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 - running Newton's method in complex arithmetic
 - examining the CUDA Compute Capability
- CUDA Program Structure
 - steps to write code for the GPU
 - code to compute complex roots
 - the kernel function and main program
 - a scalable programming model

MCS 572 Lecture 30 Introduction to Supercomputing Jan Verschelde, 31 October 2016

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computing complex square roots

To compute \sqrt{c} for $c \in \mathbb{C}$, we apply Newton's method on $x^2 - c = 0$:

$$x_0 := c$$
, $x_{k+1} := x_k - \frac{x_k^2 - c}{2x_k}$, $k = 0, 1, ...$

Five iterations suffice to obtain an accurate value for \sqrt{c} .

Suitable on GPU?

- Finding roots is relevant for scientific computing.
- Data parallelism: compute for many different c's.

Application: complex root finder for polynomials in one variable.

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CUDA Compute Capability

The compute capability of an NVIDIA GPU

- is represented by a version number in the format x.y,
- identifies the features supported by the hardware.

What does it mean for the programmer? Some examples:

- 1.3 : double-precision floating-point operations
- 2.0 : synchronizing threads
- 3.5 : dynamic parallelism
- 5.3 : half-precision floating-point operations
- 6.0 : atomic addition operation on 64-bit floats

The compute capability is not the same as the CUDA version.

checking the card with deviceQuery on kepler

\$ /usr/local/cuda/samples/1 Utilities/deviceOuerv/deviceOuerv /usr/local/cuda/samples/1 Utilities/deviceQuery/deviceQuery Starting... CUDA Device Ouery (Runtime API) version (CUDART static linking) Detected 3 CUDA Capable device(s) Device 0: "Tesla K20c" CUDA Driver Version / Runtime Version 6.0 / 5.5 CUDA Capability Major/Minor version number: 3.5 Total amount of global memory: 4800 MBytes (5032706048 bytes) (13) Multiprocessors, (192) CUDA Cores/MP: 2496 CUDA Cores GPU Clock rate: 706 MHz (0.71 GHz) Memory Clock rate: 2600 Mhz Memory Bus Width: L2 Cache Size: 1310720 bytes 1D=(65536), 2D=(65536, 65536), 3D=(4096, 4096, 4096) Maximum Texture Dimension Size (x, y, z) Maximum Layered 1D Texture Size, (num) layers 1D=(16384), 2048 layers 2D=(16384, 16384), 2048 layers Maximum Layered 2D Texture Size, (num) layers Total amount of constant memory: 65536 bytes Total amount of shared memory per block: 49152 bytes Total number of registers available per block: 65536 32 Warp size: Maximum number of threads per multiprocessor: 2048 Maximum number of threads per block: 1024 Max dimension size of a thread block (x,y,z): (1024, 1024, 64) Max dimension size of a grid size (x,y,z): (2147483647, 65535, 65535) Maximum memory pitch: 2147483647 bytes Texture alignment: 512 bytes Yes with 2 copy engine(s) Concurrent copy and kernel execution: Run time limit on kernels: Integrated GPU sharing Host Memory: Nο Support host page-locked memory mapping: Yes Alignment requirement for Surfaces: Yes Device has ECC support: Enabled Device supports Unified Addressing (UVA): Yes Device PCT Bus TD / PCT location TD: 4 / 0 Compute Mode:

< Default (multiple host threads can use ::cudaSetDevice() with device simultaneously) >

