Optimizing User Engagement through Adaptive Ad Sequencing

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Abstract

In this paper, we propose a unified dynamic framework for adaptive ad sequencing that optimizes user engagement with ads. Our framework comprises three components – (1) a Markov Decision Process that incorporates inter-temporal trade-offs in ad interventions, (2) an empirical framework that combines machine learning methods with insights from causal inference to achieve personalization, counterfactual validity, and scalability, and (3) a robust policy evaluation method. We apply our framework to large-scale data from the leading in-app ad network of an Asian country. We find that the dynamic policy generated by our framework improves the current practice in the industry by 5.76%. This improvement almost entirely comes from the increased average ad response to each impression instead of the increased usage by each user. We further document a U-shaped pattern in improvements across the length of the user's past history, with high values when the user is new or when enough data are available for the user. Next, we show that ad diversity is higher under our policy because it manages users' attention more effectively. We conclude by discussing the implications and broad applicability of our framework to settings where a platform wants to sequence content to optimize user engagement.

Keywords: advertising, personalization, adaptive interventions, policy evaluation, dynamic programming, machine learning, batch reinforcement learning

1 Introduction

1.1 Motivation for Adaptive Ad Sequencing

Consumers now spend a significant portion of their time on mobile apps. The average time spent on mobile apps by US adults has grown steadily over the past few years, surpassing 4 hours per day for the first time in the first quarter of 2021 (Kristianto, 2021). This demand expansion, in turn, has amplified marketing activities targeted towards mobile app users. In 2020, mobile advertising generated nearly $100 billion in the US, accounting for over double the share of its digital counterpart, desktop advertising (IAB, 2021). Most of this growth in mobile advertising is attributed to in-app ads (i.e., ads shown inside mobile apps), with over 80% of ad spend in the mobile advertising category (eMarketer, 2018).

Two key features of mobile in-app ads have contributed to this dramatic growth. First, the mobile app ecosystem has excellent user tracking ability, thereby allowing "personalization" of ad interventions and targeting of users based on their prior behavioral history (Han et al., 2012). Second, in-app ads are usually refreshable and dynamic in nature: each ad intervention is shown for a fixed amount of time (e.g., 30 seconds or one minute) inside the app and followed by another ad intervention. As such, a user can see multiple ad exposures within a session.[1] Refreshable ads, together with the potential for personalization, make in-app advertising amenable to "adaptive ad sequencing", that is, optimizing the sequence of ads based on real-time behavioral information.

Adaptive ad sequencing brings a forward-looking perspective to the publisher's ad allocation problem.[2] That is, sequencing not only captures the immediate user engagement when making a decision to show an ad based on the information available, but it also takes user engagement in future events and exposures into account. Figure 1 illustrates this point by differentiating between the information available from the past and the information that would be available in the future. However, most platforms do not use a forward-looking model for ad allocation because it adds to the complexity of the model substantially. This is one of the reasons why the current state of advertising practice is to use supervised learning and contextual bandit algorithms that only focus on the data available at the moment and ignore the future exposures (Li et al., 2010; Theocharous et al., 2015). Further, the returns from adopting a forward-looking model are not clear. Thus, the publisher's decision on whether to

[1] A session is an uninterrupted time that a user spends inside an app. This is in contrast with the common practice in desktop advertising, where ads remain fixed throughout a session.

[2] In this paper, we use the publisher, ad network, and platform interchangeably when we refer to the agent who makes the ad placement decision.

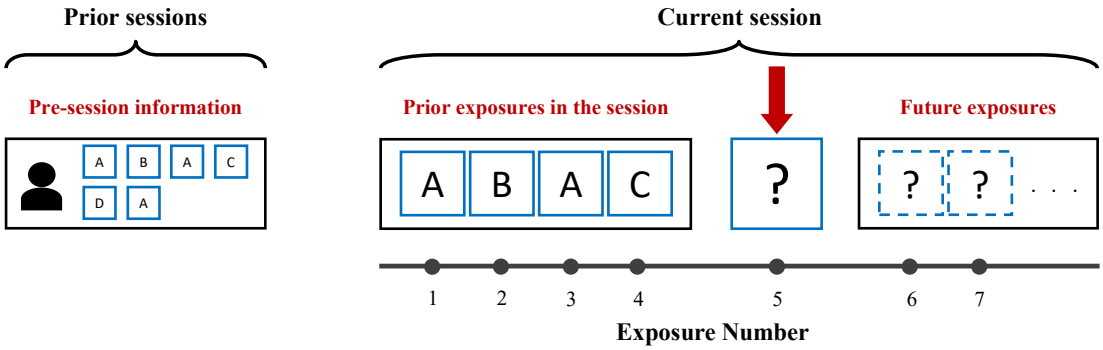


Figure 1: A visual schema for the publisher's ad sequencing decision. The user is at the fifth exposure in the session, and the publisher needs to decide which ad to show to this user. Unlike the myopic publisher that only uses the information from the past, a forward-looking publisher also accounts for the futures exposures when making the decision.

use a dynamic framework boils down to whether incorporating future information helps them achieve a better outcome.

In principle, using a forward-looking framework is only valuable when there is inter-dependence between ad exposures, i.e., the ad shown in the current exposure affects the performance of future exposures and the overall value created by the system. The impact of the publisher's current decision on the future exposures can fundamentally be of two types – (1) extensive margin, which means that the user will stay longer in the session and generate more exposures, and (2) intensive margin, which means that the engagement with each exposure in the future will be higher, on average. Prior literature on advertising offers multiple accounts that suggest a great possibility for value creation through both channels. On the one hand, sequencing can result in greater usage in light of studies on the link between advertising and subsequent usage (Wilbur, 2008; Goli et al., 2021). On the other hand, sequencing can increase the response rate to each exposure by better managing effects of carryover, spillover, temporal spacing, and variety (Rutz and Bucklin, 2011; Jeziorski and Segal, 2015; Sahni, 2015; Lu and Yang, 2017; Rafieian and Yoganarasimhan, 2021a).

1.2 Research Agenda and Challenges

The dynamic effects of advertising give rise to the inter-temporal trade-offs in ad allocation. For example, Rafieian and Yoganarasimhan (2021a) find that an increase in the variety of ads in a session results in a higher engagement with the next ad. However, it is not clear that increasing variety is the optimal decision at any point because it can come at the expense of showing an irrelevant ad. While the dynamic effects of advertising and the

resulting inter-temporal trade-offs are well-established in the literature, neither research nor practice has looked into how we can collectively incorporate these findings to optimize publisher's outcomes by dynamically sequencing ads. Our goal in this paper is to fill this gap by developing a unified framework for adaptive ad sequencing and documenting the gains from this framework.

To build such a framework, we first need to specify our objective. We view the problem through the lens of a publisher who aims to maximize the expected number of clicks per session. While our framework is general and can accommodate any measure of user engagement over any optimization horizon, we focus on clicks as our measure of user engagement because clicks are instrumental to a publisher's business model in mobile in-app advertising. With our objective in place, we seek to answer the following three questions:

1. How can we develop a unified dynamic framework that incorporates the inter-temporal trade-offs in ad allocation and designs a policy that maximizes user engagement?

2. How can we empirically evaluate the performance of the counterfactual policy identified by our adaptive ad sequencing framework?

3. What are the gains from using our adaptive ad sequencing framework over existing benchmarks? Are these gains due to increased usage (extensive margin) or increased average ad response (intensive margin)? Which session characteristics are linked to greater gains? How different is the policy identified by our framework from the benchmark policies?

1.3 Our Approach

In this paper, we present a unified three-pronged framework that addresses these challenges and develops an adaptive ad sequencing policy to maximize user engagement with ads. We present an overview of our approach in Figure 2, where the top row illustrates that we start with a theoretical framework that models the domain structure of our problem and informs us of the key empirical tasks required for policy identification and evaluation, and the bottom row describes the specifics of our approach.

For our theoretical framework, we specify a domain-specific Markov Decision Process (MDP henceforth) that characterizes the structure of adaptive ad interventions. In particular, we use a rich set of state variables that collectively incorporate the dynamic effects of advertising identified in the literature. Our MDP characterizes the reward at any exposure as well as how the state evolves in future periods, given any action taken by the publisher.

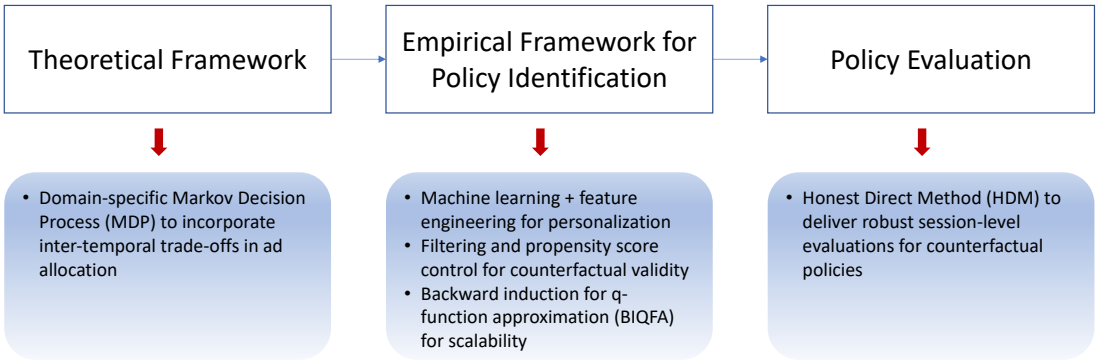


Figure 2: An overview of our approach.

Since our goal is to optimize the number of clicks per session, we define the reward as the expected probability of click, given the state variable and ad. This probability is also part of the state transition, as it helps us update the user's preference in real-time for the next period. Another probabilistic factor that affects the future state is the expected probability of the user leaving the session after an intervention, which determines with what probability the user will be available to see the next ad exposure. The combination of reward and transition functions allows us to characterize the publisher's optimization problem theoretically.

Next, to empirically identify the optimal sequencing policy, we develop an empirical framework that allows us to evaluate all possible sequencing policies for each session. As broken down by our theoretical framework, we first need to obtain personalized estimates of the primitives of our MDP – expected click and leave probabilities. We do so by using machine learning methods that can capture more complex relationships between the covariates and the outcome. In particular, we use an Extreme Gradient Boosting (XGBoost henceforth) algorithm with a rich set of features to predict click and leave outcomes. To ensure the counterfactual validity of our estimates, we use key insights from the causal inference literature and narrow down our focus on counterfactual sequences that *could have been shown* in our data. This is because machine learning algorithms can only generate accurate predictions for instances within the joint distribution of the training set used for model fitting. Further, we control for propensity scores to account for potential selection in our predictions. Lastly, for the scalability of our empirical framework, we develop an algorithm called *backward induction for q-function approximation (BIQFA)* that takes the primitive estimates and learns a function that approximates the expected sum of current and future rewards for each pair of state variables and ad. This function approximation approach avoids the exhaustive search over any pair of state and ad, thereby reducing the computational burden substantially.

While our empirical framework for policy identification separately evaluates each policy to find the optimal one, we cannot use the same evaluation approach to assess the performance of our policy, since the policy identified by our framework will always outperform other policies by construction. To address this challenge, we develop an approach called *Honest Direct Method (HDM)* that completely separates the evaluation criteria for policy identification and policy evaluation: the data and models used for identification have no overlap with the data and model used for evaluation. To increase the robustness of this approach, we use a fully held-out third data set for final evaluation that is not used for model building in either policy identification or policy evaluation stages.

1.4 Findings and Contributions

We apply our framework to the data from a leading mobile in-app ad network of a large Asian country. Our setting has notable features that make it amenable to our research goals. First, the ad network uses a refreshable ad format where ad interventions last for one minute and change within the session. Second, the ad network runs a quasi-proportional auction that employs a probabilistic allocation rule, which induces a high degree of randomization in ad allocation Together, these two features create exogenous variation in the sequences shown in the data, thereby satisfying an essential requirement for our framework.

To establish the performance of our adaptive ad sequencing framework, we evaluate the gains from *adaptive ad sequencing policy* relative to three benchmark policies: (1) *random policy*, which selects ads randomly and is often used as a benchmark in the reinforcement learning literature, (2) *single-ad policy*, which only shows a single ad with the highest reward in the session, thereby mimicking the practice of using a non-refreshable ad slot as is common in desktop advertising, and (3) *adaptive myopic policy*, which uses all the information available and selects the ad with the highest reward at any exposure but ignores the expected future rewards. The adaptive myopic policy reflects the standard practice in the advertising industry, where publishers use supervised learning or contextual bandit algorithms to estimate click-through rate (CTR) for an ad in a given impression.

We evaluate all these policies on a completely held-out test set using different metrics. First, we document a 79.59% increase in the expected number of clicks from our fully dynamic policy over the random policy. Next, we show that our fully dynamic policy results in 27.46% greater expected number of clicks per session than the single-ad policy. This finding demonstrates the opportunity cost of using a non-refreshable ad slot throughout the session, supporting the current industry trend of using refreshable ad slots. Finally, we focus on our key comparison in this paper and demonstrate a 5.76% gain in the expected number of clicks

per session from our fully dynamic policy over the adaptive myopic policy. This suggests that choosing the best match at any point will not necessarily create the best match outcome at the end of the session. Instead, the right action sometimes is to show the ad that is not necessarily the best match at the moment but transitions the session to a better state in the future. This finding provides a strong proof-of-concept for the use of our framework. It has important implications for publishers and ad networks, especially since the current practice in the industry overlooks the dynamics of ad sequencing.

We further compare our policy with the benchmark policies using two other metrics – session length and ad concentration. Focusing on the session length shows how much of the gains from our policy come from an increase in usage and the number of impressions generated (extensive margin). While our policy achieves a slightly higher session length, it is only 0.2% greater than the session length under the adaptive myopic policy, which suggests that the source for our gains is not the increase in usage, but the increase in the average ad response rate (intensive margin). We then focus on ad concentration as our next metric and use the Herfindahl–Hirschman Index (HHI) for ads shown under each policy. Our results reveal an interesting pattern: adaptive ad sequencing policy results in a lower HHI than both adaptive myopic and single-ad policies, suggesting a greater ad diversity under our policy. A greater ad diversity can have long-term implications for the competition between advertisers as well as welfare impacts for consumers.

Next, to better interpret the mechanism underlying our gains, we explore the heterogeneity in gains from our policy over the adaptive myopic policy. We document a U-shaped pattern in gains over the number of prior sessions a user has been part of, which can be a composite of two accounts. On the one hand, a higher number of prior sessions means a richer past behavioral history, which can increase gains from sequencing as the framework has more certainty about session-level dynamics. On the other hand, as suggested in Rafieian and Yoganarasimhan (2021a), users' responsiveness to some dynamic effects declines as they become more experienced. We present a series of regression models that provide further support for both accounts and document the heterogeneity in gains across other pre-session covariates.

To understand where the difference between our policy and adaptive myopic policy comes from, we use a series of descriptive approaches. We first focus on the distribution of ad allocation under each policy and measure the discrepancy in these two distributions, using different measures such as ℓ-norm and Kullback-Leibler divergence. Among pre-session characteristics, we find that a higher number of past impressions is associated with a greater

discrepancy in distributions. In contrast, a higher variety of prior ads and number of past clicks are associated with a lower discrepancy in distributions. We further compare the two policies at the session level in how they utilize frequency and spacing strategies. We find that our policy uses lower frequency and higher spacing in interventions towards the end of the session than the adaptive myopic policy. Together, these results suggest that our policy better manages the users' attention than the adaptive myopic policy.

In sum, our paper makes several contributions to the literature. First, from a methodological standpoint, we develop a unified dynamic framework that takes the past advertising data and scalably produces an optimal dynamic policy to personalize the sequence of ads in a session. A key contribution of our adaptive ad sequencing framework is that it does not impose restrictive assumptions on the dynamic structure of the problem and remains agnostic about how dynamics arise in our setting. To our knowledge, this is the first paper that takes a prescriptive approach to generate an optimal dynamic policy by collectively incorporating the dynamic effects of advertising documented in the literature. Substantively, we establish the gains from our dynamic framework over a set of benchmarks that are often used in research and practice. This proof-of-concept is particularly important as the current practice in this industry ignores the dynamics of the ad allocation problem. We further present a comprehensive analysis of the gains from our framework to provide interpretation for the mechanism underlying the gains. Our findings shed light on when and why our framework is more valuable than alternative policies. Lastly, from a managerial perspective, our framework is fairly general and can be a applied to a wide variety of domains where a platform or publisher aims to optimally sequence content to achieve better user-level outcomes, such as sequencing of articles to increase audience engagement with the content in news websites, sequencing of social media posts to increase user interaction and engagement, and sequencing of push notifications to reduce customer churn.

2 Related Literature

First, our paper relates to the marketing literature on personalization and targeting. Early papers in this stream build Bayesian frameworks that exploit behavioral data and personalize marketing mix variables (Rossi et al., 1996; Ansari and Mela, 2003; Manchanda et al., 2006). Recent papers in this domain use machine learning algorithms often combined with insights from causal inference to achieve greater personalization in different domains such as search (Yoganarasimhan, 2020), advertising (Rafieian and Yoganarasimhan, 2021b), free trial length (Yoganarasimhan et al., 2020), and product versioning through offering different ad loads to users (Goli et al., 2021). While all these papers focus on prescriptive or substantive

frameworks to study personalization, they all study this phenomenon from a static point of view. Our paper extends this literature by bringing a dynamic objective to this problem and offering a scalable framework to develop forward-looking personalized targeting policies.

Second, our work relates to both the substantive and prescriptive literature on the dynamics of advertising. Early work in this domain focuses on aggregate advertising models to understand ad responses over time and strategies such as pulsing (Little, 1979; Horsky, 1977; Simon, 1982; Naik et al., 1998; Dubé et al., 2005; Aravindakshan and Naik, 2011).[3] More recent papers in this domain use larger scale individual-level data of digital advertising and document different dynamic effects of advertising, such as effects of ad carryover or spillover, temporal spacing, and variety in search advertising (Rutz and Bucklin, 2011; Jeziorski and Segal, 2015; Lu and Yang, 2017; Sahni, 2015; Zantedeschi et al., 2017; Rafieian and Yoganarasimhan, 2021a). Inspired by the dynamics of advertising, a different stream of work brings a more prescriptive view to the problem and focuses on the optimal policy design for advertisers and platforms. Given the complexity of the problem, these papers often simplify the problem by mapping the entire space into a few segments (Urban et al., 2013), ignoring inter-temporal trade-offs through a bandit specification (Schwartz et al., 2017), or imposing some structure on the dynamics to find a closed-form solution (Wilbur et al., 2013; Kar et al., 2015; Sun et al., 2017). Our paper contributes to this literature by proposing a scalable framework that collectively incorporates all the documented dynamic effects of advertising to find the optimal dynamic policy without reducing the richness and dimensionality of the state space or imposing any structure on the dynamics of the problem.

