# Privacy and Polarization: An Inference-Based Framework

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**Abstract**

Advances in behavioral targeting allow news publishers to monetize based on advertising. However, behavioral targeting requires consumer tracking, which has heightened privacy concerns among consumers and regulators. In this paper, we examine how stricter privacy regulations that ban consumer tracking affect news publishers' content strategies. We develop a theoretical framework that captures a change in privacy policies as a shift in publishers' inference about consumer types. We consider a model where news publishers choose the content and advertising, and ideologically heterogeneous consumers select their preferred content based on their ideology and idiosyncratic shocks. We compare two salient informational environments: (1) behavioral targeting, where perfect inference about consumers is allowed, and (2) contextual targeting, where consumer tracking is banned due to privacy regulations, and publishers can only infer consumer types based on their content choice. We show that privacy regulations that ban behavioral targeting incentivize publishers to shift towards more extreme and polarizing content in both monopoly and duopoly settings, even though the shift to more extreme content can hurt both demand and consumer welfare. In summary, our research uncovers a previously unexplored relationship between privacy and polarization, shedding light on the potential unintended consequences of privacy regulations in media markets.

**JEL Codes:** M37, L82, L13, D83.
**Keywords:** advertising, targeting, privacy, polarization.

# 1 Introduction

Digital publishers increasingly use advertising as a monetization strategy. At the core of ad-based monetization is behavioral ad targeting that creates a sustainable revenue stream for publishers and keeps the online content mostly free. However, behavioral ad targeting naturally requires the collection and use of consumer-level data, thereby leading to privacy concerns among consumers. According to a recent survey by Pew Research, over 80% of US adults are concerned about how companies use the data they collect from them, making the need for privacy regulation an issue with bipartisan support (McClain et al., 2023). In response to consumers' privacy concerns in the US and globally, regulatory bodies and even some private firms have started taking actions to restrict consumer tracking online and protect consumer privacy. A few prominent examples include the California Consumer Privacy Act (CCPA) and the European Union's General Data Protection Regulation (GDPR) in the public sector, and Apple's App Tracking Transparency (ATT) and Google's Privacy Sandbox in the private sector.

Although the main intent of privacy regulations is to safeguard consumer privacy, a consistent finding from past empirical research in this domain is that privacy regulation hurts digital publishers (Goldfarb and Tucker, 2011; Alcobendas et al., 2021; Johnson et al., 2023). More specifically, prior research suggests that the revenue loss due to privacy regulations is more pronounced for general interest (vs. specialized) publishers who would have greater uncertainty about their consumer types in the absence of consumer tracking (Goldfarb and Tucker, 2011). For example, in the absence of consumer tracking, a mainstream news website like the New York Times has a harder time inferring consumer types and interests to show them relevant ads, compared to a niche, ideologically extreme website like Infowars, which has more precise information about their consumers. Thus, the negative impact of privacy regulation on news publishers largely depends on the content they create.

The content-dependent impact of privacy regulation on news publishers gives rise to an important question: does privacy regulation affect content strategies employed by news publishers? To counter the loss imposed by privacy regulations, a news website can shift from general news coverage to more specific and niche content, thereby drawing more accurate inferences about users who consume that content and showing them more relevant ads. For example, a news publisher knows more about a consumer who clicked on an ideological opinion piece than a consumer who clicked on a daily news. Such a shift in news content has important implications for consumer welfare in the news landscape, especially given the rise in political polarization and media bias. Nevertheless, the extant research on privacy has focused on measuring the impact of regulations on publishers' market *outcomes* and largely ignored their strategic response in terms of *content design*.

In this paper, we bridge this gap and endogenize content decisions by news publishers in markets with and without privacy regulations that ban consumer tracking. We explicitly model the choice of mainstream and niche content using a simple Hotelling model of product design and consumer demand. Specifically, we seek to answer the following questions:

1. How does privacy regulation affect content strategies employed by a monopolist news publisher who monetizes based on advertising? How does the equilibrium change in a duopoly news market?

2. How do the aforementioned strategies affect consumer polarization? When do news publishers have greater incentive to create polarizing content?

3. What are the implications in terms of both demand, consumer welfare, and publisher profits?

To answer these questions, we develop a model in which profit-maximizing news publishers choose their content design and advertising, and ideologically heterogeneous readers

consume it based on both its slant and their own idiosyncratic taste shocks. In our model, publishers monetize by selling ad impressions to advertisers. As such, their utility is the product of two separate components: (1) an extensive margin that captures the *quantity of impressions*, and (2) an intensive margin that captures the *quality of impressions*.

To study the impact of privacy regulations that ban consumer tracking (e.g., GDPR), we compare two different information environments: *behavioral targeting* and *contextual targeting*. The difference between the two comes from the possibility of tracking. Under *behavioral targeting*, publishers possess perfect knowledge of each consumer's type as they are allowed to track consumers. On the other hand, the contextual targeting scenario mimics the situation in the presence of privacy regulations that ban consumer tracking, where publishers are only allowed to use contextual information to target their ads to consumers. The comparison between contextual and behavioral targeting allows us to examine the downstream impact of privacy regulations on publishers' equilibrium choices and overall market outcomes.

Under behavioral targeting, since publishers can perfectly match ads to consumers on a one-to-one basis, they can achieve maximum quality of impressions regardless of what content each consumer chooses to consume. As a result, the publisher's profit-maximization problem is greatly simplified: because perfect ads-to-readers matching is possible, the profit-maximizing content choice is the one that maximizes the quantity of impressions.

Conversely, under contextual targeting, publishers no longer have access to perfect information about consumer types, so they need to rely solely on a single piece of information to infer consumer types: consumer's self-selection into content. As such, publishers may have an incentive to deviate from the demand-maximizing content strategy (equilibrium under behavioral targeting) and create content that helps increase the quality of impressions. In other words, publishers may benefit from creating content with lower demand but sharper signals about consumers, thereby achieving higher profits by balancing the quantity and quality of impressions.

We consider a monopolist news publisher and examine its equilibrium strategies under both behavioral and contextual targeting regimes. In our analysis, we show that deviating from the demand-maximizing content strategy can be an equilibrium under contextual targeting when consumers are sufficiently ideologically differentiated, and their sensitivity to imperfect ad targeting (mismatches between ad and consumer type) is sufficiently high. Interestingly, we find that the deviation is more toward extreme (vs. moderate) content as publishers can sharpen the signal about consumer types by moving towards the extreme ends of consumers' ideological preferences. Notably, this incentive persists even when such a shift towards extreme content reduces both consumer welfare and total welfare. This is a particularly important finding as it does not depend on an explicit consumer (or collective) distaste for increased polarization.

We then extend our model to a duopoly setting where two news publishers compete. In line with our main finding in the monopoly case, we show that the move from behavioral to contextual targeting due to privacy regulations results in a shift to more extreme and polarizing content. Under behavioral targeting, our setting is consistent with the canonical Hotelling model: when enough consumers are centrists, both publishers choose a moderate (centrist) position, split demand equally, and make equally precise inference about their (identically distributed) readers. The picture changes with contextual targeting: in this case, even with a relatively uniformly distributed set of consumers, one or both publishers have incentives to switch to extreme content, sacrificing demand for inferential accuracy. Depending on the consumers' ideological distribution and the importance of targeting, we identify both extreme-moderate and extreme-extreme equilibria, and show that these are always more likely to exist under contextual targeting. Overall, we find that market forces will become stronger for publishers to produce more extreme and polarizing content.

In summary, our paper makes several contributions to the literature. We study the link between privacy and polarization and highlight the possibility of an unintended consequence

of privacy regulations in increasing polarization. Our finding is important as it goes against extensive media speculations and policy memos that cite personalization as a key contributor to the increased polarization over the past few decades (Pariser, 2011). In our model, we reverse this and emphasize that it is exactly the *inability to personalize* that leads firms to move to more extreme and polarizing content to sharpen their inference about consumers. A key innovation of our framework is in explicitly modeling privacy as an inference problem. From a game theoretical standpoint, this modeling framework allows us to examine players' equilibrium responses to the privacy shifts. For empirical studies, this provides a framework to quantify the magnitude of information gain/loss in various data environments. Finally, in the context of media bias, we introduce inference motives as an important determinant of actions chosen by strategic players, which is largely ignored in the prior literature on media markets.

# 2 Literature Review

Our work relates to the literature on media markets. Early theoretical work in this domain considers the competition between broadcasters in the presence of the advertising market and shares insights into equilibrium outcomes in this market in terms of content provision and advertising strategies (Dukes and Gal-Or, 2003; Gal-Or and Dukes, 2003; Anderson and Coate, 2005; Godes et al., 2009). A separate stream of work in this literature has examined content strategies as they relate to media bias and polarization (Mullainathan and Shleifer, 2005; Gentzkow and Shapiro, 2006a; Xiang and Sarvary, 2007). With the growth of digital news consumption, a series of recent studies have focused on the specific aspects of the digital context and examined pricing and content strategies by media markets (Ambrus et al., 2016; Athey et al., 2018; Lin, 2020; Amaldoss et al., 2021; Jain and Qian, 2021; Amaldoss et al., 2023; Amaldoss and Du, 2023). Our work adds to this stream of work by studying a key

aspect of the digital context: behavioral targeting and the possibility of privacy regulations. In particular, we study the impact of privacy regulations on media markets and examine the equilibrium outcomes in terms of content strategies and their implications for media bias and polarization.

