Sensor Placement for RANS-based Data Assimilation Using Eigenspace Perturbations

Omid Bidar, PhD student, University of Sheffield

Supervisors: Dr Sean Anderson (ACSE) and Prof Ning Qin (MEC)

Data Driven Methods in Fluid Dynamics
Leeds Institute for Fluids Mechanics, March 2023

Outline

Motivations

Turbulence:

- Investigated using experiments and/or CFD
- Need to trade-off level of fidelity against costs

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*Duraisamy, Iaccarino, and Xiao, 2019, Annual Review of Fluid Mechanics

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Turbulence Reynolds-average Navier-Stokes modelling:

- Solve conservation laws and turbulence model equation(s)
- RANS widely used due to simplicity and low computational cost
- Turbulence models prone to inaccuracies in complex cases
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Can we determine, *a priori*, sparse sensor placement configurations to experimentally generate the required data?

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Test case:

- Separated flow over NASA 2D hump
- $M_{\text{ref}} = 0.1$; $Re_c = 936,000$
- Data from wall-resolved LES[†]

Uzun *et al.*, 2018, AIAA Journal

Outline

Eigenspace perturbation

‡ Iaccarino *et al.*, 2017, Physical Review Fluids

Eigenspace perturbation

Eigenvalue decomposition of the Reynolds stress tensor:

$$R_{ij} = \langle u_i u_j \rangle = 2k \left(\frac{\delta_{ij}}{3} + v_{in} \Lambda_{nl} v_{lj} \right), \tag{1}$$

with k: turbulent kinetic energy (R_{ij} amplitude), v: eigenvector matrix (R_{ij} orientation), and Λ: diagonal matrix of eigenvalues (R_{ij} shape)[‡].

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Eigenspace perturbation in practice

- Transform the eigenvalues to the barycentric triangle, and perturb to the 3 limits of R_{ij} anisotropy (i.e. vertices of the triangle). (x3 CFD simulations)

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Eigenspace perturbation in practice

- Transform the eigenvalues to the barycentric triangle, and perturb to the 3 limits of R_{ij} anisotropy (i.e. vertices of the triangle). (x3 CFD simulations)
- Perturb the eigenvectors to the 2 extremal states of the TKE production mechanism. (x2 CFD simulations)

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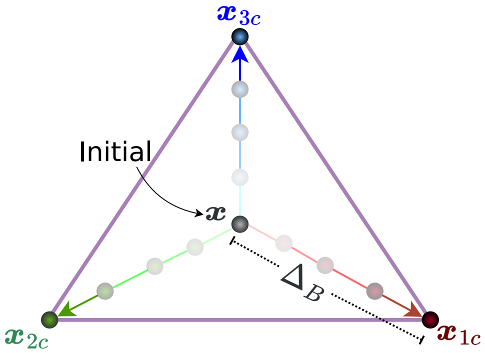
Eigenvalue perturbations

§ Banerjee *et al.*, 2007, Journal of Turbulence

Eigenvalue perturbations

Turbulence anisotropy in the Barycentric map§:

$$\boldsymbol{x} = \boldsymbol{x_{1c}}(\lambda_1 - \lambda_2) + \boldsymbol{x_{2c}}(2\lambda_2 - 2\lambda_3) + \boldsymbol{x_{3c}}(3\lambda_3 + 1),$$



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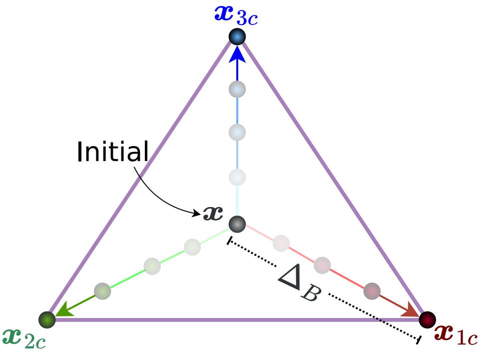

