = 12 ms^{-2}

Thus, $a = \sqrt{a_r^2 + a_t^2} = \sqrt{7.2^2 + 12^2}$

$\cong \sqrt{50 + 144} \cong \sqrt{194} \cong 14\ ms^{-2}$

29. The potential energy function for the force between two atoms in a diatomic molecule is approximately given by $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$, where a and b are constants and x is the distance between the atoms. If the dissociation energy of the molecule is $D = [U(x = \infty) - U_{\text{at equilibrium}}]$, D is

(1) $\frac{b^2}{6a}$ (2) $\frac{b^2}{2a}$ (3) $\frac{b^2}{12a}$ (4) $\frac{b^2}{4a}$

Ans (4)

$U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$

$F = -\frac{dU}{dx} = -\left[-12ax^{-13} + 6bx^{-7}\right]$

At equilibrium, F = 0 or

$-12\, ax^{-13} + 6bx^{-7} = 0$

Or $x^{-6} = \frac{b}{2a} \Rightarrow \frac{1}{x^6} = \frac{b}{2a}$

Thus, $v_{eg} = a\left(\frac{1}{x^6}\right)^2 - b\left(\frac{1}{x^6}\right)$

$= a\frac{b^2}{4a^2} - \frac{b^2}{2a} \Rightarrow U_{eq} = -\frac{b^2}{4a}$

Thus, $D = U(\infty) - U(eq) = 0 - \left(-\frac{b^2}{4a}\right)$

Or $D = \frac{b^2}{4a}$

30. Two conductors have the same resistance at 0°C but their temperature coefficients of resistance are α_1 and α_2. The respective temperature coefficients of their series and parallel combinations are nearly

(1) $\frac{\alpha_1 + \alpha_2}{2}, \frac{\alpha_1 + \alpha_2}{2}$ (2) $\frac{\alpha_1 + \alpha_2}{2}, \alpha_1 + \alpha_2$

(3) $\alpha_1 + \alpha_2, \frac{\alpha_1 + \alpha_2}{2}$ (4) $\alpha_1 + \alpha_2, \frac{\alpha_1\alpha_2}{\alpha_1 + \alpha_2}$

Ans (1)

$R_1 = R_0(1 + \alpha_1 t_1)$

$R_2 = R_0(1 + \alpha_2 t)$

$R_s = R_1 + R_2$

$2R_0(1 + \alpha_s t) = 2R_0 + R_0(\alpha_1 t + \alpha_2 t)$

$\Rightarrow \alpha_s = \frac{\alpha_1 + \alpha_2}{2}$

$R_p = \frac{R_1 R_2}{R_1 + R_2} = \frac{R_0(1+\alpha_1 t)\, R_0(1+\alpha_2 t)}{R_0(1+\alpha_1 t) + R_0(1+\alpha_2 t)}$

$\frac{R_0}{2}(1+\alpha_p t) = \frac{R_0^2\,[1+\alpha_1 t+\alpha_2 t+\alpha_1\alpha_2 t^2]}{R_0[2+\alpha_1 t+\alpha_2 t]}$

On simplifying $\alpha_p = \frac{\alpha_1 + \alpha_2}{2}$

PART – B : CHEMISTRY

31. In aqueous solution the ionisation constants for carbonic acid are

$K_1 = 4.2 \times 10^{-7}$ and $K_2 = 4.8 \times 10^{-11}$

Select the correct statement for a saturated 0.034 M solution of the carbonic acid.

(1) The concentration of H^+ is double that of CO_3^{2-}

(2) The concentration of CO_3^{2-} is 0.034 M

(3) The concentration of CO_3^{2-} is greater than that of HCO_3^-

(4) The concentrations of H^+ and HCO_3^- are approximately equal

Ans (4)

32. Solubility product of silver bromide is 5.0×10^{-13}. The quantity of potassium bromide (molar mass taken as 120 g mol^{-1}) to be added to 1 litre of 0.05 M solution of silver nitrate to start the precipitation of AgBr is

(1) 5.0×10^{-8} g (2) 1.2×10^{-10} g (3) 1.2×10^{-9} g (4) 6.2×10^{-5} g

Ans (3)

K_{sp} (AgBr) = $[Ag^+][Br^-]$

$5.0 \times 10^{-13} = (0.05)\,[Br^-]$

$$\therefore [Br^-] = \frac{5.0\times10^{-13}}{0.05} = 1\times10^{-11}\,M$$

Weight of KBr required = Molarity × GMM × Vol (in L)

$= 1 \times 10^{-11} \times 120 \times 1$

$= 1.2 \times 10^{-9}$ g.

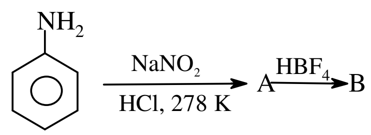
33. The correct sequence which shows decreasing order of the ionic radii of the elements is

(1) $O^{2-} > F^- > Na^+ > Mg^{2+} > Al^{3+}$ (2) $Al^{3+} > Mg^{2+} > Na^+ > F^- > O^{2-}$

(3) $Na^+ > Mg^{2+} > Al^{3+} > O^{2-} > F^-$ (4) $Na^+ > F^- > Mg^{2+} > O^{2-} > Al^{3+}$

Ans (1)

34. In the chemical reactions,



the compounds 'A' and 'B' respectively are

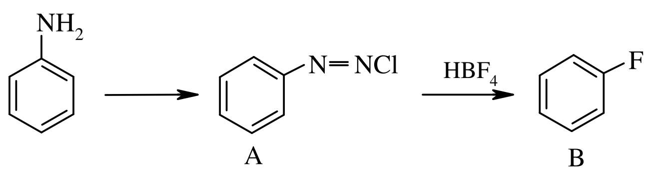
(1) Nitrobenzene and chlorobenzene

(2) Nitrobenzene and fluorobenzene

(3) Phenol and benzene

(4) Benzene diazonium chloride and fluorobenzene

Ans (4)



35. If 10^{-4} dm^3 of water is introduced into a 1.0 dm^3 flask at 300 K, how many moles of water are in the vapour phase when equilibrium is established?

(Given : Vapour pressure of H_2O at 300 K is 3170 Pa; R = 8.314 J K^{-1} mol^{-1})

(1) 1.27×10^{-3} mol (2) 5.56×10^{-3} mol (3) 1.53×10^{-2} mol (4) 4.46×10^{-2} mol

Ans (1)

$$n = \frac{PV}{RT} = \frac{3170 \times 10^{-3}}{8.314 \times 300} = 1.27 \times 10^{-3} \text{ mol}$$

36. From amongst the following alcohols the one that would react fastest with conc. HCl and anhydrous $ZnCl_2$, is

(1) 1-Butanol (2) 2-Butanol

(3) 2-Methylpropan-2-ol (4) 2-Methylpropanol

Ans (3)

Order of reactivity of alcohols towards Lucas reagent (Zn + conc. HCl) is 3° > 2° > 1°.

37. If sodium sulphate is considered to be completely dissociated into cations and anions in aqueous solution, the change in freezing point of water (ΔT_f), when 0.01 mol of sodium sulphate is dissolved in 1 kg of water, is (K_f = 1.86 K kg mol^{-1})

(1) 0.0186 K (2) 0.0372 K (3) 0.0558 K (4) 0.0744 K

Ans (1)

$\Delta T = K_f \times \text{molality}$

$= 1.86 \times 0.01 = 0.0186$ K.

38. Three reactions involving $H_2PO_4^-$ are given below

(i) $H_3PO_4 + H_2O \rightarrow H_3O^+ + H_2PO_4^-$ (ii) $H_2PO_4^- + H_2O \rightarrow HPO_4^{2-} + H_3O^+$

(iii) $H_2PO_4^- + OH^- \rightarrow H_3PO_4 + O^{2-}$

In which of the above does $H_2PO_4^-$ act as an acid?

