AUTOMATA THEORY

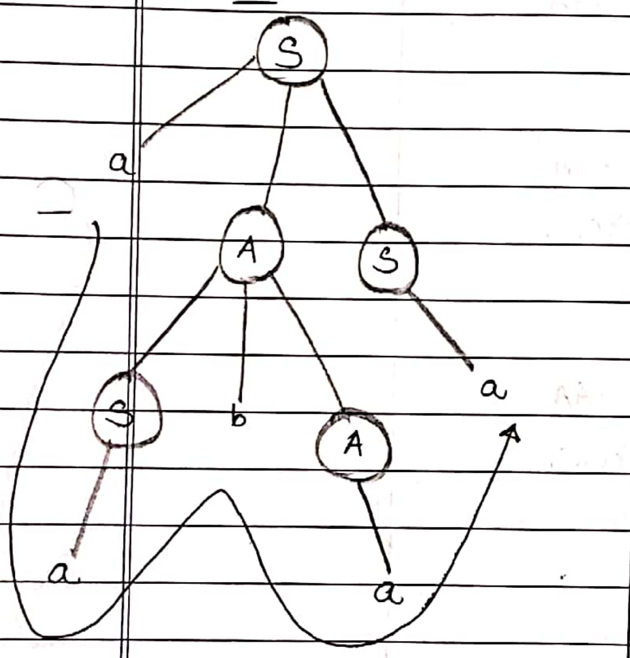
Q.1 Derive using LMD and RMD for "aabbaa".

Soln: →

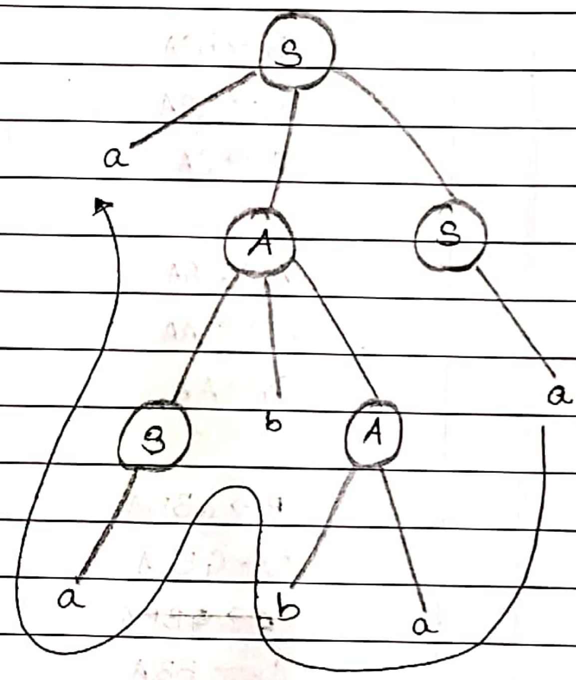
LMD:	RMD:
S ⇒ aAS using S → aAS	S ⇒ aAS using S → aAS
⇒ aSbAS using A → SbA	⇒ aAa using S → a
⇒ aabAS using S → a	⇒ aSbAa using A → SbA
⇒ aabbaS using A → ba	⇒ aSbbaa using A → ba
⇒ aabbaa using S → a	⇒ aabbaa using S → a

Parse Tree / Rule Tree / Derivation Tree

LMD:



RMD:



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Q.2. Express the below CFG in CNF:

$S \rightarrow aB \mid bA$

$A \rightarrow a \mid aS \mid bAA$

$B \rightarrow b \mid bSA \mid aBBA$.

Soln: →

Step 1: No Null, Unit & Useless Production found.
∴ Grammar is already simplified.

Step 2:

Production	Solution
$A \rightarrow a$	$A \rightarrow a$
$B \rightarrow b$	$B \rightarrow b$
$A \rightarrow aS$	$C_1 \rightarrow a$
$A \rightarrow C_1S$	$A \rightarrow C_1S$
$S \rightarrow aB$	$S \rightarrow C_1B$
$S \rightarrow bA$	$C_2 \rightarrow b$
	$S \rightarrow C_2A$
$B \rightarrow bSA$	
$B \rightarrow C_2SA$	
$C_3 \rightarrow SA$	$C_3 \rightarrow SA$
	$B \rightarrow C_2C_3$
$A \rightarrow bAA$	
$A \rightarrow C_2AA$	
$C_4 \rightarrow AA$	$C_4 \rightarrow AA$
	$A \rightarrow C_2C_4$
$B \rightarrow aBBA$	
$B \rightarrow C_1BBA$	
~~$C_5 \rightarrow aBBA$~~	
$C_5 \rightarrow BBA$	
$C_6 \rightarrow BA$	$B \rightarrow C_1C_5$
	$C_5 \rightarrow BC_6$
	$C_6 \rightarrow BA$

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$\therefore$ CNF is

$S \rightarrow C_1 B \,/\, C_2 A$

$A \rightarrow a \,/\, C_1 S \,/\, C_2 C_4$

$B \rightarrow b \,/\, C_2 C_3 \,/\, C_1 C_5$

$C_1 \rightarrow a$

$C_2 \rightarrow b$

$C_3 \rightarrow SA$

$C_4 \rightarrow AA$

$C_5 \rightarrow BC_5$

$C_6 \rightarrow BA$

Q:3. Define :

(i) Grammar :-

→ Grammar is used for specifying the syntax of a language & is defined as follows;

$G = (V, T, P, S)$

where,

V = finite set of variables / non-terminal.