- Perturbing the eigenvalues:

$$\boldsymbol{x}^* = \boldsymbol{x} + \Delta_B(\boldsymbol{x}^{(t)} - \boldsymbol{x}),$$
$$\lambda_l^* = (1 - \Delta_B)\lambda_l + \Delta_B B^{-1}\boldsymbol{x}^{(t)},$$

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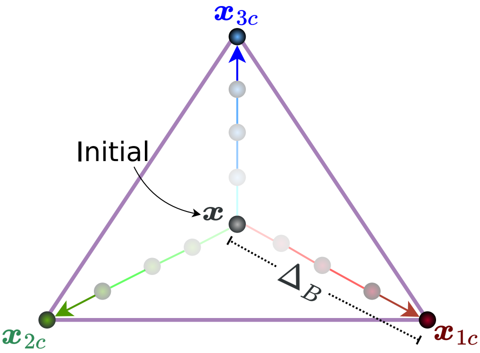

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- Bounds of turbulence anisotropy:

$$B^{-1}\boldsymbol{x}_{1c} = (2/3, -1/3, -1/3)^T$$
$$B^{-1}\boldsymbol{x}_{2c} = (1/6, 1/6, -1/3)^T$$
$$B^{-1}\boldsymbol{x}_{3c} = (0, 0, 0)^T$$

§Banerjee *et al.*, 2007, Journal of Turbulence

Eigenvector perturbations

¶ Iaccarino *et al.*, 2017, Physical Review Fluids

Eigenvector perturbations

- Turbulent kinetic energy production: $\mathcal{P}_k = -\underbrace{\langle u_i u_j \rangle}_{R} \underbrace{\frac{\partial U_i}{\partial x_j}}_{A}.$
- Modulate TKE by varying the Frobenius inner product: $\langle A, R \rangle_F = tr(AR)$.
- Bounds are given by: $\langle A, R \rangle_F \in [\lambda_1\gamma_3 + \lambda_2\gamma_2 + \lambda_3\gamma_1, \lambda_1\gamma_1 + \lambda_2\gamma_2 + \lambda_3\gamma_3]$ where $\lambda_1 \geq \lambda_2 \geq \lambda_3$ are the eigenvalues of the strain rate tensor (symmetric components of A).
- In the coordinate system defined by the eigenvectors (γ_l) of the mean strain tensor, the eigenvector bounds are¶:

$$v_{\text{min}} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \qquad v_{\text{max}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

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Streamwise velocity profiles UQ

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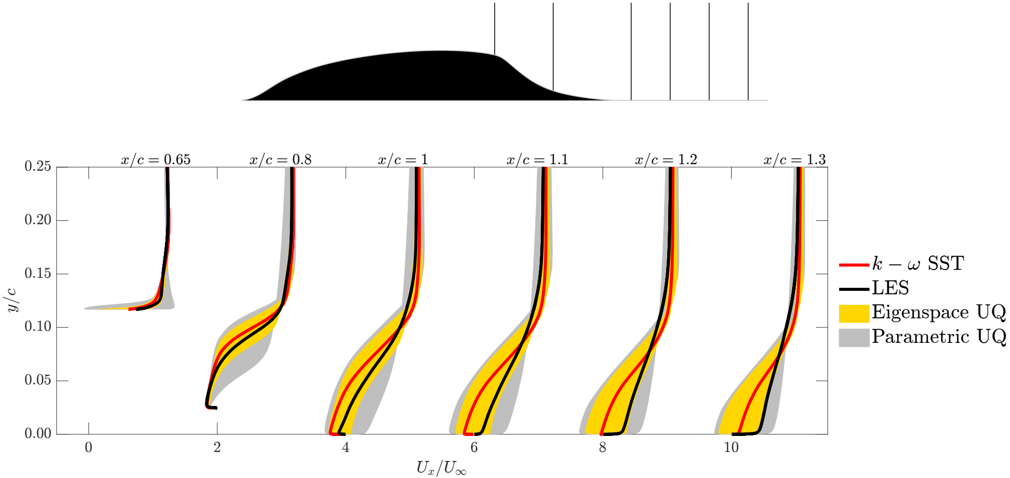