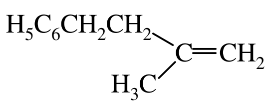
(1) (i) only (2) (ii) only (3) (i) and (ii) (4) (iii) only

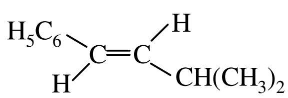
Ans (2)

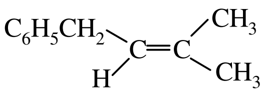
Only in equation (II), $H_2PO_4^-$ acts as proton donar.

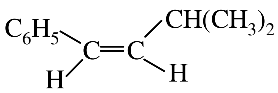
39. The main product of the following reaction is

$C_6H_5CH_2CH(OH)CH(CH_3)_2 \xrightarrow{\text{conc. } H_2SO_4}$

(1) 

(2) 

(3) 

(4) 

Ans (2)

Acid catalysed dehydration involves formation of stable carbocation and is anti-elimination.

40. The energy required to break one mole of Cl–Cl bonds in Cl_2 is 242 kJ mol^{-1}. The longest wavelength of light capable of breaking a single Cl–Cl bond is (c = 3×10^8 m s^{-1} and $N_A = 6.02 \times 10^{23}$ mol^{-1})

(1) 494 nm (2) 594 nm (3) 640 nm (4) 700 nm

Ans (1)

Energy = Nhν per mole

$$= Nh\frac{c}{\lambda}$$

$$242000 = \frac{6.02\times10^{23}\times6.62\times10^{-34}\times3\times10^{8}}{\lambda}$$

$$\therefore\ \lambda = \frac{6.02\times10^{23}\times6.62\times10^{-34}\times3\times10^{8}}{242000}$$

$= 0.000494 \times 10^{-3}$ m = 494 nm.

41. 29.5 mg of an organic compound containing nitrogen was digested according to Kjeldahl's method and the evolved ammonia was absorbed in 20 mL of 0.1 M HCl solution. The excess of the acid required 15 mL of 0.1 M NaOH solution for complete neutralisation. The percentage of nitrogen in the compound is

(1) 29.5 (2) 59.0 (3) 47.4 (4) 23.7

Ans (4)

Mass of organic compound (m) = 29.5 mg = 0.0295 g

Normality of HCl = 0.1 N

Volume of HCl used up = (20 − 15) = 5.0 cm^3

$$\text{Percentage of nitrogen} = \frac{1.4\times NV}{m}$$

$$= \frac{1.4\times0.1\times5.0}{0.0295} = 23.7\ \%$$

meq NH_3 = 0.5

mass of N in mg = 0.5 × 14

$$\%\ N = \frac{0.5}{29.5}\times14\times100 = 23.7\ \%$$

42. Ionisation energy of He^+ is 19.6×10^{-18} $atom^{-1}$. The energy of the first stationary state (n = 1) of Li^{2+} is

(1) 8.82×10^{-17} J $atom^{-1}$ (2) 4.41×10^{-16} J $atom^{-1}$

(3) -4.41×10^{-17} J $atom^{-1}$ (4) -2.2×10^{-15} J $atom^{-1}$

Ans (3)

According to Bohr's atomic model, $E \propto \frac{Z^2}{n^2}$ and $\frac{E_1 n_1^2}{Z_1^2} = \frac{E_2 n_2^2}{Z_2^2}$

For $n_1 = n_2 = 1$, we can write $\frac{E_1}{Z_1^2} = \frac{E_2}{Z_2^2}$ or $\frac{19.6\times10^{-18}}{2^2} = \frac{E_2}{3^2}$

$$\therefore\ E_2 = \frac{19.6\times10^{-18}}{4}\times9$$

$$= 4.41\times10^{-17}\,\text{J atom}^{-1}$$

∴ The energy of the first stationary state of Li^{2+} = $-\,4.41 \times 10^{-17}$ J $atom^{-1}$.

43. On mixing, heptane and octane from an ideal solution. At 373 K, the vapour pressures of the two liquid components (heptane and octane) are 105 kPa and 45 kPa respectively. Vapour pressure of the solution obtained by mixing 25.0 g of heptane and 35 g of octane will be (molar mass of heptane = 100 g mol^{-1} and of octane = 114 g mol^{-1})

(1) 144.5 kPa (2) 72.0 kPa (3) 36.1 kPa (4) 96.2 kPa

Ans (2)

We know that heptane and octane form an ideal mixture.

$$P_{total} = X_A P_A^o + X_B \; P_B^o$$
$$= 0.45 \times 105 + 0.55 \times 45$$
$$= 47.25 + 24.75$$
$$= 72.0 \text{ k } P_a$$

Where X_A is the mole fraction of heptane

$$X_A = \frac{n_A}{n_A + n_B} = \frac{\frac{25}{100}}{\frac{25}{100} + \frac{35}{114}} = \frac{0.25}{0.25 + 0.3} = \frac{0.25}{0.55} = 0.45$$

X_B is the molefraction of octane.

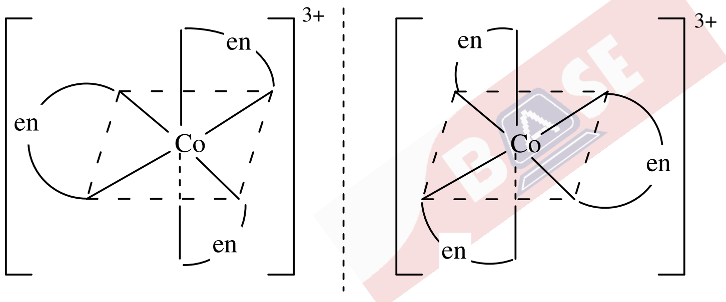
$$X_B = \frac{n_B}{n_A + n_B} = \frac{\frac{35}{114}}{\frac{25}{100} + \frac{35}{114}} = \frac{0.3}{0.25 + 0.3} = \frac{0.3}{0.55} = 0.55$$

44. Which one of the following has an optical isomer?

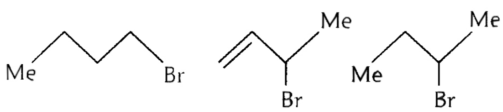
(1) $[Zn(en)_2]^{2+}$ (2) $[Zn(en)(NH_3)_2]^{2+}$ (3) $[Co(en)_3]^{3+}$ (4) $[Co(H_2O)_4(en)]^{3+}$

Ans (3)

$[Co(en)_3]^{3+}$ has an optical isomer



45. Consider the following bromides



(A) (B) (C)

The correct order of $S_N{}^1$ reactivity is

(1) A > B > C (2) B > C > A (3) B > A > C (4) C > B > A

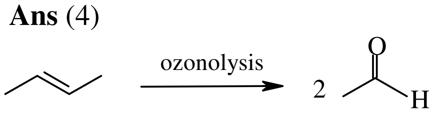
Ans (1)

Higher the stability of carbocation greater the reactivity towards S_{N^1} reactions.

46. One mole of a symmetrical alkene on ozonolysis gives two moles of an aldehyde having a molecular mass of 44 u. The alkene

(1) Ethane (2) Propene (3) 1-butene (4) 2-butene

Ans (4)



47. Consider the reaction

$Cl_2\,(aq) + H_2S\,(aq) \rightarrow S(s) + 2H^+(aq) + 2Cl^-(aq)$.

The rate equation for this reaction is rate = k $[Cl_2]\,[H_2S]$.

Which of these mechanisms is/are consistent with this rate equation?