Our work relates to the literature on behavioral targeting and personalization. With the advancements in targeting technologies in advertising markets, a series of papers have studied the impact of targeting accuracy on equilibrium market outcomes (Chen et al., 2001; Iyer et al., 2005; Levin and Milgrom, 2010; Bergemann and Bonatti, 2011; Sayedi, 2018; Rafieian and Yoganarasimhan, 2021; Shin and Yu, 2021; Shin and Shin, 2023). In the news media context, many have speculated that greater personalization results in more polarization, citing the positive correlation between the rise of political polarization in the US and the surge in personalized content delivery through online platforms (Pariser, 2011). Despite the widespread lay belief that causally connects personalization and polarization, empirical findings in this domain do not present a consistent viewpoint. Surprisingly, the demographic groups in the US that are least likely to use the Internet experienced the greatest increase in polarization (Boxell et al., 2017). Moreover, studies analyzing users' browsing histories reveal that despite social media and the Internet being associated with greater ideological divergence among users, they also increase exposure to opposing views (Flaxman et al., 2016). Investigations focused on specific platforms and their personalized features offer conflicting results concerning the link between personalization and polarization. Notably, studies investigating Facebook's news feed algorithm, Google's search personalization, and YouTube have found limited evidence suggesting that personalization contributes to content bias (Bakshy et al., 2015; Ribeiro et al., 2020; Hosseinmardi et al., 2021). In our paper, we build on the theoretical literature on ad targeting and develop a model to study the impact of personalization on the supply of polarizing content. Our work extends this literature by providing an inference-based theoretical account that presents a more nuanced view of the

link between personalization and polarization.

Our paper relates to the literature on privacy. A vast body of theoretical work has examined different issues related to consumer identification, privacy, and information markets (Villas-Boas, 1999, 2004; Taylor, 2004; Acquisti and Varian, 2005; Bergemann and Bonatti, 2015; Bergemann et al., 2018; Yang, 2022; Ke and Sudhir, 2023; Yao, 2023). Empirical papers in this domain have studied the impact of privacy regulations on market outcomes in different settings (Goldfarb and Tucker, 2011; Johnson, 2022; Johnson et al., 2023). In particular, Goldfarb and Tucker (2011) study a change in tracking and targeting regulations and document lower response rates to ads and, therefore, lower ad revenues for publishers. Notably, they demonstrate a heterogeneous effect of privacy regulation on digital publishers, with general interest publishers such as the New York Times experiencing higher revenue loss than specialized publishers such as Car and Driver Magazine. We extend this literature by endogenizing the content design decision to allow publishers to respond optimally to the change in privacy policies. We present a generic theoretical framework that characterizes privacy as an inference problem, whereby stricter privacy policies have a more negative impact on the accuracy of inference about consumers. Importantly, we identify the possibility of increased polarization as an unintended consequence of privacy regulations in digital markets.

# 3 Model

To formalize our main intuitions and derive closed form solutions for both firms' content choices, ad choices and consumer welfare, we specialize our general intuitions to an analytically tractable case. Briefly, our model borrows elements from classic ones in this space, such as Dukes (2004), Gentzkow and Shapiro (2006a) and Bergemann and Bonatti (2011), and introduces an inferential motive for firms whenever behavioral targeting is disallowed.

There are three ideological positions on each issue: left, center, and right.[1] One could think of this as politics, but the model can also be interpreted more broadly. For instance, one can think of the left as strongly in favor and the right as strongly against a given issue (e.g., animal rights versus hunting).

The (media) firm, or publisher, chooses its content, $x \in \{0, 1/2, 1\}$, where $x = 0$ and $x = 1$ correspond to two extremes of niche content and $x = 1/2$ corresponds to mainstream content.[2]

There are three distinct types of consumers, each corresponding to a different ideological position, $\theta \in \{0, 1/2, 1\}$. We assume that consumers are symmetric in their ideology: $\lambda$ proportion of consumers endorse the issue ($\theta = 0$), $\lambda$ proportion of consumers oppose the issue ($\theta = 1$), and the remaining $1-2\lambda$ proportion of consumers are centrist ($\theta = 1/2$). Thus, $\lambda \in [0, 1/2]$ can be thought of as a measure of ideological partisanship among consumers and hence, as we will see, an important determinant of the publisher's positioning.

Each consumer consumes at most one unit of content.[3] A type $\theta$ consumer's utility of consuming content $x$ consists of two parts: a deterministic term depending on the distance between the consumer's type and the content location, and a content-specific idiosyncratic term $\epsilon \sim f(\cdot)$, where $\epsilon \in [-1/2, 1/2]$ and $f(\epsilon) > 0,\ \forall \epsilon \in [-1/2, 1/2]$. Moreover, we make the standard assumption that $\mathbf{E}(\epsilon) = 0$. We assume that the idiosyncratic terms for different contents are independent. Then, the utility of a type $\theta$ consumer consuming content $x$ is:

$$U(x, \theta) = 1/2 - |x - \theta| + \epsilon$$

[1] Our monopoly case can be entirely characterized by two ideological positions. However, this is not the case for duopoly, as whenever firms choose to provide partisan content they naturally do so by choosing opposite positions. In order to have a unified framework and to match well-known phenomena such as political polarization, we use three positions throughout our model section.

[2] We stress that our use of "niche" and "mainstream" is in line with that in Johnson and Myatt (2006), and needs not have a market share interpretation. In other words, niche content is always more polarizing than mainstream one, but it might have lower demand.

[3] The assumption that only one unit of content is consumed is without loss of generality. As long as individual consumption does not depend on $\theta$, all of our findings remain unchanged.

The first term, $1/2$, is the base utility from media consumption. It can be generalized to $v$. However, fixing it at $1/2$ can convey the main insights and make the model simple. The second term, $|x-\theta|$, is the distance between the consumer's location and the media content's location. It captures the consumer's horizontal preference or ideology. We normalize the consumer's outside option to 0.

The base utility, the horizontal term, and the support of the idiosyncratic term jointly imply that the consumer located at one extreme ($\theta = 0$ or 1) will never consume the content located at the other extreme ($x = 1$ or 0), but may consume the mainstream content ($x = 1/2$) if her idiosyncratic term for the mainstream content is sufficiently high. Mainstream consumers ($\theta = 1/2$) may consume the mainstream content or either niche content, depending on the relative values of the idiosyncratic terms.

The firm's revenue comes from advertising. We assume that there is a perfectly competitive ad marketplace so that ads are priced according to their expected value to advertisers.

Ads $a$ lie in the same space as both media content and consumers' ideaologi, and thus take value between 0 and 1. We define the match between an advertisement, $a$, and the consumer type, $\theta$, as $\mathcal{M}(a, \theta)$, where $\mathcal{M} : [0, 1] \times [0, 1] \to \mathbb{R}$ is a continuous and smooth function in both of its arguments.

It is natural to assume that $\mathcal{M}(\cdot, \cdot)$ is decreasing in the distance between the advertising location, $a$, and the consumer type, $\theta$. Therefore, when it has access to each consumer's type, the firm will choose the advertiser that is closer to the consumer's type to maximize profits. For the sake of analytical simplicity, we will assume that $\mathcal{M}(a, \theta) = 1 - \gamma(a - \theta)^2$, for $\gamma > 0$.

The advertising revenue is maximized when the ad perfectly matches the consumer's type and decreases in the extent of advertising mismatch, $|a - \theta|$, convexly. The parameter $\gamma$ captures the value of targeting. The advertising effectiveness does not vary much in the extent of advertising mismatch when $\gamma$ is small, whereas it is very sensitive to the extent of

advertising mismatch when $\gamma$ is large. Consequently, the firm highly values matching their consumers' types with the right advertiser when $\gamma$ is large.

We emphasize two facts. First, we do not restrict ourselves to the case of $\gamma \leq 1$: therefore, a large ad-consumer mismatch ($(a-\theta)^2 > 1/\gamma$) can result in negative ad profits. These could be interpreted, for instance, as the ad being repulsive to consumers (e.g., a hunting rifle ad for a vegetarian consumer), alienating them in the future.

Second, $\mathcal{M}(\cdot,\cdot)$ can take a maximum value of 1. This is without loss of generality, as assuming a more or less profitable ad market for the publisher simply shifts the publisher's profits, but not its strategies.

The timeline in the monopoly case is as follows:

1. The firm chooses the content $x$.

2. Each consumer chooses between the firm's content and the outside option.

3. Given its content choice $x$ (and the information it possesses about the consumer type $\theta$), the firm selects the most profitable ad $a(\theta, x)$.

When analyzing the duopoly case, we will assume that firms' choices in 1. and 3. happen simultaneously.

## 3.1 Behavioral and Contextual Advertising

We contrast behavioral and contextual advertising by modeling both in a stark fashion.

First, we assume that, when behavioral tracking is allowed, the firm has perfect knowledge of each consumer type, $\theta$. Therefore, it utilizes this information to perfectly match the ad with (and thus personalize for) each consumer, $a(\theta, x) = \theta$. Notice how this implies that the firm need (and does) not condition its ad choice $a$ on its content choice $x$.

Clearly, this is a best-case scenario for the firm: the advertising revenue per consumer is always 1, since $\mathcal{M}(\theta,\theta) = 1 \ \forall \ \theta, \gamma$. As a result, the firm only cares about maximizing total demand when choosing its content.

In contrast, we assume the firm does not have any individual-level knowledge of consumer types when behavioral tracking is disallowed. In this case, it can only rely on contextual targeting, that is, infer the consumer types based on consumers' self-selection. Thus, there is no one-to-one personalization.

The ad choice is thus solely a function of the content: $a = a(x)$. This implies that, if two potentially very different consumers (large $|\theta_1 - \theta_2|$) end up consuming the same content ($x_1 = x_2$), they will also see the same ad ($a_1 = a_2$), because the firm cannot tell them apart.