T = finite set of terminals.

P = finite set of production rules.

S = Start Variables

(ii) Context Free Grammar (CFG) :-

A Grammar is said to be Context free Grammar (CFG) if all the productions are of the form

$A \rightarrow \alpha$

where, $A \rightarrow$ Variable

& $\alpha \rightarrow$ is in some sentential form.

eg: $G = (\{S, A\}, \{a, b\}, P, S)$

P :

$S \rightarrow aSb \mid aA \mid a$
$A \rightarrow Ab \mid b \mid \epsilon$

(iii) Unit Production :-
→ A production of the form $A \rightarrow B$ where A and B are variables are called unit production.

Q.4. Write Short Note on : Chomsky Hierarchy.

Soln: →

(i) Type 0 / Unrestricted Grammar :-
In this grammar, there are no restrictions on production rule eg: $S \rightarrow aA$, $aA \rightarrow bBC$, $C \rightarrow b$, $B \rightarrow a$.
Unrestricted grammar generates recursively Enumerable language & to recognize it turing machine can be constructed.

(ii) Type 1 / Context Sensitive Grammar :-
In this grammar, there are two restrictions on production rule (i) $|\alpha| \leq |\beta|$
(ii) Start variable(s) cannot appear on R.H.S
Eg:- $S \rightarrow bB$, $bB \rightarrow cB$, $B \rightarrow \alpha$.
It is called Context Sensitive because the replacement of capital 'A' by 'r' is allowed only in the context 'α' is preceding 'A' & 'β' is succeeding 'A'

$A \rightarrow r$

$\alpha A \beta \stackrel{*}{\Rightarrow} \alpha r \beta$

Context Sensitive grammar generates Context Sensitive language and to recognize it LBA (Linear Bounded Automata) can be constructed.

(iii) Type 2 / Context free Grammar (CFG):

In this grammar all productions should be of the form $A \to \alpha$ where 'A' is a variable & 'α' is in some sentisial form eg: $S \to aSb / bSb / a$.

CFG generates CFL & to recognize it PDA can be constructed.

(iv) Type 3 / Regular Grammar (R.E):

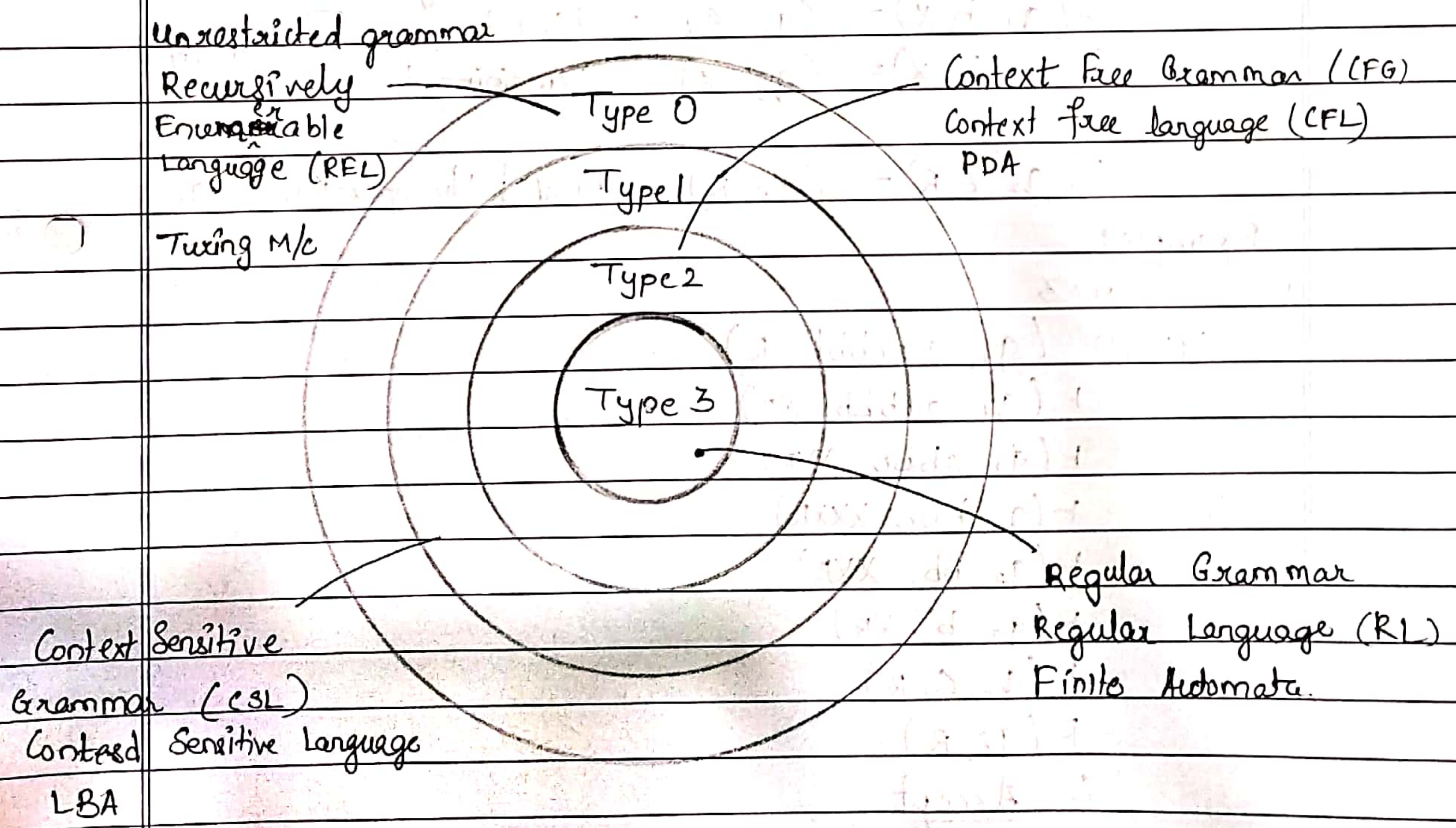
It is a CFG with a restriction i.e the R.H.S can contain at the most variable.