Uncertainty map, $var(U_x)$

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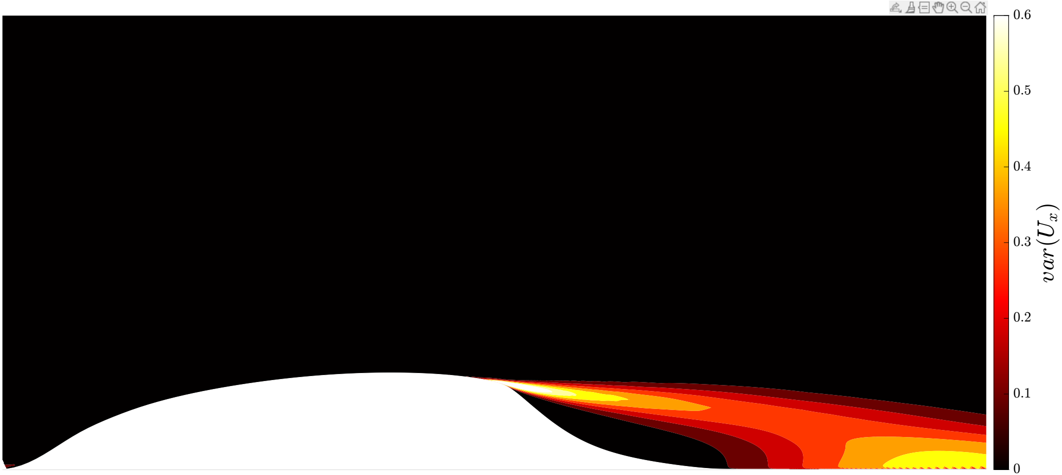
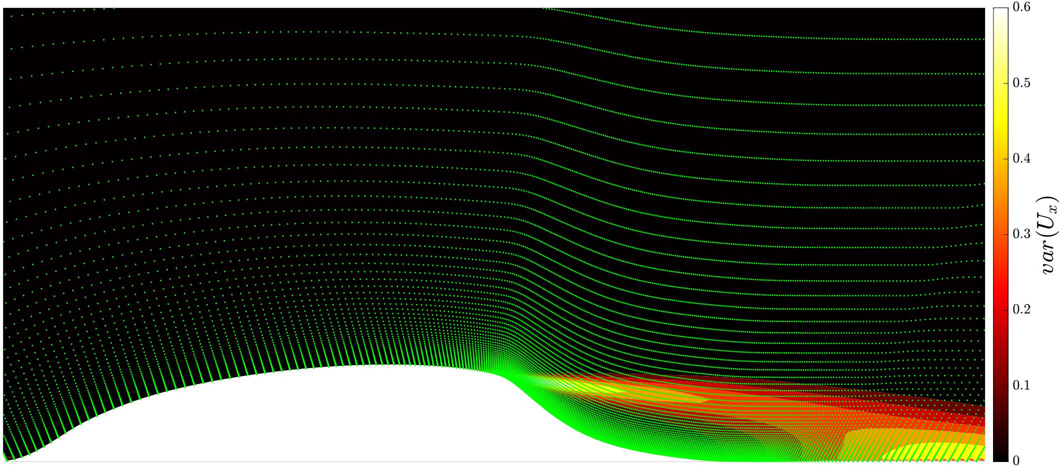