Define by $\pi^b(x)$ and $\pi^c(x)$ the profits in the behavioral and contextual case, respectively. The firm's total profit equals the total demand given the content choice, denoted by $D(x)$, multiplied by the expected advertising revenue, $\mathbf{E}_x\big(\mathcal{M}(a,\theta)\big)$. Thus, optimal profits are given by

$$\pi(x,a) = \max_{x,a} D(x) \cdot \mathbf{E}_x\big(\mathcal{M}(a,\theta)\big).$$

Since the firm can always perfectly match the ads with the consumer type under behavioral advertising, we have $\max_a \mathbf{E}_x\big(\mathcal{M}(a,\theta)\big) = \mathcal{M}(\theta,\theta) = 1, \ \forall x$. Therefore,

$$\pi^b(x) = \max_x D(x).$$

Under contextual advertising, $a = a(x)$,

$$\begin{aligned}
\pi^c(x) &= \max_{x,a} D(x) \cdot \mathbf{E}_x\big(\mathcal{M}(a(x),\theta)\big) \\
&= \max_{x,a} D(x) \cdot \mathbf{E}_x[1 - \gamma(a(x) - \theta)^2] \\
&= \max_{x,a} D(x) \cdot [1 - \gamma \mathbf{E}_x(a(x) - \theta)^2] \qquad (1)
\end{aligned}$$

Given any content choice $x$, one can see from equation (1) that the optimal advertising choice is $a^*(x) = \mathbf{E}_x(\theta)$. In other words, the optimal ad to be displayed with context $x$, $a^*(x)$, matches the expected type among $x$ readers, $\mathbf{E}_x(\theta)$. Hence,

$$\begin{aligned}\pi^c(x) =& D(x) \cdot [1 - \gamma \mathbf{E}_x(\mathbf{E}_x(\theta) - \theta)^2] \\ =& D(x) \cdot [1 - \gamma \mathbf{Var}(\theta|x)] \end{aligned} \tag{2}$$

That is, the firm's ad profits are negatively correlated with the variance of consumer types given the content choice $x$ under contextual advertising. Intuitively, the firm relies on the content choice to infer consumer type in this case. The larger the variance is, the more heterogeneous the consumers are. This makes the firm's inference task harder and increases the extent of advertising mismatch.

Importantly, we notice one fundamental difference in the firm's profit maximization under the two regimes. Under behavioral targeting, context and ads are optimized independently: first, the firm chooses demand-maximizing content, and then it simply matches ads to consumers on a one-to-one, personalized basis, independently of content. Only the *quantity* of clicks matter.

Conversely, under contextual advertising, content choices are a direct input into advertising profits per consumer, since these are negatively correlated with $Var(\theta|x)$. Thus, the firm jointly determines $x$ and $a^*(x)$, taking into account both the *quantity* and *quality* of impressions.

## 3.2 Consumer Privacy

There is a natural connection between the firm's inference task and consumer privacy.

**Definition 1** (Ex-Ante and Ex-Post Privacy)**.** *The consumer's ex-ante privacy is the firm's uncertainty about consumer type,* $Privacy(\theta) = \mathbf{Var}(\theta)$*. The consumer's ex-post privacy is*

*the firm's uncertainty about consumer type conditional on the firm choosing content $x$ and the consumer consuming it, $Privacy(\theta|x) = \mathbf{Var}(\theta|x)$.*

$Privacy(\theta)$ is the unconditional variance of consumer type. It measures the ex-ante uncertainty that the firm faces when trying to recognize the consumer type. $Privacy(\theta|x)$ is the conditional variance of consumer type given the firm's content choice and the consumer's consumption choice. It measures the ex-ante uncertainty that the firm faces when trying to recognize the consumer type among consumers who consume its content.

From equation (2), one can see that the firm's advertising revenue is negatively correlated with the ex-post privacy of the consumer. The more privacy the consumer has, the harder it is for the firm to infer individual consumer type. As a result of the tougher inference task, the extent of the advertising mismatch increases, and the advertising revenue decreases. This simple connection highlights the importance of consumer privacy for the firm's advertising revenue.

More importantly, the firm can manipulate consumer privacy through its content choice because of consumer self-selection. By choosing different content, the firm can enhance or reduce consumer privacy. The following definition summarizes the different effects the firm's content choice can have on consumer privacy.

**Definition 2.** *(Privacy Enhancing/Preserving/Reducing Choice) Define the difference between ex-post and ex-ante privacy by $\Delta(x) = Privacy(\theta|x) - Privacy(\theta)$. A content choice is privacy-enhancing if $\Delta(x) > 0$, privacy-preserving if $\Delta(x) = 0$, and privacy-reducing if $\Delta(x) < 0$.*

### 3.3 Content Choice and Inference

Suppose the firm's total demand is $D(x)$ and its posterior belief about consumer type is $\vec{\mu}(x) = (\mu_0(x), \mu_{1/2}(x), \mu_1(x)) = (\mu_0(x), 1 - \mu_0(x) - \mu_1(x), \mu_1(x))$ if it chooses content $x$.[4] Assume further that mainstream content has a higher demand than niche content. One can see that the firm will choose mainstream content and obtain a profit of $D(1/2)$ under behavioral advertising.

Now consider contextual advertising. The optimal ad choice for a given content choice is

$$\begin{aligned} a^*(x) =& \mathbf{E}_x(\theta) \\ =& \frac{1}{2}\mu_{1/2}(x) + 1 \cdot \mu_1(x) \\ =& \frac{1 - \mu_0(x) + \mu_1(x)}{2} \end{aligned}$$

The ex-post privacy is

$$\begin{aligned} \mathbf{Var}(\theta|x) =& \mathbf{E}_x(\theta^2) - [\mathbf{E}_x(\theta)]^2 \\ =& \frac{\mu_0(x) + \mu_1(x) - [\mu_0(x) - \mu_1(x)]^2}{4} \end{aligned}$$

The expected profit is

$$\begin{aligned} \pi^c(x) =& D(x) \cdot [1 - \gamma \mathbf{Var}(\theta|x)] \\ =& D(x) \cdot \left[1 - \gamma \frac{\mu_0(x) + \mu_1(x) - [\mu_0(x) - \mu_1(x)]^2}{4}\right] \end{aligned}$$

The firm will choose niche content if $\max\{\pi^c(0), \pi^c(1)\} > \pi^c(1/2)$. In the next section we show that with contextual targeting this can happen in equilibrium, even when niche content

[4] Even if the number of type 0 and type 1 consumer are identical and the demand function is symmetric, the posterior belief about niche consumers, $\mu_0(x)$ and $\mu_1(x)$, may not be symmetric. This is because the equilibrium content choices may be asymmetric when there are two firms.

is not demand maximizing.

**Advertising Revenue Loss from Privacy Regulation**

For a given content choice $x$, the firm obtains a higher advertising revenue when behavioral tracking is allowed due to the better ad match. Its advertising revenue loss from a privacy regulation that bans behavioral tracking is

$$\begin{aligned} & D(x) \cdot [\mathcal{M}(\theta, \theta) - \max_{a(x)} \mathbf{E}_x\big(\mathcal{M}(a(x), \theta)\big)] \\ =& D(x) \cdot [1 - \mathbf{E}_x\big(\mathcal{M}(\mathbf{E}_x(\theta), \theta)\big)] \\ =& \gamma D(x) \mathbf{Var}(\theta|x) \end{aligned}$$

The larger the ex-post privacy is, or the more valuable targeting is, the more advertising loss the firm faces under privacy regulation. This expression, when evaluated at $x = 1/2$, is the upper bound of the firm's overall loss from a ban in behavioral targeting because the firm may be better off by switching to niche content under contextual targeting. Mainstream content choice allows the firm to obtain more demand but also makes its inference on consumer types less accurate, hurting advertising profit per consumer. By choosing niche content, the firm trades off the total demand for a more accurate inference. This is the main quantity-quality trade-off in our model. In the next section, we will characterize when the firm prefers a higher quantity and when it prefers a better quality.

# 4 Equilibrium

## 4.1 Monopoly

### 4.1.1 Demand

When there is only one firm, the consumer compares the utility of consuming the monopoly's content with the utility of choosing the outside option. Thus, she will consume the content if and only if the utility from content consumption is positive. The following lemma calculates the consumer demand given different content choices.

**Lemma 1.** *The total demand for a monopoly's mainstream content,* $x = 1/2$*, is* $1-2F(0)\lambda$*. Among them,* $1-2\lambda$ *are mainstream consumers,* $[1-F(0)]\lambda$ *are type 0 consumers, and* $[1-F(0)]\lambda$ *are type 1 consumers.*

*The total demand for a monopoly's niche content,* $x = 0$ *or* $1$*, is* $1-F(0)+[2F(0)-1]\lambda$*. Among them,* $\lambda$ *are type* $\theta = x$ *consumers and* $[1-F(0)](1-2\lambda)$ *are mainstream consumers.*

This lemma highlights an important, general and natural property. The average distance between consumers to niche content is longer than between consumers to mainstream content. If the firm chooses mainstream content, $x = 1/2$, both types of niche consumers, $\theta = 0, 1$, may consume it because they are located not too far way from the content. In contrast, if the firm chooses niche content, say $x = 0$, mainstream consumers may consume it, while type $\theta = 1$ consumers will not, independent on their idiosyncratic taste shock, because they are too ideologically distant from it. This endogenous selection leads to the symmetry in the firm's inference ability. By choosing niche content $x = 0$, the firm knows for sure that it does not attract any type $\theta = 1$ consumers, and can better infer the consumer's type by Bayesian updating.

### 4.1.2 Content Choice

The monopoly's content choice is straightforward under behavioral advertising. It chooses the content that maximizes the total demand because there is no inference problem, and profit simply equals total demand.