Finally, our paper relates to the literature on batch reinforcement learning (RL), where the learner does not actively interact with the environment and must rely on observational data from the past to design an optimal dynamic policy. This class of problems is particularly relevant when safety guarantees are of utmost priority, and the system is not allowed to actively explore (Thomas et al., 2019). An important task in all these problems is to find a robust approach to evaluate counterfactual policies, i.e., policies that have not necessarily been implemented in the data available. This problem is often referred to as off-policy policy evaluation in the batch RL literature, and a variety of approaches are proposed that use both model-based and model-free approaches for off-policy policy evaluation (Thomas et al., 2015; Thomas and Brunskill, 2016; Le et al., 2019; Kallus and Uehara, 2020). Closely related to our empirical context, Theocharous et al. (2015) use real advertising data and extend the problem of personalized ad recommendation to a dynamic setting. However, their paper

[3]Please see Chapter 7 in Tellis (2003) for a summary of the earlier work on advertising dynamics.

only captures usage-related dynamics and ignores other dynamic ad effects such as temporal spacing, spillover, and variety. As such, the empirical results are a bit mixed with a low level of confidence in establishing gains from dynamic over myopic policies, despite their use of a high-confidence off-policy evaluation framework. Our work uses platform data with a richer state space and develops a dynamic framework that collectively incorporates dynamic effects of advertising and establishes the gains from our framework over myopic policies. More broadly, we add to the batch RL literature by presenting a model-based backward induction q-function approximation (BIQFA) algorithm and using an honest direct method that allows us to further explore the mechanism behind the gains from a dynamic policy and adds to the interpretability of our framework.

3 Setting and Data

3.1 Setting

Our data come from a leading mobile in-app advertising network of a large Asian country that had over 85% of the market share around the time of this study. Figure 3 summarizes most key aspects of the setting. We number the arrows in Figure 3 and explain each step of the ad allocation process in details below:

1. The ad network designs an auction to sell ad slots. In our setting, the ad network runs a quasi-proportional auction with a cost-per-click payment scheme. As such, for a given ad slot and a set of participating ads $\mathcal{A}$ with a bidding profile $(b_1, b_2, \ldots, b_{|\mathcal{A}|})$, the ad slot is allocated to ad a with the following probability:

$$\pi_0(b; m) = \frac{b_a m_a}{\sum_{j \in \mathcal{A}} b_j m_j}, \tag{1}$$

   where m_a is ad a's quality score, which is a measure that reflects the profitability of ad a. The ad network does not customize quality scores across auctions. The subscript 0 in π_0 refers to the fact that this is the baseline allocation policy through which our data are generated. The payment scheme is cost-per-click, similar to Google's sponsored search auctions. That is, ads are first ranked based on their product of bid and quality score, and the winning ad pays the minimum amount that guarantees their rank if a click happens on their ad.

2. Advertisers participating in the auction make the following choices: (a) design of their banner, (b) which impressions they want to target, and (c) how much to bid. Figure 3

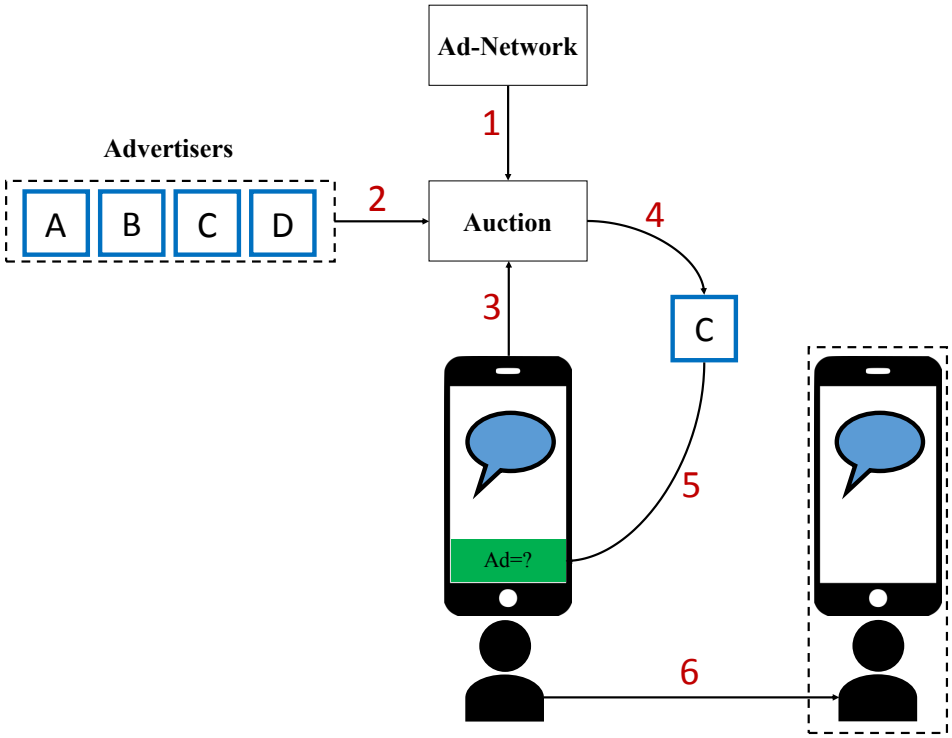


Figure 3: A visual schema of our setting

shows an example of an auction with four different ads.

3. Whenever a user starts a new session in an app (we use a messaging app in Figure 3 as an example), a new impression is being recognized, and a request is sent to the publisher to run an auction.

4. The auction takes all the participating ads into account and selects the ad probabilistically based on the weights shown in Equation (1). Note that all the participating ads have the chance to win the ad slot. This is in contrast with more widely used deterministic mechanisms like second-price auctions, where the ad with the highest product of bid and quality score always wins the ad slot.

5. The selected ad is placed at the bottom of the app, as shown in Figure 3.

6. Each ad exposure lasts one minute. During this time, the user makes two key decisions: (a) whether to click on the ad, and (b) whether to stay in the app or leave the app and end the session. If the user clicks on the ad, the corresponding advertiser has to pay the amount determined by the auction. After one minute, if the user continues using

the app, the ad network treats the continued exposure as a new impression and repeats steps 3 to 6 until the user leaves the app. We assume that a user has left the app when the time gap until the next exposure exceeds 5 minutes. Consistent with this definition, we define a session as the time interval between the time a user comes to an app and the time she leaves the app.[4]

3.2 Data

We have data on all impressions and clicks for the one month from September 30, 2015, to October 30, 2015. Overall, we observe 1,594,831,699 impressions with the following raw inputs for each impression: (1) timestamp, (2) app ID, (3) user ID (Android Advertising ID), (4) GPS coordinates, (5) targeting variables that include the province, app category, hour of the day, smartphone brand, connectivity type, and mobile service provider (MSP), (6) ad ID[5], (7) bid submitted by the winning ad, and (8) the click outcome. Importantly, our data come directly from the platform so we have access to all the information that the platform collects. Further, we observe all the variables that advertisers can possibly use for targeting. Hence, we can overcome typical issues related to unobserved confounding due to the unobservability of ad assignments.

For our study, we use a sample of our full data that reflects the main goals of this paper. Since we want to optimally sequence ads within the session, our optimal intervention depends on users' history. As such, we only focus on users for whom we can use their entire history. The challenge is that no variable in our data identifies new users. As illustrated in Figure 4, our approach is to split our data into two parts based on a date (October 22) and keep users who are active in the second part of the data (October 22 to October 30), but not in the first part (September 30 to October 22). This sampling scheme guarantees that the users who are identified as new users have not had any activity in the platform at least for the three weeks prior to that. We drop all the other users from our data.

Next, we only focus on the most popular mobile app in the platform, a messaging app that has over a 30% share of total impressions. As such, we drop new users who do not use this app. There are a few reasons why we focus on this app. First, this is the only app whose identity is known to us. Second, we expect the sequencing effects to be context-dependent, so focusing on one app helps us perform a cleaner analysis. Finally, it takes users a relatively

[4]There are obviously various ways to define a session based on the time gap between two consecutive exposures. We show that our results are robust to different definitions.

[5]We do not have the data on the banner creatives and its format, i.e., whether it is a jpeg file or an animated gif.

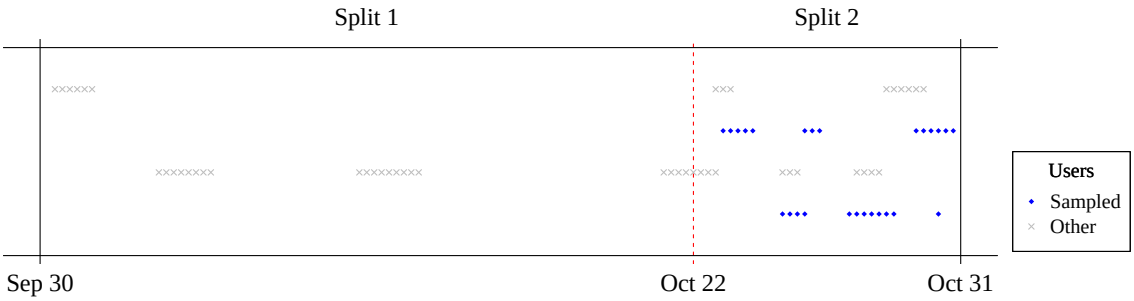


Figure 4: Schema for identification of new users.

long time to learn how to use certain apps (e.g., games), and learning effects can interfere with sequencing effects. However, this messaging app is widely popular in the country and easy to use, so we expect users to pay more attention to ads from the beginning.

Overall, our sampling procedure gives us a total of 8,031,374 impressions shown to a set of 84,306 unique new users. Over 40% of these users use other apps in addition to the messaging app. In our data, there are 1,177,422 unique sessions entirely inside the focal messaging app that correspond to 6,357,389 impressions. We only focus on the impressions shown in the messaging app for our analysis. However, we use impressions shown in other apps for feature generation. Finally, it is worth noting that our sample is almost identical to that of Rafieian and Yoganarasimhan (2021a).[6] We refer the interested reader to that paper for further description of the data.

3.3 Summary Statistics

3.3.1 User-level Variables

As discussed earlier, we sample users for whom we have the entire past history. As such, we can calculate different metrics over the entire user history and present a summary of these metrics across users. We focus on five variables and compute them using the sample of 8,031,374 impressions. We present these statistics in Table 1. We find that, on average, a user has participated in 16.23 sessions, seen 95.26 impressions and 13.97 distinct ads, and clicked 1.55 times on these impressions. Further, the average CTR for a user is roughly 2%,

[6]Our sampling procedure is almost identical to that of Rafieian and Yoganarasimhan (2021a). However, the number of impressions and sessions is slightly different because we need to drop users with missing information on latitude and longitude. Rafieian and Yoganarasimhan (2021a) use those impressions because latitude and longitude do not play a role in their analysis.

ranging from 0 to a CTR as high as 15%. Overall, we observe a large standard deviation and a wide range for all these variables. For example, while the median number of impressions a user has seen is 40 in our data, there is a user who has seen 7,259 impressions. Thus, these statistics suggest substantial heterogeneity in user behavior that we aim to understand in our framework.

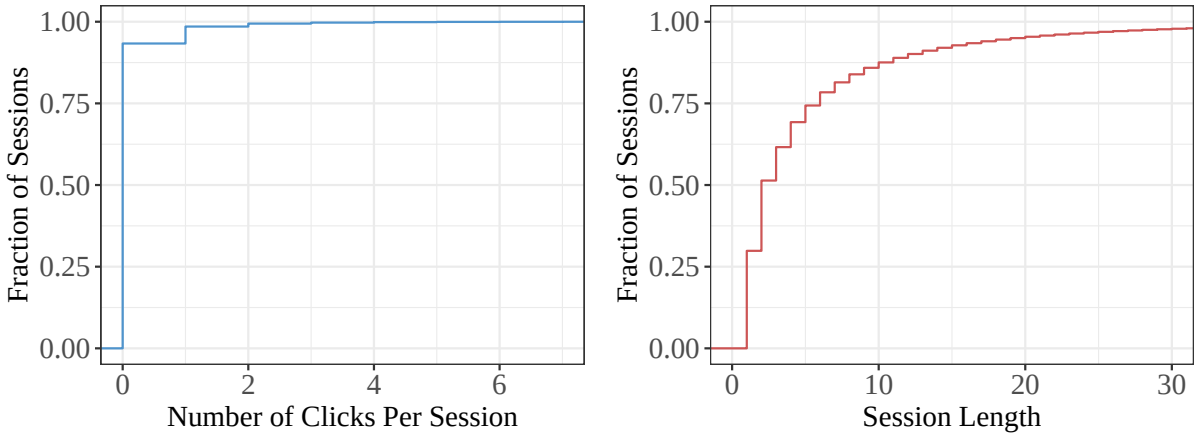
Variable	Mean	SD	Min	Median	Max
Number of Sessions	16.23	20.80	1	9	260
Number of Impressions Seen	95.26	165.62	1	40	7256
Variety of Ads Seen	13.97	11.82	1	11	114
Number of Clicks Made	1.55	2.23	0	1	20
Click-through Rate (CTR)	0.02	0.03	0	0.01	0.15

Table 1: Summary statistics of the user-level variables.

3.3.2 Distribution of Session-Level Outcomes

Our goal in this paper is to examine how much we can improve session-level user engagement through optimal sequencing of ads. As such, the key outcomes are defined at the session-level. We use the sample for the focal app to compute the empirical CDF of two main outcomes of interest in this study – the total number of clicks made in a session and session length. Figure 5a shows the empirical CDF for the total number of clicks per session, which is our primary outcome of interest. As expected, most sessions end with no clicks on ads shown within the session, and the percentage of sessions with at least one click amounts to 6.66%. This is a reasonably high percentage in this industry. Interestingly, there are sessions with more than one click. Further exploration suggests that these sessions are typically much longer than other sessions, with an average length of over 15 exposures.

In Figure 5b, we show the empirical CDF of session length, as measured by the number of exposures shown within any session. This figure shows that around 50% of all sessions end in only two exposures. Further, the empirical CDF in Figure 5b shows that the vast majority of sessions last for ten or fewer exposures, and only a tiny fraction of them last for 30 or more exposures.



(a) Number of Clicks Per Session

(b) Session Length (Number of Exposures)

Figure 5: Empirical CDF of the session length and total number of clicks per session.

4 Framework for Adaptive Ad Sequencing

We now present our dynamic framework for the sequencing of ads. We start with the theoretical setup of our model in §4.1. We then use our theoretical setup to identify and address challenges in empirically designing the optimal policy in §4.2. Next, we discuss how we evaluate a policy using the data at hand in §4.3. Finally, in §4.4, we describe the implementation of our framework and the practical challenges that may arise.

4.1 Theoretical Setup

We begin by describing the theoretical setup of our framework. Let i denote the session, and t denote each impression in that session, e.g., $t = 1$ refers to the first impression in a session. We perform our optimization at the session level, where each decision-making unit is an impression. As discussed earlier, our goal is to develop a dynamic framework that: (1) captures the inter-temporal trade-offs in a publisher's ad placement decision in a session, and (2) uses both pre-session and adaptive session-level information to personalize the sequence of ads for the user in any given session. A Markov Decision Process (MDP) gives us a general framework to characterize the publisher's problem and incorporate the two main goals. An MDP is a 5-tuple $\langle \mathcal{S}, \mathcal{A}, P, R, \beta \rangle$, where $\mathcal{S}$ is the state space, $\mathcal{A}$ is the action space, P is the transition function, R is the reward function, and β is the discount factor. We describe each of these five elements in our context as follows:

1. State Space ($\mathcal{S}$): The state space consists of all the information the publisher has about an exposure, which affects her decision at any time period. The publisher can take two

pieces of information into account: (1) pre-session information, and (2) session-level information. Pre-session information contains any data on the user up until the current session, including his demographic variables and behavioral history. For any session i, we denote the pre-session state variables by X_i. It is important to notice that the pre-session variables are not adaptive, i.e., it does not change within the session, so we can drop the t subscript. On the other hand, session-level variables are adaptive and change within the session. Unlike the conventional approach in MDP that restricts the state to represent only the previous time period, we consider the entire sequence of ads and users' decisions within the session. That is, for any exposure t in session i, we define $G_{i,t}$ as the set of session-level state variables as follows:

$$G_{i,t} = \langle A_{i,1}, Y_{i,1}, A_{i,2}, Y_{i,2}, \ldots, A_{i,t-1}, Y_{i,t-1} \rangle, \tag{2}$$

where $A_{i,s}$ denotes the ad shown in exposure number s and $Y_{i,s}$ denotes whether the user clicked on this ad ($s < t$). As a result, $G_{i,t}$ is the sequence of all ads and actions within the session up to the current time period. Overall, we define the state variables as $S_{i,t} = \langle X_i, G_{i,t} \rangle$, i.e., a combination of both pre-session and session-level variables.

2. Action Space ($\mathcal{A}$): The action space contains the set of actions the publisher can take. In our case, this action is to show one ad from the ad inventory every time an impression is recognized. As such, $\mathcal{A}$ is the entire ad inventory in our problem.

3. Transition Function (P): This function determines how the current state transitions to the future state, given the action made at that point. As such, we can define $P : \mathcal{S} \times \mathcal{A} \times \mathcal{S} \to [0, 1]$ as a stochastic function that calculates the probability $P(s' \mid s, a)$ where $s, s' \in \mathcal{S}$ and $a \in \mathcal{A}$. Note that this is a crucial component of an MDP since publishers cannot control the dynamics of the problem if the next state is not affected by the current decision. In §4.1.1, we discuss the components of the transition function in our problem in detail.

4. Reward Function (R): This function determines the reward for any action a at any state s. As such, we can define this function as $R : \mathcal{S} \times \mathcal{A} \to \mathbb{R}$. This function can take different forms depending on the publisher's objective. In our case, since the publisher is interested in optimizing user engagement, they can use different metrics that reflect user engagement, such as the probability that the user clicks on the ad. In §4.1.2, we discuss our choice of reward function in greater details.

5. Discount Factor (β): The rate at which the publisher discounts the expected future rewards. Given the short time horizon of the optimization problem, a risk-neutral publisher must value the current and expected future rewards equally, indicating that β is very close to 1.

With all these primitives defined, we can now write the publisher's maximization problem as follows:

$$\underset{a}{\operatorname{argmax}} \left[R(s,a) + \beta \mathbb{E}_{s'|s,a} V(s') \right], \tag{3}$$

where $V(s')$ is the value function incorporating expected future rewards at state s' if the publisher selects ads optimally. Following Bellman (1966), we can write this value function for any state $s \in \mathcal{S}$ as follows:

$$V(s) = \max_{a} R(s,a) + \beta \mathbb{E}_{s'|s,a} V(s') \tag{4}$$

In summary, as shown in Equation (3), the optimization problem consists of two key elements – the current period reward and the expected future rewards. The publisher chooses the ad that maximizes the sum of these two elements.