Sensor placement



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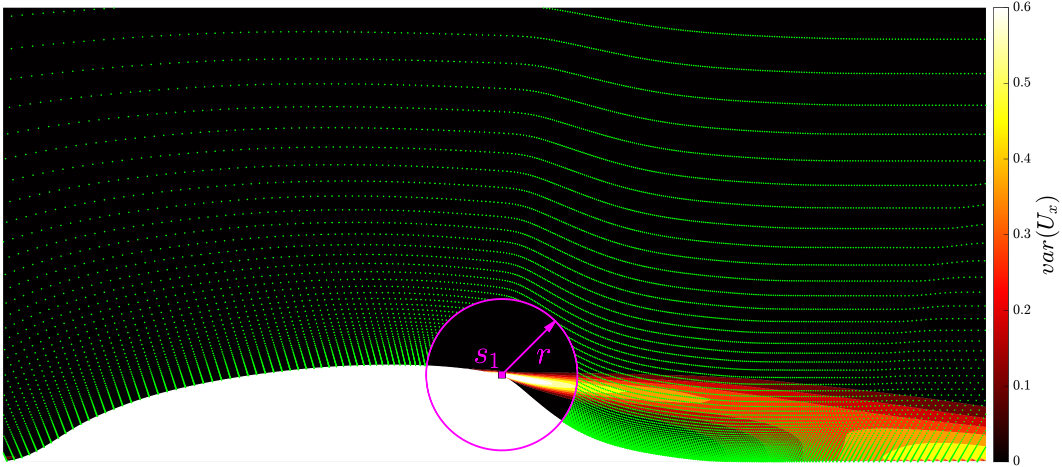

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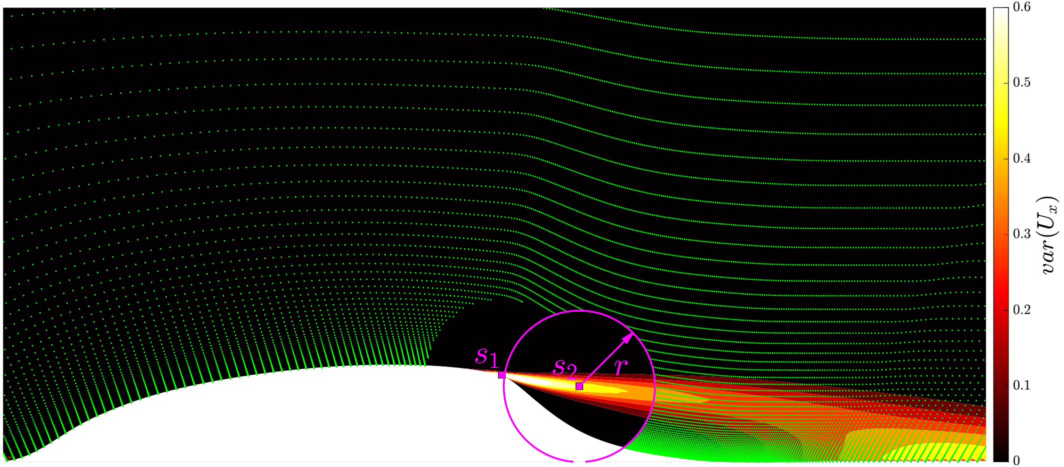

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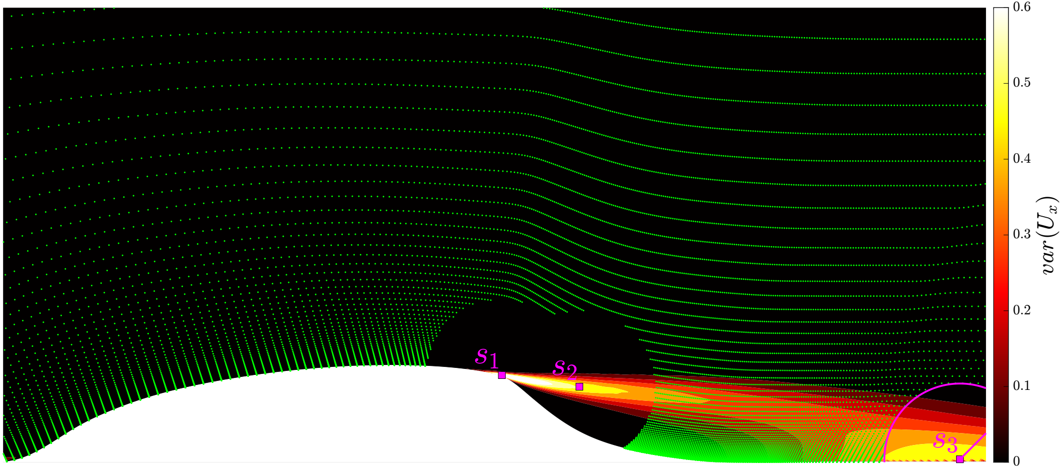