**Proposition 1.** *Under behavioral advertising, the monopoly chooses mainstream content if* $[4F(0)-1]\lambda < F(0)$ *and niche content if* $[4F(0)-1]\lambda > F(0)$.

As we have discussed, the firm can perfectly target consumers in the advertising market and does not consider inference strategically under behavioral advertising. In contrast, it relies on Bayesian updating to infer consumer type and consider inference strategically under contextual advertising. Two conditions are necessary for the inference problem to be non-trivial. First, the total demand for the mainstream content is higher than that for the niche content, $[4F(0)-1]\lambda < F(0)$. Second, the demand for mainstream content from mainstream consumers is lower than the demand for niche content from niche consumers of the same type, $\lambda > 1/3$. The firm will choose niche content under both behavioral and contextual advertising if the first condition does not hold. It will choose mainstream content under both behavioral and contextual advertising if the first condition holds but the second condition does not hold.

**Proposition 2.** *Under contextual advertising, the monopoly always chooses niche content if* $[4F(0)-1]\lambda > F(0)$, *mainstream content if* $[4F(0)-1]\lambda < F(0)$ *and* $\lambda < 1/3$. *If* $[4F(0)-1]\lambda < F(0)$ *and* $\lambda > 1/3$, *the ex-post privacy of mainstream content is higher than the ex-post privacy of niche content. The monopoly chooses niche content if* $\gamma > \gamma^m$ *and mainstream content if* $\gamma < \gamma^m$, *where* $\gamma^m = \frac{4\{F(0)+[1-4F(0)]\lambda\}\{1-F(0)+[2F(0)-1]\lambda\}}{[1-F(0)]\lambda[1-2F(0)+4F(0)\lambda]}$.

In the subsequent analyses, we will focus on the interesting case in which both conditions hold: $[4F(0)-1]\lambda < F(0)$ and $\lambda > 1/3$. By choosing niche content rather than mainstream

content, the firm faces a lower uncertainty about consumer type. Therefore, it can infer the consumer type and target consumers more accurately. Though the total demand is lower, the advertising revenue per consumer is higher. So, the firm faces a quantity and quality tradeoff: more eyeballs is equivalent (from an advertising perspective) to *worse* eyeballs. The optimal content then depends on how valuable better targeting is (the size of $\gamma$).

**Corollary 1.** *Fix $F(0) = 1/2$. Then, $\gamma^m = \frac{1-2\lambda}{\lambda^2}$. Thus, $\gamma^m$ is a decreasing function of $\lambda$.*

Intuitively, one would expect a more partisan consumer base (higher $\lambda$) to increase the likelihood the firm switches to niche content, since this is more in demand to begin with. Indeed, this is what Corollary 1 formalizes: a higher $\lambda$ leads to a lower $\gamma^m$, thus expanding the set of $\gamma$'s that lead the firm to produce niche content ($\gamma > \gamma^m$).

### 4.1.3 Consumer Welfare

We have shown that the monopoly may trade off the total demand for a more accurate inference when behavioral tracking is banned. How does the switch from mainstream content to niche content affect consumer welfare?

**Proposition 3.** *Niche content leads to lower consumer welfare if and only if $\lambda < \hat{\lambda} := \frac{1-2[1-F(0)]\mathbf{E}[\epsilon|\epsilon>0]}{3-8[1-F(0)]\mathbf{E}[\epsilon|\epsilon>0]}$. In addition, $\hat{\lambda} > 1/3$ for any distribution of $\epsilon$.*

Intuitively, niche (mainstream) consumers receive a higher surplus from consuming niche (mainstream) content. So, niche content leads to lower consumer welfare if most consumers are mainstream ones (low $\lambda$), while it leads to higher consumer welfare if most consumers are niche ones (high $\lambda$).

The cutoff threshold $\hat{\lambda}$ depends critically on the distribution of the idiosyncratic term. In particular, it is immediate to verify that $\hat{\lambda}$ is increasing in $\mathbf{E}(\epsilon|\epsilon > 0)$. A distribution of $\epsilon$ that is concentrated around 0 results in low values of $\mathbf{E}(\epsilon|\epsilon > 0)$, while the opposite is true for a bimodal distribution that concentrates mass near the extremes $-1/2$ and $1/2$.

The reason why this is a key determinant of consumer welfare is that $\mathbf{E}(\epsilon|\epsilon > 0)$ captures the average idiosyncratic match between readers and their chosen content, *conditional on them choosing mismatched content*.[5] Thus, the higher this quantity, the (absolutely and relatively) higher the consumer welfare associated with mainstream content, which attracts the highest degree of mismatches.

Proposition 3 also shows that there always exists a parameter range such that the optimal content choice by the monopoly (niche) decreases consumer welfare. Therefore, banning behavioral tracking may drive the monopolist publisher to switch from mainstream to niche content, even when doing so reduces both total demand and consumer welfare. This highlight an important unintended consequence of privacy regulation. As we will see in the next Section, competition adds nuance to this finding.

Figure 1 illustrates different content choices and consumer welfare changes under contextual advertising in non-trivial cases where $[4F(0) - 1]\lambda < F(0)$ and $\lambda > 1/3$. As we can see, all combinations of the content choice and welfare effect can occur in equilibrium.

[5]Given our utility specification, consumers always prefer ideologically aligned content to the outside option. Thus, in this case there is no positive selection in their idiosyncratic taste $\epsilon$.

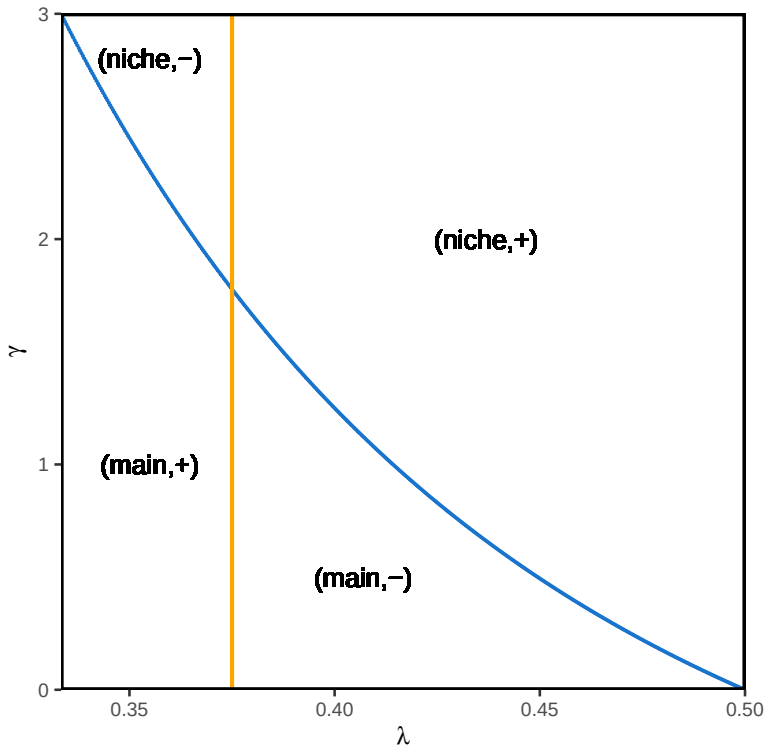



Figure 1: Monopoly's content choice and consumer welfare under contextual advertising when $\epsilon \sim U[-1/2, 1/2]$.
In each label (niche/main,+/-), the first element denotes the content choice, and the second element denotes whether the content choice increases or decreases consumer welfare compared to the opposite choice.

Before concluding our study of the monopoly case, we emphasize that the negative welfare result highlighted in Proposition 3 depends solely on consumer demand, and does not account for additional motives such as consumer polarization, disutility from decreased privacy, and the potential for consumers being exposed to offensive (that is, heavily mismatched) ads. Our negative welfare results would get even stronger when incorporating this richer preference structure.

## 4.2 Duopoly

Now suppose that the first chooses $x_1$ content, while the second chooses $x_2$ content. Denote their content choices by $(x_1, x_2)$. WLOG, $x_1 \leq x_2$. By symmetry, (0,0) is equivalent to (1,1), and (0,1/2) is equivalent to (1/2,1). So, we only consider (0,0), (0,1/2), (0,1), and (1/2,1/2). One can see that (0,1) dominates (0,0). So, (0,0) will never be an equilibrium.

### 4.2.1 Demand

We first characterize demand given different content choices. Differently from the monopoly case, consumers not only compare the utility of consuming a firm's content with the outside option, but also with the utility of consuming the other firm's content.