4.1.1 Transition Function

We now characterize the law-of-motion, i.e., how state variables transition given the publisher's action at any point. As mentioned earlier, we are interested in the probability of the next state being s', given that action a is taken in state s, i.e., $P(s' \mid a, s)$. Suppose that the user is in state $S_{i,t} = \langle X_i, G_{i,t} \rangle$ at exposure t in session i. The only time-varying factor in $S_{i,t}$ that can transition is $G_{i,t}$, which is the history of the sequence. Given the definition of $G_{i,t}$ in Equation (2), we can determine the next state if we know the user's decision to click on the current ad and/or continue staying in the session. There are three mutually exclusive possibilities for state transitions:

1. *Case 1 (click and stay):* If the user clicks on ad $A_{i,t}$ and stays in the session, we can define the next state as follows:

$$S_{i,t+1} = \langle X_i, G_{i,t}, A_{i,t}, Y_{i,t} = 1 \rangle, \tag{5}$$

   where $Y_{i,t} = 1$ indicates that the user has clicked on the ad shown in exposure number t.

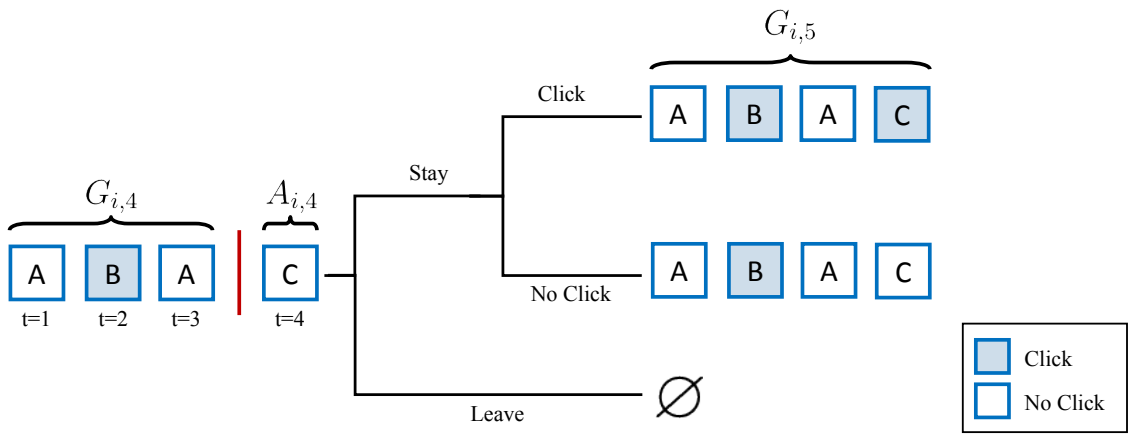


Figure 6: An example illustrating the state transitions.

2. *Case 2 (no click and stay):* If the user does not click on ad $A_{i,t}$ and stays in the session, we can similarly define the next state as follows:

$$S_{i,t+1} = \langle X_i, G_{i,t}, A_{i,t}, Y_{i,t} = 0 \rangle, \tag{6}$$

where $Y_{i,t} = 0$ indicates that the user has not clicked on the ad shown in exposure number t.

3. *Case 3 (leave):* Regardless of user's clicking outcome, if the user decides to leave, the entire session is terminated and there is no more decision to be made. Thus, we can write:

$$S_{i,t+1} = \varnothing \tag{7}$$

Figure 6 visually presents the three possibilities presented above. This figure illustrates an example where the publisher shows an ad in the fourth exposure in a session. It shows three possibilities and how each forms the next state. Based on this characterization, we can now define the transition function for any pair of action and state as follows:

$$P(S_{i,t+1} \mid a, S_{i,t}) = \begin{cases} \big(1 - P(L_{i,t} = 1 \mid a, S_{i,t})\big) P(Y_{i,t} = 1 \mid a, S_{i,t}) & \text{Case 1, Eq. (5)} \\ \big(1 - P(L_{i,t} = 1 \mid a, S_{i,t})\big)\big(1 - P(Y_{i,t} = 1 \mid a, S_{i,t})\big) & \text{Case 2, Eq. (6)} \\ P(L_{i,t} = 1 \mid a, S_{i,t}) & \text{Case 3, Eq. (7)} \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

Equation (8) illustrates the two non-deterministic components of state transitions – click and leave probabilities. As such, estimating these two outcomes would be equivalent to estimating transition functions. In §4.2, we discuss our approach to obtain these estimates.

4.1.2 Reward Function

Another piece of an MDP that needs to be defined is the reward function. The reward function can take different forms depending on the publisher's objective. We primarily focus on maximizing the total number of clicks per session as our main objective because of a few reasons. First, clicks are the main source of revenue for the publisher since the advertiser only pays when a click happens. Second, almost all ads in our study are mobile apps whose objective is to get more clicks and installs. In the literature, this type of ad is referred to as performance ads, and their match value is generally assumed to be the probability of click (Arnosti et al., 2016). Hence, clicks are particularly good measures of user engagement with ads in our setting. Third, clicks are realized immediately in the data and well-recorded without measurement error.

Given that publishers want to maximize the number of clicks made per session, we can define the reward function as the probability of click for a pair of state and action. For exposure number t in session i, we can write:

$$R(S_{i,t}, a) = P(Y_{i,t} = 1 \mid a, S_{i,t}) \tag{9}$$

This is the probability of clicking on ad a if shown in the current state.

4.2 Empirical Strategy for Policy Identification

In this section, we discuss how we can take our theoretical framework to data and identify the policy that maximizes the expected rewards for each session, as characterized in our MDP. To do so, we first formally define a policy as follows:

Definition 1. *A policy is a mapping $\pi : \mathcal{S} \times \mathcal{A} \to [0, 1]$, that assigns a probability $\pi(a \mid s)$ to any action $a \in \mathcal{A}$ taken in any given state $s \in \mathcal{S}$.*

This definition of policy allows for both deterministic and non-deterministic policies.[7] We now characterize our main goal in this section: we want to use our data to identify a policy π^* that maximizes the expected rewards for a session. That is, from the beginning to the end of a session, this policy determines which ad to show in each exposure to maximize the

[7]For a deterministic policy, $\pi(a \mid s)$ will take value one only for one ad for any given state.

expected sum of rewards in that session. Following our MDP characterization, the optimal action at any given point is determined as follows:

$$\arg\max_{a \in \mathcal{A}_{i,t}} \left[R(S_{i,t}, a) + \beta \mathbb{E}_{S_{i,t+1}|S_{i,t},a} V(S_{i,t+1}) \right], \tag{10}$$

where $\mathcal{A}_{i,t}$ is the ad inventory and $S_{i,t}$ is the state variable at exposure t in session i. Solving the optimization problem in Equation (10) for each possible state gives us the optimal policy function π^*.

To solve the dynamic programming problem defined in Equation (10), we face three key challenges. First, we need to obtain personalized estimates of the two unknown primitives in Equation (10) – click and leave probabilities. That is, for any pair of state variables and ad, we need to accurately estimate the probability of click and leave. The second challenge stems from the fact that our optimization is over the set of all ads. As such, even if we develop models that obtain personalized estimates of click and leave outcomes with high predictive accuracy for ads that are shown in our data, there is no guarantee that these models provide accurate estimates for the set of all possible ads (i.e., counterfactual ads). Thus, we need a framework with counterfactual validity. Finally, although in principle, it is sufficient to have the estimates of reward and transition probabilities in order to find value functions, such an exact solution is not computationally feasible in our setting where the state space is high dimensional and grows exponentially in the number of time periods. Hence, we need an approximate solution that is scalable. We discuss our solution to each of these three challenges in the following sections in greater detail.

4.2.1 Personalized Estimation of Model Primitives

We start with our first challenge and formalize it as follows:

Challenge 1. *Let $\mathcal{D} = \{(S_{i,t}, A_{i,t}, Y_{i,t}, L_{i,t})\}_{i,t}$ denote the sample of impressions available, where the click and leave outcomes are recorded for each impression as $Y_{i,t}$ and $L_{i,t}$ respectively. We want to estimate functions $\hat{l}$ and $\hat{y}$ that take a pair of state variable ($S_{i,t}$) and action ($A_{i,t}$) as input and returns personalized estimates of expected click and leave probabilities as follows:*

$$\hat{y}(S_{i,t}, A_{i,t}) = \mathbb{E}(Y_{i,t} \mid S_{i,t}, A_{i,t}) \tag{11}$$

$$\hat{l}(S_{i,t}, A_{i,t}) = \mathbb{E}(L_{i,t} \mid S_{i,t}, A_{i,t}) \tag{12}$$

To address this challenge, we need a function that can differentiate between impressions given the available information. Since this is an outcome prediction task, we need to use

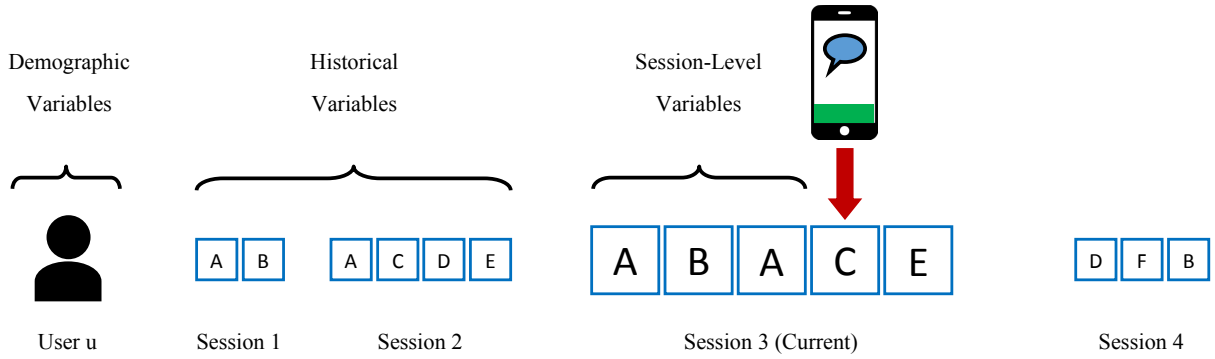


Figure 7: A visual schema for our feature categorization.

machine learning methods that do not impose restrictive parametric assumptions and capture more complex relationships between the covariates and outcomes (Mullainathan and Spiess, 2017). Further, to allow a machine-learning algorithm to differentiate between impressions, it is essential to generate a rich set of covariates or features to represent impressions. Thus, our task becomes one of feature engineering where we want to use our domain knowledge to map $\langle S_{i,t}, A_{i,t} \rangle$ to a set of meaningful features that help us predict both click and leave outcomes.

We first define four feature categories: (1) timestamp and the ad shown in the impression that constitute the contextual information about the impression, (2) demographic features that are raw inputs about the user that are recorded by the platform, such as user's location and smartphone brand, (3) historical features that contain the information about the user's behavioral history up until the current session, such as the number of impressions the user has seen in prior sessions, and (4) session-level features that only use the information from the current session, such as the variety of previous ads shown in the session. Figure 12 provides an overview of our feature categorization. In this example, the user is at her fourth exposure in her third session. The features for this particular exposure include the observable demographic features, historical features generated from the prior sessions, and session-level features that are generated from the first three exposures shown in the current session.[8]

Our feature generation framework borrows from the literature on the advertising dynamics and behavioral mechanisms underlying these dynamics. Since the raw inputs for historical and session-level features are a user's past interactions with ads, we use features that summarize each user's long- and short-term interactions with each ad in terms of frequency akin to goodwill stock models (Nerlove and Arrow, 1962; Dubé et al., 2005), recency or spacing

[8]Naturally, we cannot use any information from the future to generate a feature: at any point, we only use the prior history up to that point.

according to memory-based models (Sawyer and Ward, 1979; Sahni, 2015), and clicks that have been shown to greatly help with the task of click prediction (Rafieian and Yoganarasimhan, 2021b). While we use the literature to inform our feature generation, we take an agnostic approach and let our learning algorithm flexibly capture these relationships. We store these features in large inventory matrices where rows are sessions and columns are ads. This parsimonious yet rich inventory-based summarization allows us to generate other features such as ad variety and diversity as they are determined by the frequency of all ads. We further include other usage-based features such as average session length or time interval between sessions to predict the leave outcome more accurately based on the past data. Overall, our feature generation framework takes $\langle S_{i,t}, A_{i,t} \rangle$ and gives us a set of features $g(S_{i,t}, A_{i,t})$ for each impression that we can use as inputs of our learning algorithm. We present the details of all these features in Web Appendix §A.

4.2.2 Counterfactual Validity

Our second challenge comes from the policy aspect of our framework – not only do we need to obtain personalized estimates of click and leave outcomes for impressions shown in our data, but we also need to estimate these outcomes for counterfactual ads that are not shown in the data. One immediate solution is to apply our feature generation framework to counterfactual impressions and use our learning algorithm to estimate the outcomes. However, this approach can run into two key problems. First, while machine learning algorithms are known to do well in the task of interpolation, we need further guarantees on the feasibility of our counterfactual impressions for the task of extrapolation, i.e., counterfactual estimation. Second, suppose the ad assignment is confounded by an unobserved factor that is not in our feature set. In that case, the learning algorithm may incorrectly learn the link between the unobserved variable and outcomes as an ad effect. This is similar to the issue of endogeneity or selection on unobservables in the causal inference literature. We formally present these two challenges as follows:

Challenge 2. *Suppose the predictive models $\hat{y}$ and $\hat{l}$ are trained on data $\mathcal{D} = \{(S_{i,t}, A_{i,t}, Y_{i,t}, L_{i,t})\}_{i,t}$. Let $\mathcal{D}_c = \{\bigcup_{a \in \mathcal{A}_{i,t}} (S_{i,t}, a, Y_{i,t}, L_{i,t})\}_{i,t}$ denote the counterfactual data set. To ensure the counterfactual validity of our estimates on the couterfactual data, we need to address the following challenges:*

1. *For any ad $a \in \mathcal{A}_{i,t}$, the data point with the pair of state variable and action $(S_{i,t}, a)$ and the corresponding set of features $g(S_{i,t}, a)$ could have been generated in our training data $\mathcal{D}$, so finding values of $\hat{y}(S_{i,t}, a)$ and $\hat{l}(S_{i,t}, a)$ is a form of interpolation.*

2. *For any ad $a \in \mathcal{A}_{i,t}$, the assignment probability only depends on the observed set of features used in training models $\hat{y}$ and $\hat{l}$.*

To satisfy the first condition in Challenge 2, we need to identify the feasibility set $\mathcal{A}_{i,t}$ for each impression such that any ad $a \in \mathcal{A}_{i,t}$ *could have been shown* in that impression. This is equivalent to the *overlap* or *positivity* assumption in the causal inference literature that requires each treatment condition (ad in our case) to have a non-zero propensity score. That is, if $e(S_{i,t}, a)$ denotes the propensity of ad a to be shown in exposure t in session i, we must have $e(S_{i,t}, a) > 0$ for any $a \in \mathcal{A}_{i,t}$. While attainable in principle, this is a condition that is rarely satisfied in most non-experimental digital advertising settings since ads are selected through a deterministic allocation rule in commonly used auctions such as second-price. In our setting, however, the platform uses a quasi-proportional auction that induces randomization in ad allocation: each ad has a non-zero propensity score if and only if it participates in an auction. As such, the propensity score is zero only when the ad is not participating in an auction due to their targeting decision or campaign availability. We employ a filtering strategy similar to that in Rafieian and Yoganarasimhan (2021b), where for each impression, we filter out ads that *could have never shown*. The remaining ads constitute our feasibility set $\mathcal{A}_{i,t}$, which is generally a rich set of ads given the low level of targeting in our platform. We present the details of our filtering strategy in Appendix §B.1.

The second condition in Challenge 2 also has a strong link to the causal inference literature. While this is a predictive task, our learning algorithm may still incorrectly learn the ad effects if there is any unobserved confounding. For example, suppose ad a_1 is more likely to be shown to less-educated adults than ad a_2, but we do not observe education in our data. Now, if less-educated adults have a higher probability of click, our learning algorithm may attribute the link between education and click to ads a_1 and a_2, if it does not control for education. Unconfoundedness is what satisfies this condition. That is, conditional on observed features $g(S_{i,t}, a)$, the assignment to ads is random. We can formally show this as a proposition in our data as follows:

Proposition 1. *In a setting with a quasi-proportional auction and observable targeting, the distribution of propensity scores is fully determined by observed covariates.*

Proof. Please see Appendix §B.2 □

To provide empirical support for this proposition, we estimate propensity scores using observed features and assess covariate balance (please see Appendix §B.3). We then include these propensity scores $\hat{e}(S_{i,t}, a)$ in our feature set $g(S_{i,t}, a)$ to ensure that the assignment

probabilities are accounted for. This further guarantees the unconfoundedness assumption as the conditional independence is satisfied only by conditioning on propensity scores (Rosenbaum and Rubin, 1983).

4.2.3 Value Function Approximation

Now, we discuss the final piece of our empirical framework to develop an optimal dynamic policy. Recall the publisher's optimization problem in Equation (10):

$$\arg\max_{a \in \mathcal{A}_{i,t}} \left[R(S_{i,t}, a) + \beta \mathbb{E}_{S_{i,t+1}|S_{i,t},a} V(S_{i,t+1}) \right].$$

In §4.2.1 and §4.2.2, we show how we can get the reward $R(S_{i,t}, a)$, as well as the law of motion as captured by the expectation $\mathbb{E}_{S_{i,t+1}|S_{i,t},a}$ from the equation above. The unknown part is the value function V that captures future rewards. We can use Bellman equation to characterize this value function in a recursive relationship as follows:

$$V(S_{i,t}) = max_{a \in \mathcal{A}_{i,t}} R(S_{i,t}, a) + \beta \mathbb{E}_{S_{i,t+1}|S_{i,t},a} V(S_{i,t+1}). \tag{13}$$

Since we know the reward function and law of motion, the typical approach to find the value function is to construct a table of all states and directly find values using Equation (13). However, this task becomes infeasible when we have a high-dimensional state space, as we need to store all the corresponding values. We can formally characterize the computational intensity of this task as follows:

Challenge 3. *Let T denote the length of the horizon over which we want to perform our optimization, and let N denote the number of sessions. For each session, our state space grows exponentially in T. Specifically, for a single session i, the order of state variables would be $O((2|\mathcal{A}_{i,1}|)^{T-1})$, since we need to record the entire ad sequence as well as actions (click or not click). Thus, for all sessions the complexity order would be $O((2 \max_i |\mathcal{A}_{i,1}|)^{T-1} \times N)$, where $|\mathcal{A}|$ is the size of our ad inventory.*

To put things in perspective, even if we only have 10 ads in our inventory and want to perform the dynamic optimization for 10 periods, each session has the complexity order of 10^9. Now, if we want to that for the number of sessions in our data that is roughly one million, the order of complexity would be 10^{15}.