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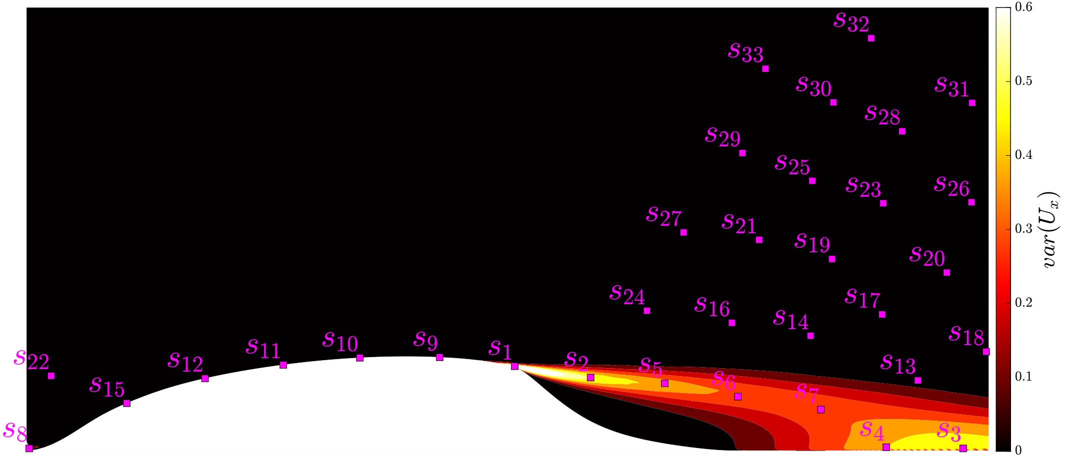

Adjoint-based field inversion

Formulation

‖ Singh *et al.*, 2017, AIAA Journal

** Bidar *et al.*, 2022, AIAA AVIATION 2022 Forum, AIAA 2022-4125

Adjoint-based field inversion

Formulation

1. Perturb turbulence transport equation(s) by a spatial field[‖] $\beta \in \mathbb{R}^{N_{\text{cells}}}$:

$$\frac{D\omega}{Dt} = \beta\left(\boldsymbol{x}\right) \cdot \text{production} + \text{transport} - \text{dissipation}.$$

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2. Find optimum β that minimises error between data Y_d and RANS predictions $Y_m(\beta)$:

$$\min_{\beta} \quad \mathcal{L} = \|Y_m(\beta) - Y_d\|_2^2 + \underbrace{\lambda\|\beta - 1\|_2^2}_{\text{regularisation}}.$$

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Pros: model-consistent; can work with limited data

Cons: highly-dimensional optimisation problem ► open-source implementation[**]

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Reconstructed streamwise velocity profiles, $r = 0.1c, N_s = 15$