**Lemma 2.** *Suppose the duopoly's content choices are (0,1). Firm 1's total demand is* $\lambda + \frac{1-F(0)^2}{2}(1-2\lambda)$. *Among them,* $\lambda$ *are type 0 consumers and* $\frac{1-F(0)^2}{2}(1-2\lambda)$ *are mainstream consumers. Firm 2's total demand is* $\lambda + \frac{1-F(0)^2}{2}(1-2\lambda)$. *Among them,* $\lambda$ *are type 1 consumers and* $\frac{1-F(0)^2}{2}(1-2\lambda)$ *are mainstream consumers.*

*Suppose the duopoly's content choices are (1/2,1/2). Each firm's total demand is* $\frac{1-2\lambda}{2} + [1-F(0)^2]\lambda$. *Among them,* $\frac{1-2\lambda}{2}$ *are mainstream consumers,* $\frac{1-F(0)^2}{2}\lambda$ *are type 0 consumers, and* $\frac{1-F(0)^2}{2}\lambda$ *are type 1 consumers.*

*Suppose the duopoly's content choices are (0,1/2). Firm 1's total demand is* $\lambda[1 - \int_{-1/2}^{0} F(\epsilon_0)f(\epsilon+1/2)d\epsilon] + (1-2\lambda)\int_{-1/2}^{0} F(\epsilon)f(\epsilon+1/2)d\epsilon$. *Among them,* $\lambda[1-\int_{-1/2}^{0} F(\epsilon)f(\epsilon+1/2)d\epsilon]$ *are type 0 consumers and* $(1-2\lambda)\int_{-1/2}^{0} F(\epsilon)f(\epsilon+1/2)d\epsilon$ *are mainstream consumers. Firm 2's total demand is* $\lambda\int_{-1/2}^{0} F(\epsilon)f(\epsilon+1/2)d\epsilon + (1-2\lambda)[1-\int_{-1/2}^{0} F(\epsilon)f(\epsilon+1/2)d\epsilon] + [1-F(0)]\lambda$. *Among them,* $\lambda\int_{-1/2}^{0} F(\epsilon)f(\epsilon+1/2)d\epsilon$ *are type 0 consumers,* $(1-2\lambda)[1-\int_{-1/2}^{0} F(\epsilon)f(\epsilon+1/2)d\epsilon]$ *are type 1/2 consumers, and* $[1-F(0)]\lambda$ *are type 1 consumers.*

### 4.2.2 Content Choice

Due to tractability reasons, we make an additional assumption that $\epsilon \sim U[-1/2, 1/2]$ to simplify the demand function. The following result summarizes the equilibria under behavioral and contextual advertising.

**Proposition 4.** *Suppose* $\epsilon \sim U[-1/2, 1/2]$. *The equilibria under behavioral and contextual advertising are summarized in the following table, where the thresholds* $\lambda_1 = 4/11, \lambda_2 = 3/7, \gamma^d = (\lambda_2 - \lambda)\frac{28(1+5\lambda)}{\lambda(44\lambda-1)}$, $\gamma^{d'} = (\lambda_1 - \lambda)\frac{44(2\lambda+3)(7-9\lambda)}{\lambda(-540\lambda^2+460\lambda-63)}$, *and* $\lambda_0$ *is the unique* $\lambda \in (1/3, \lambda_1)$ *such that* $\gamma^d = \gamma^{d'}$. $\gamma^d < \gamma^{d'}$ *if and only if* $\lambda < \lambda_0$.

| *Condition* | *Equilibrium under Behavioral Advertising* | *Equilibrium under Contextual advertising* |
|---|---|---|
| $1/3 < \lambda < \lambda_1$ & $\gamma < \min\{\gamma^d, \gamma^{d'}\}$ | *(1/2,1/2)* | *(1/2,1/2)* |
| $1/3 < \lambda < \lambda_0$ & $\gamma^d < \gamma < \gamma^{d'}$ | *(1/2,1/2)* | *(0,1/2)* |
| $\lambda_0 < \lambda < \lambda_1$ & $\gamma^{d'} < \gamma < \gamma^d$ | *(1/2,1/2)* | *(1/2,1/2) or (0,1)* |
| $1/3 < \lambda < \lambda_1$ & $\gamma > \max\{\gamma^d, \gamma^{d'}\}$ | *(1/2,1/2)* | *(0,1)* |
| $\lambda_0 < \lambda_1 < \lambda < \lambda_2$ & $\gamma < \gamma^d$ | *(1/2,1/2) or (0,1)* | *(1/2,1/2) or (0,1)* |
| $\lambda_1 < \lambda < \lambda_2$ & $\gamma > \gamma^d$ | *(1/2,1/2) or (0,1)* | *(0,1)* |
| $\lambda > \lambda_2$ | *(0,1)* | *(0,1)* |

Figure 2 also illustrates the duopoly's content choices under behavioral (left figure) and contextual (right figure) advertising. The content choices are the same under both behavioral and contextual advertising in the white region, while the extent of media polarization is larger under contextual advertising than under behavioral advertising in the solid and striped shaded region.[6]

When $\lambda > \lambda_2$, there are enough niche consumers so that the duopoly will choose the polarized content strategy (0,1) even under behavioral advertising. So, there is no room for

[6]The extent of media polarization increases from (1/2,1/2) to (0,1/2) to (0,1).

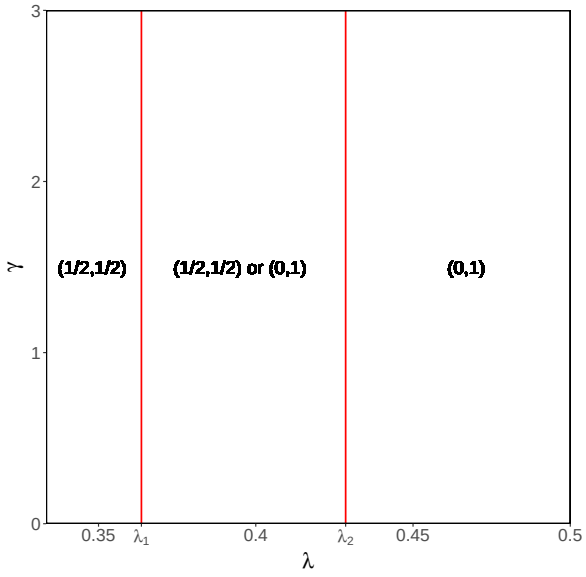
the duopoly to be more polarized when switching to contextual advertising. Now consider the interesting case in which $\lambda < \lambda_2$. In both monopoly and duopoly cases, contextual advertising leads to more polarization when $\gamma > \gamma^m$ (the striped shaded region in Figure 2b). In addition, it leads to more polarization when $\min\{\gamma^d, \gamma^{d'}\} < \gamma < \gamma^m$ and $\lambda < \lambda_1$ or when $\gamma^d < \gamma < \gamma^m$ and $\lambda_1 < \lambda < \lambda_2$ (the solid shaded region in Figure 2b) under duopoly but not under monopoly. The intuition is that, on the one hand, duopolistic firms directly cannibalize each other's demand if they choose mainstream content. In contrast, they can cover different niche consumers by choosing the opposite content locations and soften the competition. As a result, the benefit of a higher total demand from mainstream content in the duopoly case is lower than in the monopoly case. On the other hand, the benefit of a more accurate inference by choosing niche content is not affected by competition. So, firms are leaning more toward quality in the quantity-quality trade-off under duopoly than under monopoly.

The fact that an increase in privacy results in an increase in equilibrium content polarization is interesting in light of the well discussed link between content personalization (that is, lack of privacy) and polarization.

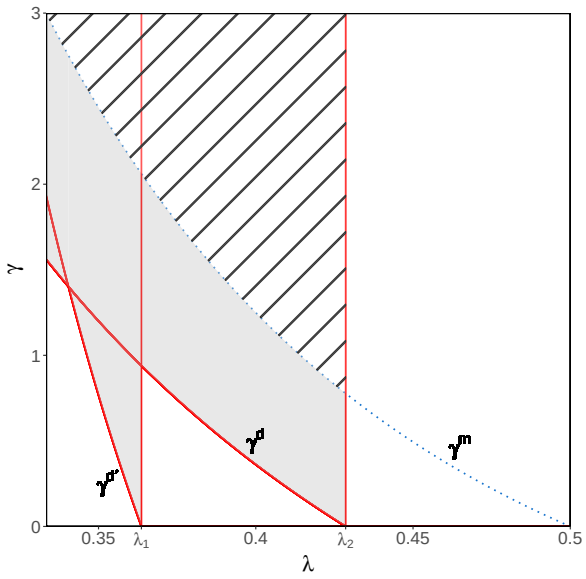
With personalization, content does not polarize consumers; rather, it simply mnatches their prior positions. One could say that in this case content positioning is the consequence, rather than cause, of polarization.

In Proposition 4, we highlight a somewhat opposite result: the desire to make precise inference to increase advertising profits pushes two competing firms away from the standard Hotelling equilibrium, $(1/2, 1/2)$, and toward content $(0, 1)$ that is more ideologically polarized than the underlying distribution of the consumers' ideological preferences, for any value of $\lambda < 1/2$.

In other words, polarization in our model is not driven by the desire to maximize the number of eyeballs by feeding consumers content that they ideologically match perfectly




(a) Behavioral advertising




(b) Contextual advertising

White region: the same equilibrium as the behavioral advertising case;
Striped shaded region: more polarized equilibrium than the behavioral advertising case, under both monopoly and duopoly;
Solid shaded region: more polarized equilibrium than the behavioral advertising case, under duopoly but not under monopoly.

Figure 2: Duopoly's content choices when $\epsilon \sim U[-1/2, 1/2]$.

with. Rather, it comes at the expense of eyeballs, with the goal of maximizing inferential accuracy.

However, with two firms, polarization becomes desirable for consumers. The next proposition shows that consumer welfare always increases in the extent of polarization under duopoly.

**Proposition 5.** *Suppose* $\lambda > 1/3$, $[4F(0)-1]\lambda < F(0)$, *and* $\epsilon \sim U[-1/2, 1/2]$. *(0,1) content choices always lead to higher consumer welfare than (0,1/2) content choices. (0,1/2) content choices always lead to higher consumer welfare than (1/2,1/2) content choices.*

# 5 Conclusion

Over the last few years, increasingly strict privacy regulations have altered the ability of online publishers to monetize on advertising.

How does this affect publishers' content strategies? We examine this question theoretically by building a simple model of media product design and consumer demand. In our baseline model, a monopolist media publisher designs content and consumers select their preferred content based on both their ideology and idiosyncratic shocks. Advertisers pay the publisher according to the expected efficacy of their ads for different pieces of content.