Our solution is to develop a function approximation algorithm that approximates the value function instead of finding all the values directly. That is, we want to learn a function $\hat{v} : \mathcal{S} \to \mathbb{R}$ with a set of parameters θ_v. This approach can significantly reduce the time

complexity since we need only an order of magnitude smaller subset of states to learn a function, and the representation of this function is only through the set of parameters θ_v.

Before we present our algorithm, we first introduce a new notation. We define a function $Q : \mathcal{S} \times \mathcal{A} \to \mathbb{R}$ to represent the entire term that the publisher maximizes in Equation (10) as follows:

$$Q(S_{i,t}, a) = R(S_{i,t}, a) + \beta \mathbb{E}_{S_{i,t+1}|S_{i,t},a} V(S_{i,t+1}). \tag{14}$$

The Q function is often referred to as the choice-specific value function in the econometrics literature (Aguirregabiria and Mira, 2002). Given the Bellman equation in Equation (13), we can write:

$$Q(S_{i,t}, a) = R(S_{i,t}, a) + \beta \mathbb{E}_{S_{i,t+1}|S_{i,t},a} \max_{a' \in \mathcal{A}_{i,t+1}} Q(S_{i,t+1}, a'). \tag{15}$$

Now, we can use our transition function in Equation 8 and plug in our estimates for click and leave probabilities to define $\tilde{Q}_t$ in a similar way to Equation (15) as follows:

$$\begin{aligned} \tilde{Q}_t(S_{i,t}, a) = \hat{y}(S_{i,t}, a) &+ (1 - \hat{l}(S_{i,t}, a))\hat{y}(S_{i,t}, a) \max_{a' \in \mathcal{A}_{i,t+1}} \tilde{Q}_{t+1}(\langle S_{i,t}, a, Y_{i,t} = 1 \rangle, a') \\ &+ (1 - \hat{l}(S_{i,t}, a))(1 - \hat{y}(S_{i,t}, a)) \max_{a' \in \mathcal{A}_{i,t+1}} \tilde{Q}_{t+1}(\langle S_{i,t}, a, Y_{i,t} = 0 \rangle, a'), \end{aligned} \tag{16}$$

where the first term $\hat{y}(S_{i,t}, a)$ is the current period reward, and the other two elements in the RHS of Equation (16) capture the two transition possibilities where the session still continues – "click and stay" and "no click and stay".

Function $\tilde{Q}_t$ represents a plugin version of our Q function in Equation (14) at time period t, where we directly plug in our reward and transition estimates to find the Q values.[9] Our goal is to estimate a function $\hat{q}_t$ that approximates $\tilde{Q}_t$. However, this task is not trivial as these functions appear in both LHS and RHS of Equation (16). We can follow the common insight in the literature to formulate an iterative procedure such as value iteration or backward induction to simplify the task to supervised learning. In our framework, we focus on backward induction as it is reasonable to assume a finite horizon because most sessions end in a few exposures. Further, for a short length of horizon T, the backward induction algorithm runs faster than a value iteration algorithm since value iteration may require far more iterations for convergence.

The logic behind backward induction for q-function approximation (BIQFA) is simple: from the set of $\{\hat{q}_1, \hat{q}_2, \ldots, \hat{q}_T\}$, we learn the functions one at a time in a backward order. We start with the last time period T where the function $\hat{q}_T$ is equivalent to our click prediction

[9]It is worth noting that the subscript t in $\tilde{Q}_t$ is only for notational simplicity.

function $\hat{y}$ since this is the last period and the future rewards are assumed to be zero.[10] We can then complete the RHS of Equation (16) and obtain the plugin outcomes for any subset of states in period $T-1$. These plugin outcomes are often referred to as Bellman backups and denoted by $\bar{Q}$ (Lee et al., 2021). Once we have these plugin outcomes, the task of estimating $\hat{q}_{T-1}$ simplifies to one of supervised learning, where we can use our set of state variables and actions to estimate the plugin outcomes or Bellman backups. We can continue this process until we have the full set of functions $\{\hat{q}_1, \hat{q}_2, \ldots, \hat{q}_T\}$. We present our algorithm in detail below:

Algorithm 1 Backward Induction for Q-Function Approximation (BIQFA)

Input: $\mathcal{D}, \hat{y}, \hat{l}, \hat{e}, T, \tilde{S}_1, \tilde{S}_2, \ldots, \tilde{S}_T$ ▷ $\tilde{S}_t \subset \mathcal{S}$ at exposure t
Output: $\hat{q}_1, \hat{q}_2, \ldots, \hat{q}_T$

1: $\hat{q}_T \leftarrow \hat{y}$
2: **for** $t = T-1 \rightarrow 1$ **do**
3: $\quad \tilde{Q}_{t+1} \leftarrow \hat{q}_{t+1}$
4: $\quad$ **for each** $s \in \tilde{S}_t, a \in \mathcal{A}$ **do**
5: $\quad\quad \bar{Q}_{s,a} \leftarrow \tilde{Q}_t(s,a)$ ▷ Create Bellman backups using Equation (16)
6: $\quad\quad$ **if** $\hat{e}(s,a) = 0$ **then**
7: $\quad\quad\quad \bar{Q}_{s,a} = 0$
8: $\quad\quad$ **end if**
9: $\quad\quad Z_{s,a} \leftarrow \{g(s,a), \hat{y}(s,a), \hat{l}(s,a)\}$ ▷ Set of inputs given to the learning algorithm
10: $\quad$ **end for**
11: $\quad \hat{q}_t \leftarrow$ ***learn***$(Z_{s,a}, \bar{Q}_{s,a})$ ▷ Any learning algorithm can be used
12: **end for**

A few points are worth noting about our BIQFA algorithm. First, $\tilde{S}_t$ should be a relatively small sub-sample of the full state space at exposure t for computational tractability. As such, the closer the sample is to the actual states that would be generated under the optimal policy, the better the function approximation learns the q-function. A good candidate is to use an adaptive myopic policy that selects the ad with the highest reward at any point, i.e., $\text{argmax}_{a \in \mathcal{A}_{i,t}} \hat{y}(S_{i,t}, a)$ for any state variable $S_{i,t}$. Second, while our set of generated features $g(s,a)$ suffices in principle for learning q-functions, we include click and leave predictions as features (line 9 of our algorithm) to help the learning algorithm capture the dynamic structure more easily. Finally, given that we use propensity scores in our feature set $Z_{s,a}$, the learning algorithm easily learns the association between zero propensity and zero Bellman backup.

[10] Formally, we can incorporate that by setting $\tilde{Q}_s(\cdot) = 0$ for any $s > T$.

4.3 Evaluation

Once we identified the optimal dynamic policy for adaptive ad sequencing using our empirical framework in §4.2, we need to evaluate this policy and compare it to other benchmarks. As such, we need an evaluation framework that takes any policy π^* and data $\mathcal{D}$ as input and evaluates the policy in terms of the outcomes of interest, specifically the expected number of clicks per session. This task is often referred to as *counterfactual policy evaluation* in the marketing and economics literature, and *off-policy policy evaluation* in the reinforcement learning literature.

The fundamental problem is that the data at hand are often generated by a *behavior policy* π^b, which is different from the policies we want to evaluate (π^*). In a case like that, there are many approaches to evaluate the policy π^*. The common approach in marketing and economics literature is to use a counterfactual simulation approach, where we simulate the data given policy π, using the estimates for reward and transition functions (Dubé et al., 2005; Simester et al., 2006). This approach is often referred to as the *direct method (DM)* in the reinforcement learning literature as it directly uses model estimates to evaluate the policy (Kallus and Uehara, 2020). An important advantage of this approach is that it can capture the heterogeneity at the most granular level, which is session-level in our case. That is, we can evaluate each session under a policy and examine which sessions have higher gains. On the other hand, the main issue with the DM is that reward and transition estimates may be largely biased in the absence of randomization, resulting in a biased policy evaluation. In our setting, we have randomization in ad allocation that satisfies the unconfoundedness assumption. Thus, the typical challenges with the DM approach are not present in our setting.

Nevertheless, there is still an important challenge in DM when it comes to policy evaluation:

Challenge 4. *Let $\mathcal{D}_{\text{Model}}$ denote the data used for policy identification, and $\mathcal{D}_{\text{Evaluation}}$ denote the data used for policy evaluation. If $\mathcal{D}_{\text{Model}} = \mathcal{D}_{\text{Evaluation}}$, then our evaluation always shows a better performance for the identified optimal dynamic policy, because our policy identification framework chooses a policy if it is best-performing given $\mathcal{D}_{\text{Model}}$ and models trained on it.*

This is an important theoretical issue, which is often unaddressed in counterfactual policy evaluation in the structural econometrics literature. To ensure that our imposed structure does not force a certain outcome, we follow the insights from the evaluation approach in Mannor et al. (2007) and double q-learning in Hasselt (2010) for de-biasing the value function estimates through sample splitting such that $\mathcal{D}_{Model} \cap \mathcal{D}_{Evaluation} = \varnothing$. We call this approach *honest direct method (HDM)* and present it in the step-by-step procedure as follows:

- *Step 1:* We split the data into three parts: $\mathcal{D}_{Model}$, $\mathcal{D}_{Evaluation}$, and $\mathcal{D}_{Test}$.
- *Step 2:* We use our modeling data $\mathcal{D}_{Model}$ to estimate functions needed for policy identification: $\hat{y}^M$, $\hat{l}^M$, and $\hat{q}_t^M$ for any t (notice that superscript M refers to the data used for estimation). We can use these functions to identify the optimal policy π^M.
- *Step 3:* We use our evaluation data to estimate the model primitives $\hat{y}^E$ and $\hat{l}^E$ needed to simulate the data under any counterfactual policy, where superscript E refers to the fact that we use the evaluation data.
- *Step 4:* For any session in $\mathcal{D}_{Test}$, we use our policy π^M from Step 2 and our estimates $\hat{y}^E$ and $\hat{l}^E$ from Step 3 to simulate the data under the policy and evaluate its outcomes. While we can run large-scale simulations to evaluate the outcome, there is an analytical derivation for our Honest Direct Method (HDM). For any exposure t, let g_t denote a t-step trajectory of states, actions, and rewards as follows:

$$g_t = \langle s_1, a_1, r_1, \ldots, s_{t-1}, a_{t-1}, r_{t-1}, s_t, a_t, r_t \rangle, \tag{17}$$

  where s, a, and r denote state, action (ad), and the reward outcome respectively. The probability of any arbitrary g_t is determined by the policy π^M and transition functions $\hat{y}^E$ and $\hat{l}^E$. For brevity, we use γ^E to denote the joint distribution of transitions. The trajectory g_t comes from the joint distribution (π^M, γ^E), where the policy comes from Step 2 and transitions come from Step 3 to satisfy our honesty criteria. Now, for any session i with initial state $S_{i,1}$ and policy π^M, we can define the policy evaluation function ρ as follows:

$$\rho(\pi^M; S_{i,1}, T) = \mathbb{E}_{g_t \sim (\pi^M, \gamma^E)} \left[\sum_{t=1}^{T} \beta^{t-1} r_t \mid s_1 = S_{i,1} \right], \tag{18}$$

  where T denotes the horizon length and the expectation is taken over all trajectories. While all trajectories is a massively large set, we can develop different algorithms to perform this task more efficiently and find $\rho(\pi^M; S_{i,1}, T)$. We describe the algorithm we use in Appendix §C.1.

Overall, by splitting our data into three sets, our HDM approach overcomes two important issues with a model-based evaluation – (1) using a separate test set to perform policy evaluation avoids the issues of overfitting, and (2) separating the modeling and evaluation data sets ensures that the imposed structure of policy evaluation does not systematically favor one policy over another. That is, any other policy can theoretically outperform our optimal dynamic policy.

4.4 Practical Considerations and Implementation

While our framework is set up more generally to be broadly applicable to other domains, there are many elements that we need to set given the context, such as the length of the horizon or the size of action space (ad inventory). We discuss these practical details in this section as follows:

- First, we need to set the length of horizon T. From our data, we observe that over 85% of sessions end in 10 or fewer exposures (Figure 5b). As such, $T = 10$ is a reasonable choice as the majority of events happen in the first ten exposures. However, it is worth emphasizing that the computational complexity increases only linearly in T in our function approximation framework.
- Second, we need to define the ad inventory. An obvious choice would be to focus on our inventory's entire set of ads. While our framework is computationally scalable to having a large action space, it would be practically difficult to obtain accurate, personalized estimates for ads with low frequency in data. As a result, we only focus on the top 15 ads with the highest frequency in our data that collectively generate over 70% of all impressions.[11]
- Third, we need to set a splitting rule for $\mathcal{D}_{Model}$, $\mathcal{D}_{Evaluation}$, and $\mathcal{D}_{Test}$. We split our data at the user level according to an approximately 40-40-20 percent rule such that $\mathcal{D}_{Test}$ contains sessions for 20% of users and $\mathcal{D}_{Model}$ and $\mathcal{D}_{Evaluation}$ each represents 40% of users. The specific details of our splitting procedure is presented in Appendix §C.2.
- Fourth, we need to choose a learning algorithm and a validation procedure for the task of estimating click and leave outcomes, i.e., functions $\hat{y}^M$, $\hat{l}^M$, $\hat{y}^E$, and $\hat{l}^E$. Generally, one could use any learning algorithm to estimate these functions. In our study, we use the Extreme Gradient Boosting (XGBoost henceforth) method developed by Chen and Guestrin (2016), which is a fast and scalable version of Boosted Regression Trees (Friedman, 2001). There are some key reasons why we use XGBoost as our main learning. First, it has been shown to outperform most existing methods in most prediction contests, especially those related to human decision-making like ours (Chen and Guestrin, 2016). Second, Rafieian and Yoganarasimhan (2021b) show that in the same context, XGBoost achieves the highest predictive accuracy compared to other methods. Following the arguments in Rafieian and Yoganarasimhan (2021b), we use the logarithmic loss as our loss function. To tune the parameters of XGBoost, we use a hold-out validation procedure to prevent the model from over-fitting. We select the hyper-parameters accurately using

[11] Each one of these top 15 ads has been shown at least in 1% of all impressions.

a grid search over a large set of hyper-parameters and select those that give us the best performance on a hold-out validation set. For more details, please see Appendix D.

- Fifth, for the task of q-function approximation in our BIQFA algorithm, we need to specify a learning algorithm. For internal consistency, we use XGBoost as our learning algorithm.

In sum, the choices above are made not because of the limitations in our framework but rather according to the specifics of our context. In a different context, one may need to change these decisions to get the best out of this framework.

5 Results

In this section, we present our results. First, in §5.1, we present some results on the predictive accuracy of our machine learning models for click and leave estimation. Next, in §5.2, we perform counterfactual policy evaluation and document the gains from adopting our adaptive ad sequencing framework over a series of benchmarks. Finally, we explore the mechanism and develop descriptive tools to explain the gains from our framework in §5.3.

5.1 Predictive Accuracy of Machine Learning Models

In this section, we examine the predictive accuracy of our click and leave estimation models. We focus on two different metrics that capture different aspects of the predictive performance:

- *Relative Information Gain (*RIG*):* This metric reflects the percentage improvement in logarithmic loss compared to a baseline model that simply predicts the average CTR for all impressions. We use *RIG* as our primary metric as it is defined based on the log loss, which is the loss function we used in our XGBoost models to estimate click and leave outcomes.
- *Area Under the Curve (*AUC*):* It determines how well we can identify *true positives* without identifying *false positives*. This score ranges from 0 to 1, and a higher score indicates better performance and greater classification.

These two metrics are commonly used to evaluate the predictive performance of click prediction models. In general, *RIG* is more relevant when we want to evaluate how well our model estimates the probabilities, whereas AUC demonstrates how good a classifier our model is. For both metrics, a higher value means better performance.

Model	Outcome	Training Sample	Metric	*Sample*		
				$\mathcal{D}_{Model}$	$\mathcal{D}_{Evaluation}$	$\mathcal{D}_{Test}$
$\hat{y}^M$	Click	$\mathcal{D}_{Model}$	*RIG*	0.2123	0.1988	0.2021
			AUC	0.8229	0.8110	0.8139
$\hat{y}^E$	Click	$\mathcal{D}_{Evaluation}$	*RIG*	0.2019	0.2175	0.2024
			AUC	0.8138	0.8283	0.8138
$\hat{l}^M$	Leave	$\mathcal{D}_{Model}$	*RIG*	0.1009	0.0882	0.0881
			AUC	0.7189	0.7055	0.7047
$\hat{l}^E$	Leave	$\mathcal{D}_{Evaluation}$	*RIG*	0.0880	0.1005	0.0877
			AUC	0.7051	0.7188	0.7045

Table 2: Predictive accuracy of XGBoost models for click and leave estimation.

5.1.1 Results from Click and Leave Estimation Models

We now evaluate the predictive performance of our click and leave estimation models. As discussed earlier in §4.3, our honest direct method estimates two separate models for each outcome – one using the modeling data $\mathcal{D}_{Model}$, and the other using the evaluation data $\mathcal{D}_{Evaluation}$. This gives us a total of four models $\hat{y}^M$, $\hat{y}^E$, $\hat{l}^M$, and $\hat{l}^E$. We present both *RIG* and *AUC* for each of these models when evaluated on modeling, evaluation, and test samples separately.

We present our results in Table 2. In the top two panels, we examine the predictive accuracy of our click models. The model achieves an over 0.20 *RIG* on the test set, which demonstrates a substantial predictive accuracy compared to the literature (Yi et al., 2013). Further, both in- and out-of-sample, our click models achieve an *AUC* of over 0.80, which shows a good classification performance by the model.

The last two panels in Table 2 show how our leave models perform. Unlike our click models, we do not expect our leave model to reach a very high predictive accuracy because app usage is less dependent on ad exposures and more driven by the app. This is particularly challenging for a messenger app where users' decision to leave primarily stems from their messaging behavior, which is unobserved to the advertising platform. Despite these limitations, both our *RIG* and *AUC* measures show information gain from our predictive model compared to average estimators. Thus, our approach to endogenize usage is advantageous over a bulk of papers in the literature that rely on simple average estimates for continuation probabilities

(Kempe and Mahdian, 2008; Kar et al., 2015).[12]

5.1.2 Value of Different Pieces of Information

We use a rich set of features to build our models. We now want to see which pieces of information contributed more towards building a better predictive model. Consistent with our feature categories in §4.2.1, we define four categories that vary in the granularity and level of personalization they can allow: (1) *ad+timestamp* which comprises the identities of ads shown and the timestamp of the impression but does not include personal information that requires any form of tracking, (2) *demographic features* that are raw characteristics about the user such as location and smartphone brand, hence containing some personal information, (3) *historical features* that are constructed based on the user's past behavioral history (e.g., ads seen and clicks), and require user tracking up until the current session, and (4) *session-level features* that goes one step beyond historical features and collects similar information about the user's current session, thereby requiring advanced real-time tracking. We want to compare the contribution of these four different pieces of information to the predictive performance of our models.