A. $Cl_2 + H_2S \rightarrow H^+ + Cl^- + Cl^+ + HS^-$ (slow)

$Cl^+ + HS^- \rightarrow H^+ + Cl^- + S$ (fast)

B. $H_2S \Leftrightarrow H^+ + HS^-$ (fast equilibrium)

$Cl_2 + HS^- \rightarrow 2Cl^- + H^+ + S$ (slow)

(1) A only (2) B only

(3) Both (A) & (B) (4) Neither (A) nor (B)

Ans (1)

48. The Gibbs energy for the decomposition Al_2O_3 at 500°C is as follows

$\frac{2}{3}Al_2O_3 \rightarrow \frac{4}{3}Al + O_2,\quad \Delta_rG = +966\,kJ\,mol^{-1}$

The potential difference needed for electrolytic reduction of Al_2O_3 at 500°C is at least

(1) 5.0 V (2) 4.5 V (3) 3.0 V (4) 2.5 V

Ans (4)

$\frac{2}{3}\,Al_2O_3 \rightarrow \frac{4}{3}\,Al + O_2 \qquad \Delta_rG = +966\ kJ\ mol^{-1}$

$2Al_2O_3 \rightarrow 4Al + 3O_2$

$Al_2O_3 \rightarrow 2Al + \frac{3}{2}O_2 \qquad \Delta_rG = +\frac{966\times 3}{2} = 1449\ kJ\ /\ mol$

We know that $\Delta G° = -nFE°$

$-\,\Delta G° = nFE°$

$1449000 = 6 \times 96500 \times E°$

$\therefore\ E° = \frac{1449000}{6\times 96500} = 2.5V$

49. The correct order of increasing basicity of the given conjugate bases ($R = CH_3$) is

(1) $RCO\overline{O} < HC \equiv \overline{C} < \overline{N}H_2 < \overline{R}$

(2) $RCO\overline{O} < HC \equiv \overline{C} < \overline{R} < \overline{N}H_2$

(3) $\overline{R} < HC \equiv \overline{C} < RCO\overline{O} < \overline{N}H_2$

(4) $RCO\overline{O} < \overline{N}H_2 < HC \equiv \overline{C} < \overline{R}$

Ans (1)

Lesser the stability of conjugate base greater the basicity and weaker be its conjugate acid.

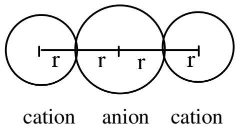
50. The edge length of a face centered cubic cell of an ionic substance is 508 pm. If the radius of the cation is 110 pm, the radius of the anion is

(1) 144 pm (2) 288 pm (3) 398 pm (4) 618 pm

Ans (1)

Edge length = 508 pm = $2(r_+ + r_-)$; $r_- = \frac{508}{2} - 110 = 144\ pm$

Radius of cation = 110 pm



$\therefore$ Diameter of anion = $508 - 2 \times 110$

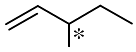
$= 288$ pm

$\therefore$ Radius of anion = $\frac{288}{2} = 144$ pm

51. Out of the following, the alkene that exhibits optical isomerism is

(1) 2-methyl-2-pentene
(2) 3-methyl-2-pentene
(3) 4-methyl-1-pentene
(4) 3-methyl-1-pentene

Ans (4)

 (3-methyl-1-pentene) has one chiral carbon, hence, exhibits optical activity.

52. For a particular reversible reaction at temperature T, ΔH and ΔS were found to be both +ve. If T_e is the temperature at equilibrium, the reaction would be spontaneous when

(1) $T = T_e$
(2) $T_e > T$
(3) $T > T_e$
(4) T_e is 5 times T

Ans (3)

$\Delta G = \Delta H(+ve) - T\Delta S\ (+ve)$

At equilibrium, $\Delta G = 0$.

ΔG becomes −ve (i.e., the reaction becomes spontaneous) only if the temperature T is greater than the temperature at equilibrium (T_e).

53. Percentages of free space in cubic close packed structure and in body centered packed structure are respectively

(1) 48% and 26%
(2) 30% and 26%
(3) 26% and 32%
(4) 32% and 48%

Ans (3)

In CCP lattice, packing efficiency is 74 %

$\therefore$ Free space = 100 − 74 = 26 %

In bcc lattice, packing efficiency is 68 %

$\therefore$ Free space = 100 − 68 = 32 %

54. The polymer containing strong intermolecular forces e.g. hydrogen bonding, is

(1) Natural rubber
(2) Teflon
(3) Nylon 6, 6
(4) Polystyrene

Ans (3)

55. At 25°C, the solubility product of $Mg(OH)_2$ is 1.0×10^{-11}. At which pH, will Mg^{2+} ions start precipitating in the form of $Mg(OH)_2$ from a solution of 0.001 M Mg^{2+} ions?

(1) 8
(2) 9
(3) 10
(4) 11

Ans (3)

$K_{sp} = [Mg^{2+}][OH^-]^2$

$1.0 \times 10^{-11} = (0.001)\ (OH^-)^2$ or $[OH^-]^2 = \frac{1.0\times10^{-11}}{0.001} \Rightarrow 1\times10^{-8}$

or $[OH^{-1}] = \sqrt{1\times10^{-8}} = 1\times10^{-4}$ M

$\therefore$ pOH = 4 or pH = 10

56. The correct order of $E^0_{\frac{M^{2+}}{M}}$ values with negative sign for the four successive elements Cr, Mn, Fe and Co is

(1) Cr > Mn > Fe > Co　　(2) Mn > Cr > Fe > Co

(3) Cr > Fe > Mn > Co　　(4) Fe > Mn > Cr > Co

Ans (2)

57. Biuret test is not given by

(1) Proteins　　(2) Carbohydrates　　(3) Polypeptides　　(4) Urea

Ans (1)

Biuret test is not answered by carbohydrates

58. The time for half life period of a certain reaction A → Products is 1 h. When the initial concentration of the reactant 'A', is 2.0 mol L^{-1}, how much time does it take for its concentration to come from 0.50 to 0.25 mol L^{-1} if it is a zero order reaction?

(1) 1 h　　(2) 4 h　　(3) 0.5 h　　(4) 0.25 h

Ans (4)

$$t_{\frac{1}{2}} = \frac{A_0}{2k} \quad k = 1 \text{ mol } L^{-1} \quad , \quad t_{\frac{1}{2}} = \frac{0.5}{2\times 1} = 0.25 \text{ h}$$

59. A solution containing 2.675 g of $CoCl_3.6NH_3$ (molar mass = 267.5 g mol^{-1}) is passed through a cation exchanger. The chloride ions obtained in solution were treated with excess of $AgNO_3$ to give 4.78 g of AgCl (molar mass = 143.5 g mol^{-1}). The formula of the complex is (At. mass of Ag = 108 u)

(1) $[CoCl(NH_3)_5]Cl_2$　　(2) $[Co(NH_3)_6]Cl_3$

(3) $[CoCl_2(NH_3)_4]Cl$　　(4) $[CoCl_3(NH_3)_3]$

Ans (2)

Number of moles of $COCl_3.6NH_3 = \dfrac{2.675g}{267.5gmol^{-1}} = 0.01$ mol

Number of moles of AgCl precipitated $= \dfrac{4.78g}{143.5gmol^{-1}} = 0.033$ mol

∴ 0.01 mole of the complex precipitates 0.033 mole of Cl^- ions in the form of AgCl. As, such the formula of the complex is $[Co(NH_3)_6]Cl_3$

60. The standard enthalpy of formation of NH_3 is –46.0 kJ mol^{-1}. If the enthalpy of formation of H_2 from its atoms is –436 kJ mol^{-1} and that of N_2 is –712 kJ mol^{-1}, the average bond enthalpy of N–H bond is NH_3 is

(1) –1102 kJ mol^{-1}　　(2) –964 kJ mol^{-1}　　(3) +352 kJ mol^{-1}　　(4) +1056 kJ mol^{-1}