We focus on the comparison of two salient informational environments: under *behavioral targeting*, publishers can perfectly track readers, and thus personalize ads on a one-to-one basis. Under *contextual targeting*, publishers can not track readers, and are therefore limited to tailoring ads to content, relying on the information contained in readers' self-selection. We show that banning behavioral targeting incentivizes publishers to shift towards niche (or polarizing) content to aid inference about readers (thus decreasing their privacy), thereby increasing the publisher's ad revenue per reader.

This holds true even when a switch to niche content decreases total demand and hurts

consumer welfare. In duopoly, contextual targeting increases polarization in equilibrium content provision, often pushing both publishers to produce niche content. In summary, our research uncovers a previously unexplored relationship between privacy and polarization, shedding light on the potential unintended consequences of privacy regulations in media markets.

The findings of this study shed light on the intricate connection between privacy and polarization. In recent years, a growing body of research has underscored the pivotal role played by increasingly precise content personalization, driven by behavioral targeting, in the surge of political polarization within the United States. This perspective posits that reduced privacy leads to heightened personalization, ultimately intensifying polarization (Pariser, 2011). However, our research takes a thought-provoking turn by offering an alternative viewpoint. In our model, the inability to personalize content compels firms to employ an alternative strategy—consumer self-selection. This strategic shift creates a unique incentive for the production of more partisan content, particularly in duopoly scenarios, where polarization intensifies (as noted in the references).

Our research underscores an important shift from viewing content polarization as a mere reflection of consumers' existing ideological polarization. Instead, we demonstrate the active role played by content polarization in several scenarios. The equilibrium content consumed becomes more extreme and polarized than the initial ideological distribution. In essence, the content itself actively contributes to the escalation of polarization, a significant finding that challenges prior assumptions about this relationship.

These findings contribute to the ongoing discourse on privacy, content personalization, and political polarization. They highlight the complex dynamics at play and encourage further exploration into the underlying mechanisms of this relationship. As privacy regulations continue to evolve and shape the digital landscape, understanding the impact of these changes on content strategies and societal polarization remains a pertinent area for both

research and policy.

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# Appendix

*Proof of Lemma 1.* One can see that consumers whose type matches exactly the content's location will consumer for sure. So, we just need to determine whether other consumers prefer consuming the content to the outside option.

1. Mainstream content $x = 1/2$

   The demand from mainstream consumers is $1 - 2\lambda$.

   Consider a type 0 consumer. Her utility from consuming the content is $U(1/2, 0) = \epsilon$. She will consume the content if and only if $\epsilon > 0$, her utility from the outside option. The consumption probability is:

   $$P(\epsilon > 0) = 1 - F(0)$$

   Hence, the demand from type 0 consumers is $\lambda[1 - F(0)]$. By symmetry, the demand from type 1 consumers is also $\lambda[1 - F(0)]$.

   In sum, the total demand is $1 - 2\lambda + 2\lambda[1 - F(0)]$.

2. Niche content $x = 0$

   The demand from type 0 consumers is $\lambda$ and the demand from type 1 consumers is 0.

   Consider a type $1/2$ consumer. Her utility from consuming the content is $U(0, 1/2) = \epsilon$. She will consume the content if and only if $\epsilon > 0$, her utility from the outside option. By the same argument in the previous case, one can see that the demand from type $1/2$ consumers is $(1 - 2\lambda)[1 - F(0)]$.

   In sum, the total demand is $\lambda + (1 - 2\lambda)[1 - F(0)]$.

3. Niche content $x = 1$

   It is symmetric to the previous $x = 0$ case.

■

*Proof of Proposition 2.* We prove a lemma first.

**Lemma 3.** *If* $[4F(0)-1]\lambda < F(0)$ *and* $\lambda > 1/3$*, the ex-post privacy of mainstream content is higher than the ex-post privacy of niche content.*

*Proof of Lemma 3.* Due to the symmetry between two types of niche contents, we only need to compare the ex-post privacy of mainstream content and type 0 content.

1. Mainstream content $x = 1/2$

   By symmetry, $\mathbf{E}[\theta|x=1/2] = 1/2$. So,

$$\begin{aligned}\mathbf{Var}[\theta|x=1/2] =& \mathbf{E}[\theta - 1/2|x=1/2]^2 \\ =& 2\cdot\frac{\lambda[1-F(0)]}{1-2\lambda F(0)}\cdot(1/2)^2 \\ =& \frac{1}{2}\cdot\frac{\lambda[1-F(0)]}{1-2\lambda F(0)}\end{aligned}$$

2. Niche content $x = 0$

$$\begin{aligned}\mathbf{Var}[\theta|x=0] =& \mathbf{E}[\theta^2|x=0] - \mathbf{E}[\theta|x=0]^2 \\ =& \frac{[1-F(0)](1-2\lambda)}{\lambda+[1-F(0)](1-2\lambda)}\cdot(\frac{1}{2})^2 - \left\{\frac{[1-F(0)](1-2\lambda)}{1-F(0)+[2F(0)-1]\lambda}\cdot\frac{1}{2}\right\}^2 \\ =& \frac{1}{4}\cdot\frac{[1-F(0)](1-2\lambda)}{\lambda+[1-F(0)](1-2\lambda)} - \frac{1}{4}\cdot\frac{[1-F(0)]^2(1-2\lambda)^2}{\{\lambda+[1-F(0)](1-2\lambda)\}^2} \\ =& \frac{1}{4}\cdot\frac{[1-F(0)](1-2\lambda)\lambda}{\{\lambda+[1-F(0)](1-2\lambda)\}^2}\end{aligned}$$

$$
\begin{aligned}
&\mathbf{Var}[\theta|x=0] < \mathbf{Var}[\theta|x=1/2] \\
\Leftrightarrow &\frac{1}{4}\cdot\frac{[1-F(0)](1-2\lambda)\lambda}{\{\lambda+[1-F(0)](1-2\lambda)\}^2} < \frac{1}{2}\cdot\frac{\lambda[1-F(0)]}{1-2\lambda F(0)} \\
\Leftrightarrow &(1-2\lambda)[1-2F(0)\lambda] < 2\{\lambda+[1-F(0)](1-2\lambda)\}^2 \\
\Leftrightarrow &\lambda+\lambda(1-2\lambda)F(0)+2\lambda(1-2\lambda)[1-F(0)]+\lambda^2+[1-F(0)]^2(1-2\lambda)^2 > \frac{1}{2}
\end{aligned}
$$

The LHS is greater than $\lambda+\lambda(1-2\lambda)F(0)+\lambda(1-2\lambda)[1-F(0)]+\lambda^2=\lambda(2-\lambda)$. Since $\lambda\in(1/3,1/2)$ and $\lambda(2-\lambda)$ increases in $\lambda$ for $\lambda<1$, we have the LHS $>\lambda(2-\lambda)>1/3(2-1/3)=5/9>1/2$. Therefore, $\mathbf{Var}[\theta|x=0]<\mathbf{Var}[\theta|x=1/2]$ always holds. By definition, the ex-post privacy of mainstream content is higher than the ex-post privacy of niche content. ■

Consider contextual advertising. By symmetry, we only need to compare the firm's expected profits from niche content and mainstream content.

$$
\begin{aligned}
&\pi^c(0) > \pi^c(1/2) \\
\Leftrightarrow\ &D(0)\cdot[1-\gamma\mathbf{Var}(\theta|x=0)] > D(1/2)\cdot[1-\gamma\mathbf{Var}(\theta|x=1/2)] \\
\Leftrightarrow\ &\gamma > \frac{\frac{D(1/2)}{D(0)}-1}{\mathbf{Var}(\theta|x=1/2)\frac{D(1/2)}{D(0)}-\mathbf{Var}(\theta|x=0)}
\end{aligned}
$$

Denote $\frac{\frac{D(1/2)}{D(0)}-1}{\mathbf{Var}(\theta|x=1/2)\frac{D(1/2)}{D(0)}-\mathbf{Var}(\theta|x=0)}$ by $\gamma^m$. Since $D(1/2)>D(0)$ and $\mathbf{Var}[\theta|x=0]<\mathbf{Var}[\theta|x=1/2]$, we have $\gamma^m>0$. So, the monopoly chooses niche content if $\gamma>\gamma^m$ and mainstream content if $\gamma<\gamma^m$ under contextual advertising. ■

*Proof of Proposition 3.* We first compute the consumer welfare for each content choice.

1. Mainstream content $x=1/2$

The consumer welfare is:

$$\begin{aligned}&(1-2\lambda)[1/2+\mathbf{E}[\epsilon]]+2[1-F(0)]\lambda\mathbf{E}[\epsilon|\epsilon>0]\\=&1/2-\lambda+2[1-F(0)]\lambda\mathbf{E}[\epsilon|\epsilon>0]\end{aligned}$$

2. Niche content $x=0$

   The consumer welfare is:

$$\begin{aligned}&\lambda[1/2+\mathbf{E}[\epsilon]]+[1-F(0)](1-2\lambda)\mathbf{E}[\epsilon|\epsilon>0]\\=&\lambda/2+[1-F(0)](1-2\lambda)\mathbf{E}[\epsilon|\epsilon>0]\end{aligned}$$

3. Niche content $x=1$

   It is symmetric to the previous $x=0$ case.

Niche content leads to lower consumer welfare if and only if

$$\begin{aligned}&\lambda/2+[1-F(0)](1-2\lambda)\mathbf{E}[\epsilon|\epsilon>0]<1/2-\lambda+2[1-F(0)]\lambda\mathbf{E}[\epsilon|\epsilon>0]\\\Leftrightarrow&\lambda<\hat{\lambda}=\frac{1-2[1-F(0)]\mathbf{E}[\epsilon|\epsilon>0]}{3-8[1-F(0)]\mathbf{E}[\epsilon|\epsilon>0]}\end{aligned}$$

In addition, $\hat{\lambda}>1/3\Leftrightarrow 2[1-F(0)]\mathbf{E}[\epsilon|\epsilon>0]>0$, which holds for any distribution of $\epsilon$. ■

*Proof of Lemma 2.* There are three candidate equilibria: (0,1/2), (1/2,1/2), (0,1).