An interesting characteristic of Gradient Boosted Trees is the ability to return the importance of features based on the number of times each feature is selected for splitting and the corresponding empirical improvement (Friedman, 2001). As such, we automatically know the importance measures from every single feature. Since the four feature categories include mutually exclusive sets of features, we sum the importance for each set to construct the total importance measure for each category. We present the results in Figure 8. We first notice that *ad+timestamp* contributes to the click model, but its contribution is modest for the leave model. This is expected as the user's decision to use a messenger app is likely not driven by the ad shown. Second, we find that *historical* and *session-level* features contribute the most to the predictive performance of both click and leave models. In contrast, the contribution of *demographic* features is modest for both models. This finding highlights the importance of user tracking in building good ad response models. Finally, while using a shorter history, *session-level* features are as powerful (if not more) than *historical* features, which is quite promising for our main framework, as it aims to exploit these *session-level* features in a dynamic fashion.

Inspired by our feature categories, we define four separate models that are progressively

[12]The closest approach to ours is Wilbur et al. (2013) that use more contextual and behavioral information to estimate continuation probabilities. We extend that approach by using a richer set of features and a more flexible learner.

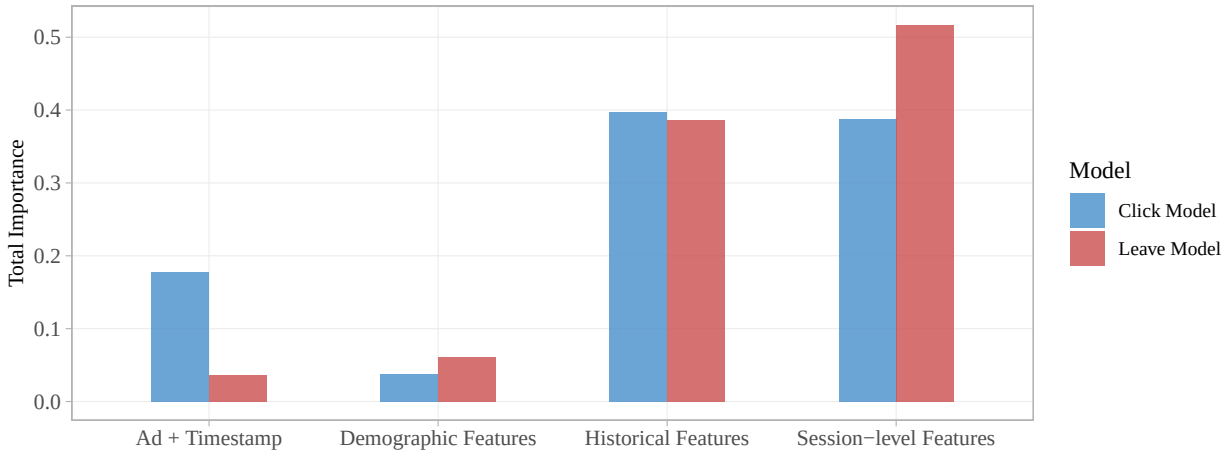


Figure 8: Feature importance of different feature categories in estimating click and leave outcomes.

Level of Personalization	*Click Model*		*Leave Model*	
	RIG	*AUC*	*RIG*	*AUC*
No Personalization	0.0288	0.6702	0.0030	0.5354
Demographic	0.0432	0.6857	0.0042	0.5423
Demographic+Historical	0.1567	0.7937	0.0798	0.6941
Adaptive (all features)	0.2021	0.8139	0.0881	0.7044

Table 3: Predictive accuracy of models with different levels of personalization.

more personalized by adding one feature category at a time: (1) *No Personalization*, which only uses *ad+timestamp* as inputs, (2) *Demographic Personalization* that adds *demographic features* to the first model, (3) *Demographic+Historical Personalization*, which adds the set of *historical* features to the second model, and uses all the features except the real-time *session-level* features, and (4) *Adaptive Personalization*, which combines all the features including the real-time *session-level* features. We estimate both click and leave outcomes using these inputs and present the predictive accuracy of these models in Table 3. The results paint a consistent picture with Figure 8: adding *historical* and *session-level* features result in a substantial performance increase. Specifically, the value of *session-level* features serves as a primary motivation for our adaptive ad sequencing framework.

5.2 Counterfactual Policy Evaluation

We now use our honest direct method (HDM) to evaluate the performance of our adaptive ad sequencing framework and compare it to competing benchmarks. We refer to the policy developed by our framework as *fully dynamic* or *adaptive forward-looking* interchangeably throughout. We now define a series of competing policies for benchmarking[13]:

- *Adaptive Myopic Policy:* This policy uses all the information available at any exposure and selects the ad that maximizes the reward at that point, i.e., the highest CTR. This policy is myopic as it ignores the expected future rewards and is equivalent to $\beta = 0$ in our MDP in Equation (10). However, this policy is adaptive because it uses the real-time updated session-level features as it moves forward. We use this policy as the main comparison point for our framework because it reflects the standard practice in the advertising industry, where the platforms use a version of contextual bandit to select the ad at any point (Theocharous et al., 2015).
- *Single-ad Policy:* This policy selects a single ad to show for the entire session. As such, this policy is not adaptive as it only uses pre-session information (demographic and historical features) to select the ad with the highest CTR. Using this policy as a benchmark is important from a managerial standpoint because it mimics the practice of using a fixed ad slot as opposed to a refreshable ad slot. Further, it highlights the value of adaptive session-level information.
- *Random Policy:* This policy randomly selects an ad from the ad inventory at any point. While this is a naïve policy, it is often used in the reinforcement learning literature as a benchmark.

We document the performance of our *fully dynamic* policy and these three benchmarks in terms of different outcomes in Table 4. We start with the main metric of interest in this paper – the expected number of clicks per session. This metric determines how many clicks each policy generates in total when we multiply it by the number of sessions. Our results in the first row of Table 4 show that the *fully dynamic* policy developed by our adaptive ad sequencing framework results in substantial gains by achieving an expected number of 0.1671 clicks per session. In particular, the *fully dynamic* policy generates 5.76%, 27.46%, and 79.59% more clicks than *adaptive myopic*, *single-ad*, and *random* policies respectively. The gains from our *fully dynamic* policy over the *single-ad* policy illustrates the opportunity cost of using a non-refreshable ad slot that only shows one ad for the entire session. More importantly, the gains from our *fully dynamic* policy over the *adaptive myopic* policy make

[13]We further formalize these benchmark policies in Appendix §E

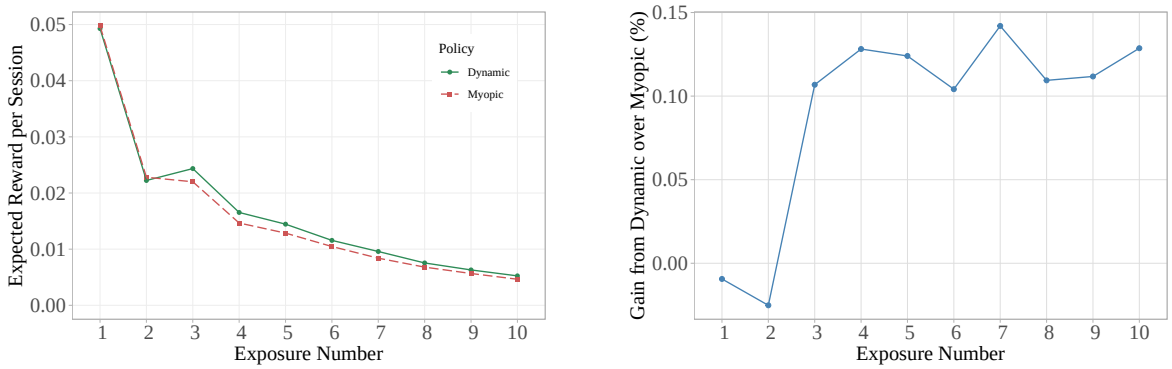
Metric	*Sequencing Policies*			
	Fully Dynamic	*Adaptive Myopic*	*Single-Ad*	*Random*
Expected No. of Clicks Per Session	0.1671	0.1580	0.1311	0.0930
– (% Click Increase over Random)	79.59%	69.81%	40.90%	0.00%
Expected CTR (per Impression)	4.26%	4.04%	3.43%	2.42%
Expected Session Length	3.9258	3.9164	3.8246	3.8518
Ad Concentration (HHI)	0.2902	0.3178	0.3480	0.1159
No. of Users	14,084	14,084	14,084	14,084
No. of Sessions	201,466	201,466	201,466	201,466

Table 4: Performance of different sequencing policies in the test data.

a compelling case for the use of dynamic optimization and reinforcement learning in the advertising domain and call for a change in the current practice of using myopic frameworks, particularly in cases like ours where users are exposed to multiple ads sequentially over a short period of time.

Next, we aim to identify the primary source for the gains from the *fully dynamic* policy. As discussed earlier, there are two channels through which adaptive ad sequencing can create value – (1) by making users stay longer, thereby increasing the total number of impressions generated (extensive margin), or (2) by making each impression more likely to receive a click (intensive margin). We test each source using two other metrics – expected CTR for each impression and expected session length. We find that each impression has a significantly higher probability of receiving a click, but the increase in usage is only 0.2% compared to the *adaptive myopic* policy. Thus, adaptive ad sequencing increases the total number of clicks in a session by increasing the response rate to each ad. Later, in §5.3, we further explore the mechanism behind the increase in response rate through sequencing.

Finally, we examine how concentrated the ad allocation is under each policy. We first calculate the average share of each ad under each policy and then use the well-known Herfindahl–Hirschman Index (HHI) to measure ad concentration. Lower HHI values indicate a lower ad concentration and more evenly distributed shares. Naturally, we expect the random sequencing policy to have a very low HHI as it most evenly distributes ad shares. We observe that in the fifth row of our table. Interestingly, we find that our *fully dynamic* policy results in a lower HHI than both *adaptive myopic* and *single-ad* policies. This is likely because the dynamic policy makes better use of synergies between ads, thereby increasing the shares for less popular ads. This is an important finding because it means that the better performance



(a) Expected reward over exposure number.

(b) Percentage gain over exposure number.

Figure 9: Expected rewards of fully dynamic and adaptive myopic policies across exposure numbers.

of the *fully dynamic* policy does not come at the expense of less popular ads. The lower ad concentration can also have welfare impacts for consumers as they are exposed to a more diverse set of ads.

5.3 Interpretation and Mechanism Analysis

In the previous section, we established the gains from our adaptive ad sequencing framework: notably, we showed that our *fully dynamic* policy, on average, generates 5.76% more clicks than the *adaptive myopic* policy, which is the common practice in the advertising industry. However, it is not clear what constitutes these gains – when these gains are higher and how the two policies are different. In this section, we seek to answer these questions and better understand the differences in the two policies in terms of both outcomes and the process.

In principle, all the differences between the *fully dynamic* and *adaptive myopic* policies stem from the fact that only the former takes into account the expected future rewards when making a decision. Figure 9 reflects this difference between the two policies by breaking down the contribution of each policy by the exposure number. Figure 9a shows the expected reward from each policy at each exposure number. As expected, we observe a decreasing pattern over time as the user's likelihood of staying in the session decreases. In Figure 9b, we focus on the relative gains from the *fully dynamic* policy over *adaptive myopic* policy across exposure numbers. As shown in this figure, the *fully dynamic* performs worse than the *adaptive myopic* policy in the first two exposures. However, the gains from the *fully dynamic* policy appear from the third exposure onwards. The existence of this pattern further highlights the value of scalability in our framework because a framework that is not scalable would not be able to extract value from the later exposures.

In summary, the observed difference in Figure 9 is because of the inter-temporal trade-off the *fully dynamic* policy makes as captured by expected future rewards in Equation (10), i.e., $\beta \mathbb{E}_{S_{i,t+1}|S_{i,t},a} V(S_{i,t+1})$. While this additional term in the equation helps achieve a better performance, it is generally very hard to interpret it as many factors go into the construction of value function. Our aim in this section is to use the domain knowledge in advertising to add to the interpretability of our framework and share insights into the possible mechanisms behind the gains from it.[14]

5.3.1 Heterogeneity in Gains Across Past Historical Features

In this section, we want to better understand the heterogeneity in gains from *fully dynamic* over *adaptive myopic* policy. As such, we need to first formalize what we mean by gains. Let π_d^M and π_m^M denote the *fully dynamic* and *adaptive myopic* policies identified using the modeling data $\mathcal{D}_{Model}$. For each session i, we use Equation (18) to define the variable $Gain_i$ as follows:

$$Gain_i = \frac{\hat{\rho}(\pi_d^M; S_{i,1}, T = 10)}{\hat{\rho}(\pi_m^M; S_{i,1}, T = 10)} - 1, \tag{19}$$

where $\hat{\rho}(\pi_d^M; S_{i,1}, T = 10)$ and $\hat{\rho}(\pi_m^M; S_{i,1}, T = 10)$ represent the expected number of clicks for session i with initial state variables $S_{i,1}$ for the first 10 exposures, under *fully dynamic* and *adaptive myopic* policies respectively. The variable gain measures the percentage improvement in expected rewards from the *fully dynamic* over *adaptive myopic* policy for any specific session. Thus, it allows us to document the heterogeneity in gains across sessions.

We first focus on a simple variable that is available prior to any session – the number of previous sessions the user has experienced. We want to see how the gains change as we have more information about prior sessions. On the one hand, a richer past history for a user can increase gains because the *fully dynamic* model obtains more accurate predictions about the session dynamics (e.g., how long the session will last), which results in employing more effective sequencing strategies. On the other hand, as a user becomes more experienced with ads, some sequencing effects may become less effective, particularly those that stem from better managing user attention to ads (Rafieian and Yoganarasimhan, 2021a). For all the sessions in our test data, we define five quintiles based on the number of prior sessions.[15] We show the average gains for each quintile in Figure 10. Interestingly, we find a U-shaped pattern consistent with both accounts presented above: the gains initially decrease as users become more experienced (quintiles 1–3), but as more data become available, the gains rise

[14] It is worth emphasizing that our approach is fully exploratory and descriptive, informed by the domain of advertising.

[15] Each quintile contains 20% of all sessions, with quintile 1 being the bottom 20% of values.

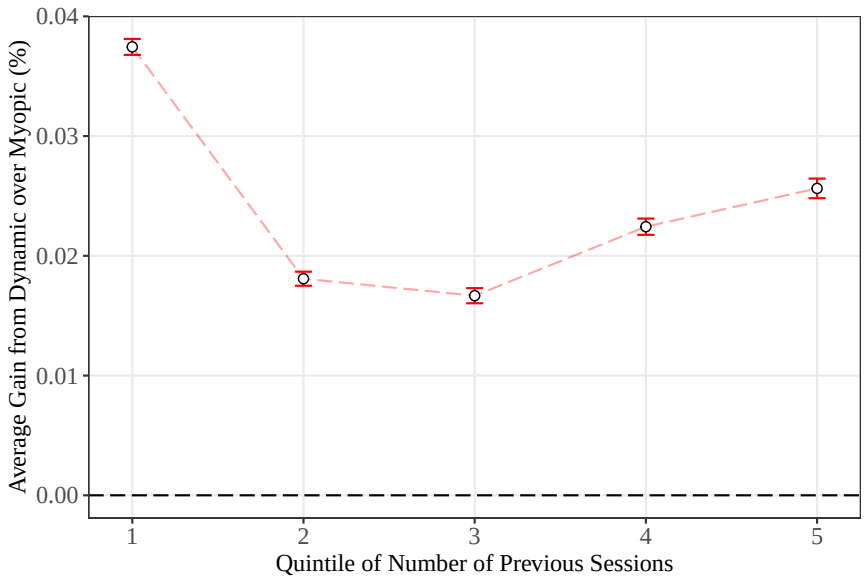


Figure 10: Average gains from dynamic policy over myopic policy across quintiles for the number of past sessions.

(quintiles 3–5), which highlights the value in having more data.

Next, we further document the heterogeneity in gains across a richer set of historical covariates. Following the results shown in Figure 10, we first include two variables that are correlated with the number of prior sessions – (1) number of past impressions, and (2) the variety of ads. The former demonstrates the abundance of data available, whereas the latter user attention – that is, the higher the variety of ads seen before, the lower the level of attention would be as ads contain lower levels of novelty. Notice that both variables are historical and generated based on the information up until the current session. We then regress gains for each session on these two covariates while controlling for user and hour fixed effects to make sure the estimates are not subject to the user- and time-specific confounds. We exclude the first session for each user because the historical features do not exist for those sessions. We present the results of this model in the first column of Table 5. Our results show that a higher number of past impressions (greater abundance of data) increases gains, whereas a higher variety of ads seen (lower attention level) decreases gains from the *fully dynamic* over the *adaptive myopic* policy. Together, these estimates support the two opposing accounts presented earlier: richer history increases model certainty but reduces the effectiveness of sequencing strategies.

In columns 2–4, we add historical features one by one. We first add the *number of past clicks* prior to the current session. A higher value of this covariate indicates a greater ad

Historical Features	*Dependent Variable:* Gain_i			
	(1)	(2)	(3)	(4)
No. of Past Impression	0.00001^{***}	0.00001^{**}	0.00001^{***}	0.00001^{*}
	(4.60)	(3.00)	(3.40)	(2.07)
Variety of Ads Seen	-0.00024^{*}	-0.00028^{**}	-0.00021^{*}	-0.00033^{**}
	(-2.43)	(-2.79)	(-2.10)	(-3.28)
No. of Past Clicks		0.00076^{***}	0.00080^{***}	0.00080^{***}
		(3.70)	(3.89)	(3.91)
Time Since Last Session			0.00008^{***}	0.00007^{**}
			(4.84)	(4.17)
Last Session Length				0.00036^{***}
				(22.23)
User Fixed Effects	✓	✓	✓	✓
Hour Fixed Effects	✓	✓	✓	✓
No. of Obs.	190,206	190,206	190,206	190,206
R^2	0.271	0.271	0.271	0.273
Adjusted R^2	0.220	0.220	0.220	0.222
Note:				*p<0.05; **p<0.01; ***p<0.001

Table 5: Heterogeneity in gains from dynamic policy over myopic policy across the past historical information available prior to the session. Numbers in parenthesis are t-statistics that are estimated using OLS.

response and overall engagement with ads. As shown in the second column of Table 5, this covariate has a positive association with gains: the *fully dynamic* policy generates greater gains for users with higher past responsiveness to ads, holding the past experience constant. Next, we include another historical feature – the *time since the last session* (in hours). Higher values of this covariate show lower recency in users' interaction with ads. In general, we expect higher recency to reduce the novelty of ad stimuli, thereby lowering the gains from the *fully dynamic* policy. We confirm this prediction by finding a positive coefficient for the *time since the last session* in column 3 of Table 5: the greater the gap is between the current session and the last session, the higher the gains are from sequencing. Finally, we include the *length of the last session* as another covariate in our model. This covariate is a signal for the length of the current session. As shown in Figure 9, gains from sequencing appear later in a session. Hence, when the session is longer, we expect the gains to be higher. The positive and statistically significant coefficient for *Last Session Length* in the fourth column of Table 5 provides support for this prediction.