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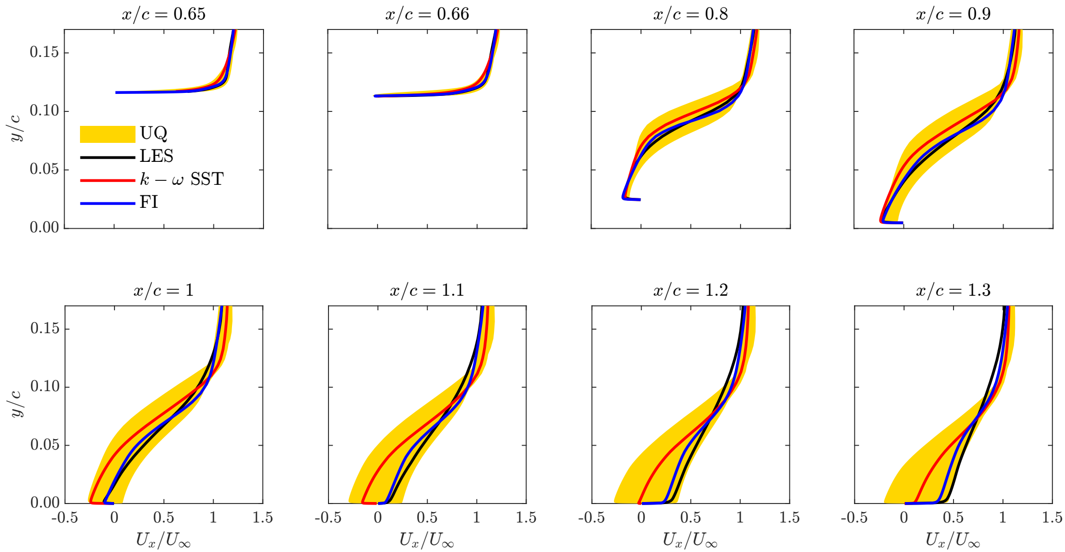

Reconstructed surface pressure

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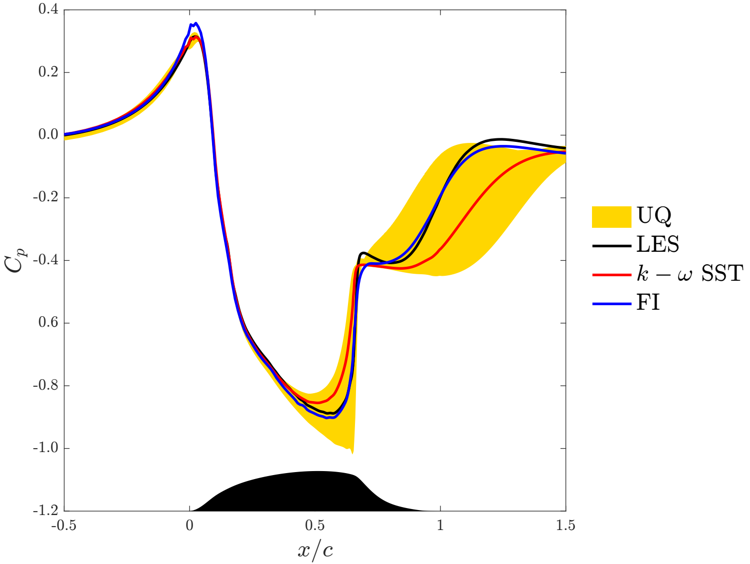
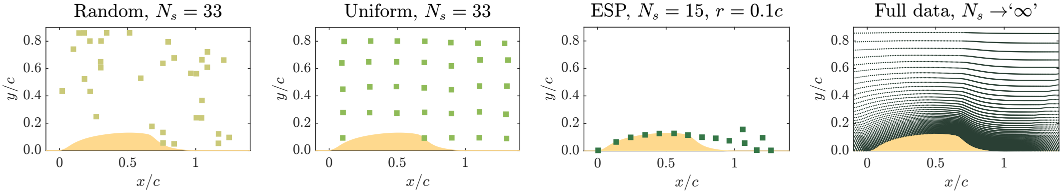
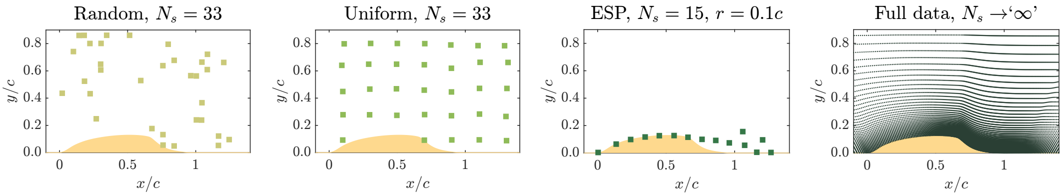


Comparison


Random, $N_s = 33$
Uniform, $N_s = 33$
ESP, $N_s = 15$, $r = 0.1c$
Full data, $N_s \to$‘∞’
y/c
x/c
0.0
0.2
0.4
0.6
0.8
0
0.5
1