1. (0,1)

   The demands for type 0 and type 1 content are symmetric. So, we only need to examine type 0 content. Each consumer will choose type 0 content if and only if her utility from type 0 content is positive (higher than the outside option) and higher than her utility from type 1 content.

One can see that all type 0 consumers will choose type 0 content. Now consider type 1/2 consumers. A mainstream consumer will consume one of the contents if and only if her utility from at least one content is positive. By symmetry, her overall probabilities of choosing type 0 and type 1 content are identical.

$$\begin{aligned} & P(\text{a mainstream consumer chooses type 0 content}) \\ &= \frac{1}{2} \cdot P(\max\{\epsilon_0, \epsilon_1\} > 0) \\ \overset{\textit{independence of } \epsilon_j}{=} & \frac{1}{2} \cdot [1 - P(\epsilon_0 \leq 0)P(\epsilon_1 \leq 0)] \\ &= \frac{1}{2} \cdot [1 - F(0)^2] \end{aligned}$$

In sum, the demand of type 0 content from type 0 consumers is $\lambda$ and from type 1/2 consumers is $\frac{1-F(0)^2}{2}(1-2\lambda)$. The total demand of type 0 content is $\lambda + \frac{1-F(0)^2}{2}(1-2\lambda)$.

2. (1/2, 1/2)

One can see that all mainstream consumers will consume one of the contents. The demands from type 0 and type 1 consumers are symmetric. Consider type 0 consumers. She will consume one of the contents if and only if her utility from at least one content is positive. By symmetry, her overall probabilities of choosing either mainstream content are identical.

$$\begin{aligned} & P(\text{a type 0 consumer chooses firm } i) \\ &= \frac{1}{2} \cdot P(\max\{\epsilon_{1/2}, \epsilon_{1/2'}\} > 0) \\ \overset{\textit{independence of } \epsilon_j}{=} & \frac{1}{2} \cdot [1 - P(\epsilon_{1/2} \leq 0)P(\epsilon_{1/2'} \leq 0)] \\ &= \frac{1 - F(0)^2}{2} \end{aligned}$$

In sum, the demand of either firm from mainstream consumers is $\frac{1-2\lambda}{2}$, from type 0

consumers is $\frac{1-F(0)^2}{2}\lambda$, and from type 1 consumers is $\frac{1-F(0)^2}{2}\lambda$. The total demand of either firm is $\frac{1-2\lambda}{2} + [1-F(0)^2]\lambda$.

3. (0, 1/2)

Consider first the demand for type 0 content. Since a type 0 consumer's utility from consuming type 0 content is always positive, we only need to compare her utility from type 0 and type 1/2 contents.

$$
\begin{aligned}
& P(\text{a type 0 consumer chooses type 0 content}) \\
&= P(1+\epsilon_0 > 1/2 + \epsilon_{1/2}) \\
&= P(\epsilon_{1/2} < \epsilon_0 + 1/2) \\
&= 1 - F(0) + \int_{-1/2}^{0} \int_{-1/2}^{\epsilon_0+1/2} f(\epsilon_{1/2}) d\epsilon_{1/2} f(\epsilon_0) d\epsilon_0 \\
&= 1 - F(0) + \int_{-1/2}^{0} F(\epsilon_0 + 1/2) f(\epsilon_0) d\epsilon_0 \\
&\overset{\text{integral by parts}}{=} 1 - \int_{-1/2}^{0} F(\epsilon_0) f(\epsilon_0 + 1/2) d\epsilon_0
\end{aligned}
$$

A type 1/2 consumer will choose type 0 content if and only if her utility of consuming it is positive and higher than her utility of consuming mainstream content. Since her utility of consuming mainstream content is always positive, we only need the condition that her utility of consuming type 0 content is higher than her utility of consuming

mainstream content.

$$
\begin{aligned}
& P(\text{a type } 1/2 \text{ consumer chooses type 0 content}) \\
&= P(1/2 + \epsilon_0 > 1 + \epsilon_{1/2}) \\
&= P(\epsilon_0 > \epsilon_{1/2} + 1/2) \\
&\overset{symmetry}{=} P(\epsilon_{1/2} > \epsilon_0 + 1/2) \\
&= 1 - P(\epsilon_{1/2} < \epsilon_0 + 1/2) \\
&\overset{previous\ case}{=} 1 - [1 - \int_{-1/2}^{0} F(\epsilon_0) f(\epsilon_0 + 1/2) d\epsilon_0] \\
&= \int_{-1/2}^{0} F(\epsilon_0) f(\epsilon_0 + 1/2) d\epsilon_0
\end{aligned}
$$

Therefore, the demand of type 0 content from type 0 consumers is $\lambda[1-\int_{-1/2}^{0} F(\epsilon_0) f(\epsilon_0 + 1/2) d\epsilon_0]$ and from type 1/2 consumers is $(1-2\lambda)\int_{-1/2}^{0} F(\epsilon_0) f(\epsilon_0 + 1/2) d\epsilon_0$. The total demand of of type 0 content is $\lambda[1-\int_{-1/2}^{0} F(\epsilon_0) f(\epsilon_0 + 1/2) d\epsilon_0] + (1-2\lambda)\int_{-1/2}^{0} F(\epsilon_0) f(\epsilon_0 + 1/2) d\epsilon_0$.

Consider now the demand for type 1/2 content.

$$
\begin{aligned}
& P(\text{a type 0 consumer chooses type 1/2 content}) \\
&= 1 - P(\text{a type 0 consumer chooses type 0 content}) \\
&= \int_{-1/2}^{0} F(\epsilon_0) f(\epsilon_0 + 1/2) d\epsilon_0
\end{aligned}
$$

$$
\begin{aligned}
& P(\text{a type 1/2 consumer chooses type 1/2 content}) \\
&= 1 - P(\text{a type 1/2 consumer chooses type 0 content}) \\
&= 1 - \int_{-1/2}^{0} F(\epsilon_0) f(\epsilon_0 + 1/2) d\epsilon_0
\end{aligned}
$$

$$
\begin{aligned}
& P(\text{a type 1 consumer chooses type } 1/2 \text{ content}) \\
&= P(U(1/2,1) > 0) \\
&= P(\epsilon_{1/2} > 0) \\
&= 1 - F(0)
\end{aligned}
$$

Therefore, the demand of type $1/2$ content from type 0 consumers is $\lambda \int_{-1/2}^{0} F(\epsilon_0) f(\epsilon_0 + 1/2) d\epsilon_0$, from type $1/2$ consumers is $(1-2\lambda)[1 - \int_{-1/2}^{0} F(\epsilon_0) f(\epsilon_0 + 1/2) d\epsilon_0]$, and from type 1 consumers is $[1 - F(0)]\lambda$. The total demand of type $1/2$ content is $\lambda \int_{-1/2}^{0} F(\epsilon_0) f(\epsilon_0 + 1/2) d\epsilon_0 + (1-2\lambda)[1 - \int_{-1/2}^{0} F(\epsilon_0) f(\epsilon_0 + 1/2) d\epsilon_0] + [1 - F(0)]\lambda$.

■

*Proof of Proposition 4.* We first simplify the demand in Lemma 2 when $\epsilon \sim U[-1/2, 1/2]$.

**Lemma 4.** *Suppose* $\epsilon \sim U[-1/2, 1/2]$.

*Suppose the duopoly's content choices are (0,1). Firm 1's total demand is* $3/8 + \lambda/4$. *Among them,* $\lambda$ *are type 0 consumers and* $3/8 - 3\lambda/4$ *are mainstream consumers. Firm 2's total demand is* $3/8 + \lambda/4$. *Among them,* $\lambda$ *are type 1 consumers and* $3/8 - 3\lambda/4$ *are mainstream consumers.*

*Suppose the duopoly's content choices are (1/2,1/2). Each firm's total demand is* $1/2 - \lambda/4$. *Among them,* $1/2 - \lambda$ *are mainstream consumers,* $3\lambda/8$ *are type 0 consumers, and* $3\lambda/8$ *are type 1 consumers.*

*Suppose the duopoly's content choices are (0,1/2). Firm 1's total demand is* $1/8 + 5\lambda/8$. *Among them,* $7\lambda/8$ *are type 0 consumers and* $(1-2\lambda)/8$ *are mainstream consumers. Firm 2's total demand is* $7/8 - 9\lambda/8$. *Among them,* $\lambda/8$ *are type 0 consumers,* $\frac{7}{8}(1-2\lambda)$ *are type 1/2 consumers, and* $\lambda/2$ *are type 1 consumers.*

Now consider the equilibrium strategy.

1. Behavioral advertising

   Under behavioral advertising, the advertising revenue per consumer is 1. So, each firm's profit equals its total demand. Suppose the equilibrium is (0,1/2). Firm 1 will deviate from $x = 0$ to $x = 1/2$ if and only if:

$$\begin{aligned} & 1/2 - \lambda/4 > 1/8 + 5\lambda/8 \\ \Leftrightarrow & \lambda < \lambda_2 = 3/7 \end{aligned} \tag{3}$$

   Firm 2 will deviate from $x = 1/2$ to $x = 1$ if and only if:

$$\begin{aligned} & 3/8 + \lambda/4 > 7/8 - 9\lambda/8 \\ \Leftrightarrow & \lambda > \lambda_1 = 4/11 \end{aligned} \tag{4}$$

   From the above conditions and noting that $\lambda_1 < \lambda_2$, one can see that (0,1/2) will never be an equilibrium.