Ans (3)

$$\frac{1}{2}N_2 + \frac{3}{2}H_2 \rightarrow NH_3$$

$$\Delta H_{r\times n} = \sum BE(\text{reactants}) - \sum BE(\text{products})$$

$$-46.0 = \frac{1}{2}(-712) + \frac{3}{2}(-436) - 3\times BE(N-H)$$

$$-46.0 = -356 - 654 - 3\times BE\,(N-H) \qquad \therefore\ 3\times BE(N-H) = -964$$

$$\therefore\ BE(N-H) = -\frac{964}{3} = -321.3 \text{ kJ mol}^{-1}$$

Average bond dissociation energy of N–H bond in NH_3 is ≈ 352 kJ mol^{-1}

PART – C : MATHEMATICS

61. Consider the following relations:

$R = \{(x, y) \mid x, y$ are real numbers and $x = wy$ for some rational number $w\}$;

$S = \left\{\left(\frac{m}{n}, \frac{p}{q}\right) \middle|\right.$ m, n, p and q are integers such that $n, q \neq 0$ and $qm = pn\}$. Then

(1) R is an equivalence relation but S is not an equivalence relation

(2) Neither R nor S is an equivalence relation

(3) S is an equivalence relation but R is not an equivalence relation

(4) R and S both are equivalence relations

Ans (3)

$R = \{(x, y) / x, y$ are real numbers and $x = \omega y$ for some rational number $\omega\}$

$S = \left\{\left(\frac{m}{n}, \frac{p}{q}\right) / m, n, p, q \text{ are integers such that } n, q \neq 0 \text{ and } qm = pn\right\}$

Let us consider the relation R

(i) R is reflexive: $\because$ for any $x \in R$, $x = 1.x$ where 1 is rational

$\Rightarrow (x, x) \in R \quad \forall x$

(ii) R is not symmetric: Clearly $(0, \sqrt{2}) \in R$ since $0 = 0.\sqrt{2}$ where 0 is rational

But $(\sqrt{2}, 0) \notin R$ since $\sqrt{2} = \omega.0$ is not true for any ω rational.

$\Rightarrow$ R is not an equivalence relation

Let us now consider the relation S

(i) S is reflexive: $\because$ for $\frac{m}{n}$ such that $m, n \in Z$ and $n \neq 0$

We have $mn = nm$

$\Rightarrow \left(\frac{m}{n}, \frac{m}{n}\right) \in S \qquad \forall$ such $\frac{m}{n}$

(ii) S is symmetric: we have $\frac{m}{n}$ and $\frac{p}{q}$ such that $n, q \neq 0$ and $m, n, p, q \in Z$

$qm = pn$ iff $np = mq$

$\Leftrightarrow \left(\frac{m}{n}, \frac{p}{q}\right) \in S$ iff $\left(\frac{p}{q}, \frac{m}{n}\right) \in S$

(iii) S is transitive: $\left(\frac{m}{n}, \frac{p}{q}\right) \in S$ and $\left(\frac{p}{q}, \frac{r}{s}\right) \in S$

Clearly $m, n, p, q, r, s \in Z$ and $n, q, s \neq 0$

And $\quad qm = pn \quad \ldots (1)$

$\quad sp = qr \quad \ldots (2)$

Take product of equation (1) and (2), we get

$pqms = pqrn$

$\Rightarrow ms = rn$

$\Rightarrow \left(\frac{m}{n}, \frac{r}{s}\right) \in S$

$\Rightarrow$ S is an equivalence relation

62. The number of complex numbers z such that $|z - 1| = |z + 1| = |z - i|$ equals

(1) 0 (2) 1 (3) 2 (4) ∞

Ans (2)

We need to find the number of complex numbers z obeying the equation $|z - 1| = |z + 1| = |z - i|$

Clearly the above equation says that z is equidistant from the points 1, −1, i. Since 1, −1, i are non-collinear points, it forms a unique triangle for which there is a unique circumcentre. Hence, there exist only one such z.

63. If α and β are the roots of the equation $x^2 - x + 1 = 0$, then $\alpha^{2009} + \beta^{2009} =$

(1) –2 (2) –1 (3) 1 (4) 2

Ans (3)

Given α and β are the roots of the equation $x^2 - x + 1 = 0$.

Let us multiply the given equation by x + 1 both sides, we get

$x^3 + 1 = 0 \Rightarrow x = (-1)^{\frac{1}{3}} \Rightarrow x = -1, -\omega, -\omega^2$

where ω is the complex cube root of unity.

$\because$ x + 1 is a factor we introduced, we do not consider its corresponding root x = – 1. $\therefore$ $x = -\omega$ or $-\omega^2$

$$\text{So, } \alpha^{2009} + \beta^{2009} = (-\omega)^{2009} + (-\omega^2)^{2009}$$

$$= -\left[\omega^{2009} + (\omega^2)^{2009}\right]$$

$$= -\left[\omega^2 + (\omega^2)^2\right] \quad (\because 2009 = 3k + 2 \text{ for some } k \in Z)$$

$$= -\left[\omega^2 + \omega\right] = 1$$

64. Consider the system of linear equations:

$x_1 + 2x_2 + x_3 = 3$

$2x_1 + 3x_2 + x_3 = 3$

$3x_1 + 5x_2 + 2x_3 = 1$

The system has

(1) Infinite number of solutions (2) Exactly 3 solutions

(3) A unique solutions (4) No solution

Ans (4)

$x_1 + 2x_2 + x_3 = 3$ . . . (1)

$2x_1 + 3x_2 + x_3 = 3$ . . . (2)

$3x_1 + 5x_2 + 2x_3 = 1$ . . . (3)

Adding equations (1) and (2), we get $3x_1 + 5x_2 + 2x_3 = 6$. But according to (3), $3x_1 + 5x_2 + 2x_3 = 1$.

Hence, no solution.

65. There are two urns. Urn A has 3 distinct red balls and urn B has 9 distinct blue balls. From each urn two balls are taken out at random and then transferred to the other. The number of ways in which this can be done is

(1) 3 (2) 36 (3) 66 (4) 108

Ans (4)

$3C_2 \times 9C_2 = 3 \times 36 = 108$

66. Let $f : (-1, 1) \rightarrow R$ be a differentiable function with $f(0) = -1$ and $f'(0) = 1$. Let $g(x) = [f(2f(x) + 2)]^2$. Then $g'(0) =$

(1) 4 (2) –4 (3) 0 (4) –2

Ans (2)

Given $f : (-1, 1) \rightarrow R$ is a differentiable function with $f(0) = -1$

And $f'(0) = 1$ and $g(x) = [f(2f(x) +2)]^2$. We need to find $g'(0)$

$g'(x) = 2[f(2f(x)+2)][f'(2f(x)+2)][2f'(x)]$

Put $x = 0$, $g'(0) = 2[f(2f(0)+2)][f'(2f(0) + 2)]\,[2f'(0)] = -4$

67. Let $f : R \rightarrow R$ be a positive increasing function with $\lim_{x\to\infty} \frac{f(3x)}{f(x)} = 1$. Then $\lim_{x\to\infty} \frac{f(2x)}{f(x)}$

(1) 1 (2) $\frac{2}{3}$ (3) $\frac{3}{2}$ (4) 3

Ans (1)

Given that $f : R \rightarrow R$ is a positive increasing function.

and $\lim_{x\to\infty} \frac{f(3x)}{f(x)} = 1$

$\because f(x)$ is an increasing function $f(x) < f(2x) < f(3x)\ \forall x > 0$

Let us divide the above inequality by $f(x)$ throughout

We get $1 < \frac{f(2x)}{f(x)} < \frac{f(3x)}{f(x)}\ \ \forall x > 0$

(note that the inequality is preserved only because $f(x) > 0\ \forall x \in R$)