5.3.2 Extent of Discrepancy Between Fully Dynamic and Adaptive Myopic Policies

The key takeaway from the previous section is that there is great heterogeneity in gains from sequencing across past historical features. These gains naturally stem from the differences between the *fully dynamic* and *adaptive myopic* policies. In this section, we want to see where the discrepancy between the two policies is more pronounced. As such, we first need to quantify the discrepancy between the two policies at the session level. For any given session i and policy π, we can determine the distribution of ad shares both analytically and through simulations. Let $\boldsymbol{\alpha}_i^{(d)}$ and $\boldsymbol{\alpha}_i^{(m)}$ denote vectors representing ad shares in session i under *fully dynamic* and *adaptive myopic* policies respectively. We quantify the discrepancy between these two distributions using five measures based on ℓ-norm and Kullback-Leibler (KL) divergence as follows:

- Outcome 1: ℓ^1-norm of the difference between shares $\|\boldsymbol{\alpha}_i^{(d)} - \boldsymbol{\alpha}_i^{(m)}\|_1$
- Outcome 2: ℓ^2-norm of the difference between shares $\|\boldsymbol{\alpha}_i^{(d)} - \boldsymbol{\alpha}_i^{(m)}\|_2$
- Outcome 3: KL divergence of $\boldsymbol{\alpha}_i^{(d)}$ from $\boldsymbol{\alpha}_i^{(m)}$, i.e., $D_{\text{KL}}(\boldsymbol{\alpha}_i^{(d)} \parallel \boldsymbol{\alpha}_i^{(m)})$
- Outcome 4: KL divergence of $\boldsymbol{\alpha}_i^{(m)}$ from $\boldsymbol{\alpha}_i^{(d)}$, i.e., $D_{\text{KL}}(\boldsymbol{\alpha}_i^{(m)} \parallel \boldsymbol{\alpha}_i^{(d)})$
- Outcome 5: Disagreement ratio, which is the fraction of ads that have non-zero share under only one of the two policies in the set of all feasible ads.

The first four measures capture the extent of difference between ad shares, whereas the fifth measure uses a more binary approach and compares distributions in the set of ads that could be shown. We use these measures of discrepancy between the two policies and regress them on the set of historical features used in the previous section. Like before, we account for user and hour fixed effects to control for the potential confounds driven by the user- and time-specific factors. These models help us understand which historical features are associated with a greater difference between the two policies. We present our results in Table 6, where each column represents the model where we use one of the discrepancy outcomes above. A few interesting patterns emerge from the results of Table 6. First, we find a consistently positive coefficient for the number of past impressions, which indicates that a richer history results in greater differentiation between policies. Next, we focus on how the variety of ads seen is associated with the discrepancy between the two policies. We find some weak negative links for the first four measures, indicating that a higher variety of previous ads makes the two policies more similar. Our estimates in the fifth column of Table 6 show a very significant negative link. This finding indicates that a higher variety of ads seen results in both policies using the same set of ads.

	DV: Discrepancy between Ad Distributions under Dynamic and Myopic				
	(1)	(2)	(3)	(4)	(5)
Number of Past Impression	0.00043***	0.00019***	0.00111***	0.00053***	0.00005***
	(45.43)	(48.44)	(46.27)	(59.47)	(23.16)
Variety of Ads Seen	−0.00099*	0.00022	0.00109	−0.00096**	−0.00180***
	(−2.57)	(1.41)	(1.10)	(−2.62)	(−19.56)
Number of Past Clicks	−0.01496***	−0.00734***	−0.02471***	−0.01568***	0.00028
	(−19.22)	(−22.85)	(−12.39)	(−21.38)	(1.52)
Time Since Last Session	−0.00008	−0.00005*	0.00006	−0.00009	−0.00002
	(−1.32)	(−2.10)	(0.38)	(−1.57)	(−1.03)
Last Session Length	−0.00095***	−0.00041***	−0.00068***	−0.00050***	0.00020***
	(−15.61)	(−16.14)	(−4.33)	(−8.65)	(13.56)
User Fixed Effects	✓	✓	✓	✓	✓
Hour Fixed Effects	✓	✓	✓	✓	✓
No. of Obs.	190,206	190,206	190,206	190,206	190,206
R^2	0.195	0.196	0.162	0.203	0.192
Adjusted R^2	0.138	0.139	0.103	0.147	0.135
Note:					*p<0.05; **p<0.01; ***p<0.001

Table 6: Discrepancy in the distribution of ad allocation between dynamic acrossthe past historical information available prior to the session. Numbers in parenthesis are t-statistics that are estimated using an OLS.

The estimates for our next covariate – the number of past clicks – shows a pattern opposite to what we saw with the variety of ads seen: while the significant and negative coefficients in the first four columns indicate that the shares become more similar under the two policies, the insignificant coefficient in the fifth column suggests that a higher number of past clicks does not necessarily make the set of ads that could be shown under these policies more or less similar. This is likely because some ads have a high probability of click under both policies, given the existence of past clicks (e.g., ads similar to the ad that is already clicked on). Next, we focus on the time since the last session, which is a measure of usage recency by the user. Overall, we mostly find insignificant associations between this variable and the outcomes.

Finally, we examine the link between the last session length and the discrepancy measures. In general, we expect a longer session to increase the discrepancy between the two policies because the *fully dynamic* policy has richer dynamics and more opportunities to differentiate. Surprisingly, we find that a higher session length is associated with more similar shares (columns 1–4) but more disagreement in the set of ads that could be shown (column 5). One potential explanation is that the discrepancy captured by our first four measures is

more pronounced if the session is short. That is, although a longer session makes the set of ads more different, the probabilities become closer as they capture the specifics of leave probabilities.

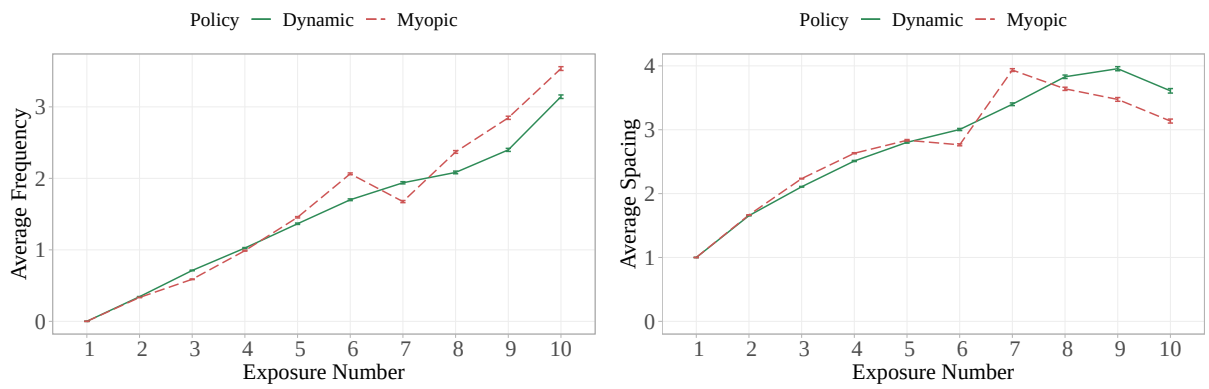
5.3.3 Distribution of Session-level Features Under Different Policies

The previous section focused on the discrepancy between the session-level ad distributions of both *fully dynamic* and *adaptive myopic* policies. While Table 6 showed how the magnitude of discrepancy varies across sessions based on their pre-session characteristic, it did not show how these sessions are different from each other. In this section, we narrow down our focus to session-level differences between the two policies. In particular, we examine how the two policies are different in their use of two features that are widely used in the advertising literature: frequency and spacing. We run a simulation to generate one data set under each policy and then compare session-level frequency and spacing between these two policies.

We plot these two session-level features under each policy across exposures in Figure 11. We first use the past session-level frequency of the ad selected at each exposure, which is the number of times that ad has been shown in the prior exposures within the session. Figure 11a shows an overall similar pattern, but a higher use of ad frequency under the *adaptive myopic* policy compared to our policy towards the end of the session. We also find that the average frequency in the full data set generated under the *adaptive myopic* policy is significantly higher than the average frequency under the *fully dynamic* policy.

Next, we focus on session-level spacing for the ad shown, i.e., the gap between the current exposure and the last time the same ad has been shown in the session, as measured by the number of exposures between the two exposures of the same ad. Our results in Figure 11b show a higher average spacing under our policy compared to the *adaptive myopic* policy towards the end of the session. When we consider the entire data sets, the average spacing is significantly higher under the *fully dynamic* policy, compared to the *adaptive myopic* policy.

In sum, we interpret the overall patterns shown in Figure 11 through the lens of the attention-based behavioral account proposed in Rafieian and Yoganarasimhan (2021a). As suggested in that paper, lower frequency and higher spacing are positively correlated with the perceived novelty of the ad stimuli (Helson, 1948). Hence, one possible conclusion is that our *fully dynamic* policy better manages users' attention compared to the *adaptive myopic* policy, particularly towards the end of the session. The lower use of ad frequency under our policy can also explain the lower ad concentration found in Table 4.



(a) Session-level frequency of ad shown

(b) Session-level spacing of ad shown

Figure 11: Distribution of session-level features at different time periods under different sequencing policies. Error bars refer to 95% confidence intervals when we compare the two samples.

6 Implications

Our findings in this paper have several implications for marketing practitioners. The most direct set of implications is for publishers and advertising platforms. We document the substantial opportunity cost of using a non-refreshable ad slot, which is a common design in many digital ad platforms. The more important implication for the publisher comes from our comparison of gains from the *fully dynamic* policy and the *adaptive myopic* policy, which is the current state of practice in the advertising industry. Our results suggest that publishers can increase user engagement by adopting a dynamic framework, thereby calling for a change in the current practice in the industry, particularly because the computational cost of our framework is only slightly higher than an adaptive myopic framework. Finally, our comprehensive analysis of the mechanism underlying the gains from our framework provides publishers and platforms with new insights into when and why our framework creates the most value.

It is worth emphasizing that the framework is general and all ad platforms can use our framework to measure the gains in user engagement from adopting a fully dynamic framework, as long as there is some level of randomization in ad allocation that satisfies the unconfoundedness assumption. Although ad platforms often use deterministic auctions such as first- or second-price auctions for ad allocation, they still incorporate some level of randomization as a common practice, which satisfies this assumption.[16] Similarly, platforms

[16]Some ad platforms implement ϵ-greedy approaches that allocate the winner of the auction with $1-\epsilon$

can use other measures of user engagement as the reward function and different optimization horizon, depending on the context. Thus, the applicability of our framework does not depend on the specific empirical setting in this paper.

Our sequencing framework can also be readily implemented in cases where a platform or publisher wants to sequence content to achieve optimal user-level outcomes. In particular, the improvement in ad response as a result of sequencing motivates a wide range of marketing applications, such as sequencing articles in news websites to increase audience engagement, sequencing social media posts to enhance user experience, sequencing promotional emails in an online retail context, and sequencing push notifications for churn management.[17] More broadly, our framework can be extended to other contexts where we want to use persuasive messaging through adaptive interventions. For example, in the context of mobile health, a growing body of work focuses on Just-In-Time Adaptive Interventions (JITAI) in mobile apps and studies their impact in shaping consumers' health behavior, including physical fitness and activity, smoking, alcohol use, and mental illness (Nahum-Shani et al., 2017). Similarly, in the context of education, these adaptive interventions can be used to improve students' motivation and outcomes (Mandel et al., 2014). These showcases can also inspire the public sector to use these tools in cases where collective action is required, such as environmental protection and political participation.

7 Conclusion

Mobile in-app advertising has grown exponentially over the last few years. The ability to exploit the time-varying information about a user to personalize ad interventions over time is a key factor in the growth of in-app advertising. Despite the dynamic nature of the information, publishers often use myopic decision-making frameworks to select ads. In this paper, we examine whether a dynamic decision-making framework benefits the publisher in terms of the user engagement with ads, as measured by the number of clicks generated per session. Our dynamic framework has three main components: (1) a theoretical framework that models the domain structure such that it captures inter-temporal trade-offs in the ad allocation decision, (2) an empirical framework that breaks the policy identification problem into a combination of machine learning tasks that achieve sequence personalization, counterfactual validity, and

probability, and the rest of ads randomly with an ϵ probability. Similarly, some platforms leave a small fraction of impressions for full experimentation (Ling et al., 2017).

[17] It is worth noting that adversarial cases where other agents influence the outcomes strategically require strategy-proof mechanism design. For example, if we want to optimize revenues through adaptive ad sequencing, advertisers' bidding behavior needs to be taken into account. Rafieian (2020) focuses on such strategy-proof sequencing frameworks and solves for the revenue-optimal auction.

scalability, and (3) a policy evaluation method that is separate from policy identification, thereby allowing a robust counterfactual policy evaluation. We apply our framework to large-scale data from the leading in-app ad network of an Asian country. Our results indicate that our adaptive ad sequencing policy results in significant gains in the expected number of clicks per session compared to a set of benchmark policies. In particular, we show that our policy results in 5.76% more clicks, on average, compared to the adaptive myopic policy that is the current state of practice. Almost all these gains stem from an increase in average response to each impression instead of increased usage by each user. Next, we document extensive heterogeneity in gains from adaptive ad sequencing and find a U-shaped pattern for gains over the length of users' past history, indicating that gains are highest for either new users or those whose past data are rich. As for the policy difference between adaptive ad sequencing and adaptive myopic, we find that our policy results in a greater ad diversity, which can be because our policy better manages user attention by showing a more diverse set of ads.

Our paper makes several contributions to the literature. First, from a methodological point-of-view, we develop a unified dynamic framework that starts with a theoretical framework that specifies the domain structure in mobile in-app advertising and an empirical framework that breaks the problem into tasks that can be solved using a combination of machine learning methods and causal inference tools. Our framework achieves scalability without imposing simplifying assumptions on the dynamics of the problem. Second, from a substantive standpoint, we document the gains from adopting an adaptive forward-looking sequencing policy compared to the adaptive myopic policy. This comparison is of particular importance as the adaptive myopic policy is currently the standard approach in the industry. We further present a comprehensive study of heterogeneity and document key differences between our policy and adaptive myopic policy, which is of great value to managers who need to interpret the gains and understand when and why the framework is most valuable.

Nevertheless, our study has some limitations that serve as excellent avenues for future research. First, our counterfactual policy evaluation is predicated on the assumption that users do not change their behavior in response to sequencing policies. While we exploit randomization to obtain our counterfactual estimate, it would be important to validate these findings in a field experiment. Further, we use the training data offline to learn counterfactual estimates for click and leave outcomes. Extending our framework to an online setting that captures exploration/exploitation trade-offs is important since offline evaluation can be costly. Finally, we use the entire within-session history to update state variables. Future research

can look into more parsimonious frameworks that can be scalable to longer time horizons.

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Appendices

A Feature Generation

As discussed earlier, our goal is to estimate click and leave outcomes for any combination of ad and state variables, as shown in Equations (11) and (12). A major challenge in estimating these equations is that the set of inputs is quite large, containing the entire sequence of prior ads shown to the user. In this section, we present a feature generation framework that maps a combination of state variables and ads ($\langle S_{i,t}, a \rangle$) to a set of meaningful features $g(S_{i,t}, a)$ that we can give as inputs to our learning algorithm. Ideally, we need our final set of features to fully represent $\langle S_{i,t}, a \rangle$ in a lower dimension without any information loss. Thus, we generate a set of features that help us predict users' clicking behavior and app usage based on the prior literature on advertising.

We categorize these features into three groups: (1) ad+timestamp, (2) demographic features, (3) historical features, and (4) session-level features. The first group contains the contextual information about the impression as it captures the exact timestamp of the impression. Demographic and historical features relate to the pre-session state variables (X_i), whereas session-level features relate to the session-level variables ($G_{i,t}$). Figure 12 provides an overview of our feature generation and categorization. In this example, the user is at her fourth exposure in her third session. The features for this particular exposure include the observable demographic features, historical features generated from the prior sessions, and session-level features that are generated from the first three exposures shown in the current session. Clearly, we do not use any information from the future to generate a feature: at any point, we only use the prior history up to that point. In the following sections, we describe all these features in detail.

A.1 Ad+Timestamp

This group of features contains the non-personal information about the impression: the timestamp of an impression and the ad shown in that impression. As such, this category of features does not require any user-level tracking.

A.2 Demographic Features

This includes the variables that we already observe in our data (see §3.2), such as the province, latitude, longitude, smartphone brand, mobile service provider (MSP), and connectivity type. For any session i, we use D_i to denote the set of demographic features. These features do not transition based on the ad that the publisher shows at any time period. As such, we

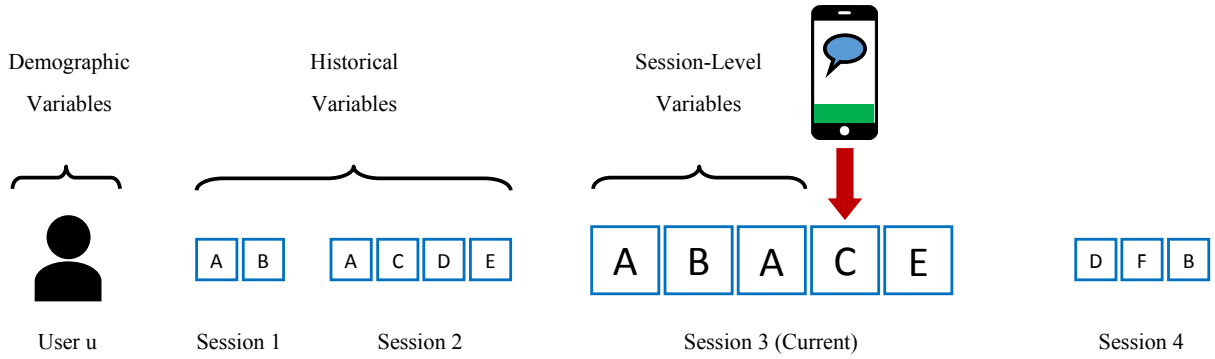


Figure 12: A visual schema for our feature generation and categorization.

do not use subscript t for them.[18] We include these features because of two reasons. First, these features help predict both users' clicking behavior and app usage. Second, the targeting variables are the main confounding source, and controlling them guarantees that we control for propensity score of ads when estimating the outcomes. In light of our discussion in §4.2.2 and Proposition 1, it is sufficient to control for these demographic features because conditional on these features, the ad allocation is random.

A.3 Historical Features

Historical features reflect the user's past activity prior to the current session. While demographic features are available in the data, we need to generate historical features based on the pre-session information. These features are not adaptive because we only use the pre-session information to generate them. As such, these features are part of X_i and remain unchanged within the session.