checking the card with deviceQuery on pascal

\$ /usr/local/cuda/samples/1 Utilities/deviceOuerv/deviceOuerv /usr/local/cuda/samples/1 Utilities/deviceQuery/deviceQuery Starting... CUDA Device Ouery (Runtime API) version (CUDART static linking) Detected 2 CUDA Capable device(s) Device 0: "Tesla P100-PCIE-16GB" CUDA Driver Version / Runtime Version 8.0 / 8.0 CUDA Capability Major/Minor version number: 6.0 Total amount of global memory: 16276 MBytes (17066885120 bytes) (56) Multiprocessors, (64) CUDA Cores/MP: 3584 CUDA Cores GPII Max Clock rate: 405 MHz (0.41 GHz) Memory Clock rate: 715 Mhz Memory Bus Width: 4096-hit L2 Cache Size: 4194304 bytes 1D=(131072), 2D=(131072, 65536), 3D=(16384, 16384, 16384) Maximum Texture Dimension Size (x, y, z) Maximum Layered 1D Texture Size, (num) layers 1D=(32768), 2048 layers Maximum Layered 2D Texture Size, (num) layers 2D=(32768, 32768), 2048 layers Total amount of constant memory: 65536 bytes Total amount of shared memory per block: 49152 bytes Total number of registers available per block: 65536 32 Warp size: Maximum number of threads per multiprocessor: 2048 Maximum number of threads per block: 1024 Max dimension size of a thread block (x,y,z): (1024, 1024, 64) Max dimension size of a grid size (x,y,z): (2147483647, 65535, 65535) Maximum memory pitch: 2147483647 bytes Texture alignment: 512 bytes Yes with 2 copy engine(s) Concurrent copy and kernel execution: Run time limit on kernels: Integrated GPU sharing Host Memory: Nο Support host page-locked memory mapping: Yes Alignment requirement for Surfaces: Yes Device has ECC support: Enabled Device supports Unified Addressing (UVA): Yes Device PCT Domain TD / Bus TD / location TD: 0 / 2 / 0 Compute Mode:

4 D > 4 D > 4 D > 4 D >

< Default (multiple host threads can use ::cudaSetDevice() with device simultaneously) >

running bandwidthTest on kepler

\$ /usr/local/cuda/samples/1 Utilities/bandwidthTest/bandwidthTest [CUDA Bandwidth Test] - Starting ... Running on ... Device 0: Tesla K20c Ouick Mode Host to Device Bandwidth, 1 Device(s) PINNED Memory Transfers Transfer Size (Bytes) Bandwidth (MB/s) 33554432 5819.5 Device to Host Bandwidth, 1 Device(s) PINNED Memory Transfers Transfer Size (Bytes) Bandwidth (MB/s) 33554432 6415.8 Device to Device Bandwidth, 1 Device(s) PINNED Memory Transfers Transfer Size (Bytes) Bandwidth (MB/s) 33554432 143248.0

Result = PASS

running bandwidthTest on pascal

```
$ /usr/local/cuda/samples/1_Utilities/bandwidthTest/bandwidthTest
[CUDA Bandwidth Test] - Starting...
Running on ...
Device 0: Tesla P100-PCIE-16GB
Ouick Mode
Host to Device Bandwidth, 1 Device(s)
PINNED Memory Transfers
  Transfer Size (Bytes) Bandwidth (MB/s)
   33554432 11530.1
Device to Host Bandwidth, 1 Device(s)
 PINNED Memory Transfers
  Transfer Size (Bytes) Bandwidth (MB/s)
   33554432 12848.3
Device to Device Bandwidth, 1 Device(s)
PINNED Memory Transfers
  Transfer Size (Bytes) Bandwidth (MB/s)
  33554432 444598.8
Result = PASS
```

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steps to write code for the GPU

Five steps to get GPU code running:

- O and C++ functions are labeled with CUDA keywords __device__, __global___, or __host___.
- 2 Determine the data for each thread to work on.
- Transferring data from/to host (CPU) to/from the device (GPU).
- Statements to launch data-parallel functions, called kernels.
- Compilation with nvcc.

step 1: CUDA extensions to functions

Three keywords before a function declaration:

```
__host__ : The function will run on the host (CPU).
```

__device__ : The function will run on the device (GPU).

__global__ : The function is called from the host but

runs on the device. This function is called a kernel.

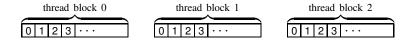
CUDA extensions to C function declarations:

	executed on	callable from
device double D()	device	device
global void K()	device	host
host int H()	host	host

step 2: data for each thread

The grid consists of N blocks, with blockIdx.x $\in \{0, N-1\}$.