Now we use sandwich theorem to the above inequality and conclude that $\lim_{x\to\infty} \frac{f(2x)}{f(x)} = 1$

68. Let $p(x)$ be a function defined on R such that $\lim_{x\to\infty} \frac{f(3x)}{f(x)} = 1$, $p'(x) = p'(1-x)$, for all $x \in [0, 1]$, $p(0) = 1$ and $p(1) = 41$. Then $\int_0^1 p(x)\,dx$ equals

(1) $\sqrt{41}$ (2) 21 (3) 41 (4) 42

Ans (2)

Given, $p(x)$ is function defined on 1R such that

$p'(x) = p'(1-x)\quad \forall x \in [0, 1]$

and $p(0) = 1$ $p(1) = 41$

We need to find $\int_0^1 p(x)dx$

Since, we have $p'(x) = p'(1-x)\ \ \forall x \in [0, 1]$

Let us take definite integral over the interval $[0, y]$ where $y \in [0, 1]$

On both sides, we get

$$\int_0^y p'(x) = \int_0^y p'(1-x)dx$$

$\Rightarrow [p(x)]_0^y = [-p(1-x)]_0^y$ [by fundamental theorem of calculus]

$\Rightarrow p(y) - p(0) = -p(1 - y) + p(1)$

$\Rightarrow p(y) + p(1 - y) = p(0) + p(1) = 42\ \forall\ y \in [0, 1]$

Again take definite integral over the interval [0, 1], we get

$$\int_0^1 p(y)dy + \int_0^1 p(1-y)dy = \int_0^1 42dy$$

$$\Rightarrow 2\int_0^1 p(y)dy = 42 \qquad \left[\because \int_a^b f(x)dx = \int_a^b f(a+b-x)dx\right]$$

$$\Rightarrow \int_0^1 p(y)dy = 21$$

69. A person is to count 4500 currency notes. Let a_n denote the number of notes he counts in the n^{th} minute. If $a_1 = a_2 = \ldots = a_{10} = 150$ and $a_{10}, a_{11}, \ldots$ are in an AP with common difference –2, then the time taken by him to count all notes is

(1) 24 minutes (2) 34 minutes (3) 125 minutes (4) 135 minutes

Ans (2)

A person is to count 4500 current notes.

a_n denotes the number of notes he counts in the n^{th} minute

if $a_i = 150 \ \forall \ i = 1, 2, \ldots 10$ and $a_{10}, a_{11} \ldots$ are in A.P. with common difference -2

Now, we need to find the time taken by him to count all the notes.

Let it take 'p' minutes for him to count all the notes.

Clearly $p > 10$. Otherwise if $p \le 10$ he can count maximum of 1500 notes only

$$\therefore \sum_{i=1}^{P} a_i = \sum_{i=1}^{10} a_i + \sum_{i=11}^{P} a_i = \sum_{i=1}^{10} 150 + \sum_{i=11}^{P} [150-(i-10)2]$$

$$= \sum_{i=1}^{10} 150 + \sum_{i=11}^{P} (170-2i) = 1500 + 170(p-10)$$

$$= -p(p+1) + 10$$

$$\Rightarrow 4500 = \sum_{i=1}^{P} a_i = -p(p+1) + 170p - 90$$

$$\Rightarrow p(p+1) - 170p + 4590 = 0$$

$$\Rightarrow p^2 - 169p + 4590 = 0$$

$$\Rightarrow (p-135)(p-34) = 0$$

Clearly, we can seen that $170 - 2i < 0$ of $i > 85$ and $170 - 2i$ represents the number of notes counted on i^{th} minute when $i > 10$. So, $170 - 2i$ cannot be negative.

$\Rightarrow p = 34$ minutes.

70. The equation of the tangent to the curve $y = x + \frac{4}{x^2}$, that is parallel to the x-axis, is

(1) $y = 0$ (2) $y = 1$ (3) $y = 2$ (4) $y = 3$

Ans (4)

The given curve is $y = x + \frac{4}{x^2}$ and it is understood that $x \neq 0$. A tangent of the above curve will be parallel to x-axis iff its derivative is zero as the given function is a differentiable function.

$$y' = 1 - \frac{8}{x^3} = 0 \Rightarrow x^3 = 8 \Rightarrow x = 2 \Rightarrow y = 3$$

$\therefore$ The point of contact of the tangent with zero slope is (2, 3)

$\Rightarrow$ Equation of the tangent is $y = 3$.

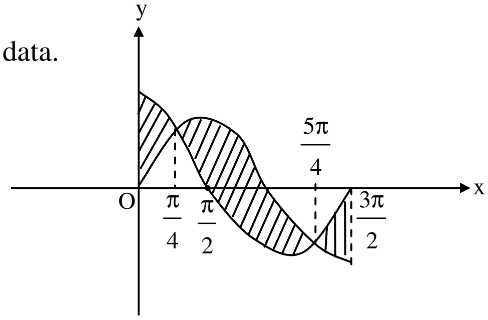
71. The area bounded by the curves y = cos x and y = sin x between the ordinates x = 0 and $x = \frac{3\pi}{2}$ is

(1) $4\sqrt{2} - 2$ (2) $4\sqrt{2} + 2$ (3) $4\sqrt{2} - 1$ (4) $4\sqrt{2} + 1$

Ans (1)

We need to find the area bounded by the curves y = cos x and y = sin x between the ordinates x = 0 and $x = \frac{3\pi}{2}$

Let us draw a rough diagram of the graph as per the given data.



clearly, $\cos x \geq \sin x \ \forall \ x \in \left[0, \frac{\pi}{4}\right]$

$\cos x \leq \sin x \ \forall \ x \in \left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$

$\cos x \geq \sin x \ \forall \ x \in \left[\frac{5\pi}{4}, \frac{3\pi}{2}\right]$

Required area = $\int_0^{\frac{3\pi}{2}} |\cos x - \sin x| dx$

$= \int_0^{\frac{\pi}{4}} |\cos x - \sin x| dx + \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} |\cos x - \sin x| dx + \int_{\frac{5\pi}{4}}^{\frac{3\pi}{2}} |\cos x - \sin x| dx$

$= \int_0^{\frac{\pi}{4}} (\cos x - \sin x) dx + \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} (\sin x - \cos x) dx + \int_{\frac{5\pi}{4}}^{\frac{3\pi}{2}} (\cos x - \sin x) dx$

$= [\sin x + \cos x]_0^{\frac{\pi}{4}} + [-\cos x - \sin x]_{\frac{\pi}{4}}^{\frac{5\pi}{4}} + [\sin x + \cos x]_{\frac{5\pi}{4}}^{\frac{3\pi}{2}}$

$= \left(\sqrt{2} - 1\right) + 2\sqrt{2} + \left(-1 + \sqrt{2}\right)$

$= 4\sqrt{2} - 2$

72. Solution of the differential equation $\cos x dy = y(\sin x - y)dx, \ 0 < x < \frac{\pi}{2}$ is

(1) secx = (tanx + c)y (2) ysecx = tanx + c

(3) ytanx = secx + c (4) tanx = (secx + c)y

Ans (1)

Given differential equation is

cos x dy = y (sin x − y)dx

cos x dy = y sinx dx − y^2 dx

cosx dy − ysinx dx = $-y^2$ dx

dividing throughout by $-y^2 \cos^2 x$, we get

$-\frac{1}{y^2} \sec x dy + \frac{1}{y} \sec x \tan x dx = \sec^2 x dx$

$$d\left(\frac{\sec x}{y}\right) = d(\tan x)$$

$$\Rightarrow \frac{\sec x}{y} = \tan x + c \Rightarrow \sec x = (\tan x + c)y$$

73. Let $\vec{a} = \hat{j} - \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$. Then the vector $\vec{b}$ satisfying $\vec{a} \times \vec{b} + \vec{c} = \vec{0}$ and $\vec{a} \cdot \vec{b} = 3$ is