If all the variable appear at the left most position then it is called Left Linear grammar.

If all the variable appear at the right most position then it is called Right linear grammar.

eg: R.L.G $S \to aB$, $B \to cB / d$.

R.G. generates R.L and to recognize it finite automata can be constructed.



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Q.5. Design a PDA to recognize the foll language:
$L = \{a^n b^n / n \geq 1\}$

Soln: →

Logic :

For each 'a', PUSH one X on the stack

For each 'b', POP one X from the stack.

Implementation:

$M = (Q, \Sigma, \Gamma, \partial, q_0, Z_0, F)$

$Q = \{q_1, q_2, q_f\}$

$\Sigma = \{a, b\}$, $\Gamma = \{X, R\}$

$q_0 = \{q_1\}$; $Z_0 = \{R\}$ $F = \{q_f\}$

∂ :

$\partial(q_1, a, R) = (q_1, XR)$ – First a] PUSH q_1

$\partial(q_1, a, X) = (q_1, XX)$ – Remaining a's

$\partial(q_1, b, X) = (q_2, \epsilon)$ – First b] Pop q_2

$\partial(q_2, b, X) = (q_2, \epsilon)$ – Remaining b's

$\partial(q_2, \epsilon, R) = (q_f, R)$ – Final] No operation q_f

Example:

n = 3

eg: (1) $(q_1, aaabbb, R)$

$\vdash (q_1, aabbb, XR)$

$\vdash (q_1, abbb, XXR)$

$\vdash (q_1, bbb, XXXR)$

$\vdash (q_2, bb, XXR)$

$\vdash (q_2, b, XR)$

$\vdash (q_2, \epsilon, R)$

$\vdash (q_f, R)$

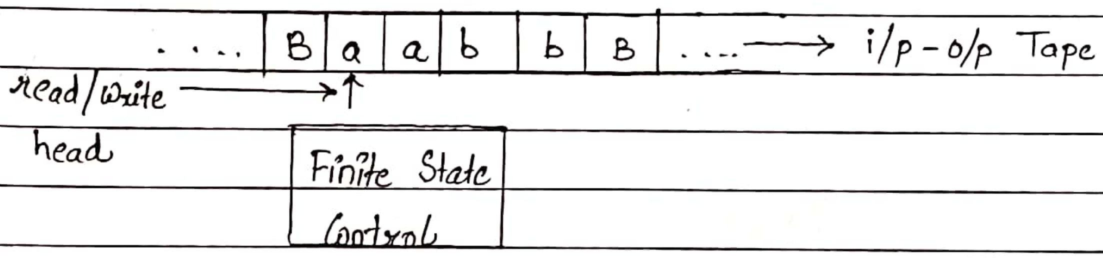
∴ Accept

Q.6. What is Turing Machine? Explain the components, model and working of Turing Machine.

Soln: →

Turing machine (TM) is considered to be a simple model of a computer & is the most powerful machine.
Turing machine can perform:
(i) Language Recognition.
(ii) Evaluation of some function

- **Model of TM:-**



Components of TM: Turing machine consists of finite set of states, input-output tape and read/write head.

- **Working of T.M:**

Depending upon the state & the tape symbol,
(i) TM can change the state / remain in the same state.
(ii) TM can change the tape symbol / keep it the same.
(iii) TM moves the head {left, right, same}

TM is mathematically represented by 7-tuple relation:-

$$M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$$

where,

Q = finite set of states
Σ = Input alphabet

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δ = Transition function

$\delta : Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R, S\}$

q_0 = start/initial symbol , $q_0 \in Q$

B = Symbol to represent blank $B \in \Gamma$

F = Finite set of final state , $F \subseteq Q$