Within each block, threadIdx. $x \in \{0,blockDim.x-1\}$.



```
int threadId = blockIdx.x *
    blockDim.x + threadIdx.x
...
float x = input[threadID]
float y = f(x)
output[threadID] = y
...
```

step 3: allocating and transferring data

```
cudaDoubleComplex *xhost = new cudaDoubleComplex[n];
// we copy n complex numbers to the device
size t s = n*sizeof(cudaDoubleComplex);
cudaDoubleComplex *xdevice;
cudaMalloc((void**)&xdevice,s);
cudaMemcpy(xdevice, xhost, s, cudaMemcpyHostToDevice);
// allocate memory for the result
cudaDoubleComplex *ydevice;
cudaMalloc((void**)&ydevice,s);
// copy results from device to host
cudaDoubleComplex *yhost = new cudaDoubleComplex[n];
cudaMemcpy(yhost,ydevice,s,cudaMemcpyDeviceToHost);
```

step 4: launching the kernel

The kernel is declared as

```
__global__ void squareRoot
  ( int n, cudaDoubleComplex *x, cudaDoubleComplex *y )
// Applies Newton's method to compute the square root
// of the n numbers in x and places the results in y.
{
   int i = blockIdx.x*blockDim.x + threadIdx.x;
   ...
```

For frequency f, dimension n, and block size w, we do:

```
// invoke the kernel with n/w blocks per grid
// and w threads per block
for(int i=0; i<f; i++)
    squareRoot<<<n/w, w>>>(n, xdevice, ydevice);
```

step 5: compiling with nvcc

Then if the makefile contains

typing make runCudaComplexSqrt does

```
nvcc -o /tmp/run_cmpsqrt -arch=sm_13 runCudaComplexSqrt.cu
```

The -arch=sm_13 is needed for double arithmetic, for the K20C.

The option <code>-arch=sm_13</code> is no longer recognized on the new P100.

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defining complex numbers

```
#ifndef __CUDADOUBLECOMPLEX_CU__
#define __CUDADOUBLECOMPLEX_CU__

#include <cmath>
#include <cstdlib>
#include <iomanip>
#include <vector_types.h>
#include <math_functions.h>

typedef double2 cudaDoubleComplex;
```

We use the <code>double2</code> of <code>vector_types.h</code> to define complex numbers because <code>double2</code> is a native CUDA type allowing for coalesced memory access.

random complex numbers

```
host cudaDoubleComplex randomDoubleComplex()
// Returns a complex number on the unit circle
// with angle uniformly generated in [0,2*pi].
{
   cudaDoubleComplex result;
   int r = rand();
   double u = double(r)/RAND MAX;
   double angle = 2.0 \times M PI*u;
   result.x = cos(angle);
   result.y = \sin(angle);
   return result;
```

calling sqrt of math_functions.h

```
__device__ double radius ( const cudaDoubleComplex c )
// Returns the radius of the complex number.
{
   double result;
   result = c.x*c.x + c.y*c.y;
   return sqrt(result);
}
```

overloading for output

defining complex addition

```
__device__ cudaDoubleComplex operator+
  ( const cudaDoubleComplex a, const cudaDoubleComplex b )
// Returns the sum of a and b.
  {
    cudaDoubleComplex result;
    result.x = a.x + b.x;
    result.y = a.y + b.y;
    return result;
}
```

The rest of the arithmetical operations are defined in a similar manner.

All definitions related to complex numbers are stored in the file <code>cudaDoubleComplex.cu</code>.

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the kernel function

```
#include "cudaDoubleComplex.cu"
__global__ void squareRoot
( int n, cudaDoubleComplex *x, cudaDoubleComplex *y )
// Applies Newton's method to compute the square root
// of the n numbers in x and places the results in y.
   int i = blockIdx.x*blockDim.x + threadIdx.x;
   cudaDoubleComplex inc;
   cudaDoubleComplex c = x[i];
   cudaDoubleComplex r = c;
   for (int j=0; j<5; j++)
      inc = r + r;
      inc = (r*r - c)/inc;
      r = r - inc;
   v[i] = r;
```

the main function — command line arguments

```
int main (int argc, char*argv[])
   if(argc < 5)
      cout << "call with 4 arguments : " << endl;
      cout << "dimension, block size, frequency, and check (0 or 1)"
           << endl:
   else
      int n = atoi(argv[1]); // dimension
      int w = atoi(argv[2]); // block size
      int f = atoi(argv[3]); // frequency
      int t = atoi(argv[4]); // test or not
      // we generate n random complex numbers on the host
      cudaDoubleComplex *xhost = new cudaDoubleComplex[n];
      for(int i=0; i<n; i++) xhost[i] = randomDoubleComplex();</pre>
```