(1) $-\hat{i} + \hat{j} - 2\hat{k}$ (2) $2\hat{i} - \hat{j} + 2\hat{k}$ (3) $\hat{i} - \hat{j} - 2\hat{k}$ (4) $\hat{i} + \hat{j} - 2\hat{k}$

Ans (1)

Given $\vec{a} = \hat{j} - \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$

we need to find $\vec{b}$ such that $(\vec{a} \times \vec{b}) + \vec{c} = \vec{0}$ and $\vec{a}.\,\vec{b} = 3$

Let $\vec{b} = x\hat{i} + y\hat{j} + z\hat{k}$

Now $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & -1 \\ x & y & z \end{vmatrix} = -\hat{i} + \hat{j} + \hat{k} = -\vec{c}$

$\Rightarrow (y + z)\hat{i} - x\hat{j} - x\hat{k} = -\hat{i} + \hat{j} + \hat{k}$

$\Rightarrow x = -1,\ y + z = -1$ … (1)

Now, $\vec{a} \cdot \vec{b} = 3$

$\Rightarrow (\hat{j} - \hat{k}).\,(x\hat{i} + y\hat{j} + z\hat{k}) = 3$

$\Rightarrow y - z = 3$ … (2)

From (1) and (2), we get y = 1, z = –2

$\therefore\ \vec{b} = -\hat{i} + \hat{j} - 2\hat{k}$

74. If the vectors $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$, $\vec{b} = 2\hat{i} + 4\hat{j} + \hat{k}$ and $\vec{c} = \lambda\hat{i} + \hat{j} + \mu\hat{k}$ are mutually orthogonal, then $(\lambda, \mu) =$

(1) (–3, 2) (2) (2, –3) (3) (–2, 3) (4) (3, –2)

Ans (1)

Let $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$, $\vec{b} = 2\hat{i} + 4\hat{j} + \hat{k}$ and $\vec{c} = \lambda\hat{i} + \hat{j} + \mu\hat{k}$ and it is given that $\vec{a}, \vec{b}, \vec{c}$ are mutually orthogonal.

$\Rightarrow \vec{a} \times \vec{b} = k\,\vec{c}$ for some scalar

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ 2 & 4 & 1 \end{vmatrix} = k\left(\lambda\hat{i} + \hat{j} + \mu\hat{k}\right)$$

$$-9\hat{i} + 3\hat{j} + 6\hat{k} = k\left(\lambda\hat{i} + \hat{j} + \mu\hat{k}\right)$$

By comparing the x, y, z components, we get

$k = 3,\ \lambda = -3,\ \mu = 2$

$\Rightarrow (\lambda, \mu) = (-3, 2)$

75. If two tangents drawn from a point P to the parabola $y^2 = 4x$ are at right angles, then the locus of P is

(1) $x = 1$ (2) $2x + 1 = 0$ (3) $x = -1$ (4) $2x - 1 = 0$

Ans (3)

Given P is a point such that two tangents drawn from P to the parabola $y^2 = 4x$ are at angles or in other words, P is a point of intersection a pair of perpendicular tangents of a parabola $y^2 = 4x$.

We know that the locus of points of intersection of perpendicular tangents of a parabola is a line and that line is the directrix.

$\Rightarrow$ P is a point on the directrix of $y^2 = 4x$

$\Rightarrow$ locus of P is $x = -1$

76. The line L given by $\frac{x}{5}+\frac{y}{b}=1$ passes through the point (13, 32). The line K is parallel to L and has the equation $\frac{x}{c}+\frac{y}{3}=1$. Then the distance between L and K is

(1) $\frac{23}{\sqrt{15}}$ (2) $\sqrt{17}$ (3) $\frac{17}{\sqrt{15}}$ (4) $\frac{23}{\sqrt{17}}$

Ans (4)

Equation of line L is $\frac{x}{5}+\frac{y}{b}=1$ and it passes through (13, 32) $\Rightarrow \frac{13}{5}+\frac{32}{b}=1 \Rightarrow b=-20$

Equation of line K is $\frac{x}{c}+\frac{y}{3}=1$ and it is parallel to line L

$\Rightarrow \frac{-b}{5}=\frac{-3}{c} \Rightarrow -4=\frac{3}{c} \Rightarrow c=\frac{-3}{4}$

Equation of line L can be written as $4x - y = 20$

Equation of line K can be written as $4x - y = -3$

$\therefore$ Distance between lines L and K is $\frac{|20-(-3)|}{\sqrt{4^2+(-1)^2}}=\frac{23}{\sqrt{17}}$

77. A line AB in three-dimensional space makes angles 45° and 120° with the positive x-axis and the positive y-axis respectively. If AB makes an acute angle θ with the positive z-axis, then θ equals

(1) 30° (2) 45° (3) 60° (4) 75°

Ans (3)

Given that the direction angles of a line segment AB is 45°, 120°, θ° where $0 < \theta < 90$

We need to find θ.

$\Rightarrow$ DCs of the line segment AB is cos 45°, cos 120°, cos θ°

$\Rightarrow \cos^2 45° + \cos^2 120° + \cos^2\theta = 1$

$\Rightarrow \frac{1}{2}+\frac{1}{4}+\cos^2\theta=1 \Rightarrow \cos^2\theta=\frac{1}{4} \Rightarrow \cos\theta=\frac{1}{2}$

$\Rightarrow \theta=\frac{\pi^c}{3}=60° \quad \left(\cos\theta\neq-\frac{1}{2} \because \theta \text{ is acute}\right)$

78. Let S be a non-empty subset of R Consider the following statement :

P : There is a rational number $x \in S$ such that $x > 0$.

Which of the following statements is the negation of the statement P?

(1) There is a rational number $x \in S$ such that $x \leq 0$

(2) There is no rational number $x \in S$ such that $x \leq 0$

(3) Every rational number $x \in S$ satisfies $x \leq 0$

(4) $x \in S$ and $x \leq 0 \Rightarrow x$ is not rational

Ans (3)

Given that S is a non-empty subset of R.

P : There is a rational number $x \in S$ such that $x > 0$

Now, we need to find the negation of P

Clearly, P is equivalent to saying that "There is a positive rational number in S"

So, its negation – P is "There exist no positive rational in S"

– P: There exist no positive rational number in S

$\Leftrightarrow$ –P: Every rational number $x \in S$ satisfies $x \le 0$.

79. Let $\cos(\alpha+\beta)=\frac{4}{5}$ and let $\sin(\alpha-\beta)=\frac{5}{13}$, where $0 \le \alpha, \beta \le \frac{\pi}{4}$. Then $\tan 2\alpha =$

(1) $\frac{25}{16}$ (2) $\frac{56}{33}$ (3) $\frac{19}{12}$ (4) $\frac{20}{7}$

Ans (2)

Given that $\cos(\alpha+\beta)=\frac{4}{5}$ and $\sin(\alpha+\beta)=\frac{5}{13}$, $0 \le \alpha, \beta \le \frac{\pi}{4}$ we need to find $\tan 2\alpha$.

First let us try to write $\cos(\alpha+\beta)$ and $\sin(\alpha-\beta)$ both in terms of tan.