To generate historical features, we use the insights from the prior literature on dynamics of advertising on the effects of prior ad frequency (Nerlove and Arrow, 1962; Dubé et al., 2005), recency or spacing according to memory-based models (Sawyer and Ward, 1979; Naik et al., 1998; Sahni, 2015; Aravindakshan and Naik, 2015), and ad response (Rafieian and Yoganarasimhan, 2021b). We build large inventory matrices to contain the information regarding the past frequency, spacing, and ad response of each ad. For session i, let u_i denote the user in that session. Below, we present the detailed set of our historical features along with their definition:

- $HistFreqAd_i^{(a)}$: For any ad $a \in \mathcal{A}$, this feature counts the number of times ad a has been

[18] One could argue that features such as latitude and longitude may change within the session. While this is possible, it is unlikely to happen as a result of the publisher's ad interventions. Further, the sessions are usually short, and we rarely observe such a change in our data.

shown to user u_i in the prior sessions. Together, with all ads, these features contain the frequency inventory for the prior history.

- $HistSpaceAd_i^{(a)}$: For any ad $a \in \mathcal{A}$, this feature counts the space (in terms of number of exposures) between the first impression in session i and the last time ad a has been shown. Together, with all ads, these features contain the spacing inventory for the prior history.
- $HistClickAd_i^{(a)}$: For any ad $a \in \mathcal{A}$, this feature counts the number of times ad a has been clicked by user u_i in the prior sessions. Together, with all ads, these features contain the click inventory for the prior history.
- $HistImp_i$: The total number of impressions user u_i has seen prior to session i, i.e., $HistImp_i = \sum_{a \in \mathcal{A}} HistFreqAd_i^{(a)}$.
- $HistClick_i$: The total number of clicks user u_i has made prior to session i, i.e., $HistClick_i = \sum_{a \in \mathcal{A}} HistClickAd_i^{(a)}$.
- $HistImpApp_i$: The total number of impressions user u_i has seen in the top app prior to session i. This feature may differ from Imp_i because the user may have used other apps.
- $HistClickApp_i$: The total number of impressions user u_i has clicked in the top app prior to session i. This feature may differ from $Click_i$ because the user may have used other apps.
- $ExposureImp_i^{(t)}$: For any $t \leq 10$, the feature counts the number of times user u_i has seen at exposure number t in prior sessions. In other words, it counts the number of times in prior sessions that the user u_i stayed in the session to receive exposure t. As such, this feature captures usage patterns in the user's behavior.
- $ExposureClick_i^{(t)}$: For any $t \leq 10$, the feature counts the number of times user u_i has clicked at exposure number t in prior sessions. This feature captures if there is any temporal pattern in user's clicking behavior.
- $LastSessionLength_i$: The length of last session (in number of exposures) that user u_i was exposed to prior to session i. If session i is the user's first session, this feature takes value zero. This feature thus captures the most recent usage behavior by the user.
- $AvgSessionLength_i$: The average length of the sessions (in number of exposures) that user u_i was exposed to prior to session i. This feature thus reflects the average usage behavior by the user.
- $LastGap_i$: The gap or free time (in minutes) user u_i has had between her last session and session i. This feature captures the usage recency by the user.
- $AvgGap_i$: The average gap or free time (in minutes) user u_i has had between her sessions prior to session i. This feature captures the overall usage patterns by the user in prior

sessions.

- $HistVariety_i$: The total number of distinct ads that user u_i has seen prior to session i, i.e., $HistVariety_i = \sum_{a \in \mathcal{A}} \mathbb{1}(HistFreqAd_i^{(a)} > 0)$.
- $HistGiniSimpson_i$: The Gini-Simpson index for ads that user u_i has seen prior to session i (Simpson, 1949). This metric captures the diversity of prior ad exposures by calculating the probability that two random exposures from the past showed different ads. A higher Gini-Simpson index means that the user has seen a more diverse set of ads. We can write the Gini-Simpson index as follows:

$$HistGiniSimpson_i = 1 - \sum_{a \in \mathcal{A}} \frac{HistFreqAd_i^{(a)}(HistFreqAd_i^{(a)} - 1)}{Imp_i(Imp_i - 1)} \tag{20}$$

- $HistShannon_i$: This feature calculates the Shannon entropy of ad frequencies prior to session i (Shannon, 1948). This metric also captures the amount of information in prior ad exposures, which takes a higher value when the frequencies are more evenly distributed. We can define the Shannon entropy as follows:

$$HistShannon_i = -\sum_{a \in \mathcal{A}} \frac{HistFreqAd_i^{(a)}}{Imp_i} \log\left(\frac{HistFreqAd_i^{(a)}}{Imp_i}\right) \tag{21}$$

We further define five impression-specific historical features primarily to aid the learning algorithm that use these features for prediction. Suppose that the impression is an ad a shown in exposure t in the session. We can define the following features:

- $ThisHistFreqAd_i$, which is equal to $HistFreqAd_i^{(a)}$ if ad a is shown in the impression.
- $ThisHistSpaceAd_i$, which is equal to $HistSpaceAd_i^{(a)}$ if ad a is shown in the impression.
- $ThisHistClickAd_i$, which is equal to $HistClickAd_i^{(a)}$ if ad a is shown in the impression.
- $ThisExposureImp_i$, which is equal to $ExposureImp_i^{(t)}$ if the impression is the t^{th} exposure in the session.
- $ThisExposureClick_i$, which is equal to $ExposureClick_i^{(t)}$ if the impression is the t^{th} exposure in the session.

Please note that none of these five extra features do not contain any extra information over the prior set. As such, most advanced learning algorithm can automatically use the information in these five features without explicitly including them in the feature set. However, we included these features to ensure that this relationship will be captured by our models. It is also worth emphasizing that we consider these five features historical despite the fact that we use the information about the current impression, i.e., which exposure number it is and which ad it

shows. This is because we only use the pre-session data to generate these features. Together, we denote the full list of historical features by H_i.

A.4 Session-Level Features

The session-level features are key to our analysis because we are interested in optimal sequencing of ads within the session. These are the features that transition from a time period to the next. That is, depending on the prior exposures within the session, these features will evolve. We follow a procedure similar to historical features to generate session-level features. As such, we still have large inventory matrices for frequency, spacing, and ad response within the session. Below is the full list of session-level temporal features:

- $SessFreqAd_{i,t}^{(a)}$: For any ad $a \in \mathcal{A}$, this feature counts the number of times ad a has been shown within the current session. Together, with all ads, these features contain the frequency inventory for the ongoing session.
- $SessSpaceAd_{i,t}^{(a)}$: For any ad $a \in \mathcal{A}$, this feature counts the space (in terms of number of exposures) between the current exposure and last time ad a has been shown within the current session. This feature takes value 0 if there is no prior exposure of ad a in prior sessions. Together, with all ads, these features contain the spacing inventory for the ongoing session.
- $SessClickAd_{i,t}^{(a)}$: For any ad $a \in \mathcal{A}$, this feature counts the number of times ad a has been clicked within the current session. Together, with all ads, these features contain the click inventory for the ongoing session.
- $SessImp_{i,t}$: The total number of impressions the user has seen in session i prior to exposure number t. For any exposure number t, this feature is equal to $t-1$.
- $SessClick_{i,t}$: The total number of clicks the user has made in session i prior to exposure number t, i.e., $SessClick_{i,t} = \sum_{a \in \mathcal{A}} SessClickAd_{i,t}^{(a)}$
- $SessVariety_{i,t}$: The total number of distinct ads that the user has seen within session i prior to exposure number t. We can define this feature as follows:

$$SessVariety_{i,t} = \sum_{a \in \mathcal{A}} \mathbb{1}(SessFreqAd_{i,t}^{(a)} > 0) \tag{22}$$

- $SessChanges_{it}$: The total number of consecutive changes of ads prior to the exposure number t within the session i. We can write:

$$SessChange_{i,t} = \sum_{j=2}^{t-1} \mathbb{1}(A_{i,j} \neq A_{i,j-1}), \tag{23}$$

where $A_{i,j}$ is the ad shown at exposure number j in session i.

- $SessGiniSimpson_{i,t}$: The Gini-Simpson index for the ads shown within session i prior to exposure number t. Following the same logic in Equation (20), we can write:

$$SessGiniSimpson_{i,t} = 1 - \sum_{a \in \mathcal{A}} \frac{SessFreqAd_{i,t}^{(a)}(SessFreqAd_{i,t}^{(a)} - 1)}{(t-1)(t-2)} \tag{24}$$

- $SessShannon_{i,t}$: The Shannon entropy for the ads shown within session i prior to exposure number t. Following the same logic in Equation (21), we can write:

$$SessShannon_{i,t} = -\sum_{a \in \mathcal{A}} \frac{SessFreqAd_{i,t}^{(a)}}{t-1} \log\left(\frac{SessFreqAd_{i,t}^{(a)}}{t-1}\right) \tag{25}$$

Like historical features, we generate impression-specific features based on the frequency, spacing, and click inventory information within the session. We can generate the following three features:

- $ThisSessFreqAd_{i,t}$, which is equal to $SessFreqAd_{i,t}^{(a)}$ if ad a is the ad shown in exposure t in session i.
- $ThisSessSpaceAd_{i,t}$, which is equal to $SessSpaceAd_{i,t}^{(a)}$ if ad a is the ad shown in exposure t in session i. It takes value 0 when there is no prior exposure of ad a in the session.
- $ThisSessClickAd_{i,t}$, which is equal to $SessClickAd_{i,t}^{(a)}$ if ad a is the ad shown in exposure t in session i.

For any session i and exposure number t, we denote all session-level features by $O_{i,t}$. As such, this is the only set of features that has subscript t, indicating that it changes within the session. Therefore, the publisher's actions affect the transition of these features in the session. One could argue that historical features also change within the session as user's history accumulates after each exposure. It is worth noting that we do not update the history within the session because session-level temporal features capture that information. As a result, not updating historical features will not result in any information loss.

B Counterfactual Validity

B.1 Filtering Strategy

To address the first part of the Challenge 2, we employ a filtering strategy similar to that in Rafieian and Yoganarasimhan (2021b). In our filtering strategy, our goal is to identify the set of ads that *could have never been shown* in a given impression. If an ad is not targeting one of the targeting characteristics of an impression or is not available around the time that the

impression happens, that ad *could have never been shown* in that impression. As such, the feasibility of an ad in an impression depends on two characteristics of that impression: (1) targeting characteristics, and (2) timestamp. The targeting characteristics of an impression are province, hour of the day, smartphone brand, connectivity type, mobile service provider (MSP), and app category. If the ad is not targeting one of these characteristics, we do not observe this ad in any impression corresponding to that targeting characteristic. For example, suppose that our focal impression is from a Samsung user. If ad a is not targeting Samsung users (i.e., excluded Samsung from the targeting set), then no Samsung impression shows ad a. Alternatively, if ad a has been shown in a Samsung impression, it means that ad a is targeting Samsung users. Our goal is to develop a function f that takes the combination of state variable $S_{i,t}$ and ad a as inputs and return a binary outcome that indicates whether ad a *could have been shown* in impression with state variables $S_{i,t}$. Once we get the outcomes of $f(S_{i,t}, a)$ for all ads, this gives us the feasibility set $\mathcal{A}_{i,t}$.

To develop function f, we first introduce a few notations. First, for any ad a and targeting characteristic c, we define the function $\omega_{c,a}$ that takes timestamp τ as the input and returns value one if ad a includes targeting characteristic c in his targeting criteria. It is easy to empirically estimate this ω function using our data. We first need to discretize the timestamp by a certain unit (e.g., by an hour), and then check if the set of impressions with targeting characteristic c and ad a is non-empty in each unit of timestamp. Given the abundance of our data, we can use very granular discretization, especially for more popular ads. We empirically find that an hourly unit works well in our setting and more granular discretization does not generate different results. In this empirical approach, if there is at least one instance of ad a shown in an impression with targeting characteristic c for the hour of timestamp τ, we have $\hat{\omega}_{c,a}(\tau) = 1$.

Let $C_{i,t}$ denote the full set of targeting characteristics for exposure t in session i with state variables $S_{i,t}$. Further, let $\tau_{i,t}$ denote the timestamp of this exposure. We can define our feasibility function f as follows:

$$f(S_{i,t}, a) = \prod_{c \in C_{i,t}} \hat{\omega}_{c,a}(\tau_{i,t}). \tag{26}$$

This equation indicates that if ad a excludes only one of the targeting characteristics from his targeting criteria, it *could have never been shown* in the impression. Now, we can construct the full feasibility set of the set of ads that *could have been shown*, i.e., ads that have non-zero

propensity scores as follows:

$$\mathcal{A}_{i,t} = \{a \mid f(S_{i,t}, a) = 1\}. \tag{27}$$

It is important to notice that for our main task in the paper, we need to construct the feasibility set for an impression that has not been shown in the data. For example, session i may have ended in only three exposures, but when we want to identify the optimal ad sequencing policy, we need to identify the optimal action in any time period (e.g., optimal policy in the fifth exposure for a session that ended in three exposures). Finding the feasibility set for these counterfactual exposures is straightforward because the function f only uses the information about the targeting characteristics $C_{i,t}$ and the timestamp $\tau_{i,t}$. From our targeting characteristics $C_{i,t}$, only the hour of the day varies with t. Thus, the only element we need to impute for these counterfactual exposures is the timestamp $\tau_{i,t}$. Since each exposure lasts one minute, the task of imputing these timestamps is easy: we just need to add one minute to the timestamp from the point the session ended. For example if the session ends at 5:12 PM, we assume the timestamp for the next exposure would have been 5:13 PM. This guarantees that we can identify the right feasibility set. From an empirical standpoint, however, the feasibility set is almost identical throughout the session.

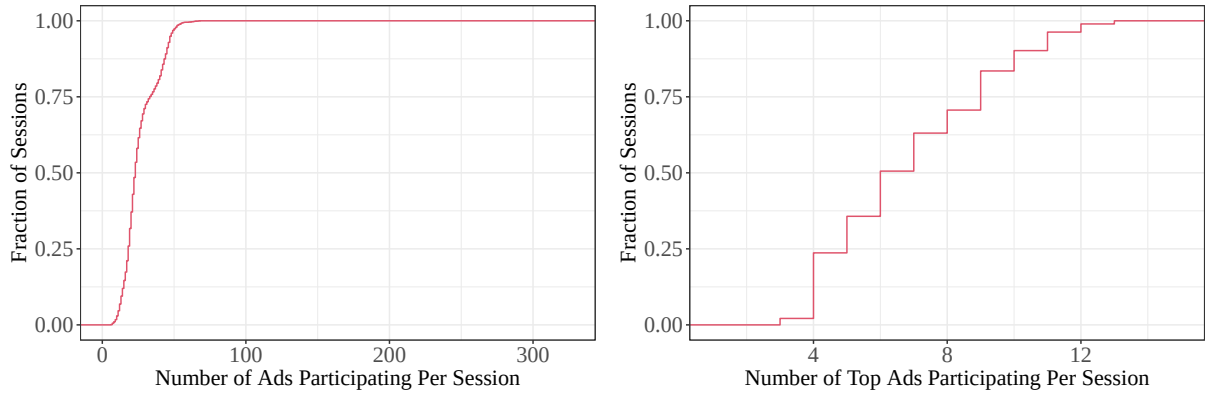
As defined in Equation (27), the size of the feasibility set $\mathcal{A}_{i,t}$ can potentially vary across sessions based on their timestamp and targeting characteristics. In Figure 13, we show the empirical CDF of the size of feasibility set, once when we consider all ads (Figure 13a), and once when we only consider the top 15 ads that we use for our main analysis (Figure 13b). As shown in these figures, the number of ads competing for each impression is quite variable across sessions. More importantly, we also find that for each session, there are many ads that *could have been shown*, which indicates a high degree of variation in our data and a low degree of customization. This extent of variation is often missing in ad platforms that provide micro-level targeting, since only a few ads often participates in the auction for each impression.

B.2 Proof for Proposition 1

Proof. We want to show that for any exposure t in session i, the propensity score $e(S_{i,t}, a)$ is fully determined by observed covariates. According to the allocation rule in the quasi-proportional auction, we know that the propensity scores are determined as follows:

$$e(S_{i,t}, a) = \mathbb{1}(a \in \mathcal{A}_{i,t}) \frac{b_{i,t,a} m_{i,t,a}}{\sum_{k \in \mathcal{A}_{i,t}} b_{i,t,k} m_{i,t,k}}, \tag{28}$$

where $\mathcal{A}_{i,t}$ is the feasibility set for the exposure, and $b_{i,t,a}$ and $m_{i,t,a}$ respectively denote the bid and quality score for ad a at exposure t in session i. If we know all $b_{i,t,a}$ and $m_{i,t,a}$ for



(a) Empirical CDF of the number of ads competing per session.

(b) Empirical CDF of the number of ads among top 15 ads competing per session.

Figure 13: Empirical CDF of the session length and total number of clicks per session.

all the ads in all exposures, the proof would be complete since we have shown how we can identify the set of competing ads $\mathcal{A}_{i,t}$ in Appendix §B.1. The main challenge is that quality scores are unknown to us. However, we can use a feature of our setting to address this challenge: every ad a has only one bid and quality score at any time. That is, bids and quality scores are not customized at the impression-level, and for any impression shown at a specific timestamp τ, each ad's bid and quality score is the same across impressions. As such, we can re-write the propensity score as follows:

$$e(S_{i,t}, a) = \mathbb{1}(a \in \mathcal{A}_{i,t}) \frac{b_a(\tau_{i,t}) m_a(\tau_{i,t})}{\sum_{k \in \mathcal{A}_{i,t}} b_k(\tau_{i,t}) m_k(\tau_{i,t})}, \tag{29}$$

where $b_a(\tau_{i,t})$ and $m_a(\tau_{i,t})$ are ad a's single bid and quality score at timestamp $\tau_{i,t}$, which is the timestamp for exposure t in session i. We can now use the fact that we observe timestamps for all impressions and resolve the issue of not observing quality scores. If bids and quality scores are only functions of time, we can identify the propensity scores in a local neighborhood of any timestamp. For example, consider the exposure t in session i at timestamp $\tau_{i,t}$. If we use the data from other impressions with the same $\mathcal{A}_{i,t}$ in the local neighborhood around $\tau_{i,t}$, the propensity score for ad a would be the proportion of times ad a has been shown in this set of impressions. More formally, the LHS of Equation (29) will be identified by only having the information about actual ad assignments in addition to $\mathcal{A}_{i,t}$ and $\tau_{i,t}$ from the RHS. Thus, the propensity scores are theoretically identified given the observed covariates and our proof is complete.

It is worth noting that this is a theoretical identification proof. In reality, we may face some practical challenges in estimating propensity scores. We discuss these practical challenges in Appendix §B.3.