   Now suppose the equilibrium is (1/2,1/2). Equation 3 implies that the firm will deviate to 0 or 1 if $\lambda > \lambda_2$. Similarly, if the equilibrium is (0,1), Equation 4 implies that the firm will deviate to 1/2 if $\lambda < \lambda_1$.

   In sum, the equilibrium is (1/2,1/2) if $\lambda < \lambda_1$, (1/2,1/2) or (0,1) if $\lambda_1 < \lambda < \lambda_2$, and (0,1) if $\lambda > \lambda_2$.

2. Contextual advertising

   We first characterize the equilibrium ad choices and profits for each firm.

   (a) (1/2,1/2)

By symmetry, $a = \mathbf{E}(\theta) = 1/2$. $\mathbf{Var}(\theta) = \mathbf{E}[\theta - \mathbf{E}(\theta)]^2 = \frac{3\lambda}{4(2-\lambda)}$.

$$\begin{aligned}\pi(1/2, 1/2) =& D(1/2, 1/2) \cdot [1 - \gamma \mathbf{Var}(\theta)] \\ =& \frac{2-\lambda}{4} - \frac{3\gamma\lambda}{16}\end{aligned} \tag{5}$$

(b) (0,1)

By symmetry, we only need to consider firm 1. Firm 2's ad choice will be $1 - a_1$, and firm 2's profit will be identical to firm 1's.

$a_1 = \mathbf{E}(\theta) = \frac{3/8-3\lambda/4}{\lambda/4+3/8} \cdot \frac{1}{2} = \frac{3(1-2\lambda)}{2(3+2\lambda)}$. $\mathbf{Var}(\theta) = \mathbf{E}[\theta - \mathbf{E}(\theta)]^2 = \frac{6\lambda(1-2\lambda)}{(3+2\lambda)^2}$.

$$\begin{aligned}\pi(0, 1) =& D(0, 1) \cdot [1 - \gamma \mathbf{Var}(\theta)] \\ =& \frac{2\lambda+3}{8} - \frac{3\gamma\lambda(1-2\lambda)}{4(2\lambda+3)}\end{aligned} \tag{6}$$

(c) (0,1/2)

Consider firm 1 first. $a_1 = \mathbf{E}_1(\theta) = \frac{1-2\lambda}{2(1+5\lambda)}$. $\mathbf{Var}_1(\theta) = \mathbf{E}_1[\theta - \mathbf{E}_1(\theta)]^2 = \frac{7\lambda(1-2\lambda)}{4(1+5\lambda)^2}$.

$$\begin{aligned}\pi_1(0, 1/2) =& D_1(0, 1/2) \cdot [1 - \gamma \mathbf{Var}_1(\theta)] \\ =& \frac{1+5\lambda}{8} - \frac{7\gamma\lambda(1-2\lambda)}{32(1+5\lambda)}\end{aligned} \tag{7}$$

Consider firm 2 then. $a_2 = \mathbf{E}_2(\theta) = \frac{7-6\lambda}{2(7-9\lambda)}$. $\mathbf{Var}_2(\theta) = \mathbf{E}_2[\theta - \mathbf{E}_2(\theta)]^2 = \frac{35\lambda-54\lambda^2}{4(7-9\lambda)^2}$.

$$\begin{aligned}\pi_2(0, 1/2) =& D_2(0, 1/2) \cdot [1 - \gamma \mathbf{Var}_2(\theta)] \\ =& \frac{7-9\lambda}{8} - \frac{\gamma(35\lambda-54\lambda^2)}{32(7-9\lambda)}\end{aligned} \tag{8}$$

Suppose the equilibrium is (0,1/2). Firm 1 will deviate from $x = 0$ to $x = 1/2$ if and

only if:

$$\begin{aligned} &\pi(1/2,1/2) > \pi_1(0,1/2) \\ \Leftrightarrow &\gamma < \gamma^d = (\lambda_2 - \lambda)\frac{28(1+5\lambda)}{\lambda(44\lambda-1)} \end{aligned} \tag{9}$$

An immediate implication is that this deviation never happens if $\lambda \geq \lambda_2$.

Firm 2 will deviate from $x = 1/2$ to $x = 1$ if and only if:[7]

$$\begin{aligned} &\pi(0,1) > \pi_2(0,1/2) \\ \Leftrightarrow &\gamma > \gamma^{d'} = (\lambda_1 - \lambda)\frac{44(2\lambda+3)(7-9\lambda)}{\lambda(-540\lambda^2+460\lambda-63)} \end{aligned} \tag{10}$$

An immediate implication is that deviation always happens if $\lambda \geq \lambda_1$.[8] Therefore, (0,1/2) may only be an equilibrium if $\lambda < \lambda_1$. Now suppose $\lambda < \lambda_1$. In this case, we have shown that firm 1 will deviate if $\gamma < \gamma^d$ and firm 2 will deviate if $\gamma > \gamma^{d'}$. Some calculation yields that there exists a unique $\lambda_0 \in (1/3, \lambda_1)$ such that $\gamma^d < (>)\gamma^{d'}$ if $\lambda < (>)\lambda_0$. Therefore, at least one deviation happens and (0,1/2) will not be an equilibrium if $\lambda > \lambda_0$. There is no deviation and (0,1/2) is an equilibrium if $\lambda < \lambda_0$.

Now suppose the equilibrium is (1/2,1/2). Equation 9 implies that the firm will deviate to 0 or 1 if $\gamma > \gamma^d$, which may hold if $\lambda < \lambda_2$ and always holds if $\lambda > \lambda_2$. Similarly, if the equilibrium is (0,1), Equation 10 implies that the firm will deviate to 1/2 if $\gamma < \gamma^{d'}$, which may hold if $\lambda < \lambda_1$ and never holds if $\lambda > \lambda_1$.

In sum, (0,1) is an equilibrium if $\lambda > \lambda_1$ or if $\lambda < \lambda_1$ & $\gamma > \gamma^{d'}$. (1/2,1/2) is an equilibrium if $\lambda < \lambda_2$ & $\gamma < \gamma^d$.

■

[7] Firm 2 has a stronger incentive to deviate to $x = 1$ rather than $x = 0$ to soften competition. So, we only need to consider its deviation to $x = 1$.

[8] $\frac{44(2\lambda+3)(7-9\lambda)}{\lambda(-540\lambda^2+460\lambda-63)} > 0$ for $\lambda \in (1/3, 1/2)$.

*Proof of Proposition 5.* We first compute the consumer welfare for each content choice.

1. (0,1)

   The consumer welfare from each firm is:

$$\begin{aligned}
&\lambda(1/2+\mathbf{E}[\epsilon_0])+(3/8-3\lambda/4)\mathbf{E}[\epsilon_0|\epsilon_0>0 \text{ and } \epsilon_0>\epsilon_1]) \\
=&\lambda/2+(3/8-3\lambda/4)\cdot 5/18 \\
=&\frac{5-\lambda}{18}
\end{aligned}$$

2. (1/2,1/2)

   The consumer welfare from each firm is:

$$\begin{aligned}
&\frac{1-2\lambda}{2}(1/2+\mathbf{E}[\epsilon_{1/2}])+2(\frac{1-F(0)^2}{2})\lambda\mathbf{E}[\epsilon_{1/2}|\epsilon_{1/2}>0 \text{ and } \epsilon_{1/2}>\epsilon_{1/2'}]) \\
=&\frac{1-2\lambda}{4}+\frac{3\lambda}{4}\frac{5}{18} \\
=&\frac{6-7\lambda}{24}
\end{aligned}$$

3. (0,1/2)

   The consumer welfare from firm 1 is:

$$\begin{aligned}
&\frac{7\lambda}{8}(1/2+\mathbf{E}[\epsilon_0|\epsilon_0+1/2>\epsilon_{1/2}])+\frac{1-2\lambda}{8}\mathbf{E}[\epsilon_0|\epsilon_0>0 \text{ and } \epsilon_0>1/2+\epsilon_{1/2}] \\
=&\frac{7\lambda}{8}\cdot(\frac{1}{2}+\frac{1}{21})+\frac{1-2\lambda}{8}\cdot\frac{1}{3} \\
=&\frac{2+19\lambda}{48}
\end{aligned}$$

The consumer welfare from firm 2 is:

$$\begin{aligned}
&\frac{\lambda}{8}\mathbf{E}[\epsilon_{1/2}|\epsilon_{1/2}>0 \text{ and } \epsilon_{1/2}>1/2+\epsilon_0]+\frac{7(1-2\lambda)}{8}(1/2+\mathbf{E}[\epsilon_{1/2}|\epsilon_{1/2}>1/2+\epsilon_0)+\\
&\frac{\lambda}{2}\mathbf{E}[\epsilon_{1/2}|\epsilon_{1/2}>0]\\
=&\frac{\lambda}{8}\cdot\frac{1}{3}+\frac{7(1-2\lambda)}{8}(1/2+1/3)+\frac{\lambda}{2}\cdot\frac{1}{4}\\
=&\frac{35-62\lambda}{48}
\end{aligned}$$

The consumer's total welfare is $\frac{2+19\lambda}{48}+\frac{35-62\lambda}{48}=\frac{37-43\lambda}{48}$.

The total consumer welfare under (0,1) content choice is higher than that under (0,1/2) content choice if and only if $2\cdot\frac{5-\lambda}{18}>\frac{37-43\lambda}{48}\Leftrightarrow\lambda>31/113$, which always holds. The total consumer welfare under (0,1/2) content choice is higher than that under (1/2,1/2) content choice if and only if $\frac{37-43\lambda}{48}>2\cdot\frac{6-7\lambda}{24}\Leftrightarrow\lambda<13/15$, which always holds.

■