$\cos(\alpha+\beta)=\frac{4}{5} \Rightarrow \tan(\alpha+\beta)=\frac{3}{4}$ $\left(\text{by triangle method also } \because \alpha+\beta \in \left[0, \frac{\pi}{2}\right]\right)$

$\sin(\alpha-\beta)=\frac{5}{13} \Rightarrow \tan(\alpha-\beta)=\frac{5}{12}$ $\left(\begin{array}{l}\text{by triangle method also, } \because \alpha-\beta \in \left[0, \frac{\pi}{4}\right] \\ \text{Note: } \alpha-\beta \notin \left[-\frac{\pi}{4}, 0\right] \because \sin(\alpha-\beta)>0\end{array}\right)$

$\therefore \tan 2\alpha = \tan[(\alpha+\beta)+(\alpha-\beta)]$

$$= \frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta)\tan(\alpha-\beta)} = \frac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}.\frac{5}{12}} = \frac{56}{33}$$

80. The circle $x^2+y^2=4x+8y+5$ intersects the line $3x-4y=m$ at two distinct points if

(1) $-85 < m < -35$ (2) $-35 < m < 15$ (3) $15 < m < 65$ (4) $35 < m < 85$

Ans (2)

Given circle is $x^2+y^2=4x+8y+5$

Which can be rewritten as $(x-2)^2+(y-4)^2=25$

Now we need to find the possible values of m for which the line $3x-4y=m$ intersects the circle at two distinct points.

A straight line will intersect a given circle at two distinct points iff the distance of the line from the centre of the circle is strictly less than the radius of the circle.

$$\Rightarrow \frac{|3(2)-4(4)-m|}{\sqrt{3^2+(-4)^2}} < 5$$

$\Rightarrow |m+10| < 25 \Rightarrow -25 < m+10 < 25 \Rightarrow -35 < m < 15$

81. For two data sets, each of size 5, the variances are given to be 4 and 5 and the corresponding means are given to be 2 and 4, respectively. The variance of the combined data set is

(1) $\frac{5}{2}$ (2) $\frac{11}{2}$ (3) 6 (4) $\frac{13}{2}$

Ans (2)

$E(X^2)-(E(X))^2=4$

$\therefore E(X^2)=4+4=8$

$\sum X_i^2 = 40$

$E(Y^2) - (E(Y))^2 = 5$

$E(Y^2) = 5 + 16 = 21$

$\therefore \sum Y_i^2 = 105$

$\sum X_i = 10,\ \sum Y_i = 20$

$\therefore\ \sum(X_i + Y_i) = 30$

$\sum(X_i^2 + Y_i^2) = 145.$

$\therefore$ Variance (combined data) = $\frac{145}{10} - 9 = \frac{55}{10} = \frac{11}{2}$

82. An urn contains nine balls of which three are red, four are blue and two are green. Three balls are drawn at random without replacement from the urn. The probability that the three balls have different colours is

(1) $\frac{1}{3}$ (2) $\frac{2}{7}$ (3) $\frac{1}{21}$ (4) $\frac{2}{23}$

Ans (2)

$$\text{Probability} = \frac{{}^3C_1 \times {}^4C_1 \times {}^2C_1}{{}^9C_3} = \frac{24 \times 6}{9 \times 8 \times 7} = \frac{2}{7}$$

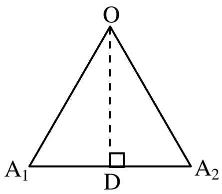
83. For a regular polygon, let r and R be the radii of the inscribed and the circumscribed circles. A false statement among the following is

(1) There is a regular polygon with $\frac{r}{R} = \frac{1}{2}$ (2) There is a regular polygon with $\frac{r}{R} = \frac{1}{\sqrt{2}}$

(3) There is a regular polygon with $\frac{r}{R} = \frac{2}{3}$ (4) There is a regular polygon with $\frac{r}{R} = \frac{\sqrt{3}}{2}$

Ans (3)

Consider a regular n-polygon, from its centre to each of the vertex draw a line segment, those n-line segments will divide the n-polygon into n number of similar triangles which will look as in the following diagram.



$OA_1 = OA_2 = R$

OD is a median

Also OD = r

And $\angle A_1OA_2 = \frac{2\pi}{n} \Rightarrow \angle DOA_2 = \frac{\pi}{n}$

$\because$ OD is also an angle bisector of top angle.

So, from right triangle ODA_2, we get $\cos\frac{\pi}{n} = \frac{r}{R}$

If n = 4 then $\frac{r}{R} = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

If n = 6 then $\frac{r}{R} = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

If n = 3 then $\frac{r}{R} = \cos\frac{\pi}{3} = \frac{1}{2}$

$\because \frac{1}{2} < \frac{2}{3} < \frac{1}{\sqrt{2}} \Rightarrow \cos\frac{\pi}{3} < \cos\theta < \cos\frac{\pi}{4}$ where $\theta = \cos^{-1}\frac{2}{3}$

$\Rightarrow \frac{\pi}{3} > \theta > \frac{\pi}{4}$ $\quad\because$ cos x is decreasing over $\left(\theta, \frac{\pi}{2}\right)$

But $\exists$ no. $n \in Z$ such $\theta = \frac{\pi}{n}$ because if $\exists$ such a n, then $\frac{\pi}{3} > \frac{\pi}{n} > \frac{\pi}{4}$ $\Rightarrow 3 < n < 4$ and $n \in Z$ which is a contradiction.

84. The number of 3×3 non-singular matrices, with four entries as 1 and all other entries as 0, is

(1) Less than 4 (2) 5 (3) 6 (4) At least 7

Ans (4)

We need to find the number of 3×3 non-singular matrices with four entries as 1 and all other entries as 0.

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ Here we can fill any one of those 6 boxes with 1 and rest all with leading diagonal elements being 1. Hence these are the matrices with four 1 and five 0 and it is non-singular. We see that there are six possible matrices in this particular format.

And consider $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$, this is another matrix with four 1 and five 0 and it is non-singular.

Hence, there are at least 7 such matrices.

85. Let $f : R \to R$ be defined by $f(x) = \begin{cases} k - 2x, & \text{if } \quad x \le -1 \\ 2x + 3, & \text{if } \quad x > -1 \end{cases}$

If f has a local minimum at $x = -1$, then a possible value of k is

(1) 1 (2) 0 (3) $-\frac{1}{2}$ (4) -1

Ans (4)

Given f: $R \to R$ is defined by

$$f(x) = \begin{cases} k - 2x, & \text{if } \; x \le -1 \\ 2x + 3, & \text{if } \; x > -1 \end{cases}$$

And f has a local minimum at $x = -1$

$\Rightarrow \exists\ \delta > 0$ such that $f(-1) \le f(x) \quad \forall\ x \in (-1 - \delta, -1 + \delta)$

$\Rightarrow k + 2 \le f(x) \quad \forall\ x \in (-1-\delta, -1+\delta)$

Clearly $\quad k + 2 \le f(x) \quad \forall\ x \in (-1-\delta, -1] \quad \because f(x) = k - 2x$

And $\quad k + 2 \le f(x) \quad \forall\ x \in (-1, -1 + \delta)$

$\Rightarrow \quad k + 2 \le 2x + 3 \quad \forall\ x \in (-1, -1 + \delta)$

$\Rightarrow k + 2 \le \lim_{x \to -1^+} 2x + 3 = 1$

$\Rightarrow k \le -1$

Directions : **Questions number 86 to 90 are Assertion – Reason type questions**. Each of these questions contains two statements.

Statement-1 : (Assertion) and **Statement-2 : (Reason)**.

Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select the correct choice.

86. Four numbers are chosen at random (without replacement) from the set $\{1, 2, 3, \ldots, 20\}$.

Statement-1 : The probability that the chosen numbers when arranged in some order will form an AP is $\frac{1}{85}$

Statement-2 : If the four chosen numbers from an AP, then the set of all possible values of common difference is $\{+1, +2, +3, \quad +4, +5\}$.