□

B.3 Propensity Score Estimation and Covariate Balance

Although propensity scores are theoretically identified given observables, there may still be some practical challenges that we need to address in order to obtain accurate propensity estimates. Our goal is to estimate the function $e(S_{i,t}, a)$ using data. From a practical standpoint, a few characteristics of our setting help achieve this goal. One issue with the identification argument above is that it assumes infinite data. However, if we do not have enough impressions with the same $\mathcal{A}_{i,t}$ in a local neighborhood of $\tau_{i,t}$, we may run into small sample problems. Two features of our setting help address this practical challenge. First, advertisers can only target on a few broad targeting categories, so there are potentially many impressions with the same targeting characteristics at any point in time. Second, even if the targeting categories were narrower, the scale and scope of our data is large enough to satisfy the requirement of finding a large number of impression with the same targeting characteristics around the same time.

Another potential practical challenge is when advertisers bid and quality score constantly change over time. That is, even though each ad has a single bid and quality score at a specific timestamp, these two values can vary every second. A useful characteristic of our setting is that quality scores are only updated once a day. Further, for all top 15 ads in our study in terms of share, we do not observe a bid change in the period of our study. Thus, the product $b_a(\tau)m_a(\tau)$ is the same for all timestamps in an entire day. This makes the process of learning propensity scores easier for a machine learning algorithm.

The outcome in the task of propensity score estimation is the actual ad assignment. This variable is a categorical variable with multiple classes, where each class represents an ad. Consistent with our empirical analysis, we only focus on top 15 ads and estimate the propensity scores for these ads in all impressions. We use the following set of covariates to estimate the propensity scores: (1) timestamp, (2) targeting variables that contain province, hour of the day, smartphone brand, connectivity type, and mobile service provider (MSP), (3) exact GPS coordinates, and (4) filtering outcome $f(S_{i,t}, a)$ for all ads. While (1) and (2) are necessary for this estimation task, we include (3) and (4) to help the algorithm learn the propensity scores more efficiently.

We use a multi-class XGBoost with a multi-class logarithmic loss as the evaluation metric that uses a softmax objective to estimate the propensity scores. We estimate the propensity scores for all impressions. The details of our procedure is similar to that of (Rafieian and Yoganarasimhan, 2021b). Like that paper, we focus on covariate balance to show evidence for the existence of imbalance in the raw data and how we can assess balance by using our

estimated propensity scores to weight the impressions. To do so, we follow the norm in the literature to measure the standardized bias with and without incorporating the inverse propensity weights (McCaffrey et al., 2013). For each variable X, we define the unweighted mean of this variable when assigned to ad a as $\bar{X}_a^{unweighted}$, which is simply the average value of variable X in the data when for impressions that show ad a. We can formally define $\bar{X}_a^{unweighted}$ as follows:

$$\bar{X}_a = \frac{\sum_i^N \sum_{t=1}^{T_i} \mathbb{1}(A_{i,t} = a) X_{i,t}}{\sum_i^N \sum_{t=1}^{T_i} \mathbb{1}(A_{i,t} = a)}, \tag{30}$$

where N is the total number of impressions, and T_i is the length of session for session i. If ads have been randomized properly across impressions, we should not see any discernible difference between of $\bar{X}_a^{unweighted}$ and the average value of this variable $\bar{X}$. To quantify this difference, we follow the norm in the literature and use the notion of standardized bias for variable X when assigned to ad a as follows:

$$SB(X, \bar{X}_a) = \frac{|\bar{X}_a - \bar{X}|}{\sigma_X}, \tag{31}$$

which is the absolute mean difference between the unweighted mean of this variable when assigned to ad a and mean of this variable for the full population, divided by the standard deviation of this variable for the population. The numerator is the general bias in the unweighted average of X when assigned to ad a, and the denominator standardizes this bias. The higher the standardized bias is, the greater the covariate imbalance in assignment to ads. In the literature, a threshold of 0.2 is often used to assess balance: if the standardized bias is greater than 0.2, we say that there is imbalance. Hence, we can define a balance function for variable X and averages when assigned to different ads as follows:

$$Balance(X, \{\bar{X}_a\}_a) = \mathbb{1}\left(\max_a \frac{|\bar{X}_a - \bar{X}|}{\sigma_X} < 0.2\right), \tag{32}$$

where $Balance(X, \{\bar{X}_a\}_a) = 1$ if and only if the maximum standardized bias for variable X when assigned to ad a from the set of all ads is lower than the threshold 0.2. As such, if $Balance(X, \{\bar{X}_a\}_a) = 1$, we can say there is balance for covariate X, and if $Balance(X, \{\bar{X}_a\}_a) = 0$, it means that there is at least one ad for which there is imbalance in X when assigned to that ad.

The existence of imbalance is generally a sign of selection in assignment to ads. One way to check if this selection is only on observables is to estimate propensity scores based on observables and then use the weight-adjusted averages for variables when assigned to each ad.

This approach allows us to check covariate balance after weight adjustment. The existence of balance is a necessary condition if we have *unconfoundedness* or *selection on observables* in the data. We can define the inverse probability weight-adjusted (IPW) average values of X when assigned to a as follows:

$$\bar{X}_a^{IPW} = \frac{\sum_i^N \sum_{t=1}^{T_i} \frac{\mathbb{1}(A_{i,t}=a)}{\hat{e}(S_{i,t},a)} X_{i,t}}{\sum_i^N \sum_{t=1}^{T_i} \frac{\mathbb{1}(A_{i,t}=a)}{\hat{e}(S_{i,t},a)}}, \tag{33}$$

where each impression is weighted by its inverse propensity score. Now, we can follow the definitions of standardized bias function to measure $SB(X, \bar{X}_a^{IPW})$, and then assess balance after weight adjustments by measuring $Balance(X, \{\bar{X}_a^{IPW}\}_a)$. Ideally, we want to have $Balance(X, \{\bar{X}_a^{IPW}\}_a) = 1$ for all pre-treatment variables X in our data.

We now empirically examine covariate balance with and without IPW adjustments. We consider all the features defined in Appendix §A that are not ad specific. This excludes the ad fixed effects and features that starts with *This*. Since we only focus on top 15 ads, this gives us a total of 69 demographic features, 76 historical features, and 51 session-level features. Of all these 196 features, 33 covariates exhibit imbalance. However, after adjusting for inverse propensity weights, we have balance for all 196 covariates. This finding provides an evidence for unconfoundedness in our data.

C Counterfactual Policy Evaluation

In this section, we present the details of our policy evaluation framework and supplement the content presented in the main text of the paper.

C.1 Direct Policy Evaluation Algorithm

We start by describing how we evaluate a policy given the initial state and our estimates of the primitives. As discussed in the §4.3, there are different algorithms that we can use to perform this task. The simplest and most common solution is often to use large-scale simulations using the policy and primitive estimates to measure the performance of the session. Another approach is to analytically derive the expected number of clicks per for each session. We presented the expectation we need to take in Equation (18). However, that expectation is over all trajectories. We use the fact that only a few of these trajectories can happen under a deterministic policy π. More precisely, it is only the past sequence of clicks by the user that creates variation in trajectories that can happen. In the first exposure, there is only one ad that can be shown under the deterministic policy π. Next, in the second exposure, there are two possibilities that we need to consider: whether the previous exposure

resulted in a click or not. Similarly, in the third exposure, the total number of possibilities would be $2^2 = 4$, and more generally, in any exposure t, the number of possibilities is 2^{t-1}. As such, we can only take the expectation over these viable trajectories and avoid considering all trajectories. Below, we present a generic direct policy evaluation algorithm that takes a policy π and a set of primitive estimates $\hat{y}$ and $\hat{l}$, and returns the expected reward for any session with initial state variables $S_{i,1}$ for any specific length of horizon T.

The key idea behind Algorithm 2 is to consider all the states s_t^* that may occur under policy π and their corresponding probabilities w_t. Since we can estimate the reward for each state s_t^* and optimal action a_t^* using our click estimation model $\hat{y}$, the value generated under the policy is the probability of being at that state times the probability of click on the impression with that state and the action selected by the policy. As shown in Algorithm 2, we start with the initial state s_1^*, which is the same as $S_{i,1}$ which happens with probability 1. For the second exposure, we take the following steps: (1) we first find the ad a_1^* to be shown under the policy π at state s_1^* such that $\pi(a_1^* \mid s_1^*) = 1$, (2) we then estimate the expected reward for the ad selected for state s_1^*, using our click estimation model $\hat{y}$, which gives us $\hat{r}_1^*$, (3) we then find the dot product of these expected rewards and the probability of being at each state to identify the total contribution to the expected reward per session, i.e., $\hat{v}_1^*$, (4) we then update the next session by considering the only two possibilities of click or no click on the ad shown, and update the next state s_2^* and (5) finally, we use our transition estimates to find the probability of being in the next state w_2, using a recursive relationship based on the click and leave probabilities, and the probability of prior states. We repeat this process for all exposure numbers from 1 to T and then sum the contribution at each step to the expected rewards per session to find the total expected reward per session. It is worth noting that from the second exposure, the s_t^* becomes a full vector and all the operations inside the for loop are implemented on a vector, thereby returning vector values for a_t^*, $\hat{r}_t^*$, for $\hat{v}_t^*$.

The fact that we do not consider impossible trajectories makes Algorithm 2 fast and scalable. We can easily use this algorithm to evaluate the policy for all the sessions in our test data. It is worth noting that Algorithm 2 does not necessarily satisfy the honesty criteria defined in §4.3. However, we can ensure honesty by setting the right inputs for this function. We need to make sure that the policy is developed using the modeling data $\mathcal{D}_{Model}$, whereas the primitive estimates used for evaluation are trained on the evaluation data $\mathcal{D}_{Evaluation}$. For example, π^M is the policy generated only using the modeling data $\mathcal{D}_{Model}$, while $\hat{y}^E$ and $\hat{l}^E$ are primitive estimation models that are trained on the evaluation data $\mathcal{D}_{Evaluation}$. As such, policy evaluation through the function $\hat{\rho}(\pi^M; S_{i,1}, T, \hat{y}^E, \hat{l}^E)$ ensures honesty, because the data

Algorithm 2 Direct Policy Evaluation Algorithm

Input: $\pi, S_{i,1}, T, \hat{y}, \hat{l}$

Output: $\hat{\rho}(\pi; S_{i,1}, T, \hat{y}, \hat{l})$

$s_1^* \leftarrow S_{i,1}$

$w_1 \leftarrow 1$ ▷ w_t is the probability of being at state s_t^*.

for $t = 1 \rightarrow T$ **do**

  $a_t^* \leftarrow \text{argmax}_a \, \pi(a \mid s_t^*)$ ▷ A vector of ads selected by policy π at state(s) s_t^*

  $\hat{r}_t^* \leftarrow \hat{y}^E(s_t^*, a_t^*)$

  $\hat{v}_t^* \leftarrow w_t \cdot \hat{r}_t^*$ ▷ Dot product of w_t and $\hat{r}_t^*$.

  $s_{t+1}^* \leftarrow \begin{pmatrix} \langle s_t^*, y_t^* = 0 \rangle \\ \langle s_t^*, y_t^* = 1 \rangle \end{pmatrix}$ ▷ y_t^* is the actual click outcome.

  $w_{t+1} \leftarrow \begin{pmatrix} \langle w_t \odot (1 - \hat{l}(s_t^*, a_t^*)) \odot (1 - \hat{y}(s_t^*, a_t^*)) \rangle \\ \langle w_t \odot (1 - \hat{l}(s_t^*, a_t^*)) \odot \hat{y}(s_t^*, a_t^*) \rangle \end{pmatrix}$ ▷ $\odot$ is the element-wise product.

end for

$\hat{\rho}(\pi; S_{i,1}, T, \hat{y}, \hat{l}) \leftarrow \sum_{t=1}^{T} \hat{v}_t^*$

used for policy identification do not overlap with the one used for policy evaluation.

C.2 Details of Model, Evaluation, and Test Data

An important part of our honest direct method is splitting the data into three parts that are used for modeling ($\mathcal{D}_{Model}$), evaluation ($\mathcal{D}_{Evaluation}$), and testing ($\mathcal{D}_{Test}$). We now share the details of this splitting. From our set of 84,306 unique users, we randomly select two separate samples of 35,000 users for the modeling and evaluation data sets. The remaining 14,306 users make the test data $\mathcal{D}_{Test}$.

Table A1 summarizes some key metrics for these three data sets: number of impressions and session in the full data and in the focal messenger app.

	$\mathcal{D}_{Model}$	$\mathcal{D}_{Evaluation}$	$\mathcal{D}_{Test}$
Number of Impressions	3,251,996	3,259,750	1,359,858
Number of Impressions in the Focal App	2,612,647	2,651,038	1,093,705
Number of Sessions	558,222	560,515	231,322
Number of Sessions in the Focal App	486,586	489,370	201,466

Table A1: Summary statistics of the user-level variables.

D Learning Algorithm and Parameter Tuning

We now discuss the details of our learning algorithm and how we tune hyper-parameters of the XGBoost model. In general, hyper-parameters need to be set by the researcher because these parameters cannot be inferred from the data like other model parameters. For the XGBoost

model, these hyper-parameters include the maximum depth of each tree (*max_depth*), learning rate (*eta*), etc. In total, we have two sets of models $\{\hat{y}^M, \hat{l}^M\}$ and $\{\hat{y}^E, \hat{l}^E\}$ to be estimated on two separate sets of data $\mathcal{D}_{Model}$ and $\mathcal{D}_{Evaluation}$.

We present a generic approach to hyper-parameter tuning in XGBoost. Suppose that you want to learn an XGBoost model $\hat{h}$ using data $\mathcal{D}^*$. We use a validation procedure whereby we split the data into two parts and use one for training the model and one for validation. We split at the user level. That is, from K users available in our data, we randomly select $0.8K$ for our training and the other $0.2K$ for validation. This is consistent with our original data splitting presented in Appendix §C.2, and ensures that we do not use impressions for the same user to validate our model selection. Let $\mathcal{D}^*_{train}$ and $\mathcal{D}^*_{validation}$ respectively denote the resulting training and validation data sets for data $\mathcal{D}^*$. For any set of specific hyper-parameters, we estimate the model on $\mathcal{D}^*_{train}$ and then evaluate its performance on $\mathcal{D}^*_{validation}$. In the end, we choose the set of hyper-parameters that give us the best performance in the validation set.

We present the full set of hyper-parameters in Table A2. These are the parameters we want to tune for our XGBoost model. Since we use the R package "xgboost", we use the same name for the hyper-parameters. Some of these parameters, we can set a prior. For example, we set the learning rate $eta = 0.1$, which is commonly used for learning algorithms. Likewise, we use 0.5 for both row and column sub-sampling factors because the optimal choice of these parameters does not significantly improve the model performance. As shown in Table A2, for each parameter, we consider a few values. Any combination of values for our hyper-parameters constitutes one full set of hyper-parameters. Since we have 8 values of *max_depth* (maximum depth of each tree), 4 values of *gamma* (the minimum loss reduction required to make a split), 4 values of *alpha* (ℓ^1-norm regularization parameter to leaf weights), and 2 values of *early_stop_round* (stopping rule that stops the iterations after we do not see improvement in the performance for a number of iterations), it gives us a total of 256 different sets of hyper-parameters to evaluate. We use *early_stop_round* instead of the setting *nround* to avoid overfitting. We choose the set that has the lowest log loss on the validation set.

	Set of Values
max_depth	{3,4,5,6,7,8,9,10}
gamma	{5,7,9,11}
alpha	{3,5,7,9 }
early_stop_round	{1,2}

Table A2: Hyper-parameters of an XGBoost model and values considered.

We use this validation procedure four times separately to learn our four models $\{\hat{y}^M, \hat{l}^M, \hat{y}^E, \hat{l}^E\}$.

E Benchmark Policies

In this section, we aim to further formalize the benchmark policies defined in §5.2. We discuss these policies below and formally characterize them:

- *Adaptive Myopic Policy:* This sequencing policy does not take into account the expected future rewards when making the decision at any point. This is equivalent to the our adaptive ad sequencing policy with $\beta = 0$ that turns off the weight on the future rewards. Thus, we can write the objective function for adaptive myopic sequencing as follows:

$$a_{i,t}^{myopic} = \arg\max_{a \in \mathcal{A}_{i,t}} \hat{y}^M(S_{i,t}, a) \tag{34}$$

  Now, we can define this policy as π_m^M as follows:

$$\hat{\pi}_m^M(a \mid S_{i,t}) = \begin{cases} 1 & a = a_{i,t}^{myopic} \\ 0 & a \neq a_{i,t}^{myopic} \end{cases} \tag{35}$$

  In this policy, the publisher selects the ad that maximizes CTR in the current period. It is worth noting that this policy is adaptive, as it uses the session-level information that is time-varying. However, it is myopic in the sense that it ignores future information. This case reflects the common practice of using contextual bandits in the industry.

- *Single-ad Policy:* This policy only uses the pre-session information. Since it does not use adaptive information, this policy allocates all the impressions to a single ad that has the highest average CTR. This is similar to the practice of using a fixed or non-refreshable ad slot where the whole session is allocated to one ad. The objective in this case is the same as Equation (36) only for $t = 1$. We can formally write this policy as follows:

$$a_{i,t}^{single\text{-}ad} = \arg\max_{a \in \mathcal{A}_{i,1}} \hat{y}^M(S_{i,1}, a) \tag{36}$$

  We can now define this policy as π_s^M as follows:

$$\hat{\pi}_s^M(a \mid S_{i,t}) = \begin{cases} 1 & a = a_{i,t}^{single\text{-}ad} \\ 0 & a \neq a_{i,t}^{single\text{-}ad} \end{cases} \tag{37}$$

  This policy provides some insight into the ad sequencing problem because it has two distinct features. First, it captures the potential gains from using a short-lived ad slot as compared to the fixed ad slot. Second, it demonstrates the value of adaptive session-level information. One could argue that the optimal single-ad that is selected for the entire

session may be different from the optimal ad for the first exposure. We acknowledge this issue and check the robustness of our results by using a dynamic optimization constrained by a single ad to be shown for the entire session. In the main text, however, we use the more straightforward approach of allocating the entire session to the ad with the highest CTR in the first exposure.

- *Random Policy:* In this sequencing policy, the publisher randomly selects ads from the ad inventory. We call this policy π_r and define it as follows:

$$\hat{\pi}^r(a \mid S_{i,t}) = \begin{cases} \frac{1}{|\mathcal{A}_i|} & a \in \mathcal{A}_{i,t} \\ 0 & a \notin \mathcal{A}_{i,t} \end{cases}, \tag{38}$$

where the probability of being shown is uniformly distributed across all the ads participating in the exposure. We drop the superscript M from this policy because this superscript denotes the use of a model that is trained on the modeling data $\mathcal{D}_{Model}$. While this is a naive policy, it can serve as a benchmark showing how well we can do without any model. Moreover, the reinforcement learning literature uses this policy as a conventional benchmark.