The main program generates n random complex numbers with radius 1.

transferring data and launching the kernel

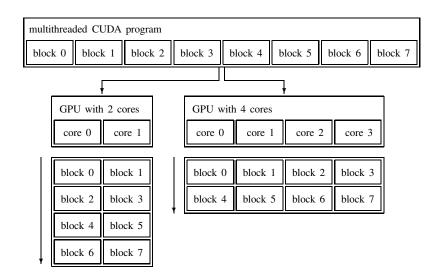
```
// copy the n random complex numbers to the device
size t s = n*sizeof(cudaDoubleComplex);
cudaDoubleComplex *xdevice;
cudaMalloc((void**)&xdevice,s);
cudaMemcpy(xdevice, xhost, s, cudaMemcpyHostToDevice);
// allocate memory for the result
cudaDoubleComplex *ydevice;
cudaMalloc((void**)&ydevice,s);
// invoke the kernel with n/w blocks per grid
// and w threads per block
for (int i=0; i < f; i++)
   squareRoot <<< n/w, w>>> (n, xdevice, ydevice);
// copy results from device to host
cudaDoubleComplex *yhost = new cudaDoubleComplex[n];
cudaMemcpy(yhost,ydevice,s,cudaMemcpyDeviceToHost);
```

testing one random number

```
if (t == 1) // test the result
      int k = rand() % n;
      cout << "testing number " << k << endl;</pre>
      cout << "
               x = " << xhost[k] << endl;
      cout << " sqrt(x) = " << yhost[k] << endl;
      cudaDoubleComplex z = Square(yhost[k]);
      cout << "sqrt(x)^2 = " << z << endl;
return 0;
```

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running the code on kepler

A test on the correctness:

On 64,000 numbers, 32 threads in a block, doing it 10,000 times:

changing #threads in a block

```
$ time /tmp/run cmpsqrt 128000 32 100000 0
    0m17.345s
real
    0m9.829s
user
sys 0m7.303s
$ time /tmp/run cmpsgrt 128000 64 100000 0
    0m10.502s
real
    0m5.711s
user
sys 0m4.497s
$ time /tmp/run_cmpsqrt 128000 128 100000 0
real 0m9.295s
user 0m5.231s
sys 0m3.865s
```

running the code on pascal

```
$ time /tmp/run cmpsqrt 128000 32 100000 0
    0m2.516s
real
    0m1.250s
user
sys 0m1.236s
$ time /tmp/run cmpsgrt 128000 64 100000 0
    0m2.521s
real
    0m1.245s
user
sys 0m1.234s
$ time /tmp/run_cmpsqrt 128000 128 100000 0
real 0m2.496s
user 0m1.288s
sys 0m1.195s
```

summary and references

In five steps we wrote our first complete CUDA program.

We started chapter 3 of the textbook by Kirk & Hwu, covering more of the CUDA Programming Guide.

Available in /usr/local/cuda/doc are

- CUDA C Best Practices Guide
- CUDA Programming Guide

Also available online at nvdia.com.

Many examples of CUDA applications are available in /usr/local/cuda/samples.

exercises

- Instead of 5 Newton iterations in runCudaComplexSqrt.cu use k iterations where k is entered by the user at the command line. What is the influence of k on the timings?
- Modify the kernel for the complex square root so it takes on input an array of complex coefficients of a polynomial of degree d. Then the root finder applies Newton's method, starting at random points. Test the correctness and experiment to find the rate of success, i.e.: for polynomials of degree d how many random trials are needed to obtain d/2 roots of the polynomial?