(1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

(2) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1

(3) Statement-1 is true, Statement-2 is false

(4) Statement-1 is false, Statement-2 is true

Ans (3)

Let d be common difference. Let x be numbers of arrangements with common difference d. Let d = 1.

$\{\{1, 2, 3, 4\}, \ldots, \{17, 18, 19, 20\}\}$ $\quad\therefore$ x = 17

Let d = 2

$\{\{1, 3, 5, 7\}, \ldots, \{14, 16, 18, 20\}\}$ $\quad$ x = 14

Let d = 3

$\{\{1, 4, 7, 10\}, \ldots, \{11, 14, 17, 20\}\}$ $\quad$ x = 11

Let d = 4

$\{\{1, 5, 9, 13\}, \ldots, \{8, 12, 16, 20\}\}$ $\quad$ x = 8

Let d = 5

$\{\{1, 6, 11, 16\}, \ldots, \{5, 10, 15, 20\}\}$ $\quad$ x = 5

Let d = 6

$\{\{1, 7, 13, 19\}, \{2, 8, 14, 20\}\}$ $\quad$ x = 2

$$\therefore \text{Probability} = \frac{57}{{}^{20}C_4} = \frac{57 \times 24}{20 \times 19 \times 18 \times 17} = \frac{1}{85}$$

Since d can be ±6 also, statement 2 is false.

87. Let $S_1 = \sum_{j=1}^{10} j(j-1)\,{}^{10}C_j$, $S_2 = \sum_{j=1}^{10} j\,{}^{10}C_j$ and $S_3 = \sum_{j=1}^{10} j^2 C_j$

Statement-1 : $S_3 = 55 \times 2^9$

Statement-2 : $S_1 = 90 \times 2^8$ and $S_2 = 10 \times 2^8$

(1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

(2) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1

(3) Statement-1 is true, Statement-2 is false

(4) Statement-1 is false, Statement-2 is true

Ans (3)

$$S_3 = \sum_{j=1}^{10} j^2\, 10C_j = \sum_{j=1}^{10} j^2 \frac{10!}{j!\,(10-j)!} = 10\sum_{j=1}^{10} j \frac{9!}{(j-1)!\,(10-j)!}$$

$$S_3 = 10\sum_{j=1}^{10} (j-1) \frac{9!}{(j-1)!\,(10-j)!} + \sum_{j=1}^{10} 9C_{j-1}$$

$$S_3 = 10\left[9\sum_{j=1}^{10} \frac{8!}{(j-2)!\,(10-j)!} + \sum_{j=1}^{10} 9C_{j-1}\right]$$

$$S_3 = 10\left[9\sum_{j=1}^{10} 8C_{j-2} + \sum_{j=1}^{10} 9C_{j-1}\right]$$

$$S_3 = 10\left[9\left(2^8\right) + 2^9\right] = 110.2^8 = 55.2^9$$

$$S_2 = \sum_{j=1}^{10} j10C_j = \sum_{j=1}^{10} j^2 \frac{10!}{j!\,(10-j)!} = 10\sum_{j=1}^{10} 9C_{j-1} = 10.2^9$$

$$S_1 = \sum_{j=1}^{10} j(j-1)10C_j = S_3 - S_2 = 45.2^9 = 90.2^8$$

Clearly, statement 1 is true and statement 2 is false.

88. **Statement-1** : The point A(3, 1, 6) is the mirror image of the point B(1, 3, 4) in the plane $x - y + z = 5$.
Statement-2 : The plane $x - y + z = 5$ bisects the line segment joining A(3, 1, 6) and B(1, 3, 4).
(1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1
(2) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1
(3) Statement-1 is true, Statement-2 is false
(4) Statement-1 is false, Statement-2 is true
Ans (2)
Statement 1: The point A (3, 1, 6) is the mirror image of the point B (1, 3, 4) in the plane $x - y + z = 5$.
Let M be the midpoint AB $\Rightarrow$ M = (2, 2, 5)
Clearly M is a point of the given plane as it satisfies the equation $x - y + z = 5$.
D.Rs of the line segment joining AB is (2, –2, 2) and D.Rs of the normal of the given plane is (1, –1, 1)
$\because$ these two D.Rs are proportional and the plane bisects AB.
We can conclude that A is the mirror image of B in the given plane.
Statement 1 is true.
Statement 2: The plane $x - y + z = 5$ bisects the plane segment joining A (3, 1, 6) and B (1, 3, 4)
Statement 2 is also true.
But, statement 2 by itself cannot imply statement 1.
$\therefore$ statement 1 is true, statement 2 is true; statement 2 is not correctly explanation for statement 1.

89. Let f : R $\rightarrow$ R be a continuous function defined by $f(x) = \dfrac{1}{e^x + 2e^{-x}}$

Statement-1 : $f(c) = \dfrac{1}{3}$, for some $c \in R$.

Statement-2 : $0 < f(x) \le \dfrac{1}{2\sqrt{2}}$, for all $x \in R$.

(1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1
(2) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1
(3) Statement-1 is true, Statement-2 is false
(4) Statement-1 is false, Statement-2 is true
Ans (1)
Let f : R $\rightarrow$ R be a continuous function defined as follows

$$f(x) = \frac{1}{e^x + 2e^{-x}}$$

We know that if a, b $\ge$ 0 then $a + b \ge 2\sqrt{ab}$
Put $a = e^x$, $b = 2e^{-x}$

Clearly a, b > 0

$\Rightarrow\ e^x + 2e^{-x} \geq 2\sqrt{e^x.2g^{-x}} = 2\sqrt{2} \qquad \forall\ x\in R$

$\Rightarrow\ 0 < \dfrac{1}{e^x+2e^{-x}} \leq \dfrac{1}{2\sqrt{2}}$

$\Rightarrow\ 0 < f(x) \leq \dfrac{1}{2\sqrt{2}} \qquad \forall\ x\in R$

$\therefore$ Statement 2 is true and $\because\ \dfrac{1}{3} \in \left(0, \dfrac{1}{2\sqrt{2}}\right]$

$\Rightarrow \exists\ c\in R$ such that $f(c) = \dfrac{1}{3}$

$\therefore$ so statement 1 is also true

$\therefore$ Statement 1 is true, statement 2 is true; statement 2 is a correct explanation for statement 1.

90. Let A be a 2 × 2 matrix with non-zero entries and let $A^2 = I$, where I is 2 × 2 identity matrix. Define Tr(A) = sum of diagonal elements of A and |A| = determinant of matrix A.

Statement-1 : Tr(A) = 0.

Statement-2 : |A| = 1.

(1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

(2) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1

(3) Statement-1 is true, Statement-2 is false

(4) Statement-1 is false, Statement-2 is true

Ans (3)

Given A is a 2×2 matrix with non-zero entries and $A^2 = I$

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad a, b, c, d \neq 0$

And $A^2 = I \Rightarrow \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} a^2+bc & b(a+d) \\ c(a+d) & bc+d^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow a + d = 0 \qquad \because\ b, c \neq 0$

$\Rightarrow Tr(A) = 0$

Now, $A^2 = I \Rightarrow |A^2| = |I|$

$\Rightarrow |A|^2 = 1$

All that we can conclude is $|A| = \pm 1$

We cannot say $|A| = \pm 1$. In fact $\exists$ a counter example to this statement 2.

Let $A = \begin{bmatrix} \dfrac{1}{\sqrt{2}} & \dfrac{1}{\sqrt{2}} \\ \dfrac{1}{\sqrt{2}} & -\dfrac{1}{\sqrt{2}} \end{bmatrix}$

Clearly, $A^2 = I$ and A has only non-zero entries.

But $|A| = -1$

$\therefore$ statement 1 is true, statement 2 is false.

* * *