Comparison



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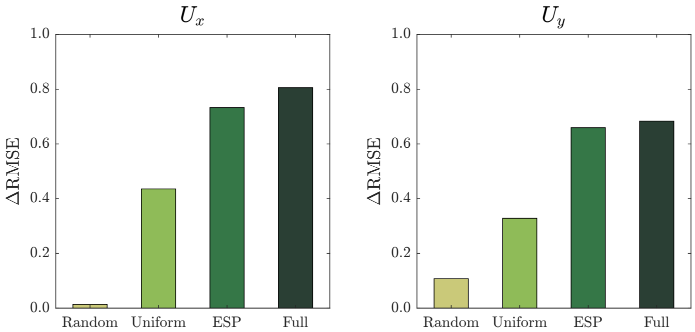
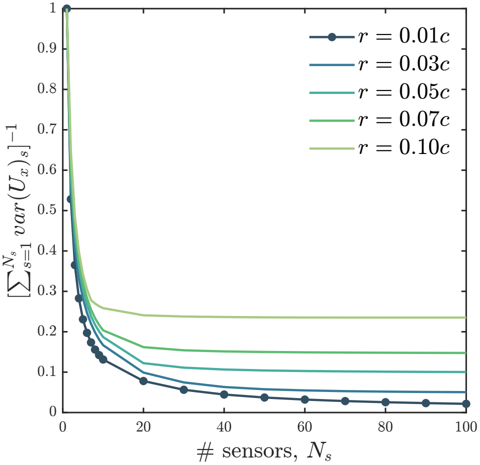
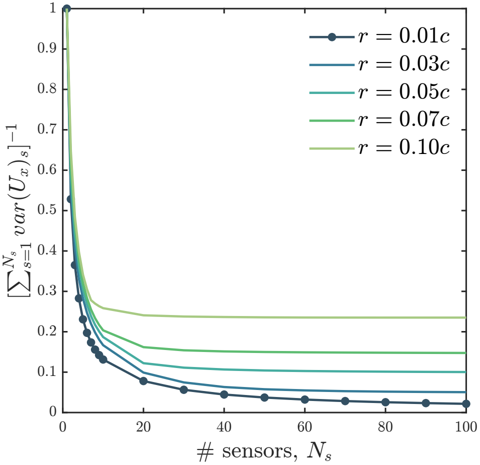


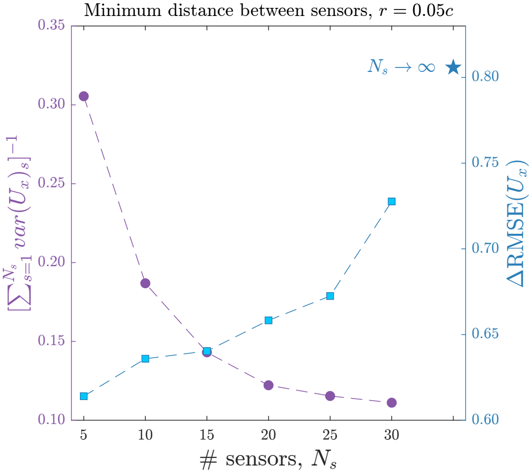
U_x
U_y
ΔRMSE
0.0
0.2
0.4
0.6
0.8
1.0
Random
Uniform
ESP
Full

sensors N_s vs. minimum distance r



\# sensors N_s vs. minimum distance r





Summary and future work

- Introduced an an optimal sensor placement strategy for adjoint-based data assimilation of turbulent flows.
- The method uses the eigenspace perturbation approach to generate an uncertainty map, and a heuristic greedy algorithm approach is used to place sensors targeting the regions of highest uncertainty.
- Advantages: a priori method; low computational costs; strategy tailored to the primary source of uncertainty in RANS modelling.
- Limitations: only investigated a constant minimum distance hard-constraint between sensors; sensor placement step is decoupled from the RANS environment, thus do not consider sensitivities of the RANS states w.r.t. the uncertainty metric.
- **Future work:** non-linear minimum distance constraint as a function of variance; deep learning.

THANK YOU!

obidar1@sheffield.ac.uk

ADDITIONAL SLIDES

Parametric vs. structural uncertainty

Uncertainty in turbulence models can be categorised as[††]:

- *Parametric*: due to the model constants
- *Functional*: due to the form of transport equation(s) used
- *Structural*: due to the modelling assumptions in derivations, e.g. eddy-viscosity hypothesis

Therefore, we can introduce parametric, functional, and/or structural perturbations to map uncertainty.

1. Parametric perturbations ▶ varying model constants between certain bounds[‡‡]
2. Structural perturbations ▶ eigenspace approach

‡‡ Deng *et al.*, 2021, Physics of Fluids

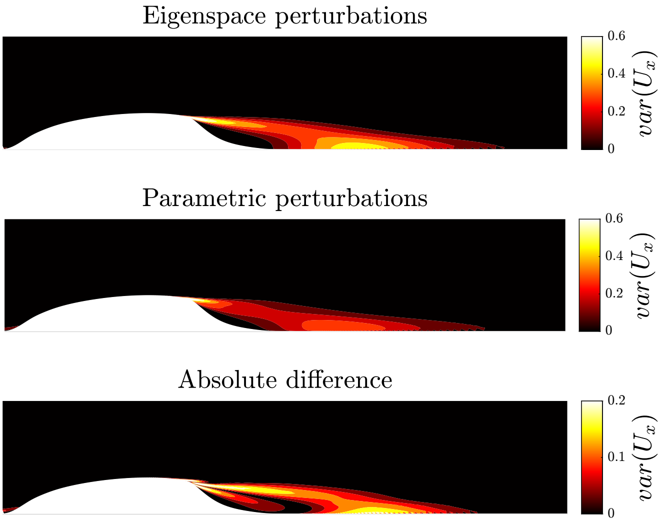
†† Xiao and Cinnella, 2019, Progress in Aerospace Sciences

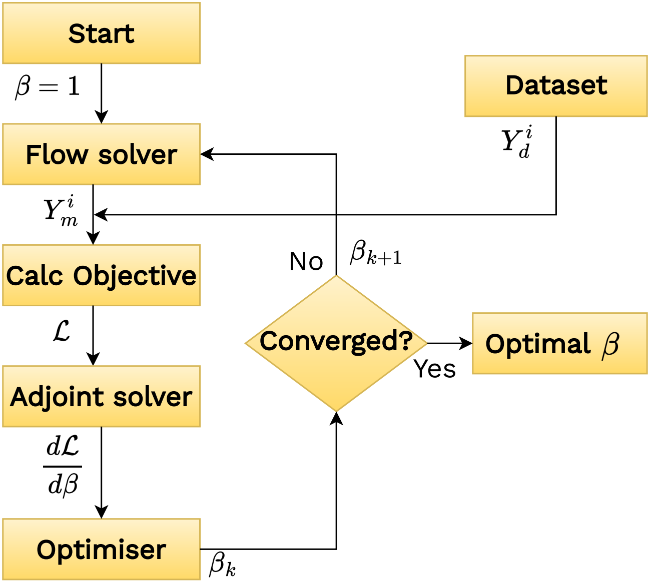
Eigenspace vs. parametric perturbations

Parametric perturbations: Twelve $k - \omega$ SST constants were varied between 80-120% of their original values in increments of 5%.

Perturbing individual constants: 12×8
Perturbing all constants simultaneously: 8

- 104 (parametric) vs. 5 (eigenspace) CFD simulations!
- Parametric perturbations are ad hoc while eigenspace perturbations is performed in a principled manner.




Start
$\beta = 1$
Flow solver
Dataset
Y_d^i
Y_m^i
Calc Objective
No
β_{k+1}
$\mathcal{L}$
Converged?
Yes
Optimal β
Adjoint solver
$\frac{d\mathcal{L}}{d\beta}$
Optimiser
β_k